LIÈGE université
Applied Sciences
Structural & Stochastic Dynamics
Urban & Environmental Engineering

A framework for the efficient spectral analysis of large wind- and wave-loaded structures

Margaux Geuzaine
Vincent Denoël

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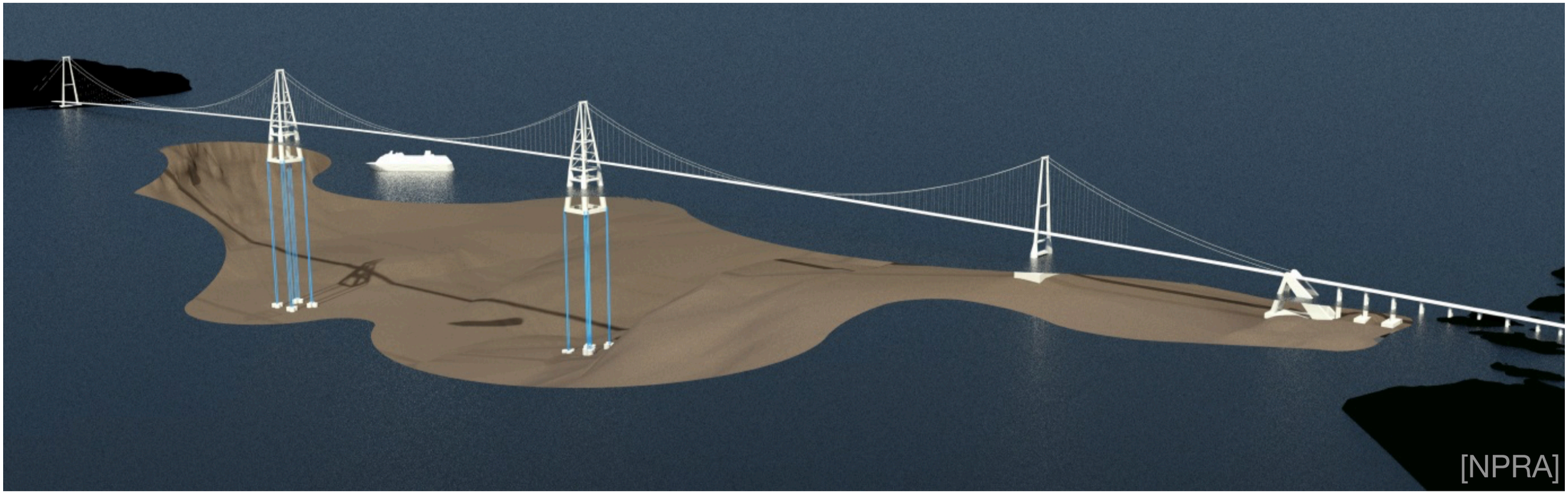
15th International Conference on Wind Engineering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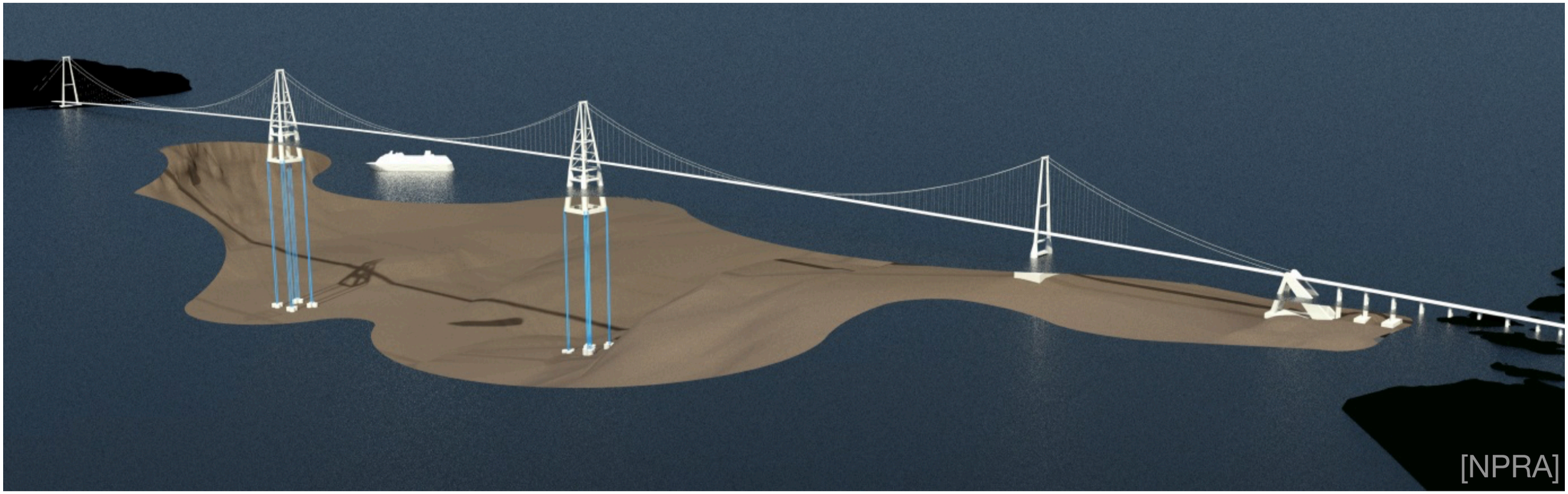
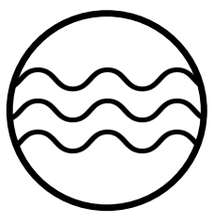


The Multiple Timescale Spectral Analysis method is able to divide CPU time by 100

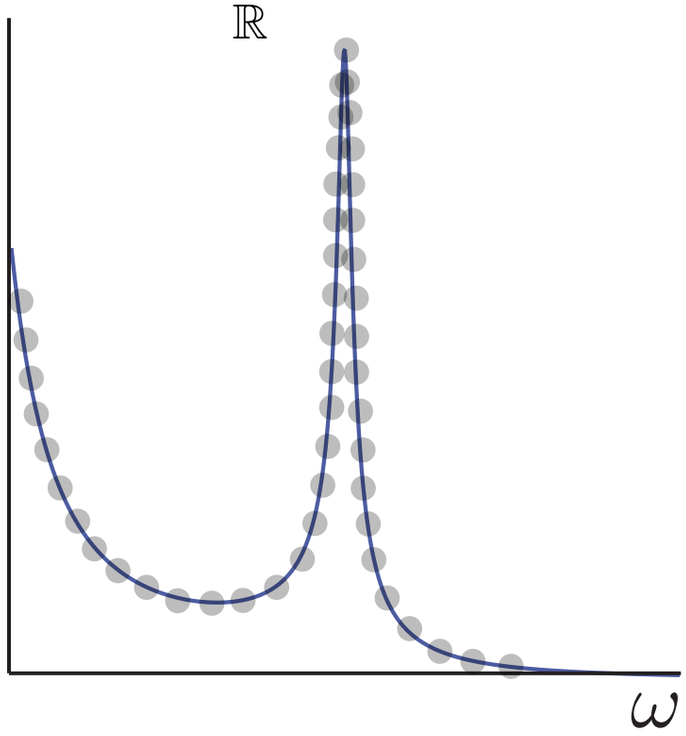
+



?

MTSA = fast computation of response statistics [Denoël, 2015]

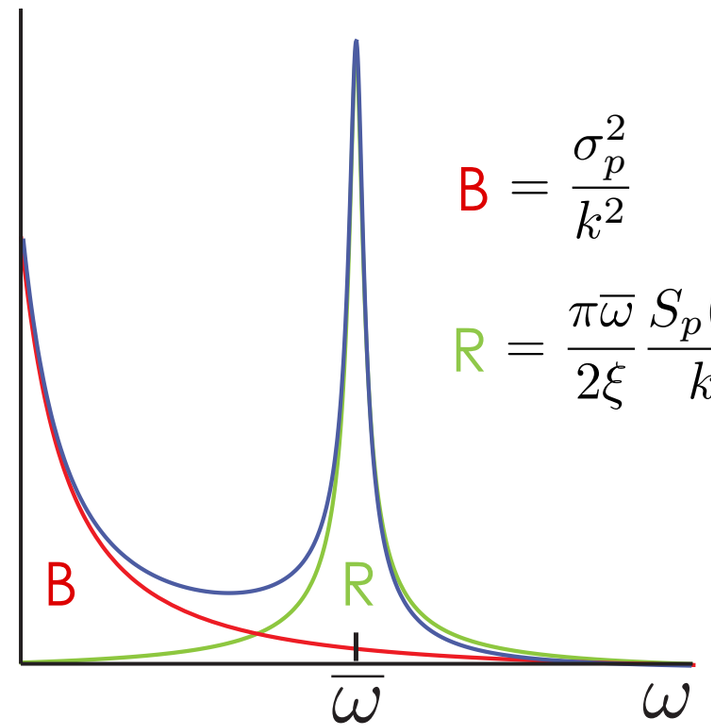
$$\sigma_x^2 = \int_{\mathbb{R}} S_x(\omega)\,\mathrm{d}\omega$$



$$\sigma_x^2 \approx \mathrm{B} + \mathrm{R}$$

$$\mathrm{B} = \frac{\sigma_p^2}{k^2}$$

$$\mathrm{R} = \frac{\pi\overline{\omega}}{2\xi}\frac{S_p(\overline{\omega})}{k^2}$$



- Existence of small parameters
- Analytical approximations
- Controllable discrepancy
- ↘ order of integration by 1

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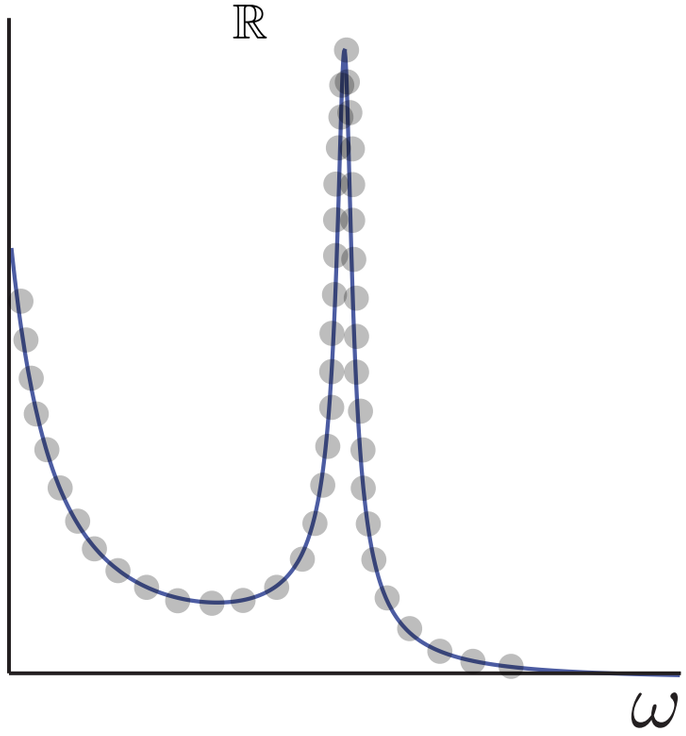

