New stellar seismic probing method: WhoSGlAd

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Study stars ?

- Heavy elements factory,
- Stellar ages $\rightarrow$ galactic history,
- Exoplanetary masses, radius and ages,
- ...

Credits: NASA

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Why asteroseismology ?

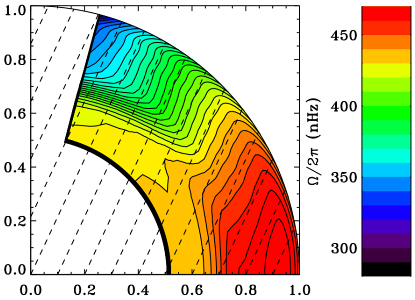
Stellar models need improvement :

- Chemical composition : He in low mass stars, solar reference;
- Mixing processes : extent of mixed regions;
- Angular momentum transport;
- ...

$\rightarrow$ Information about internal structure needed

Why asteroseismology ?

- But 'classical' methods : mainly superficial information (T_{eff}, $[Fe/H]$,..)
- Asteroseismology probes stellar interiors



Credits: GONG

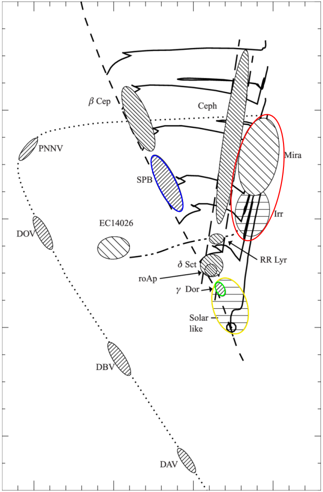
$\rightarrow$ $c(r)$, internal rotation, chemical composition profiles,...

Asteroseimology in a Nutshell

- Stellar structure may oscillate around an equilibrium state
- Stellar oscillation frequencies directly linked stellar internal structure
- Many successes : helioseismology, constraints about stellar structure, asteroseismology of red giants,...
- Need of very precise data but also methods

Pulsating Stars

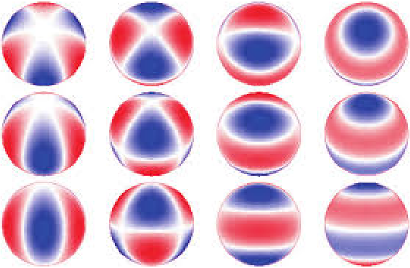
- Solar-like ($P \sim 2 - 15min$),
- γ Dor ($P \sim 0.5 - 3days$),
- SPB ($P \sim 0.8 - 3days$),
- Red giants and subgiants ($P \sim 3 - 30days$),
- ...

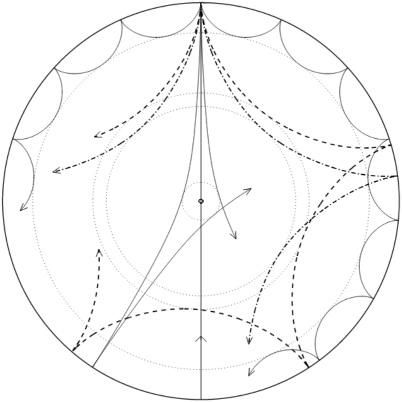


Credits: Christensen-Dalsgaard J.

Asteroseimology in a nutshell

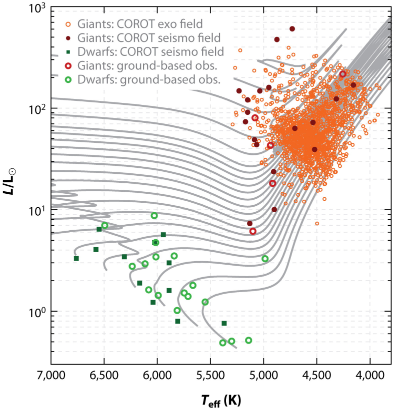
Frequencies, ν, characterised by 2 integer n and l.



