Asteroseismology of hot subdwarf and white dwarf stars

The successes of forward modeling approach with parametrized static models

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Outline

I. Pulsations in hot subdwarf and white dwarf stars
   a. Introduction
   b. Some marking results from space observations

II. Models and method for asteroseismic modeling

III. Testing the seismic results with GAIA

IV. Seismic modeling of white dwarfs

V. Conclusions & prospects
**Pulsations in hot subwarfs and white dwarfs**

**Hot subdwarfs** (sdB & sdO): He-burning objects with $T_{\text{eff}} > 20,000$ K and $\log g \sim 5 - 6$

**White dwarfs**: cooling objects, fate of $\sim 98\%$ of the stars in Universe
Pulsations in hot subwarfs and white dwarfs

Various classes of pulsators

**sdB stars (V~15):**
- short-periods (P ~ 80 - 600 s), A ≤ 1%, p-modes (envelope), discovered in 1997
- long-periods (P ~ 30 min - 3 h), A ≤ 0.1%, g-modes (core), 2003. **Space observations required!**
  + hybrid pulsators

**sdO stars (V~15):**
- short-periods (80-140s), p-modes, no successful seismic modeling so far

**White dwarfs (V~18):**
- GW Vir (He/C/O atmospheres), 1979, T_{eff} ~125,000 K
- DBV (He atmo), 1982, T_{eff} ~25,000 K
- Hot-DQV (C-rich/He atmo), 2007, T_{eff} ~20,000 K
- DAV (H atmo) or ZZ Ceti, 1968, T_{eff} ~12,000 K

All g-mode pulsators
(few hundreds to few thousands seconds periods)
Pulsations in compact stars: the space input

**sdB stars:**

> **Kepler**: discovery of 18 sdB pulsators, K2: discovery/observations of 36 sdB pulsators, g-modes and hybrids + 1 g-mode pulsator observed by CoRoT

> **TESS (WG8)**: Per sector, ≈100 sdB (plus a dozens of sdOs) among which ≈ 10 sdB pulsators (discovery and observations); g-, p-mode and hybrid pulsators
Pulsations in compact stars: the space input

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**Marking results:**

2. Observations of g-modes up to \( l=12 \) ! (Telting et al. 2014, Kern et al. 2018, Silvotti et al. 2019)
**Pulsations in compact stars: the space input**

**sdB stars:**

- **Kepler:** discovery of 18 sdB pulsators, K2: discovery/observations of 36 sdB pulsators, g-modes and hybrids + 1 g-mode pulsator observed by CoRoT
- **TESS (WG8):** \(\approx 100\) sdB (plus a dozens of sdOs) among which \(\approx 10\) sdB pulsators (discovery and observations) per sector; g-, p-mode and hybrid pulsators

**Marking results:**

3. single sdB stars are all slow rotators (Charpinet et al. 2018), in direct line with core rotation of Red Clump stars (Mosser et al. 2012) => indication of similar evolution (post-RGB stars)
Pulsations in compact stars: the space input

**White dwarfs:**
> Kepler: 5 DAVs (only 2 for more than 1 quarter) and 1 DBV (23 months)
> K2: 22 DAVs, 1 DBV
> TESS (WG8): observations/discovery of several DAV, DBV & GW Vir pulsators (Bognar et al., Bell et al., Sowicka et al. in prep.)

**Marking results:**
4. Aperiodic, sporadic outbursting events in cool DAVs (but not the coolest): increase of stellar flux up to ~15% (Bell et al. 2017, Hermes et al. 2017)
Pulsations in compact stars: the space input

**White dwarfs:**
> Kepler: 5 DAVs (only 2 for more than 1 quarter) and 1 DBV (23 months)
> K2: 22 DAVs, 1 DBV
> TESS (WG8): observations/discovery of several DAV, DBV & GW Vir pulsators (Bognar et al., Bell et al., Sowicka et al. in prep.)