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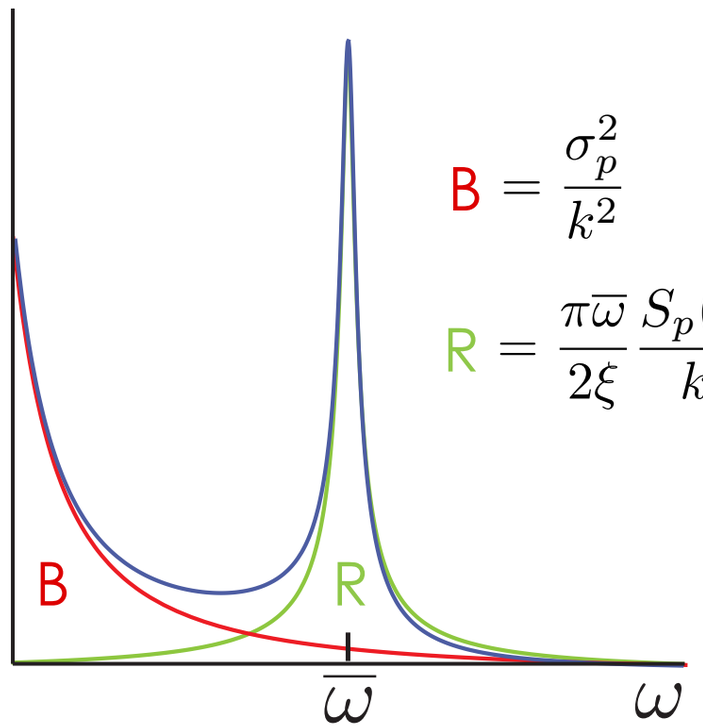

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MTSA

Fully linear 1DOF & MDOF σ^2

NL loading, NL structure } 1DOF $\sigma^2, \gamma_3, \gamma_4$

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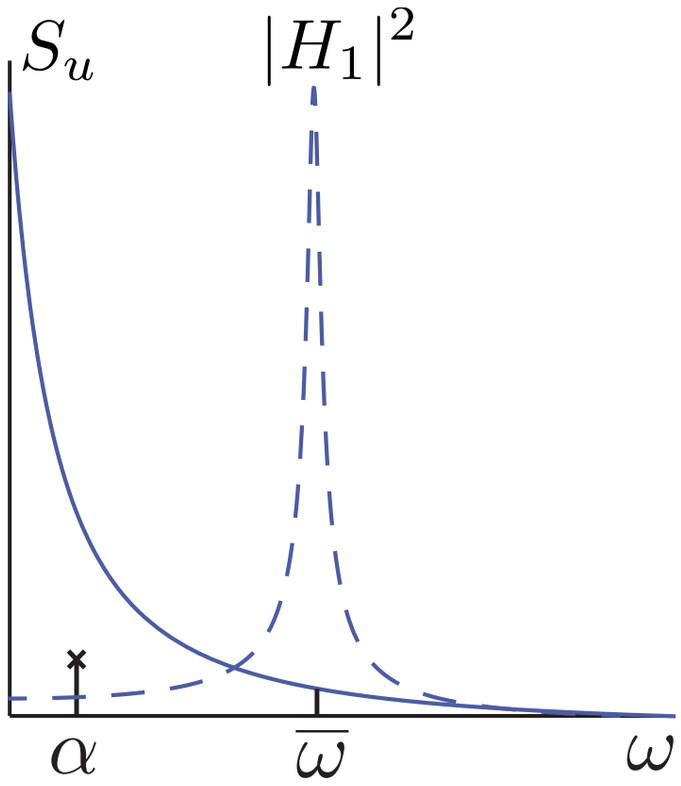
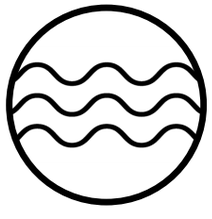
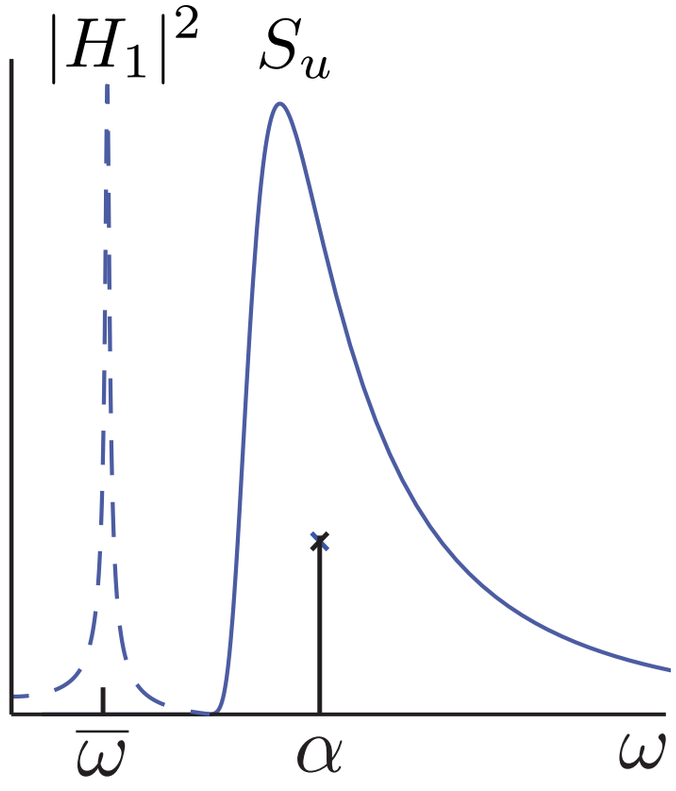
Main differences between wind and wave loadings



$\alpha \ll \overline{\omega}$ for buffeting response

$$f_u = k_d(U + u - \dot{x})^2$$





$\alpha \gg \overline{\omega}$ for surge response

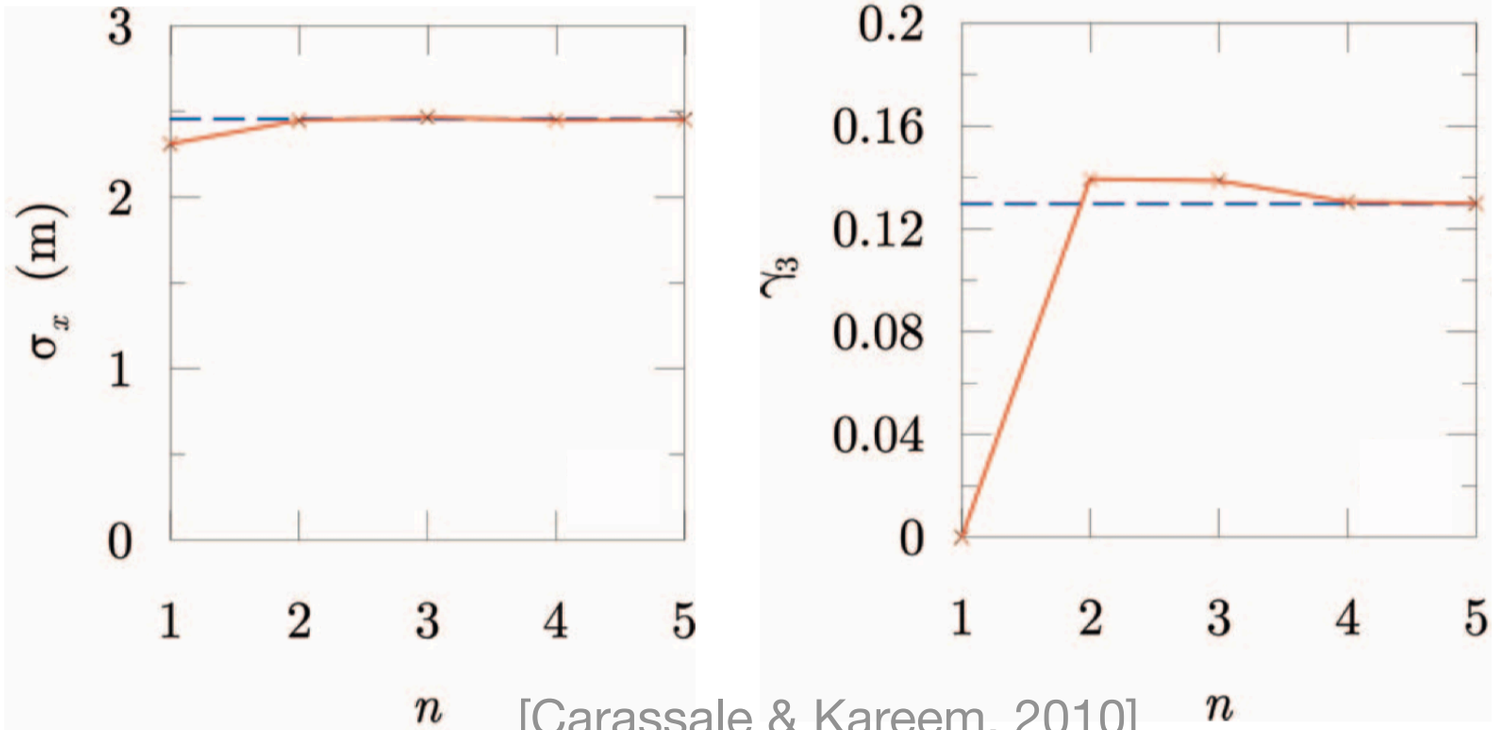
$$f_u = k_m\dot{u} + k_d(U + u - \dot{x})|U + u - \dot{x}|$$

Equivalence between time and frequency domains

1. $|U + u - \dot{x}|(U + u - \dot{x}) \approx \sum_{r=0}^{n} a_r (u - \dot{x})^r$

2. Volterra-Series expansion

3. Truncation of the series



[Carassale & Kareem, 2010]