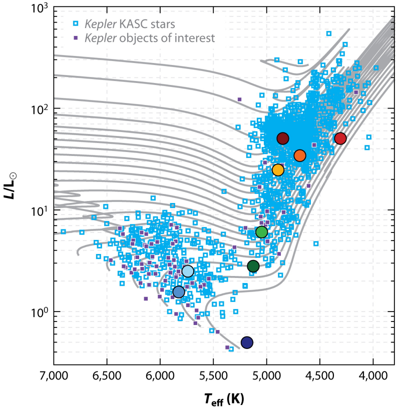


Space Missions

CoRoT (2006-2014)



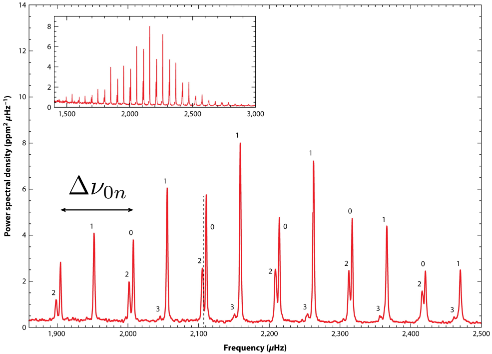
Kepler (2009-2018)



Chaplin WJ, Miglio A. 2013.
Annu. Rev. Astron. Astrophys. 51:353–92

Oscillation spectra

$$\nu_{n,l} \simeq \left(n + \frac{l}{2} + \epsilon\right)\Delta\nu$$ Gough (1986)



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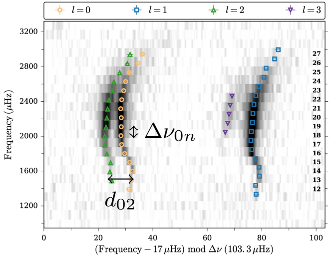
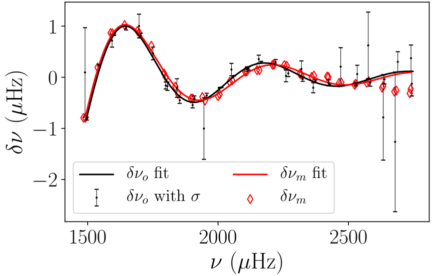


Credits: Lund et al. (2017)

Acoustic Glitches

$$\delta\nu \;=\; \nu - \nu_{\text{smooth}}$$

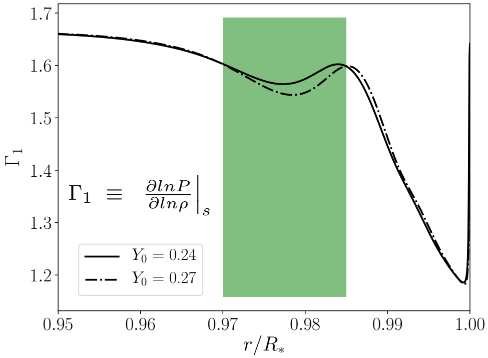
- Oscillation spectrum $\rightarrow$ smooth and glitch parts
- Glitches : due to sharp features in the stellar structure
- Provide local information



Helium Glitch

Model He-M anti-correlation (Lebreton & Goupil 2014)

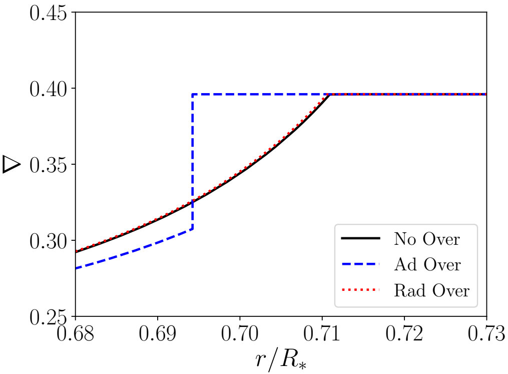
→ Helium glitch : second helium ionisation region Γ_1 dip $\Rightarrow Y_{\mathrm{surf}}$ inferences



Convection Zone Glitches

Mixing processes badly constrained

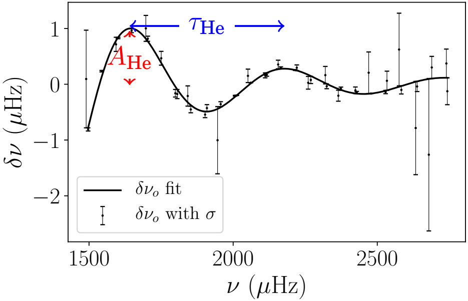
→ Convection zone glitch : radiative - convective regions transition
⇒ Transition localisation



Glitches Analysis

- First studies: Vorontsov (1988) (direct use of frequencies) and Gough (1990) (second differences);

- Glitches localisation : Monteiro et al. (2000), Mazumdar et al. (2014);
- Helium / metal content vs glitch amplitude : Basu et al. (2004);



- 16 Cygni helium content calibration : Verma et al. (2014).