**Marking results:**
5. In DAVs, dichotomy in mode line widths at weighted mean period of ~800 s (Hermes et al. 2017)
Pulsations in compact stars: the space input

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> Kepler: 5 DAVs (only 2 for more than 1 quarter) and 1 DBV (23 months)
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**Marking results:**
6. Rotation rates for white dwarfs, slow rotation + rotation rate as a function of mass (current & progenitor) (Hermes et al. 2017). No more Angular Momentum loss from HB phase (Charpinet et al. 2018)
Pulsations in compact stars: the space input

**White dwarfs:**
- Kepler: 5 DAVs (only 2 for more than 1 quarter) and 1 DBV (23 months)
- K2: 22 DAVs, 1 DBV
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Charpinet et al. 2018
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Method for forward asteroseismic modeling

- \( \log g \)
- \( T_{\text{eff}} \)
- \( M_*, R_* \)
- \( M_{\text{env}} \)
- \( M_{\text{core}} \)
- H, He, C, O profiles
- ...

Optimal model = seismic solution

\[
S^2(a_1, a_2, ..., a_N) = \sum_{i=1}^{N_{\text{obs}}} (P_{\text{obs}}^{(i)} - P_{\text{th}}^{(i)})^2
\]

Genetic algorithms
Parameterized static models

Complete static equilibrium structures, *independent* of stellar evolution

**sdB stars**

\[
\log q \equiv \log \left(1 - \frac{M(r)}{M^*}\right)
\]

- **H (+Fe) envelope**
- **He mantle**
- **He/C/O core** (with He+C+O=1)

**Complete static equilibrium structures, independent of stellar evolution**

- **H (+Fe) envelope**
- **He mantle**
- **He/C/O core** (with He+C+O=1)
Parameterized static models

Complete static equilibrium structures, *independent of stellar evolution*

**sdB stars**

- Envelope with double transition:
  - Pure H envelope
  - H/He envelope (+Fe) (Henv,diff)
  - 0 - 8% of C (C_flash)
  - in the He mantle on 0 - 100% (lq_flash) of the mantle produced by He-flash
  - He_{core}, O_{core}

- + chemical transition profiles (smooth to steep): pf_diff, pf_env, pf_flash & pf_core
- + total mass of the star M_∗

= 4th generation (4G) models of sdB stars, up to 13 parameters
Parameterized static models

Complete static equilibrium structures, independent of stellar evolution

White dwarfs

Typical configurations predicted by standard evolution models can be reproduced by our static ones and can be found in our optimization computations... if optimal
**An illustrative example**

**Modeling TESS target PG 0342+026 (Sector 5)**

- Frequency analysis with FELIX (Charpinet et al. 2010), above 4.6σ: **27 independent g-modes** (1674 s – 10331 s)

- Atmospheric parameters (D. Schneider 2019):
  - \( T_{\text{eff}} = 25,700 \pm 300 \text{ K} \)
  - \( \log g = 5.48 \pm 0.03 \)
Search the stellar model(s) whose theoretical periods best fit the observed ones, in order to minimize

\[ S^2(a_1, a_2, \ldots, a_N) = \sum_{i=1}^{N_{\text{obs}}} (P_{\text{obs}}^{(i)} - P_{\text{th}}^{(i)})^2 \]

> **Optimization procedure:** Efficient optimization codes (based on *Genetic Algorithms*) to thoroughly explore the parameter space and find the minima of \( S^2 \)

Under *external* constraints from spectroscopy in the 13-parameters space:

- \( 0.3 \leq M_* \leq 0.7 \, M_{\odot} \) (Han et al. 2002, 2003)
- \(-3.0 \leq lq_{\text{env}} \leq -1.5\)
- \(-0.40 \leq lq_{\text{core}} \leq -0.10\)
- \(0 \leq \text{He}_{\text{core}} \leq 1\)
- \(0 \leq \text{O}_{\text{core}} \leq 1\)
- \(lq_{\text{env, diff}} \): 60-100% + location of the transition \( lq_{\text{diff}} \)
- \(lq_{\text{flash}} \): 0-8% + location of the transition \( lq_{\text{flash}} \)
- Steep to smooth profiles (pro_fac parameters: \( pf_{\text{diff}} \), \( pf_{\text{env}} \), \( pf_{\text{flash}} \) and \( pf_{\text{core}} \))
- \( T_{\text{eff}}, \log g @ 3\sigma \) spectroscopy
One clear solution emerged:

$$S^2=0.20, \, \bar{dP}/P \sim 0.07\%, \, \bar{dP}=3.5\text{s}, \, \bar{dv}=0.17 \, \mu\text{Hz}$$
Modeling TESS target PG 0342+026

\[ M_\star - \log q_{\text{env}} \]

\[ M_\star = 0.452^{+0.007}_{-0.005} M_{\odot} \]
\[ \log q_{\text{env}} = -2.28^{+0.27}_{-0.07} \]

\[ \text{He}_{\text{core}} - \log q_{\text{core}} \]
\[ \text{He}_{\text{core}} = 0.68^{+0.05}_{-0.09} \]
\[ \log q_{\text{core}} = -0.27^{+0.02}_{-0.01} \]
Modeling TESS target PG 0342+026

\[ O_{\text{core}} - Pf_{\text{diff}} \]

\[ H_{\text{env,diff}} - lq_{\text{diff}} \]

\[ O_{\text{core}} = 0.27^{+0.02}_{-0.16} \]

Pf\_diff unconstrained

\[ H_{\text{env,diff}} = 0.86^{+0.07}_{-0.09} \]

lq\_diff = -3.41^{+0.03}_{-2.57}
Modeling TESS targets: TIC 169285097

Pf_core - Pf_env

C flash - lq flash

Pf_core = 121\,^{+40}_{-10}

Pf_env = 4.9\,^{+1.6}_{-1.8}

C_{\text{flash}} = 4\pm 2\%

lq_{\text{flash}} = -0.75\,^{+0.03}_{-1.26}

+ Pf_{\text{flash}}\text{ unconstrained}
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Testing the seismic results

New possibilities with GAIA

- GAIA provides high quality parallaxes/distance estimates =>

1. Possibility to cross-check with **distance** derived based on seismic stellar parameters
2. Combined to spectroscopy, possibility to cross-check with **mass** derived from asteroseismology

To date: asteroseismic solutions available for a sample of **18** sdB pulsators
Test 1: Method for deriving asteroseismic distances

Asteroseimology

\[ \begin{align*}
\text{log g} & \\
T_{\text{eff}} & \\
M_\ast & 
\end{align*} \]

N(He)/N(H)

N(He)/N(H)

Apparent magnitude \( a \)

Absolute magnitude \( M_a \)

Spectroscopic fitting

Bandpass of filter \( a \)

Absorption coefficient: Bandpass+E(B-V)

