Combined use of finite volume and network modelling for Stokes flow and permeability tensor computation in porous media

N. Combaret[a], D. Bernard[b], E. Plougonven[c]

[a]*Formerly CNRS, Université de Bordeaux, ICMCB; presently VSG, Visualization Sciences Group – Mérignac, France*
[b]*CNRS, Université de Bordeaux, ICMCB – Pessac, France*
[c] *Formerly CNRS, Université de Bordeaux, ICMCB; presently University of Nebraska Lincoln, Department of Engineering Mechanics – Lincoln, NE 68588, USA*

Key words: porous media, permeability tensor, Stokes flow, network model

1 Introduction

Recent developments in imaging techniques such as X-ray computed micro-tomography make possible 3D imaging of porous media micro-geometry at unrivalled resolutions. An alternative to the direct computation of pressure and velocity fields (DCPV) in the 3D image is the use of network models (NM) where the complex pore structure is represented by an equivalent network of pore bodies (PB) connected via pore throats (PT). This method produces a system of linear equations of moderate size that can be easily solved. Applicability of NM is not limited by computational costs, but by difficulties in building, from a discrete 3D image, an equivalent network summarizing the relevant topological (connectivity of pore bodies) and geometrical (resistance associated to each pore throat) aspects of a porous medium.

2 Construction of the graph

In this work, several crucial steps of this process are analysed in depth. Starting from a 3D binary image of a porous sample, a skeleton of the pore space is built. Direct skeletonization of a binary image often leads to a noisy skeleton, in which artefacts (closed surfaces, multiple branches connecting pore centres) appear. E. Plougonven [1] proposed a new and optimal method to avoid those artefacts, which topologically detects the voxels creating artefacts and removes them. Analysing this skeleton, pore centres are localized at the voxels having more than two connections. A graph is then defined where nodes correspond to PB and branches to connections intersecting PT. However, there exist clusters of nodes where each node does not represent a single pore. We proposed a merging criterion based on overlapping volumes in order to prevent over partitioning of simple pores. Finally, the partitioning itself is performed to precisely delineate the PB associated to the nodes and the PT, using a watershed process where the nodes locations are markers.

