Combined use of finite volume and network modelling for Stokes flow and permeability tensor computation in porous media

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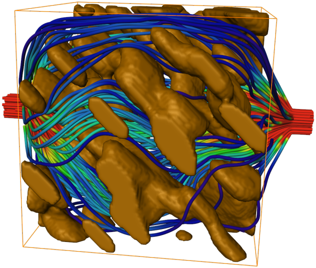
From 3D image to permeability computation

- 3D image: direct numerical processing
- Network modeling:
  - Structured network model
  - Unstructured network model



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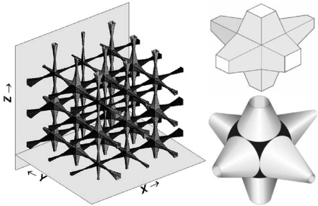

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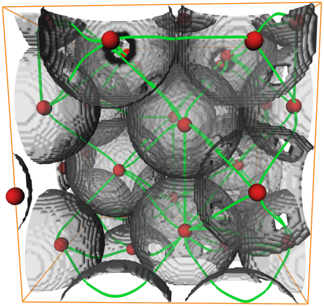

From the Stokes equations to a linear system

- Electric resistances network equivalence?
- Stokes equations

$$\begin{cases} \vec{\nabla}.\vec{v} = 0 \\ \nabla^2\vec{v} - \vec{\nabla}p = 0 \end{cases}$$

$$\text{B.C.: } \vec{v} = 0 \text{ on } \mathcal{A}_{fs}$$

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$$\begin{cases} \sum_{j=0}^{M} Q_{ij} = 0 \\ P_i - P_j = \lambda_{ij}.Q_{ij} \end{cases}$$

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Outline

- Graph construction: from the pore space to a graph relating pores
  - Pore positioning: what is a pore?
  - Pores separation: where are the pores?
- Network equations and parameters: what are the relations between the pores?
  - Stokes equations equivalence on a graph
  - Direct numerical computation of the resistances
- Conclusion & future work

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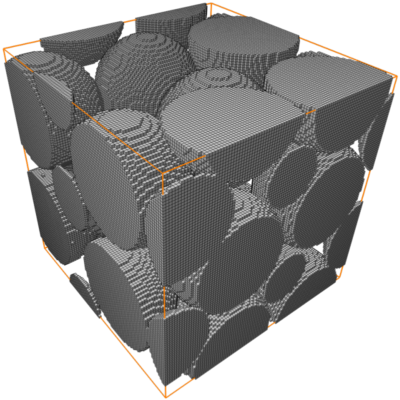
(Almost) three steps method to get the graph

- Skeletonisation: homotopic thinning
  - Digitisation artefact removal method
  - Boundary conditions
- Skeleton points characterisation: curves and intersection points
- Nodes fusion criterion: relative volume overlapping



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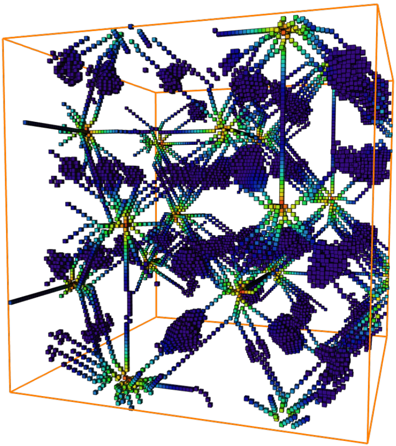

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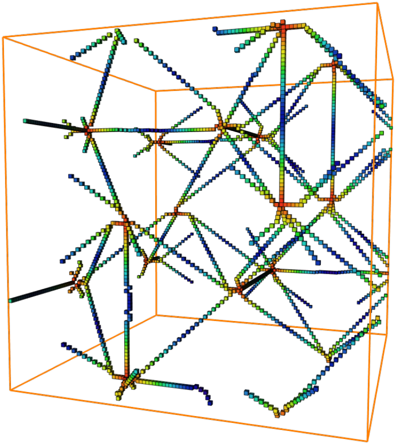