Synthetic spectrum
## Test 1: The sdB asteroseismic sample

18 sdB stars modeled by asteroseismology

<table>
<thead>
<tr>
<th>Name</th>
<th>log $g$ (cm s$^{-2}$)</th>
<th>$T_{\text{eff}}$ (K)</th>
<th>$M$ ($M_{\odot}$)</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>PG 1047+003</td>
<td>5.800±0.006</td>
<td>33000±1600</td>
<td>0.490±0.014</td>
<td>Charpinet et al. (2003)</td>
</tr>
<tr>
<td>PG 0014+067</td>
<td>5.775±0.009</td>
<td>33940±3520</td>
<td>0.477±0.024</td>
<td>Charpinet et al. (2005a)</td>
</tr>
<tr>
<td>PG 1219+534</td>
<td>5.807±0.006</td>
<td>33640±1360</td>
<td>0.457±0.012</td>
<td>Charpinet et al. (2005b)</td>
</tr>
<tr>
<td>Feige 48</td>
<td>5.437±0.006</td>
<td>29580±370</td>
<td>0.460±0.008</td>
<td>Charpinet et al. (2005c)</td>
</tr>
<tr>
<td>EC 05217–3914</td>
<td>5.730±0.008</td>
<td>32000±1800</td>
<td>0.490±0.020</td>
<td>Billères &amp; Fontaine (2005)</td>
</tr>
<tr>
<td>PG 1325+101</td>
<td>5.811±0.004</td>
<td>35050±220</td>
<td>0.499±0.011</td>
<td>Charpinet et al. (2006a)</td>
</tr>
<tr>
<td>PG 0048+091</td>
<td>5.711±0.010</td>
<td>33300±1700</td>
<td>0.447±0.027</td>
<td>Charpinet et al. (2006a)</td>
</tr>
<tr>
<td>EC 20117–4014</td>
<td>5.856±0.008</td>
<td>34800±2000</td>
<td>0.540±0.040</td>
<td>Randall et al. (2006a)</td>
</tr>
<tr>
<td>PG 0911+456</td>
<td>5.777±0.002</td>
<td>31940±220</td>
<td>0.390±0.010</td>
<td>Randall et al. (2007)</td>
</tr>
<tr>
<td>BAL 090100001</td>
<td>5.383±0.004</td>
<td>28000±1200</td>
<td>0.432±0.015</td>
<td>Van Grootel et al. (2008)</td>
</tr>
<tr>
<td>EC 09582–1137</td>
<td>5.788±0.004</td>
<td>34805±230</td>
<td>0.485±0.011</td>
<td>Randall et al. (2009)</td>
</tr>
<tr>
<td>KPD 1943+4058</td>
<td>5.520±0.030</td>
<td>28050±470</td>
<td>0.496±0.002</td>
<td>Van Grootel et al. (2010a)</td>
</tr>
<tr>
<td>KPD 0629–0016</td>
<td>5.450±0.034</td>
<td>26290±530</td>
<td>0.471±0.002</td>
<td>Van Grootel et al. (2010b)</td>
</tr>
<tr>
<td>KIC02697388$^a$</td>
<td>5.489±0.033</td>
<td>25622±420</td>
<td>0.463±0.009</td>
<td>Charpinet et al. (2011)</td>
</tr>
<tr>
<td>KIC02697388$^b$</td>
<td>5.499±0.049</td>
<td>25555±520</td>
<td>0.452±0.011</td>
<td>Charpinet et al. (2011)</td>
</tr>
<tr>
<td>PG 1336–018</td>
<td>5.739±0.002</td>
<td>32780±200</td>
<td>0.459±0.005</td>
<td>Van Grootel et al. (2013)</td>
</tr>
<tr>
<td>TIC 278659026</td>
<td>5.572±0.056</td>
<td>23738±640</td>
<td>0.391±0.013</td>
<td>Charpinet et al. (2019)</td>
</tr>
</tbody>
</table>

+ PB8783 (Van Grootel et al. 2019) + PG 0342+026 (this work)
Test 1: distance dependence on $M_*$, $T_{\text{eff}}$ and $\log g$

- Representative sdB model: $T_{\text{eff}} = 32,000$ K, $\log g = 5.8$, $M = 0.47 \, M_\odot$ -> reference distance $d_{\text{ref}}$
- Distance $d$ computed by varying one of the seismic parameters and keeping the other two constant
- Considering “expected” parameter ranges for sdBs, changing the mass can account for a ± 10% variation in derived asteroseismic distance
- $\log g$ and $T_{\text{eff}}$ have a larger effect on the distance if considering the full ranges where sdBs are found (but small effect over typical measurement uncertainties)