4. Evaluation of Volterra Frequency-Response Functions, $H_1(\omega)$ and $H_2(\omega_1, \omega_2)$

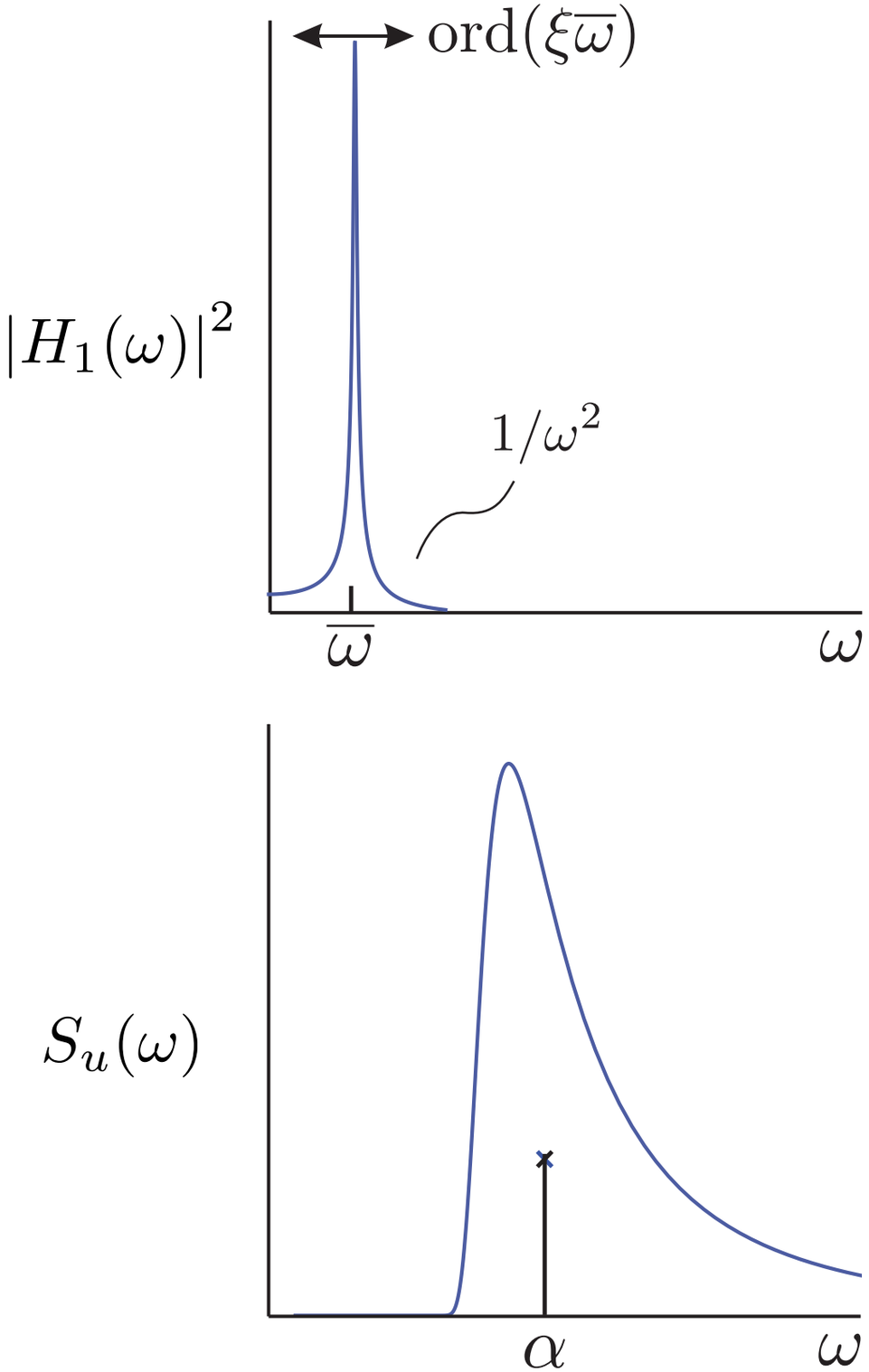
5. Integration of the spectrums

$$S_{\tilde{x}}(\omega) = S_{\tilde{x}_1}(\omega) + S_{\tilde{x}_2}(\omega)$$

$$B_{\tilde{x}}(\omega_1, \omega_2) = 3\, B_{\tilde{x}_{112}}(\omega_1, \omega_2) + B_{\tilde{x}_2}(\omega_1, \omega_2)$$

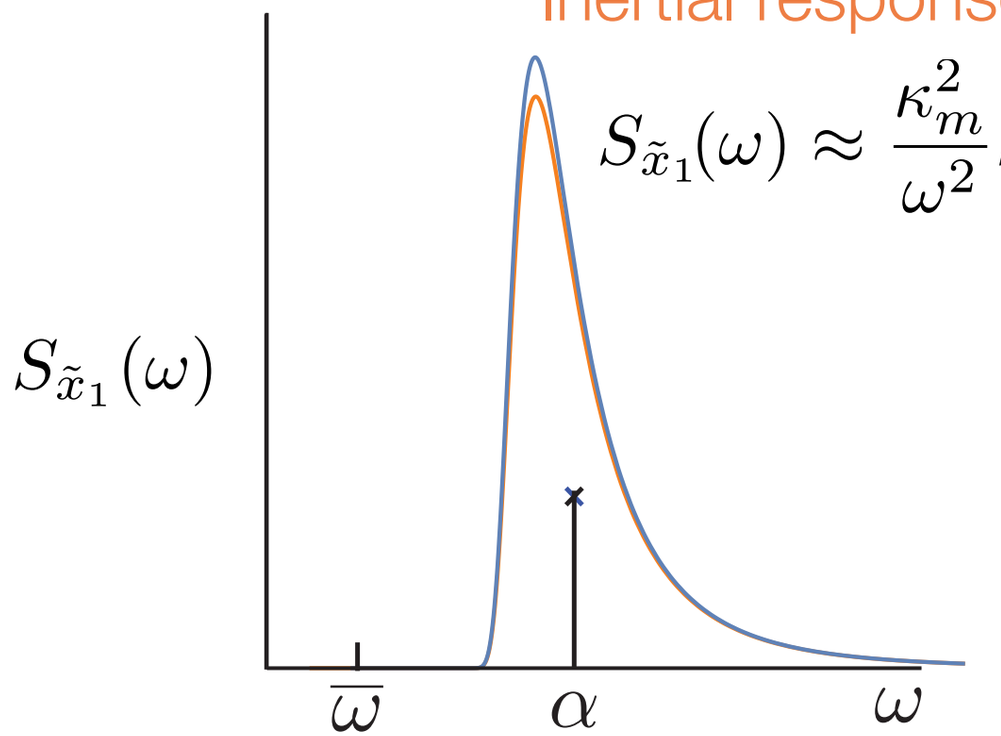
Variance of the surge response of a 1-DOF structure

1st order response $S_{\tilde{x}_1}(\omega) = |H_1(\omega)|^2 S_u(\omega)$



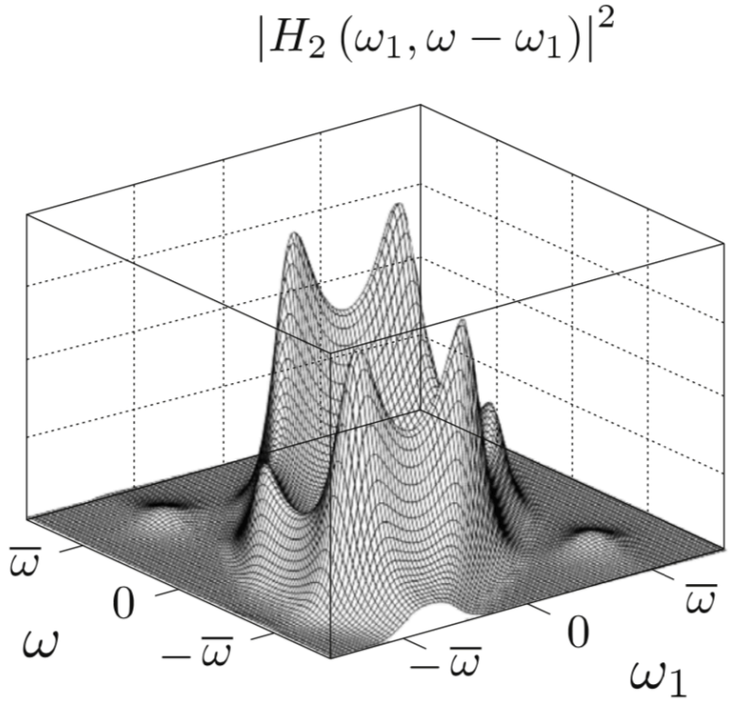
Inertial response only

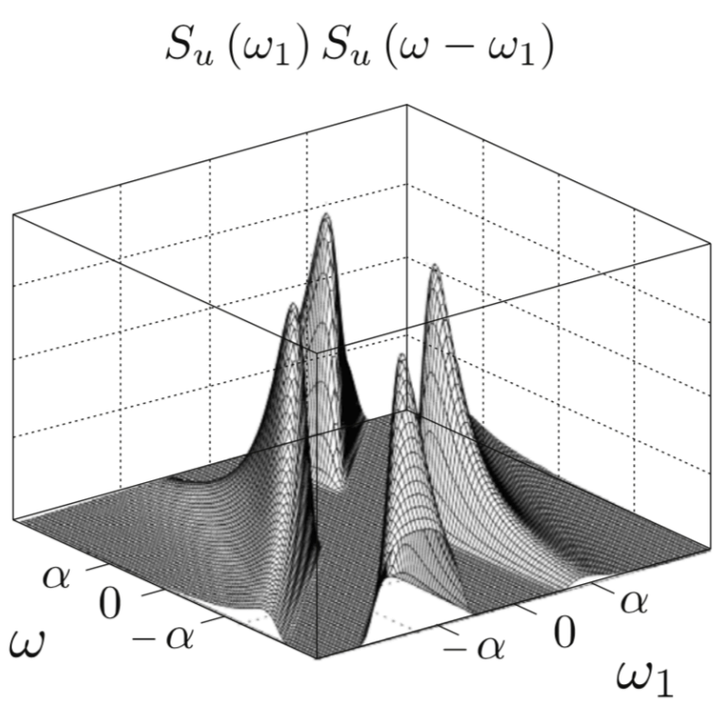
$$S_{\tilde{x}_1}(\omega) \approx \frac{\kappa_m^2}{\omega^2} S_u(\omega)$$