Challenges

Need to achieve high precision in the data

→ CoRoT (Baglin et al. 2009);

→ Kepler (Borucki et al. 2010);

→ TESS (Ricker et al. 2014);

→ PLATO (Rauer et al. 2014).

But also need of precise and robust seismic analysis methods

→ WhoSGlAd (Farnir et al. 2019).

Why improve methods ?

- Often, info used is correlated : individual frequencies, seismic indicators,..
- Smooth and glitch often treated separately
  $\rightarrow$ Y_{surf} determined separately;
- Providing finer methods leads to more precise inferences.

GOAL : Method to analyse solar-like pulsation spectra as a whole and provide statistically relevant inferences $\Rightarrow$ uncorrelated indicators to use in stellar modelling.

Starting point

Verma et al. (2014, ApJ 790, 138):

$$f(n,l) = \overbrace{\sum_{k=0}^{4} A_{k,l} n^k}^{\text{Smooth}} + \overbrace{\mathcal{A}_{He} \nu e^{-c_2 \nu^2} \sin(4\pi\tau_{He}\nu + \phi_{He})}^{\text{He Glitch}} + \underbrace{\frac{\mathcal{A}_{CZ}}{\nu^2} \sin\left(4\pi\tau_{CZ}\nu + \phi_{CZ}\right)}_{\text{CZ Glitch}} \quad (1)$$

Limitations :

- Non linear formulation,
- Smooth part regarded as dispensable,
- Correlated indicators,
- Regularisation term needed.

Principle

WhoSGlAd - Whole Spectrum and Glitches Adjustment - method

Analyses oscillation spectrum as a whole

$\Rightarrow$ proper correlations are derived;

Consider the frequencies vector space:

1. Build orthonormal basis of functions (Gram-Schmidt);
2. Project the frequencies on the basis $\rightarrow$ get independent coefficients;
3. Combine the coefficients into indicators as uncorrelated as possible;
4. Use the indicators to obtain best fit stellar models.

Gram-Schmidt

Construction of orthonormal basis elements

1. Subtract from current element its projection on the previous orthonormal elements,
2. Normalise it.

$$\boldsymbol{u}_{j_0} = \boldsymbol{p}_{j_0} - \sum_{j=1}^{j_0-1} \left\langle \boldsymbol{p}_{j_0} | \boldsymbol{q}_j \right\rangle \boldsymbol{q}_j, \tag{2}$$

$$\boldsymbol{q}_{j_0} = \frac{\boldsymbol{u}_{j_0}}{\left\| \boldsymbol{u}_{j_0} \right\|}. \tag{3}$$

Frequency Adjustment

Via projection on the basis:

$$\nu_f(n,l) = \sum_j a_j q_j(n,l), \tag{4}$$

with $a_j = \langle \boldsymbol{\nu} | \mathbf{q}_j \rangle$ independent and q_j the orthonormal basis elements.

Basis Elements

$$\nu = \nu_{smooth}(n) + \nu_{glitch}(\tilde{n})$$

Smooth part :

- Represented by polynomials in n,

Glitches :

- Verma et al (2014)
  $\propto f\left(\tau_{\mathrm{He,CZ}}\nu\right)$

$\rightarrow$ Linearised functions of $\tilde{n}\Delta\nu$ with $\tilde{n} = n + l/2$,

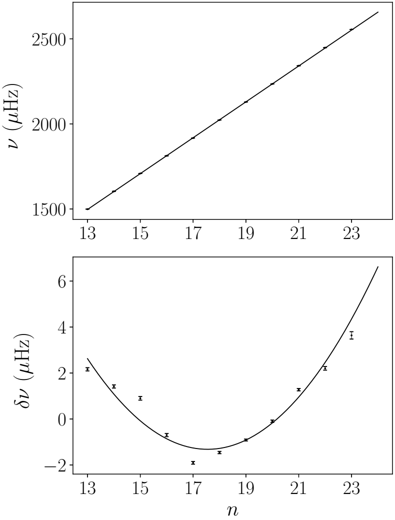
$\rightarrow$ Fixed $T_{\mathrm{He,CZ}} = \tau_{\mathrm{He,CZ}}\Delta\nu$

An Illustrative Example : Smooth

At a given degree, projection of the frequencies on the successive basis elements.