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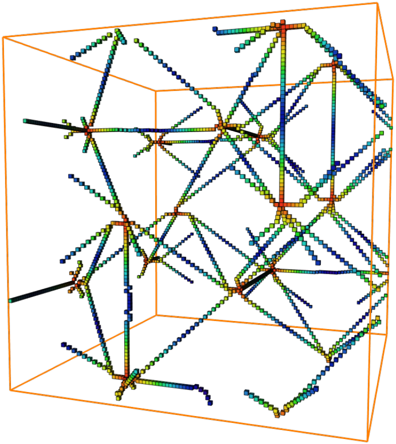

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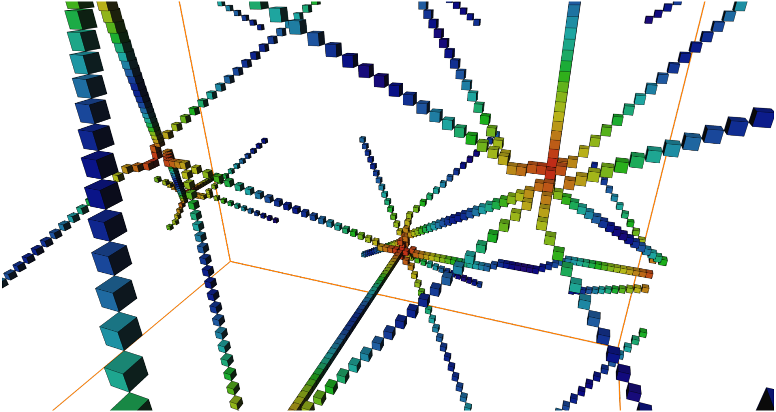

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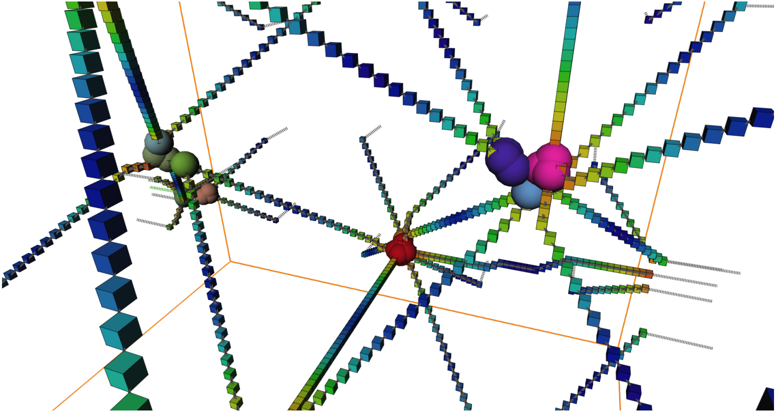

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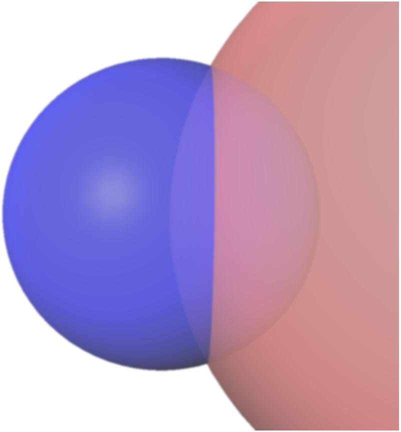

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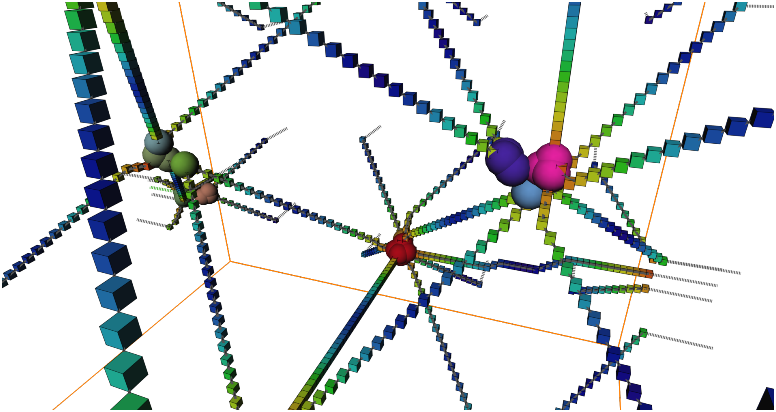