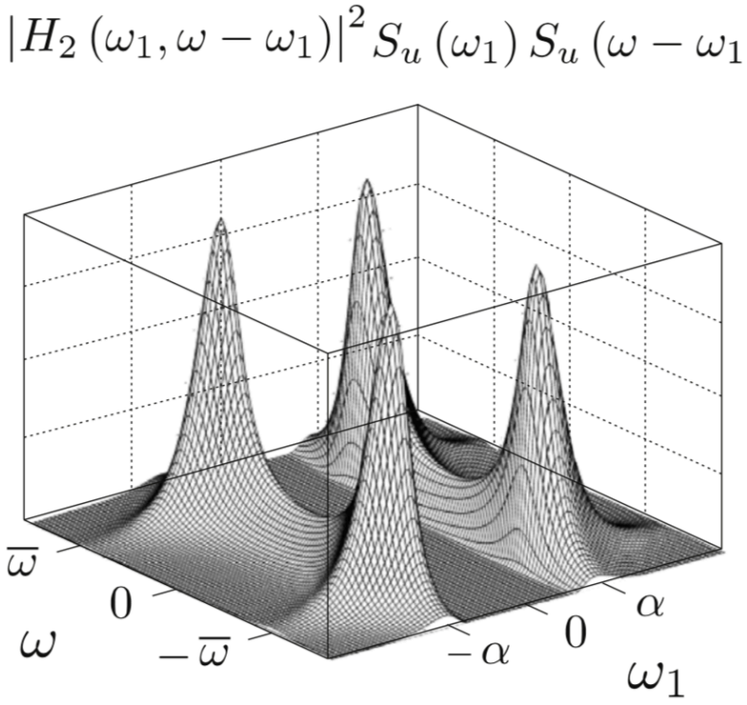


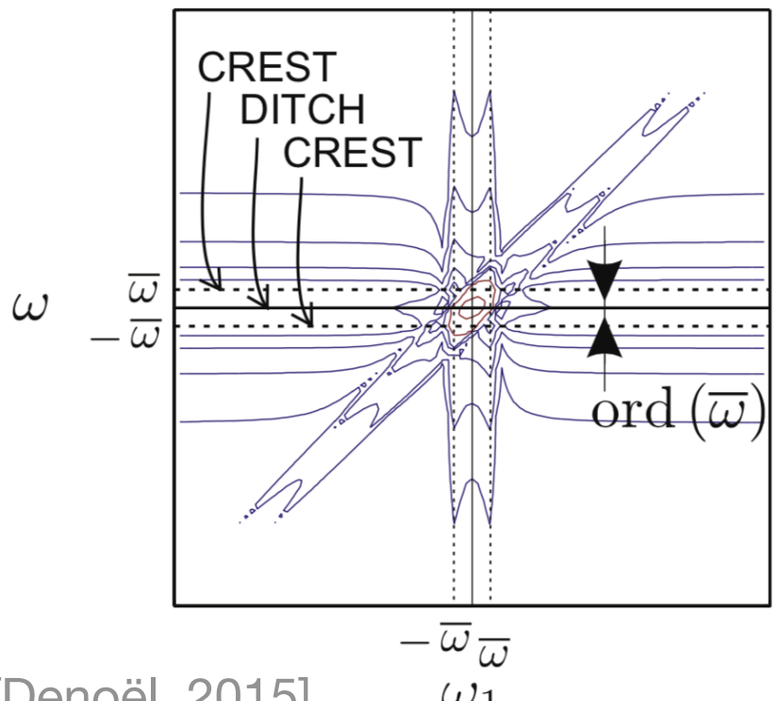
Variance of the surge response of a 1-DOF structure

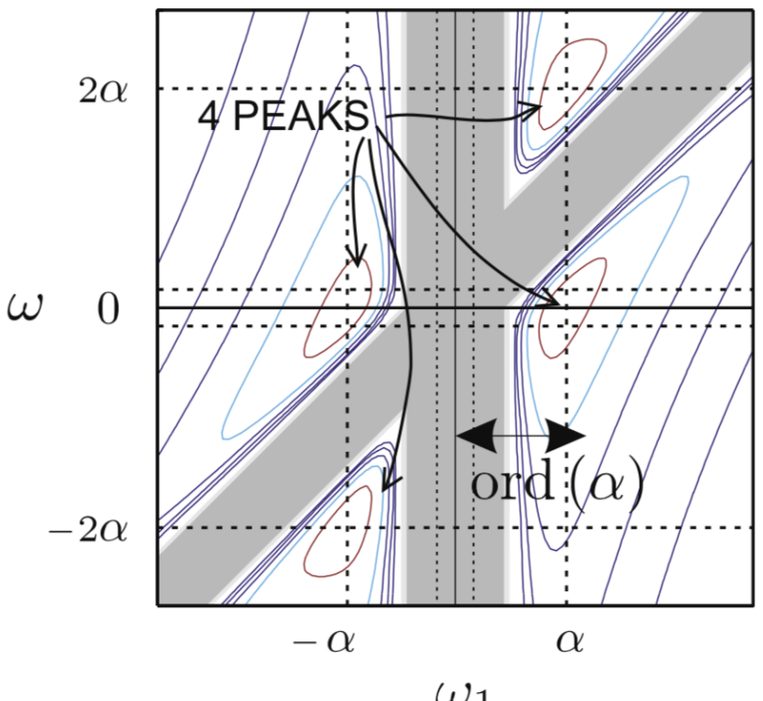
2nd order response $S_{\tilde{x}_2}(\omega) = 2\int_{\mathbb{R}} |H_2(\omega_1, \omega - \omega_1)|^2 S_u(\omega_1) S_u(\omega - \omega_1) \, \mathrm{d}\omega_1$

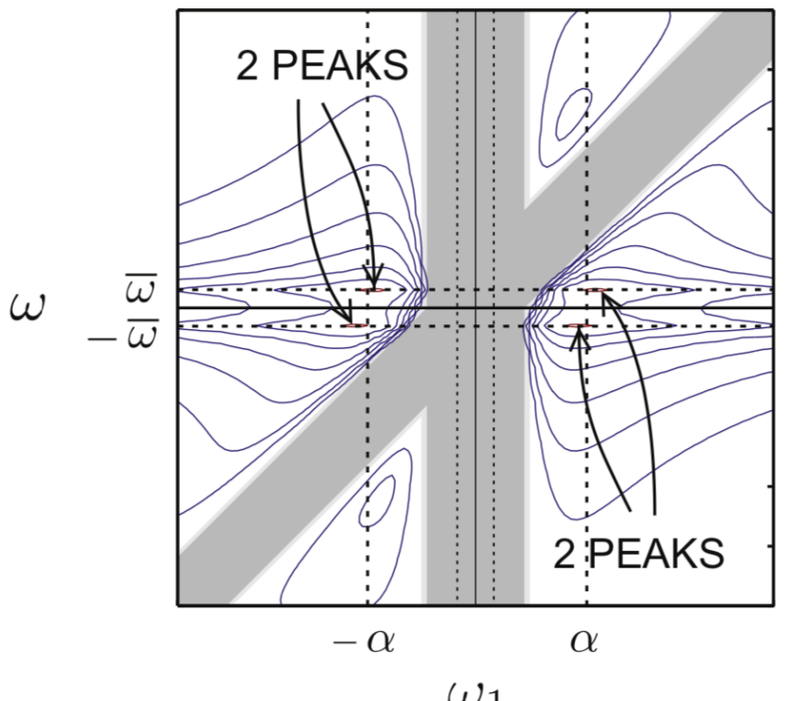








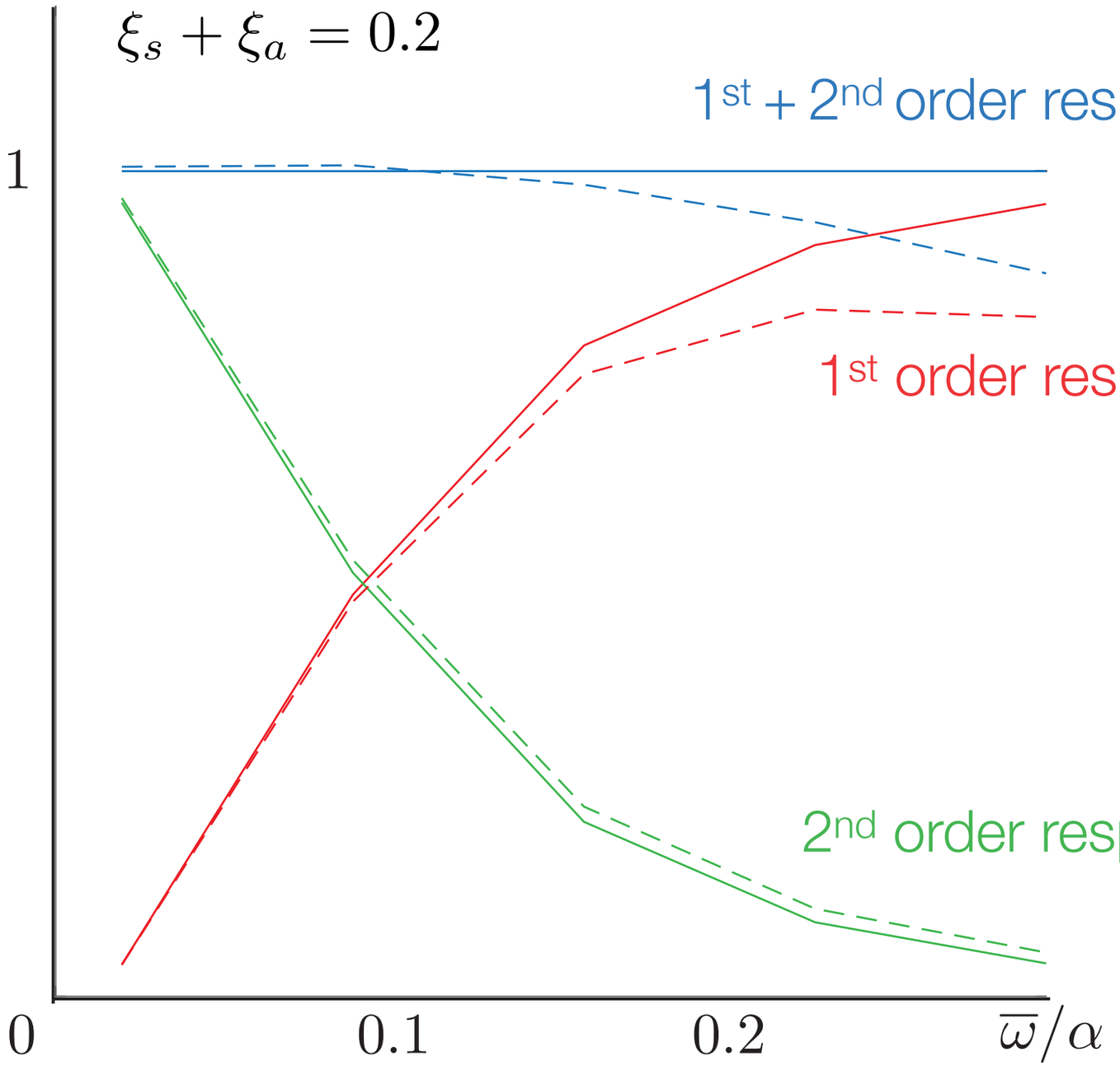




[Denoël, 2015]