### Results of Test 1: seismic vs GAIA distances

<table>
<thead>
<tr>
<th>Name</th>
<th>$d$(Gaia) (pc)</th>
<th>$d(G_{BP}G_{RP})$ (pc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PG 1047+003</td>
<td>687±37</td>
<td>609±49</td>
</tr>
<tr>
<td>PG 0014+067</td>
<td>2794±1037</td>
<td>1812±277</td>
</tr>
<tr>
<td>PG 1219+534</td>
<td>549±14</td>
<td>532±35</td>
</tr>
<tr>
<td>Feige 48</td>
<td>822±27</td>
<td>769±28</td>
</tr>
<tr>
<td>EC 05217−3914</td>
<td>1590±108</td>
<td>1636±154</td>
</tr>
<tr>
<td>PG 1325+102</td>
<td>862±51</td>
<td>820±23</td>
</tr>
<tr>
<td>PG 0048+091</td>
<td>1058±48</td>
<td>...</td>
</tr>
<tr>
<td>EC 20117−4014</td>
<td>587±13</td>
<td>...</td>
</tr>
<tr>
<td>PG 0911+456</td>
<td>1035±75</td>
<td>958±27</td>
</tr>
<tr>
<td>BAL 090100001</td>
<td>365.6±8.6</td>
<td>386±31</td>
</tr>
<tr>
<td>EC 09582−1137</td>
<td>1553±139</td>
<td>1376±40</td>
</tr>
<tr>
<td>KPD 1943+4058</td>
<td>1274±51</td>
<td>1242±73</td>
</tr>
<tr>
<td>KPD 0629−0016</td>
<td>1011±50</td>
<td>1022±67</td>
</tr>
<tr>
<td>KIC 02697388$^b$</td>
<td>1262±64</td>
<td>1328±76</td>
</tr>
<tr>
<td>KIC 02697388$^c$</td>
<td>1262±64</td>
<td>1295±79</td>
</tr>
<tr>
<td>PG 1336−018</td>
<td>552±19</td>
<td>541±26</td>
</tr>
<tr>
<td>TIC 278659026</td>
<td>203.7±2.1</td>
<td>221±21</td>
</tr>
</tbody>
</table>

All distances agree within 1sigma
Test 2: Method for deriving “spectroscopic” masses

- \( \log g = GM/R^2 \)
- \( T_{\text{eff}} \)
- Distance \( d/\text{parallax} \ \varpi \)

\[ \text{Angular diameter} \ \theta \approx 2R/d \text{ (if } \theta << 1) \]

\[ \langle \text{Spectroscopic} \rangle \text{ mass} \]

\[ M = g\theta^2 / 4G\varpi^2 \]
Results of test 2: seismic vs spectroscopic masses

$\Delta M/M \text{ seismology} \sim 10\%$

$\Delta M/M \text{ spectroscopy} \sim 25\%$

Fontaine et al., in prep.
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• DBV pulsator, observed during 23 months in the original *Kepler* mission
• 8 well-secured independent modes in the range 143.2 – 376.1 s
• Close to hot edge: $T_{\text{eff}} = 29,360 \pm 780$ K, log g=$7.89 \pm 0.05$

Seismic optimization with static parametrized models, based on $L(r) \alpha M(r)$ profiles (Giammichele et al. 2018, Nature, 554, 73):

✓ **Reproduction of the 8 periods at the precision of the observations** ($\Delta \nu \sim 0.6$ nHz or $\Delta P \sim 38 \mu$s)
✓ **Core ~15% richer in O (in mass) and ~40% more massive than expected from standard evolutionary models**
• **A legitimate problem**: in hot DBVs, still active neutrino cooling makes $L(r) \propto M(r)$ (Timmes et al. 2018) -> shift in frequencies about $30 \mu$Hz (up to $70 \mu$Hz)

• Charpinet et al. 2019 (A&A, 628, L2): corrected $L(r)$ profiles => new asteroseismic solution and chemical profiles

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**Global parameters and main conclusions on core size and composition are unchanged**
## Preliminary results for five DAV pulsators