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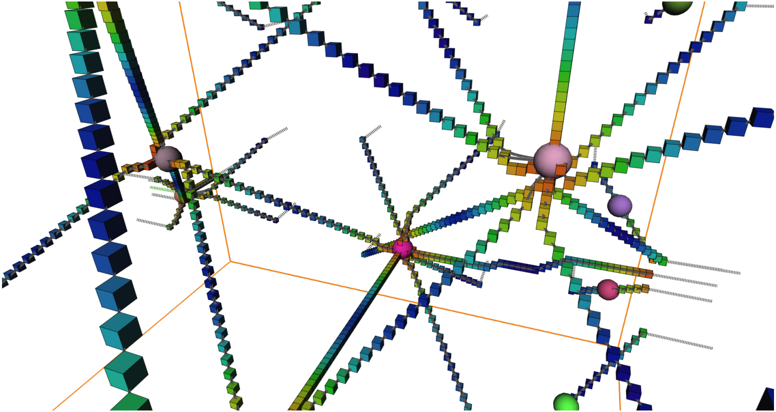

Pore space partitioning

- Pore delimitation:
  - Pore throats positioning
  - Pore volume invasion: watershed
- Unexpected configurations

⇒ Skeleton: good for positioning the pores, bad for representing their interconnexions

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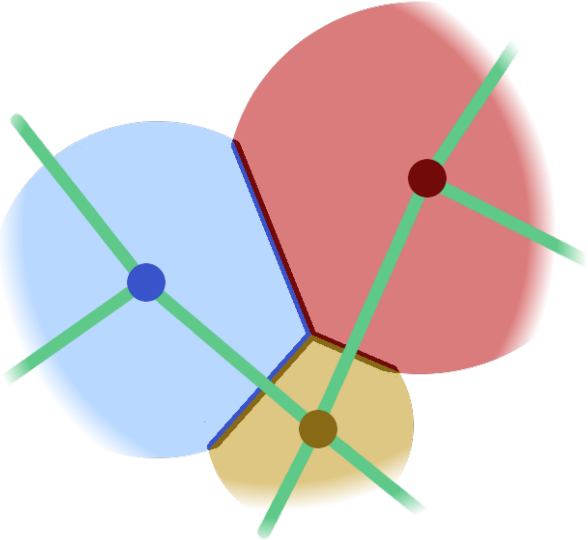

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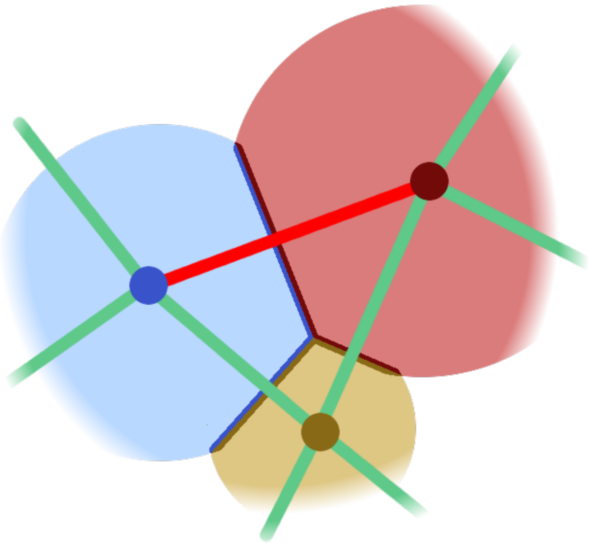

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Integration on the volume of a pore H_i

$$\int_{\mathcal{V}_{H_i}} \vec{\nabla}.\vec{v}\; dV = 0$$

$$\int_{\mathcal{A}_{H_i}} \vec{v}.\overrightarrow{n_{H_i}} dS = 0$$

$$\int_{\mathcal{A}_{H_i H}} \vec{v}.\overrightarrow{n_{H_i H}} dS = 0$$

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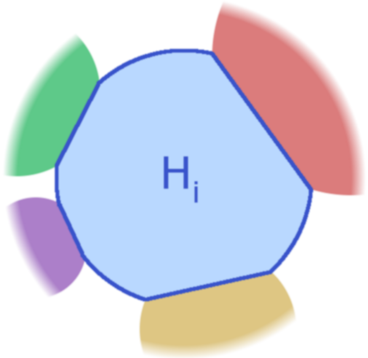