Variance of the surge response of a 1-DOF structure

Compromise between loss of accuracy and time saving



1st + 2nd order response $\sigma^2_{\tilde{x}} = \sigma^2_{\tilde{x}_1} + \sigma^2_{\tilde{x}_2}$

tCPU / 2000

1st order response $\sigma^2_{\tilde{x}_1} = \kappa^2_m \sigma^2_{\int u}$

tCPU / 10

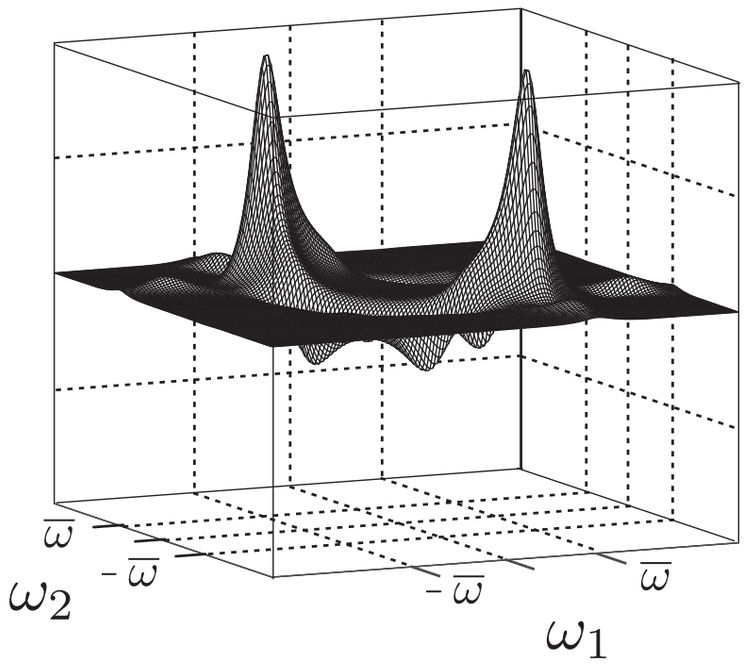
2nd order response $\sigma^2_{\tilde{x}_2} = a_2^2 \left(1 - \kappa_m\right)^4 \dfrac{\pi \overline{\omega} S_{u^2}(\overline{\omega})}{\xi_s + \xi_a}$

tCPU / 4000

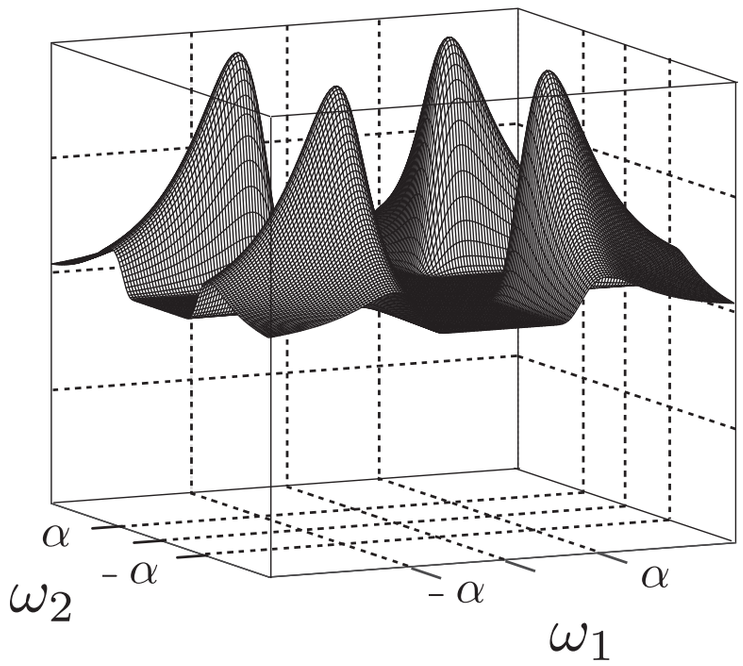
Skewness of the surge response of a 1-DOF structure

$$B_{\tilde{x}_{112}}(\omega_1,\omega_2) = 3\,H_1(\omega_1)H_1(\omega_2)\overline{H_2}(\omega_1,\omega_2)\,S_u(\omega_1)S_u(\omega_2)$$

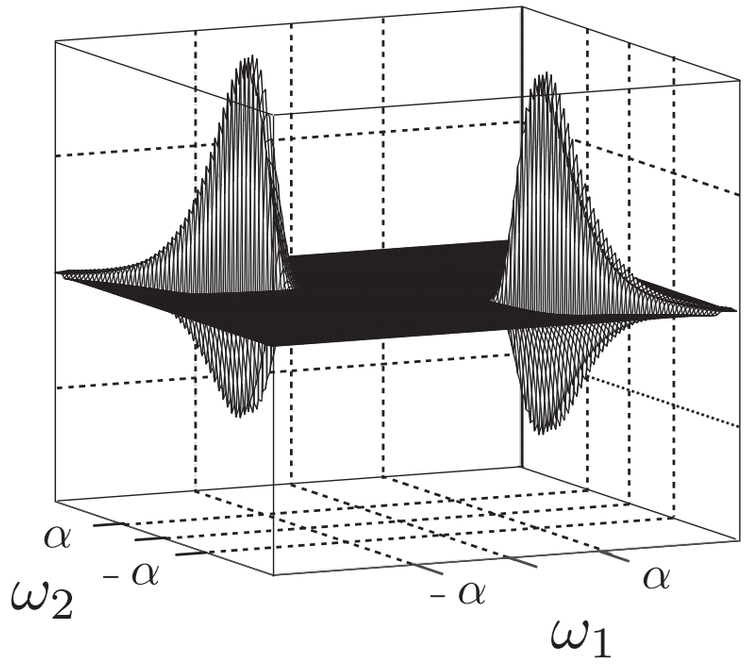
$\Re[H_1(\omega_1)H_1(\omega_2)\overline{H_2}(\omega_1,\omega_2)]$

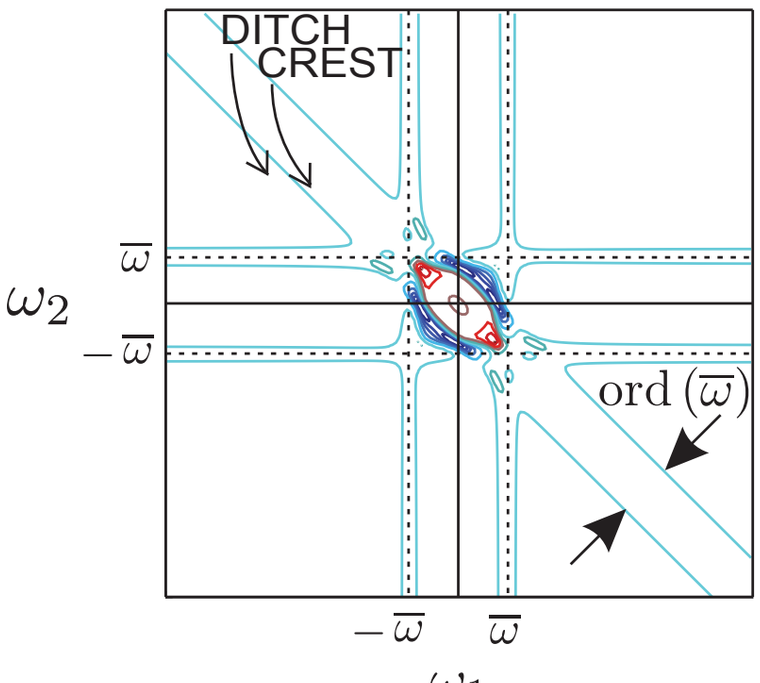


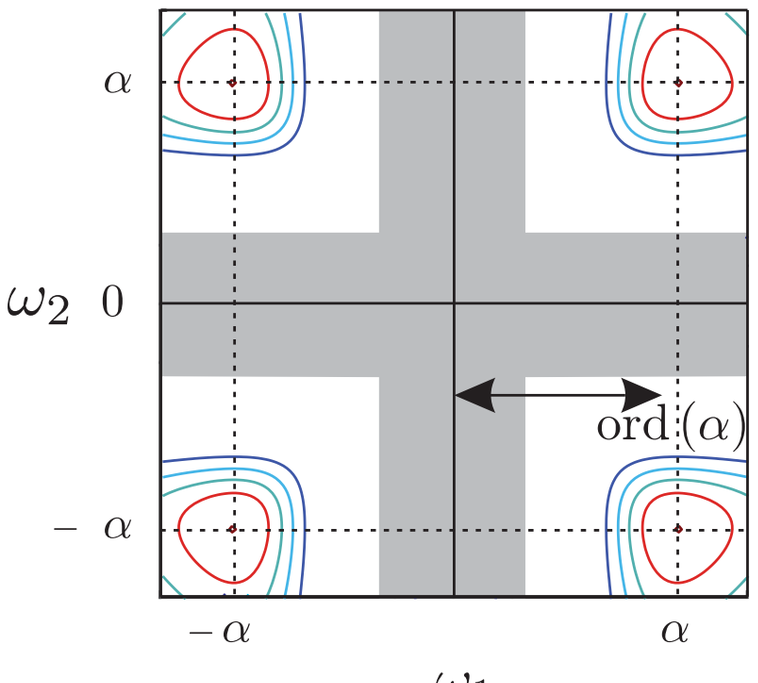
$S_u(\omega_1)S_u(\omega_2)$

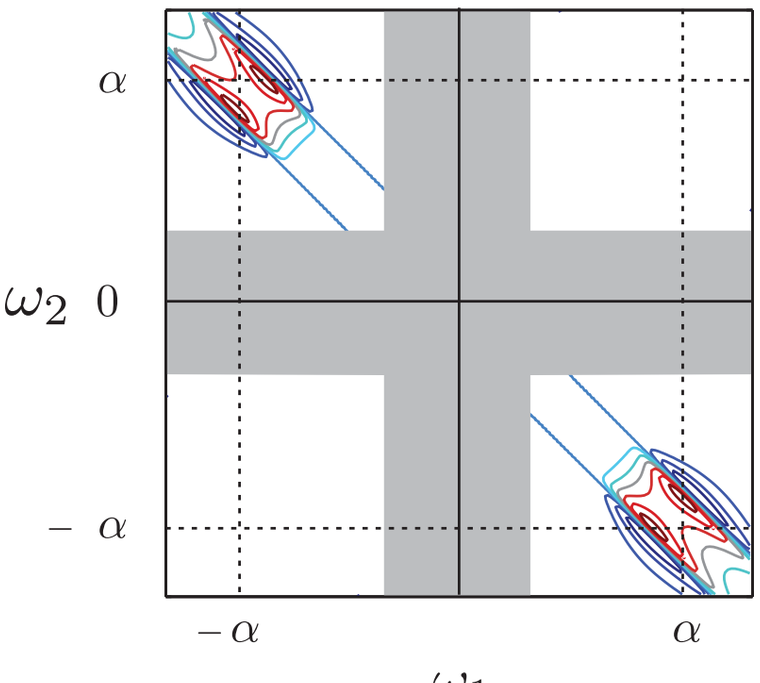


$B_{\tilde{x}_{112}}(\omega_1,\omega_2)$









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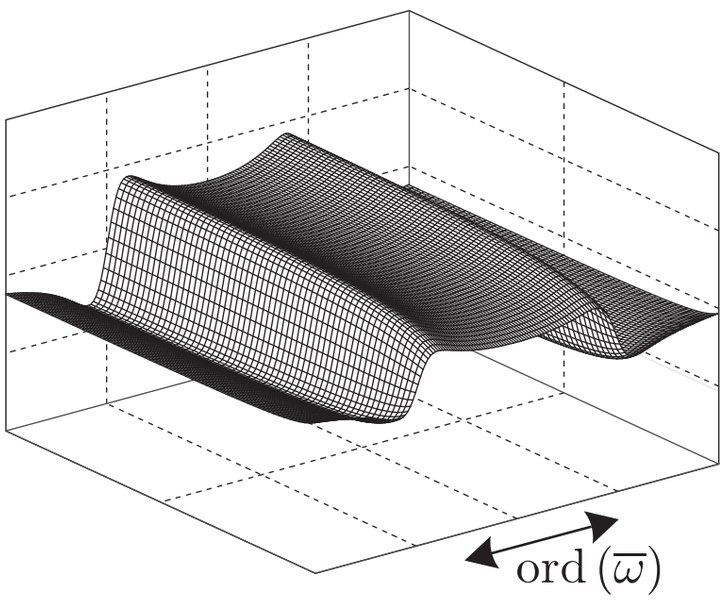
$\Re[\overline{H_2}(\omega_1,\omega_2)]$