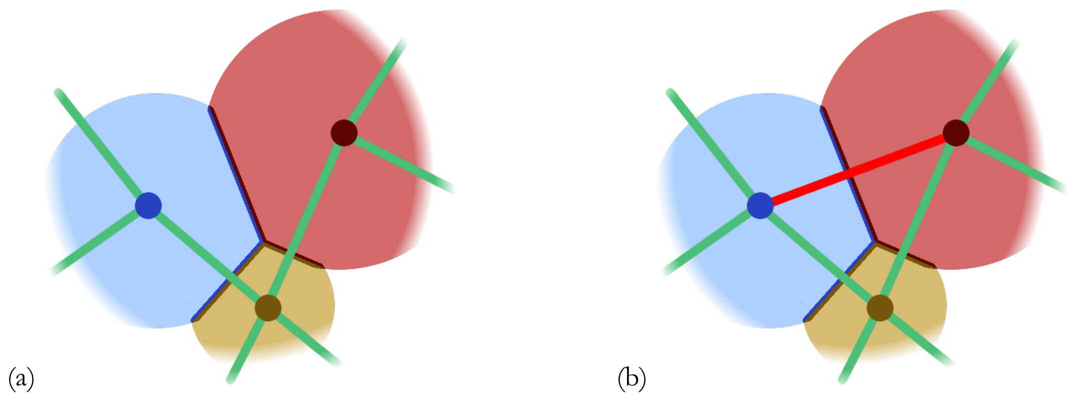


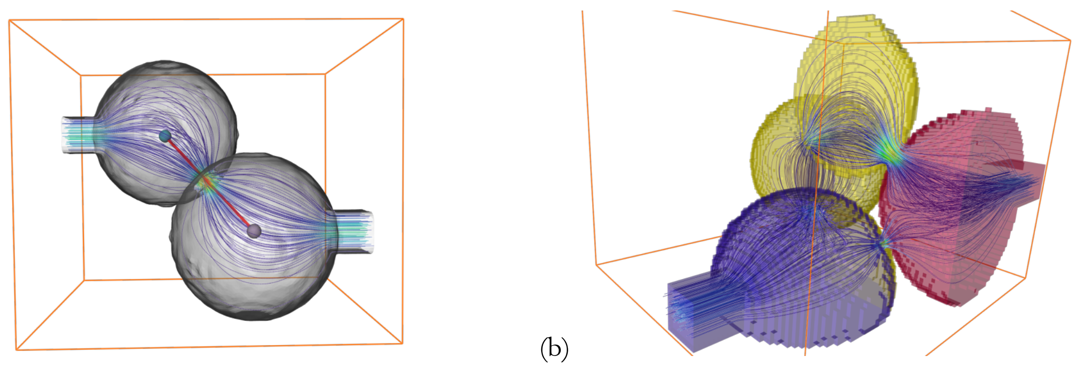
Figure 1 – Example of unexpected configuration. Three pores are identified from the skeleton but only two edges exist. The third (in red) must be added to represent the connection between the red pore and the blue pore.

Nevertheless, unexpected configurations appear where the PT of a PB is connected with more than one PB (figure 1a). Various post treatments of the graph have been tested in order to make it coherent with the partition. The only one that solves the problem implies a modification of the graph topology: branches are added so that all the PT surfaces in contact have a corresponding edge in the graph (figure 1b).

At all the steps, boundary conditions are considered with particular interest since the NM has to be used for flow modelling and permeability tensor estimation. In the first case, anchor points must be kept on the image boundaries, so that it is possible to impose a pressure gradient or a given flow rate to perform the fluid flow simulation. In the second case, periodic boundary conditions are used for skeletonization, so that a skeleton branch going out an image face is in correspondence with another branch on the opposite side.

3 Equations and resistance values

Once a representative graph on the pore space is obtained, the relations between nodes and the value of the associated parameters must be defined. Classically, an equivalent electrical resistances network is built assuming a constant pressure in the PB and a pressure variation localized at the PT region. The local resistance linking the pressure variation to the fluid flow is often estimated through an analytical expression derived from a simplified geometrical model of the PT region (series of cylinders for instance). In this work a general form of the linear system of equations to be solved was obtained performing two integrations of Stokes equations in the pore space (a volume integration and curvilinear one). With this formulation, pore pressure is not necessarily constant (nodes position is implicitly taken into account) and the flow rate, linearly related to pressure drop between two nodes, might be calculated for an arbitrarily selected surface. This last point allows us to propose an original solution to handle the cases where more than two pores share a PT. The resistance associated to each PT is thus evaluated solving a local flow problem that is theoretically introduced (figure 2).



(a) (b)

Figure 2 – a) Each couple of pore (grey transparent shape) associated to an edge (red line) is extracted from the geometry. Input and output setups are added and a local Stokes flow problem is solved in this very simple geometry. It is then easy to get the resistance between the two nodes from the pressure values at the nodes and the velocity filed. b) The method can be easily extended to non-pairwise connected pores: the resistance associated to the edge connecting the blue and red pores is calculated taking into account the fact that the yellow pore shares connection surfaces with both.

To compute the permeability tensor of the sample we consider the closure problem obtained by applying the volume averaging method to the local Stokes' equations [2]. The resulting local problem has the same form as Stokes' equations and the same methods can be used to solve it. Having built a periodic graph for which the resistances between nodes are known, the closure problem can be solved by the NM. A proper post-processing method is presented to estimate the complete permeability tensor from three solutions of the closure problem.

Comparisons of the local solutions (nodes pressure and flow rates) obtained by NM and DCPV are presented for simple synthetic cases and two real material samples.

References

[1] E. Plougonven, Link between the microstructure of porous materials and their permeability, PhD thesis, Bordeaux University (2009)
[2] S. Whitaker, The method of volume averaging, Kluwer Acad. Pub., Dordrecht (1999)