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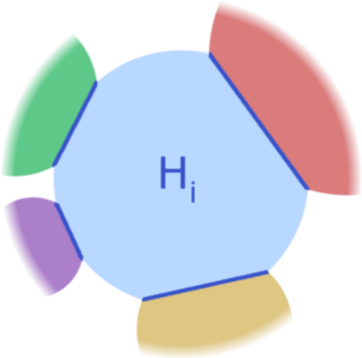

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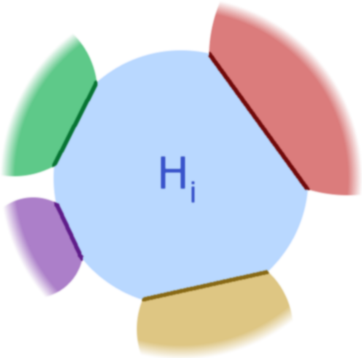

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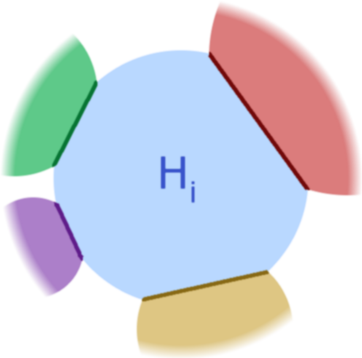

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Integration of the second Stokes equation

Integration along a path C_{ij} between two pores H_i and H_j

$$\underbrace{\int_{C_{ij}} \vec{\nabla} p.\vec{c}\, dC}_{①} - \underbrace{\int_{C_{ij}} \nabla^2 \vec{v}.\vec{c}\, dC}_{②} = 0$$

①:

$$\int_{C_{ij}} \vec{\nabla} p.\vec{c}\, dC = p\left(H_j\right) - p\left(H_i\right)$$

②:

- independent from the path (see ①)
- $Q \propto \vec{v} \propto \nabla^2 \vec{v}$

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- We need something on the pores to support the pressure values

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Local computation of the resistances

- Extracting the pores connected to a surface ($\Leftrightarrow$ an edge)
- Finite volume computation of local Stokes flow
- Resistance determination from the second Stokes equation in a linear system:

$$\lambda_{ij} = \frac{P_i^{local} - P_j^{local}}{Q_{ij}^{local}}$$

- Linear system solution: pressure value at nodes, flow rate through edges

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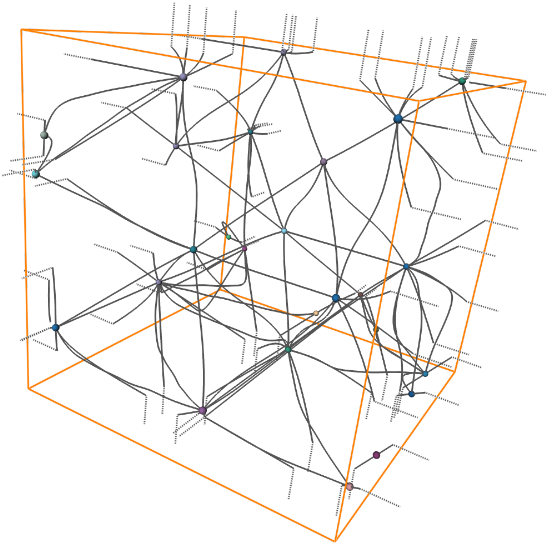

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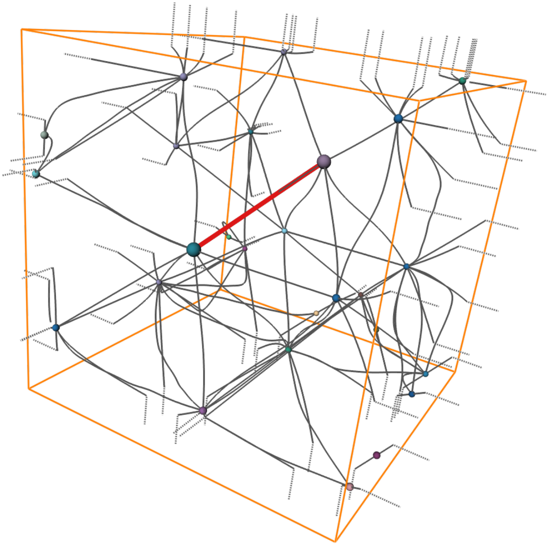