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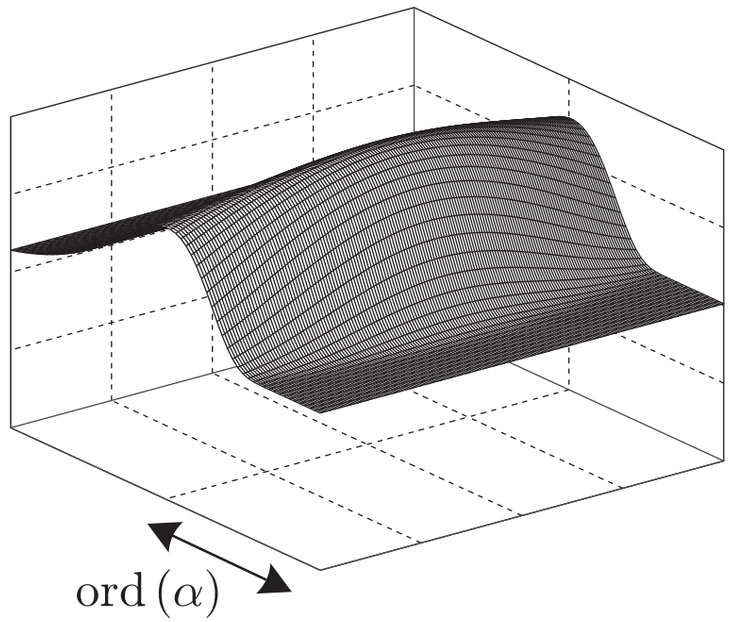
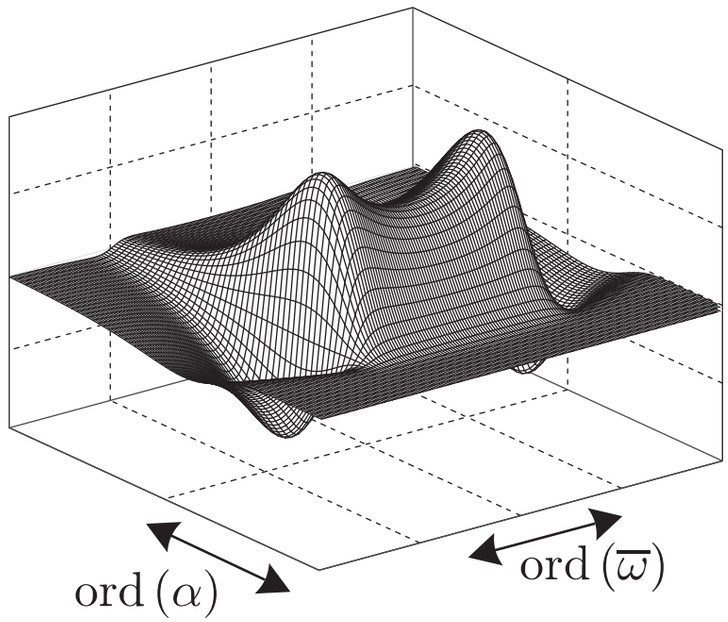


$B_{\tilde{x}_{112}}(\omega_1,\omega_2)$



Skewness of the surge response of a 1-DOF structure

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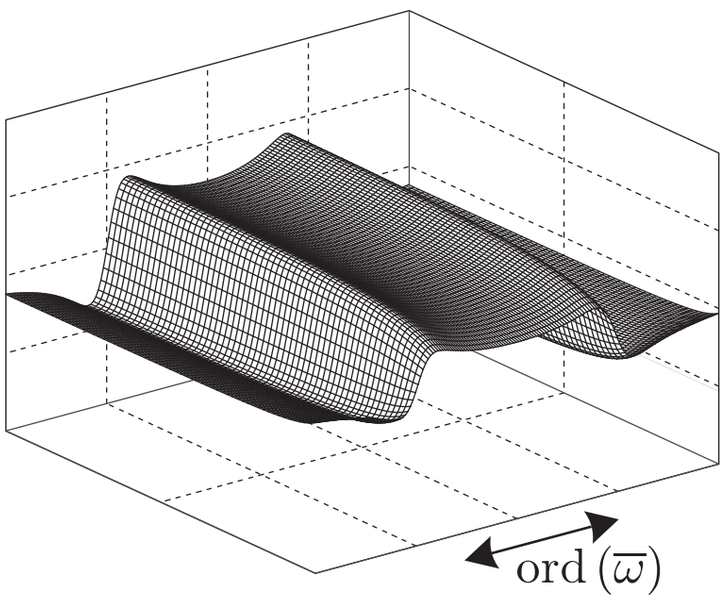

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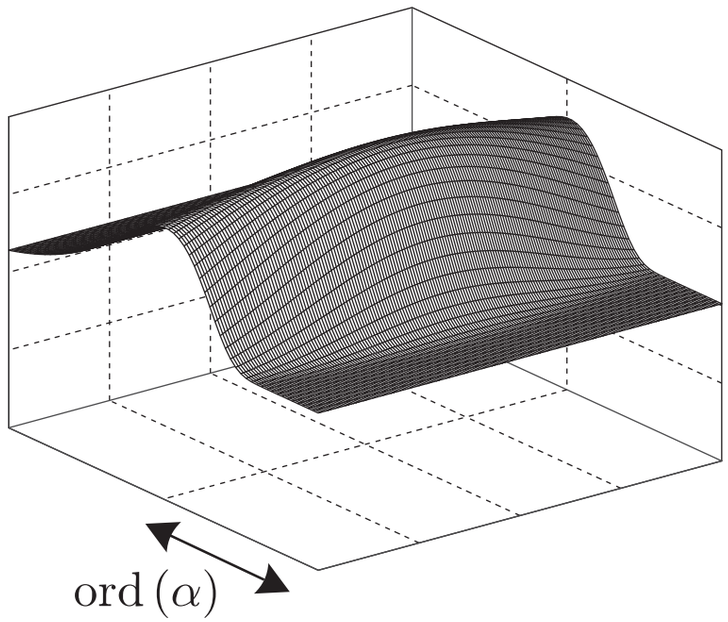

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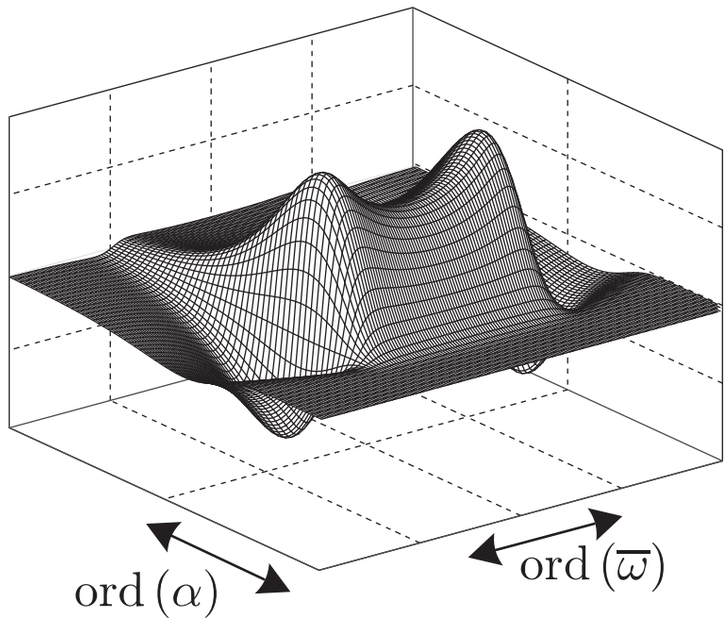

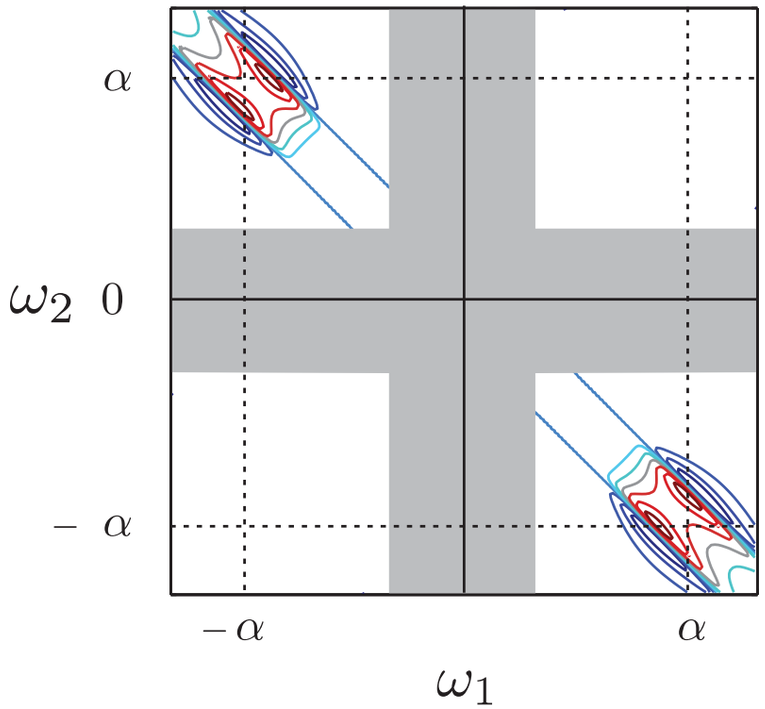

Skewness of the surge response of a 1-DOF structure

$$B_{\tilde{x}_{112}}(\omega_1, \omega_2) = 3\, H_1(\omega_1) H_1(\omega_2) \overline{H_2}(\omega_1, \omega_2)\, S_u(\omega_1) S_u(\omega_2)$$

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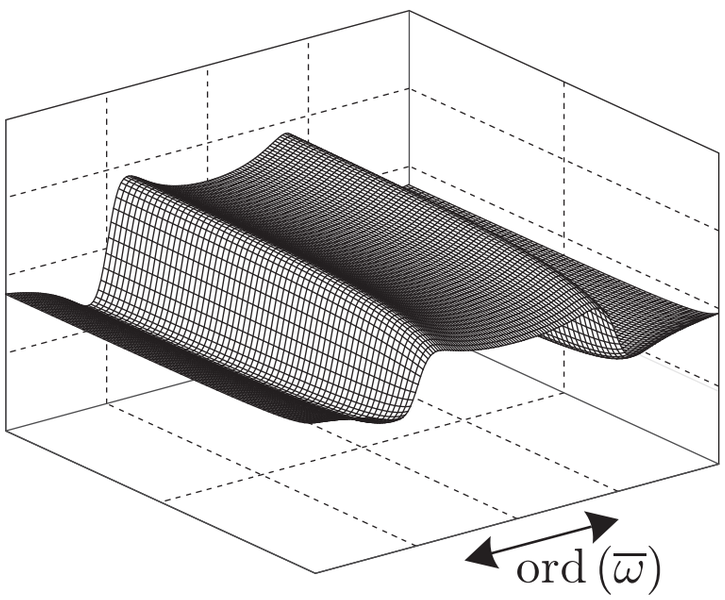