<table>
<thead>
<tr>
<th></th>
<th>KIC11911480</th>
<th>L19-2</th>
<th>SDSSJ1136+0409</th>
<th>EPIC220258806</th>
<th>EPIC220347759</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{\text{eff}}$ (spectro)</td>
<td>12,026 ± 195</td>
<td>12,058 ± 184</td>
<td>12,330 ± 260</td>
<td>12,807 ± 219</td>
<td>12,692 ± 214</td>
</tr>
<tr>
<td>$T_{\text{eff}}$ (astero)</td>
<td>12,300 ± 485</td>
<td>12,021 ± 717</td>
<td>12,297 ± 195</td>
<td>12,163 ± 417</td>
<td>12,478 ± 329</td>
</tr>
<tr>
<td>log $g$ (spectro)</td>
<td>8.00 ± 0.05</td>
<td>8.11 ± 0.05</td>
<td>7.99 ± 0.06</td>
<td>8.1 ± 0.05</td>
<td>8.09 ± 0.05</td>
</tr>
<tr>
<td>log $g$ (astero)</td>
<td>8.01 ± 0.04</td>
<td>8.13 ± 0.06</td>
<td>8.06 ± 0.04</td>
<td>8.07 ± 0.02</td>
<td>8.11 ± 0.05</td>
</tr>
<tr>
<td>$d$ (astero)</td>
<td>184.2 ± 14.6</td>
<td>21.0 ± 0.5</td>
<td>127.7 ± 7.2</td>
<td>79.6 ± 3.0</td>
<td>141.7 ± 7.0</td>
</tr>
<tr>
<td>$d$ (parallax)</td>
<td>182.9 ± 4.1</td>
<td>20.93 ± 0.01</td>
<td>130.7 ± 1.9</td>
<td>80.56 ± 0.54</td>
<td>151.8 ± 3.5</td>
</tr>
</tbody>
</table>

Other relevant parameters derived from asteroseismology (see text for details):

<p>| | | | | | |</p>
<table>
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<tbody>
<tr>
<td>Mass ($M_{\odot}$)</td>
<td>0.63</td>
<td>0.68</td>
<td>0.63</td>
<td>0.64</td>
<td>0.67</td>
</tr>
<tr>
<td>log $q$(H)</td>
<td>-3.12</td>
<td>-4.39</td>
<td>-5.55</td>
<td>-4.23</td>
<td>-4.42</td>
</tr>
<tr>
<td>log $q$(He)</td>
<td>-1.42</td>
<td>-1.87</td>
<td>-1.75</td>
<td>-1.91</td>
<td>-2.40</td>
</tr>
<tr>
<td>log $q$(core)</td>
<td>-0.66</td>
<td>-0.80</td>
<td>-0.77</td>
<td>-0.37</td>
<td>-0.74</td>
</tr>
<tr>
<td>O(core)</td>
<td>0.85</td>
<td>0.82</td>
<td>0.75</td>
<td>0.88</td>
<td>0.85</td>
</tr>
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Charpinet et al. 2019, EuroWD21, in press
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<td>12,300 ± 485</td>
<td>12,021 ± 717</td>
<td>12,297 ± 195</td>
<td>12,163 ± 417</td>
<td>12,478 ± 329</td>
</tr>
<tr>
<td>log $g$ (spectro)</td>
<td>8.00 ± 0.05</td>
<td>8.11 ± 0.05</td>
<td>7.99 ± 0.06</td>
<td>8.1 ± 0.05</td>
<td>8.09 ± 0.05</td>
</tr>
<tr>
<td>log $g$ (astero)</td>
<td>8.01 ± 0.04</td>
<td>8.13 ± 0.06</td>
<td>8.06 ± 0.04</td>
<td>8.07 ± 0.02</td>
<td>8.11 ± 0.05</td>
</tr>
<tr>
<td>$d$ (astero)</td>
<td>184.2 ± 14.6</td>
<td>21.0 ± 0.5</td>
<td>127.7 ± 7.2</td>
<td>79.6 ± 3.0</td>
<td>141.7 ± 7.0</td>
</tr>
<tr>
<td>$d$ (parallax)</td>
<td>182.9 ± 4.1</td>
<td>20.93 ± 0.01</td>
<td>130.7 ± 1.9</td>
<td>80.56 ± 0.54</td>
<td>151.8 ± 3.5</td>
</tr>
</tbody>
</table>