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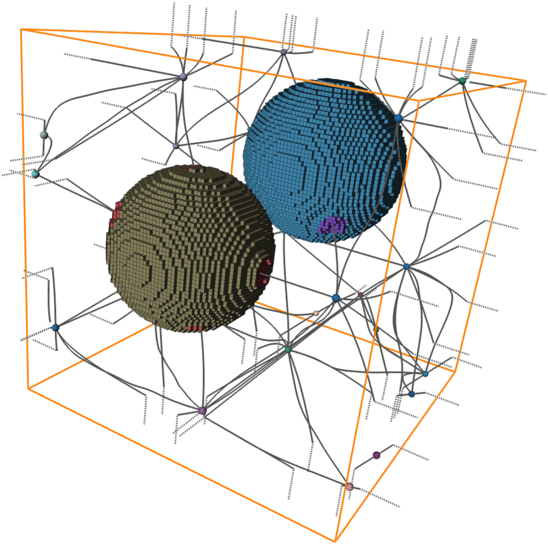

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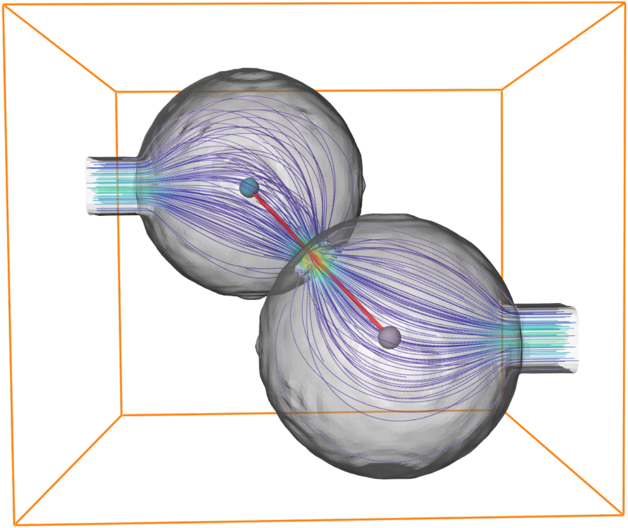

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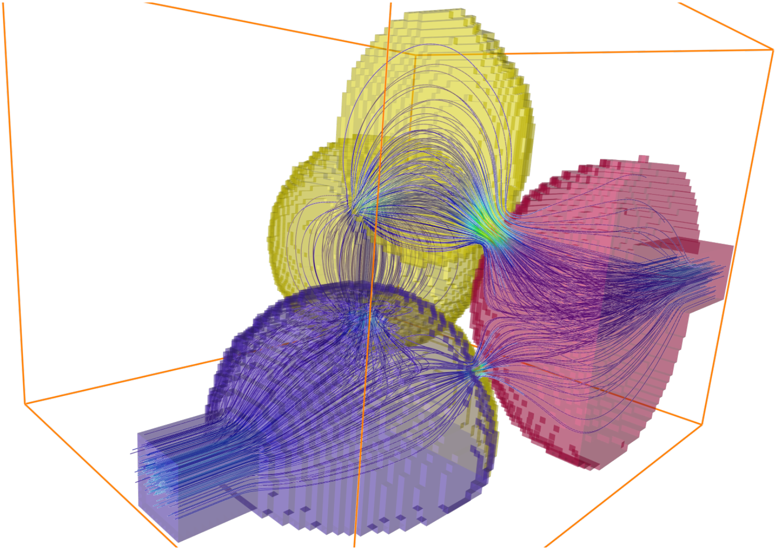

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- Strong graph construction method
  - Digitisation artefacts removal prior to skeletonisation
  - Nodes merging criterion
  - Pores delimitation methods comparison
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Future work

- Compare the linear system solution to the real numerical computation
- Finalise the implementation of the permeability tensor computation

$$\begin{cases} \vec{\nabla}.\overrightarrow{D_k} = 0 \\ \nabla^2\overrightarrow{D_k} - \vec{\nabla}d_k = -\vec{I_k} \end{cases}$$

B.C.: $\overrightarrow{D_k} = 0$ on $\mathcal{A}_{fs}$

B.C.: $\overrightarrow{D_k}$ and d_k are periodic

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