- $\rightarrow$ 0 order : mean value;
- $\rightarrow$ $1\mathrm{st}$ order : straight line approximation;
- $\rightarrow$ $2\mathrm{nd}$ order : parabola approximation.

Follow the proper ordering to define seismic indicators

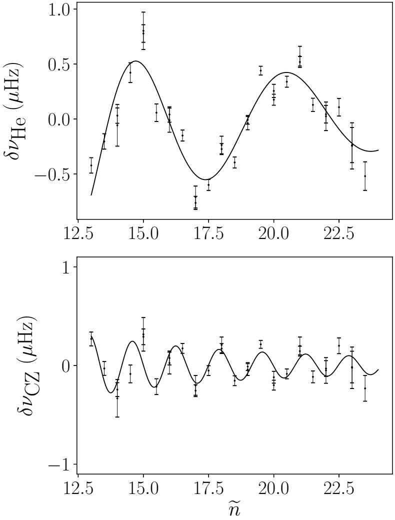


An Illustrative Example : Glitch

Simultaneous projection of the frequencies for every spherical degree on the successive basis elements.

→ First for the helium;

→ Then for the convection zone.



Indicators

Indicators built as a combination of uncorrelated coefficients a_j
$\rightarrow$ As independent as possible $\&$ proper correlations are known

- Large separation Δ (mean) and of a fixed degree Δ_l;
- Large separation differences Δ_{0l};
- Small separation ratios $\hat{r}_{0l}$;
- Epsilon estimator $\hat{\epsilon}$;
- Glitches amplitudes A_{He},A_{CZ};
- ...

Large Separation

Slope in n of the frequencies in the asymptotic formulation:

$$\nu(n,l) \simeq (n + l/2 + \epsilon)\,\Delta\nu \tag{5}$$

A different value for each degree l : Δ_l

- $\rightarrow$ Combined into mean value : Δ;
- $\rightarrow$ Normalised differences : $\Delta_{0l} = \frac{\Delta_l}{\Delta_0} - 1$;

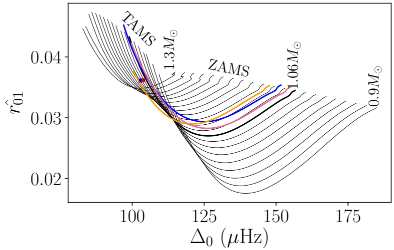
Small Separation ratios

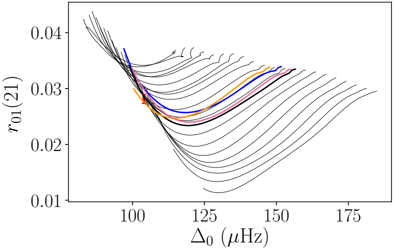
WhoSGlAd

$$\hat{r}_{01} = \frac{\overline{\nu_0} - \overline{\nu_1}}{\Delta_0} + \overline{n_1} - \overline{n_0} + \frac{1}{2}$$

Roxburgh & Vorontsov (2003)

$$r_{01}(n) = \frac{\nu_{n-1,1} - 2\nu_{n,0} + \nu_{n,1}}{2(\nu_{n,1} - \nu_{n-1,1})}$$