Other relevant parameters derived from asteroseismology (see text for details):

<table>
<thead>
<tr>
<th>Parameter</th>
<th>KIC11911480</th>
<th>L19-2</th>
<th>SDSSJ1136+0409</th>
<th>EPIC220258806</th>
<th>EPIC220347759</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass ($M_\odot$)</td>
<td>0.63</td>
<td>0.68</td>
<td>0.63</td>
<td>0.64</td>
<td>0.67</td>
</tr>
<tr>
<td>log $q$(H)</td>
<td>-3.12</td>
<td>-4.39</td>
<td>-5.55</td>
<td>-4.23</td>
<td>-4.42</td>
</tr>
<tr>
<td>log $q$(He)</td>
<td>-1.42</td>
<td>-1.87</td>
<td>-1.75</td>
<td>-1.91</td>
<td>-2.40</td>
</tr>
<tr>
<td>log $q$(core)</td>
<td>-0.66</td>
<td>-0.80</td>
<td>-0.77</td>
<td>-0.37</td>
<td>-0.74</td>
</tr>
<tr>
<td>O(core)</td>
<td>0.85</td>
<td>0.82</td>
<td>0.75</td>
<td>0.88</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Charpinet et al. 2019, EuroWD21, in press

**Similar pattern for 4 DAVs:** core ~15% richer in O and ~40% more massive than expected from standard evolutionary models

**This provides strong constraints on processes shaping stellar cores during the preceding He-burning phase**
### He-burning cores in sdB stars

<table>
<thead>
<tr>
<th>Star</th>
<th>$M^*$ ($M_\text{sun}$)</th>
<th>Mcore ($M_\text{sun}$)</th>
<th>Hecore</th>
<th>Ocore</th>
</tr>
</thead>
<tbody>
<tr>
<td>KPD0629-0016</td>
<td>0.471±0.002</td>
<td>0.22±0.01 (47% $M_\star$)</td>
<td>0.59±0.01</td>
<td>-</td>
</tr>
<tr>
<td>(Van Grootel et al. 2010)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KIC02697388</td>
<td>0.452$^{+0.018}_{-0.005}$</td>
<td>0.225$^{+0.011}<em>{-0.016}$ (49% $M</em>\star$)</td>
<td>0.73$^{+0.07}_{-0.12}$</td>
<td>-</td>
</tr>
<tr>
<td>(Charpinet et al. 2011)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TIC278659026</td>
<td>0.391±0.009</td>
<td>0.198±0.001 (50% $M_\star$)</td>
<td>0.58$^{+0.06}_{-0.03}$</td>
<td>0.16$^{+0.13}_{-0.05}$</td>
</tr>
<tr>
<td>(Charpinet et al. 2019)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PG 0342+026</td>
<td>0.452$^{+0.007}_{-0.005}$</td>
<td>0.209$^{+0.011}<em>{-0.016}$ (46% $M</em>\star$)</td>
<td>0.68$^{+0.05}_{-0.09}$</td>
<td>0.27$^{+0.02}_{-0.16}$</td>
</tr>
<tr>
<td>(This work)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Very similar cores, all significantly bigger (and more O-rich ?) than expected from stellar evolution
I. Pulsations in hot subdwarf and white dwarf stars
   a. Introduction
   b. Some marking results from space observations
II. Models and method for asteroseismic modeling
III. Testing the sdB seismic results with GAIA
IV. Seismic modeling of white dwarfs
V. Conclusions
Conclusions

✓ Space observations are crucial for g-modes of compact pulsators
✓ Some marking results from Kepler:
  • Observations of g-modes in sdB stars up to l=12!
  • Consistent picture of Angular Momentum along evolution: loss during/before RGB phase, no more after

✓ Forward modeling approach with static parametrized models is successful for sdB/WD asteroseismology:
  ✓ Global parameters and internal structures determined with a very high precision
  ✓ sdB distances and masses fully consistent with GAIA results
  ✓ He-burning cores are bigger (and more enriched in O) than predicted from stellar evolution