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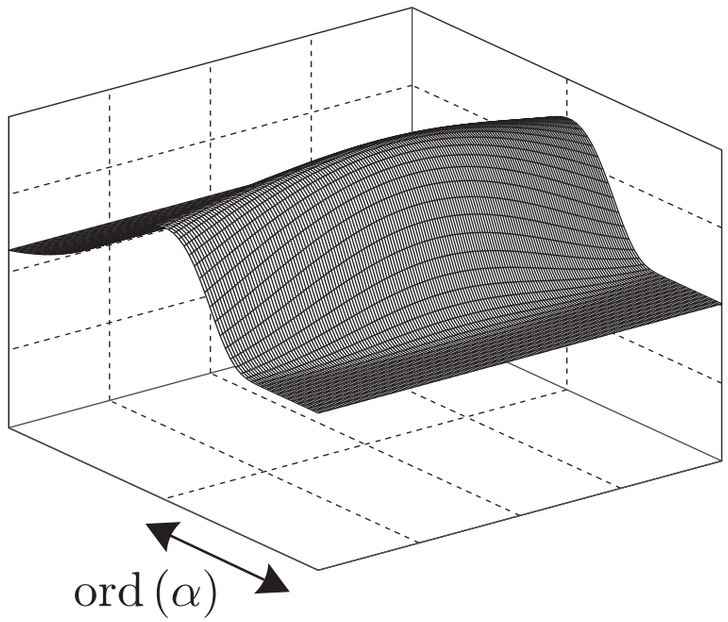

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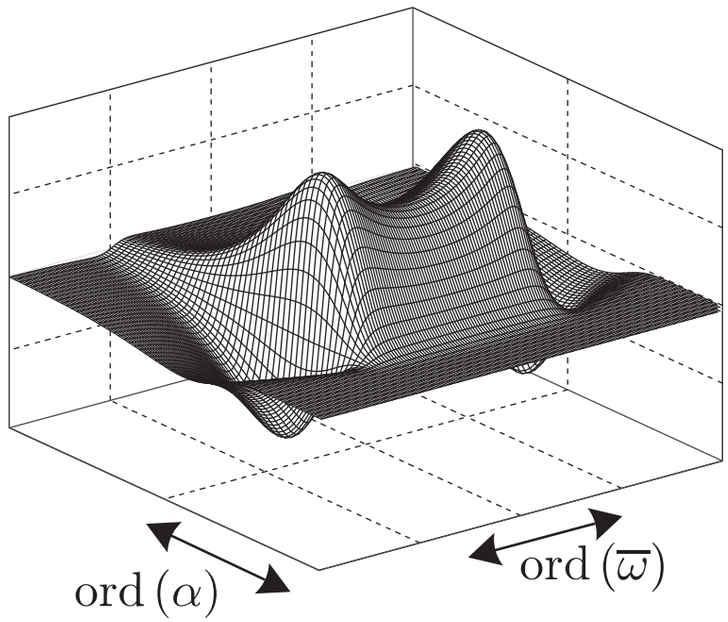

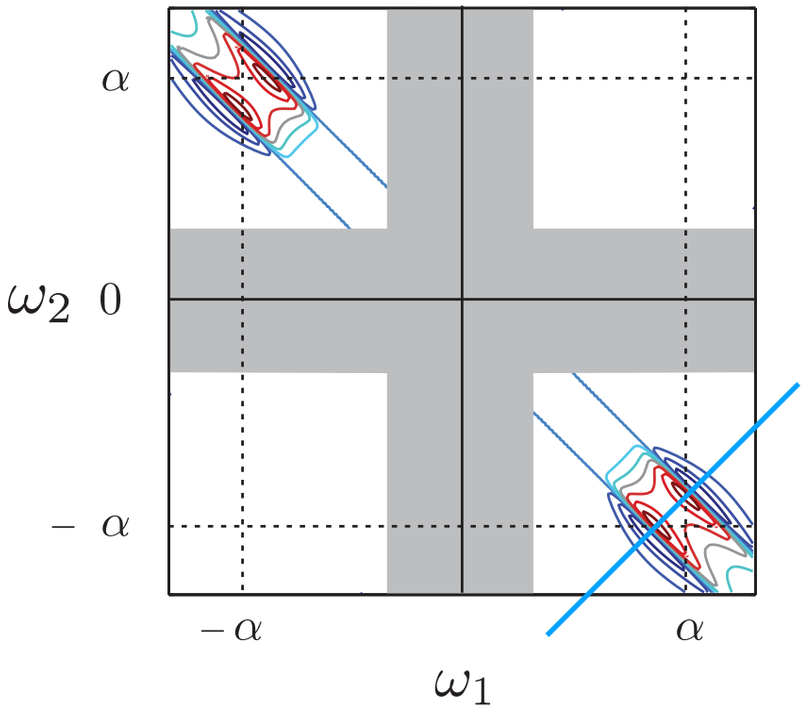

Skewness of the surge response of a 1-DOF structure

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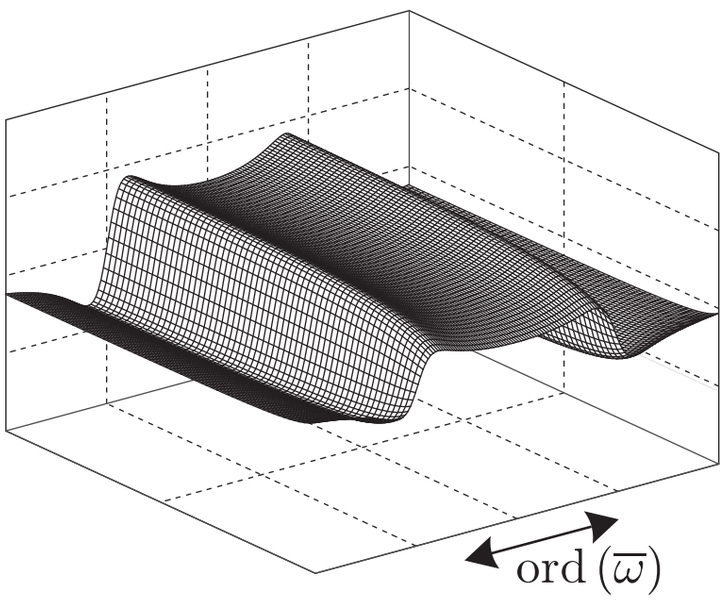

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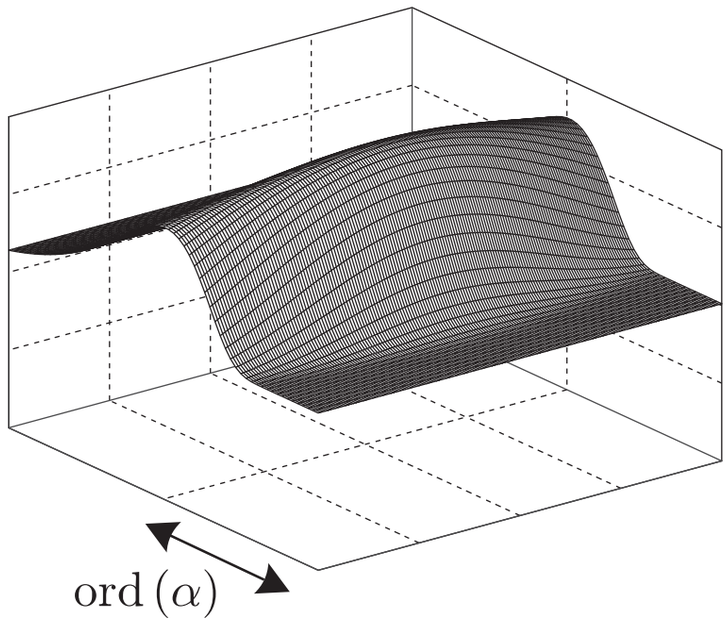
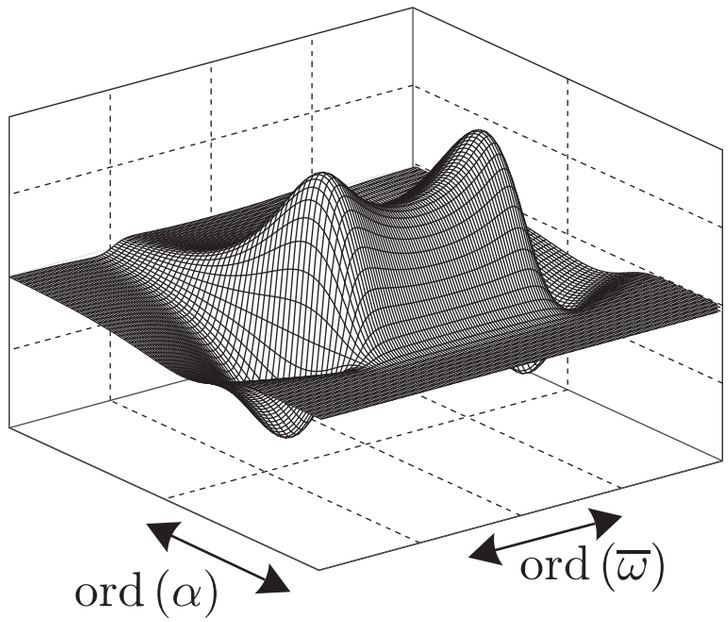
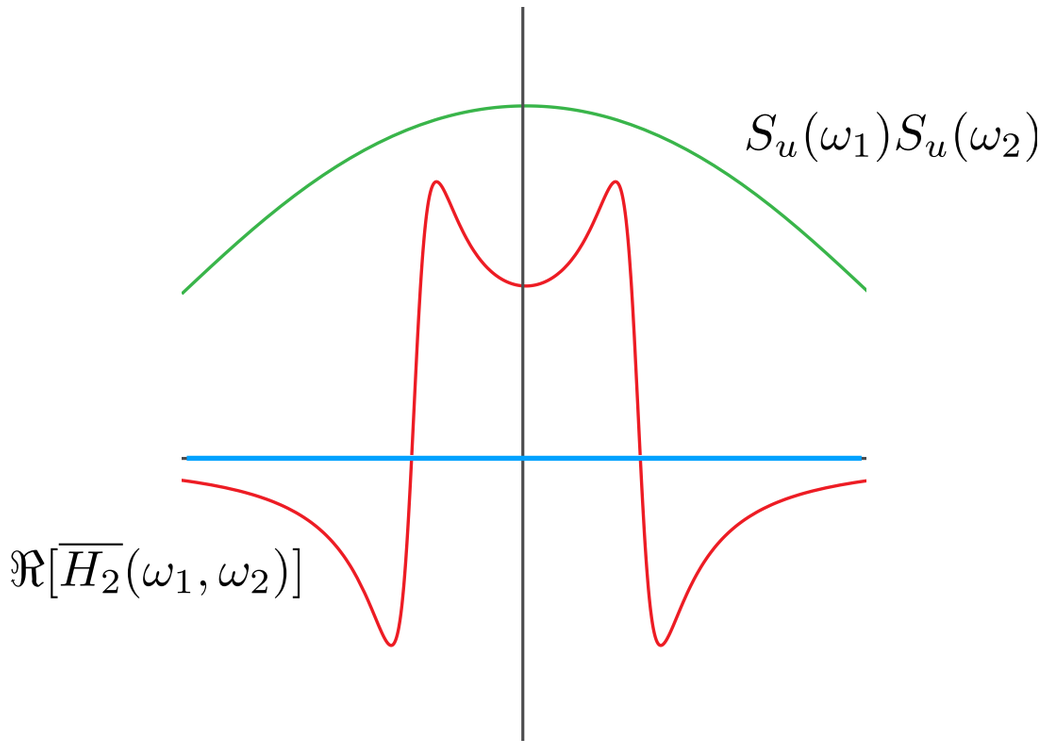
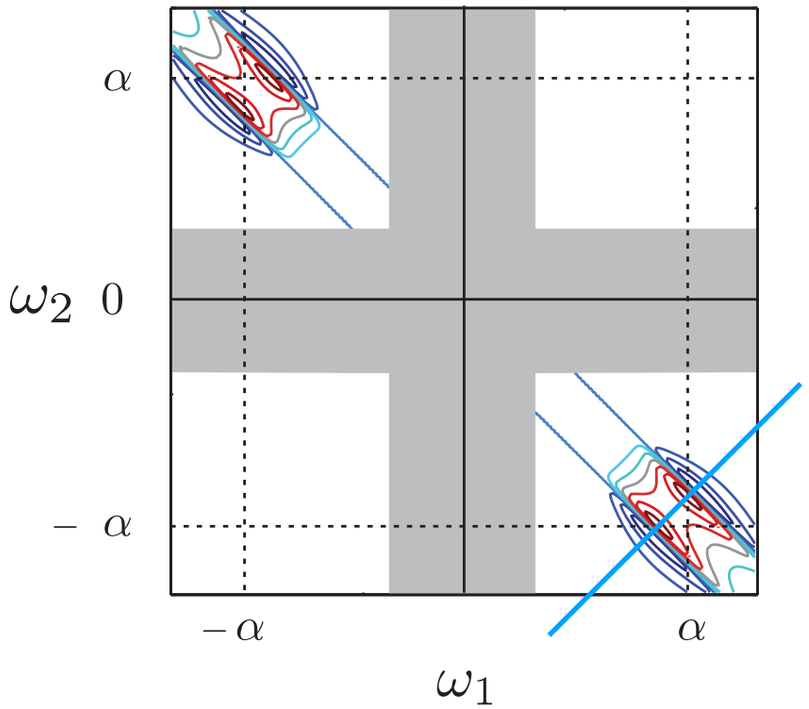