16 Cyg A : $\Delta\hat{r}_{01}/\hat{r}_{01} = 0.7\%$

$\Delta r_{01}(21)/r_{01}(21) = 2.9\%$

$(Z/X)_0 = 0.0218 \quad \alpha_{\mathrm{MLT}} = 1.82 \quad Y_0 = 0.25$

$(Z/X)_0 = 0.018 \quad \alpha_{\mathrm{MLT}} = 1.5 \quad Y_0 = 0.27$

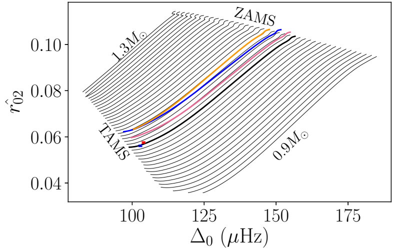
Small Separation ratios

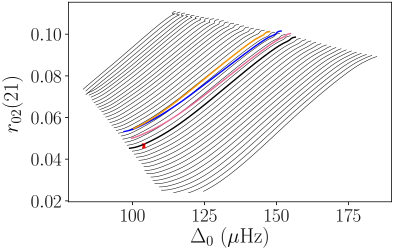
WhoSGlAd

$$\hat{r}_{02} = \frac{\overline{\nu_0} - \overline{\nu_2}}{\Delta_0} + \overline{n_2} - \overline{n_0} + \frac{2}{2}$$

Roxburgh & Vorontsov (2003)

$$r_{02}(n) = \frac{\nu_{n,0} - \nu_{n-1,2}}{(\nu_{n,1} - \nu_{n-1,1})}$$





16 Cyg A : $\Delta\hat{r}_{02}/\hat{r}_{02} = 0.6\%$

$\Delta r_{02}(21)/r_{02}(21) = 2.1\%$

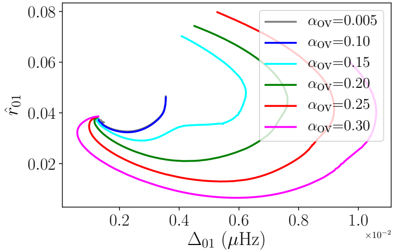
$(Z/X)_0 = 0.0218$ $\alpha_{\mathrm{MLT}} = 1.82$ $Y_0 = 0.25$

$(Z/X)_0 = 0.018$ $\alpha_{\mathrm{MLT}} = 1.5$ $Y_0 = 0.27$

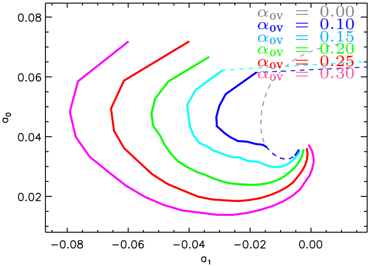
Large Separations Differences

WhoSGlAd

$$\Delta_{0l} = \frac{\Delta_l}{\Delta_0} - 1$$

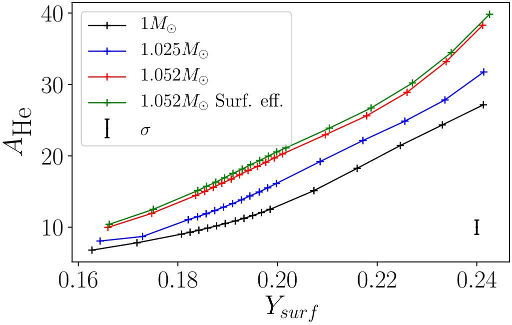


Deheuvels et al. (2016)



Helium Glitch Amplitude

Defined as helium glitch norm.
At fixed $\Delta\nu$ and $(Z/X)_0$, we get:

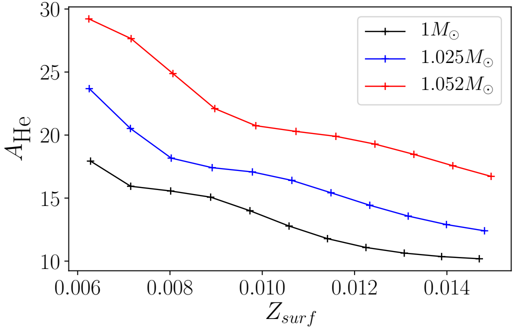


- Increasing trend with helium content (as observed Basu et al. 2004)

Helium Glitch Amplitude

Defined as helium glitch norm.
At fixed $\Delta\nu$ and Y_0, we get:

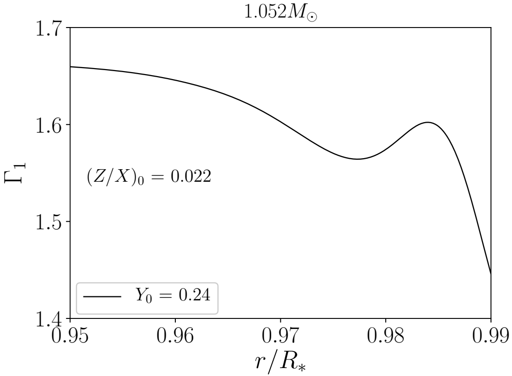
- Decreasing trend with metal abundance (as observed Basu et al. 2004)



Degeneracy

- Correlated with Y_{surf};
- Anti-correlated with Z_{surf};

$\rightarrow$ Γ_1 toy model provides an explanation.



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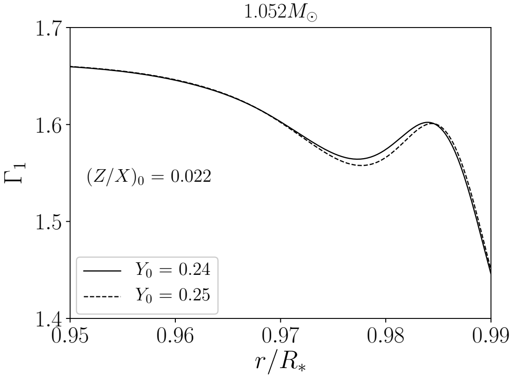

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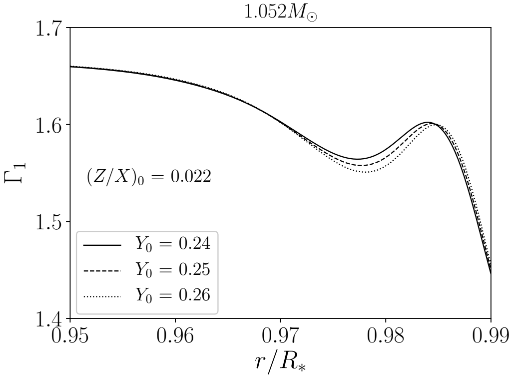

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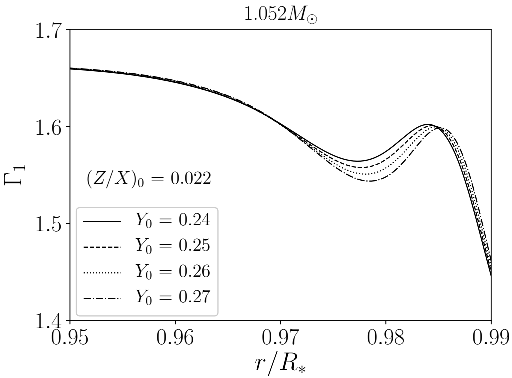

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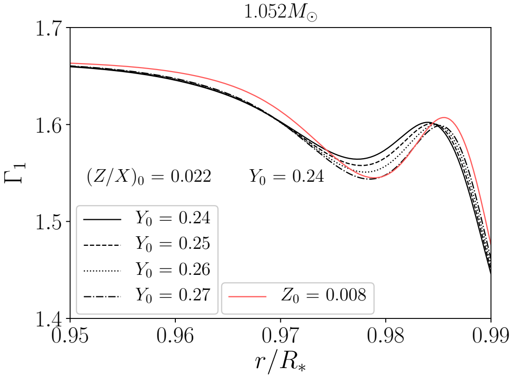

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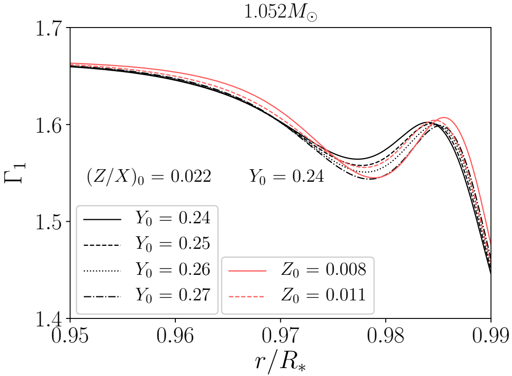