$B_{\tilde{x}_{112}}(\omega_1,\omega_2)$







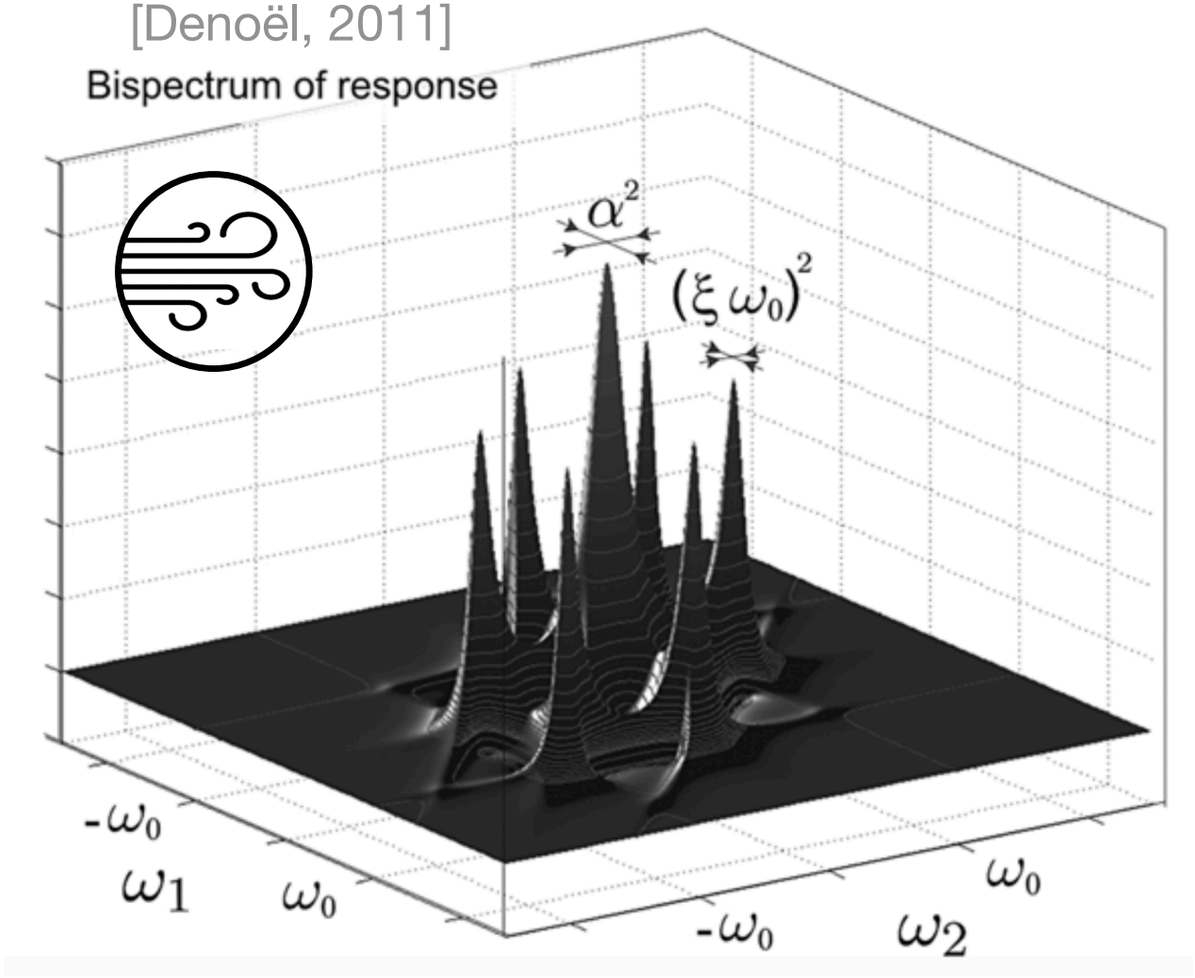
Skewness of the surge response of a 1-DOF structure

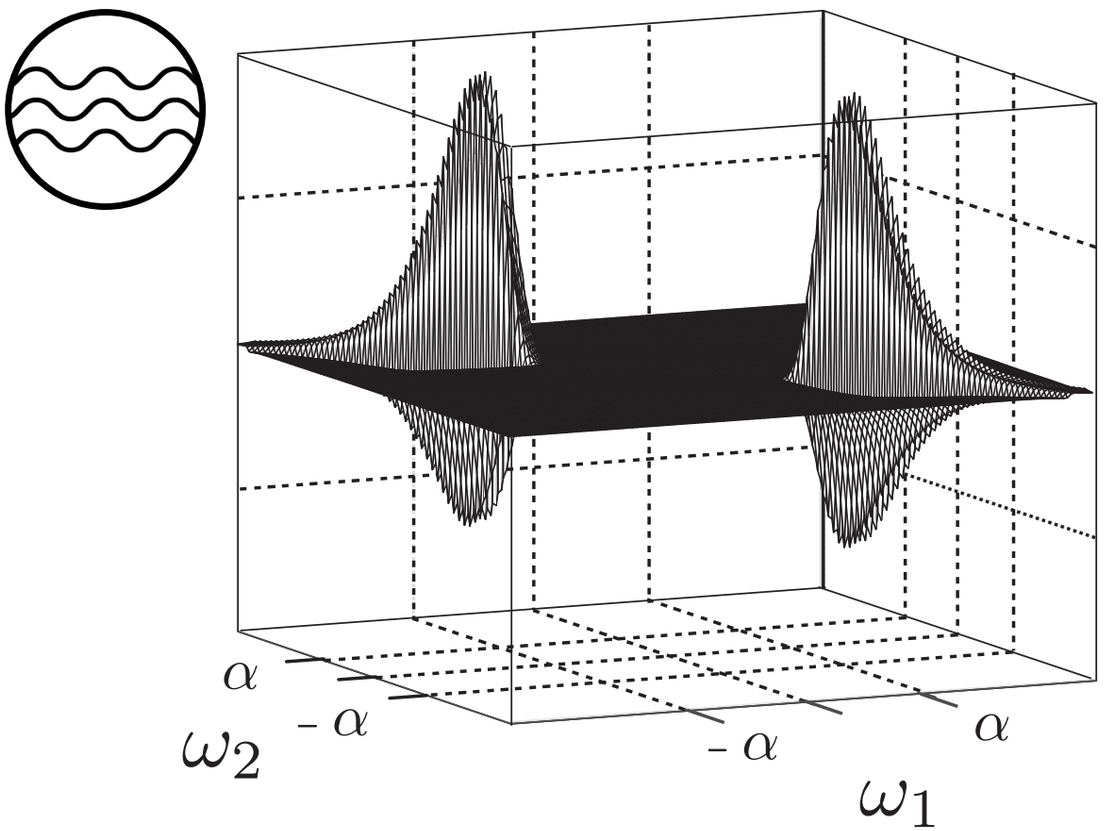
$$B_{\tilde{x}_{112}}(\omega_1,\omega_2) = 3\,H_1(\omega_1)H_1(\omega_2)\overline{H_2}(\omega_1,\omega_2)\,S_u(\omega_1)S_u(\omega_2)$$

$$\approx \frac{C\,S_u(\omega_1)\,S_u(\omega_2)}{\overline{\omega} - (\omega_1+\omega_2)^2 + 2\,i\,\overline{\omega}(\xi_s+\xi_a)(\omega_1+\omega_2)}$$

10 % error but no time saving… yet !

[Denoël, 2011]





Conclusions and perspectives

MTSA is a powerful tool :

- Meaningful analytical approximations
- Large reduction of CPU time
- Small loss of accuracy

Skewness of the surge response of a 1DOF structure is ongoing !

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Thank you !
Questions ? Comments ?

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