Degeneracy

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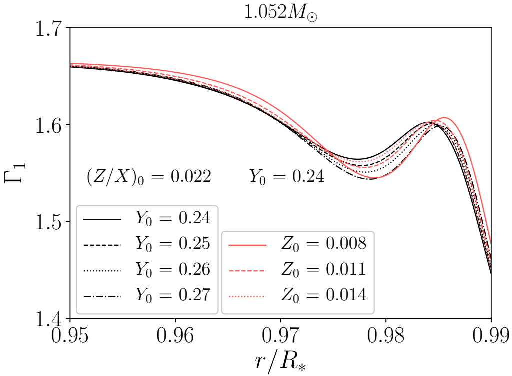

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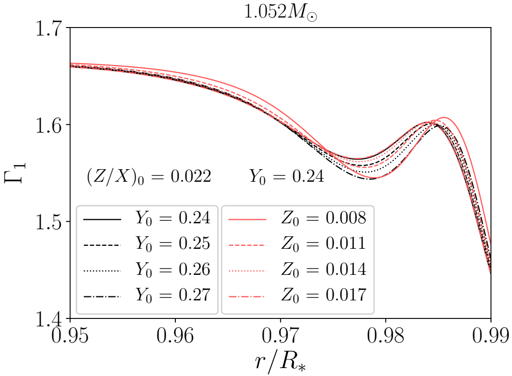
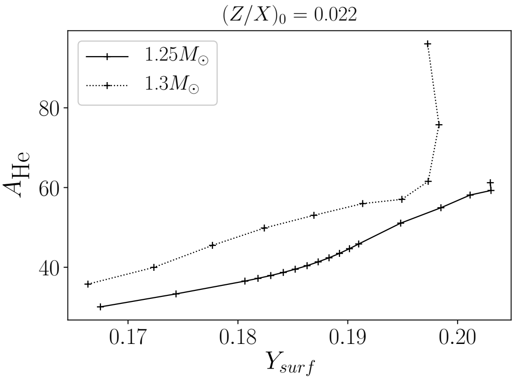


High Masses / Surface Helium

At fixed $\Delta\nu$, we get:
Linear trend of amplitude with Y_{surf} is not preserved at high masses ($\gtrsim 1.25M_{\odot}$) and Y_{surf}.



- High M and Y_0 : shallower convective envelope

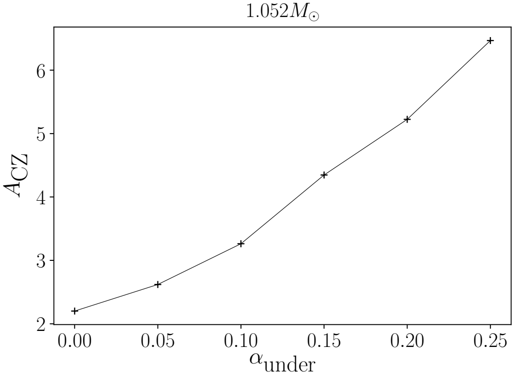
$\rightarrow$ More efficient microscopic diffusion

$\rightarrow$ Lower Y_{surf} and Z_{surf}

$\Rightarrow$ Higher amplitude

Convection Zone Glitch Amplitude

Defined as convection zone glitch norm.
At fixed $\Delta\nu$, we get:



- CZ transition sharpness proxy

Convection Zone Glitch Amplitude

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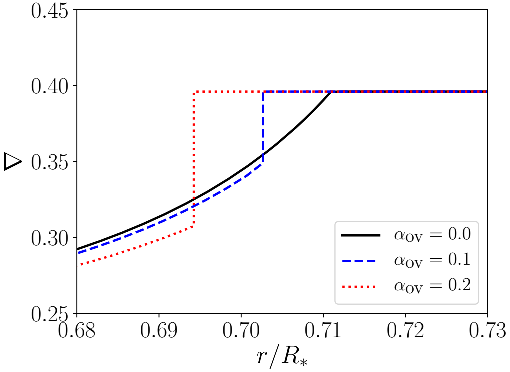
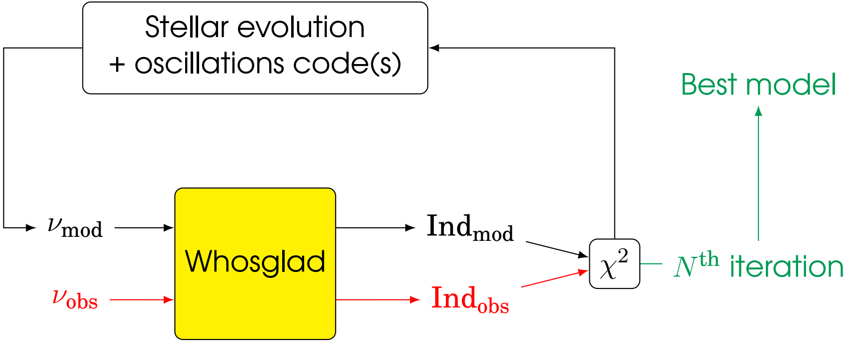
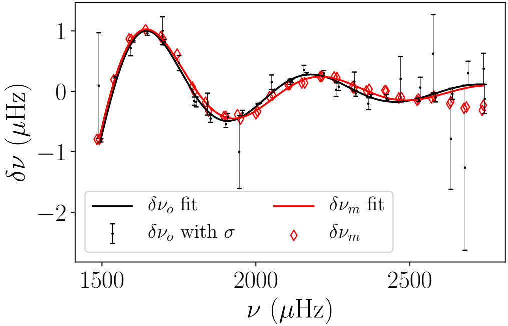


- CZ transition sharpness proxy

Stellar Modelling

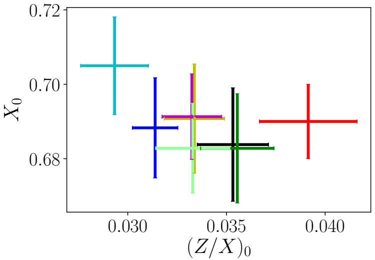


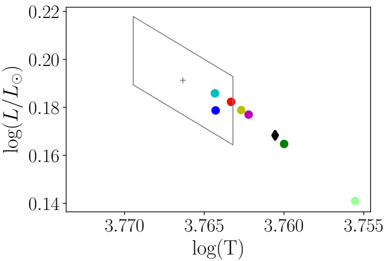
16 Cygni Glitch & Model Fitting



- Constraints
  1. $\Delta = 104.088 \pm 0.005 \mu Hz$
  2. $r_{01} = 0.0362 \pm 0.0002$
  3. $r_{02} = 0.0575 \pm 0.0003$
  4. $A_{\mathrm{He}} = 30.3 \pm 1.0$
- Fitted parameters
  1. $M = 1.06 \pm 0.02 M_\odot$
  2. $t = 6.8 \pm 0.2 Gyr$
  3. $X_0 = 0.684 \pm 0.015$
  4. $(Z/X)_0 = 0.035 \pm 0.002$

16 Cygni A Best Fit Models



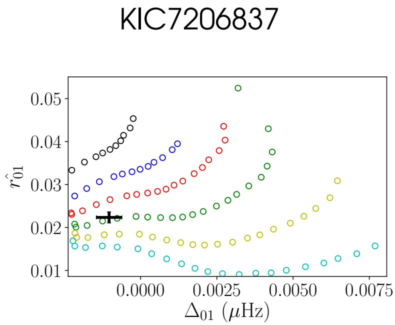


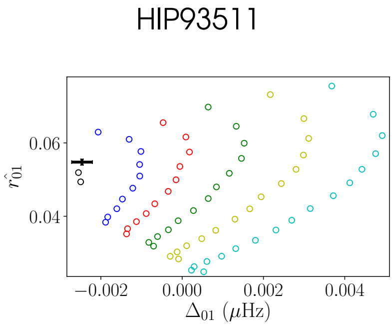
Ref: AGSS09, OPAL, FREE, $\alpha_{\mathrm{MLT}} = 1.82$

Legend: AGSS09 GN93 OP LANL

CEFF OPAL05 Dturb α_{conv}

Overshoot





$\alpha_{\mathbf{ov}} = 0.005$ $\alpha_{\mathbf{ov}} = 0.10$ $\alpha_{\mathbf{ov}} = 0.15$

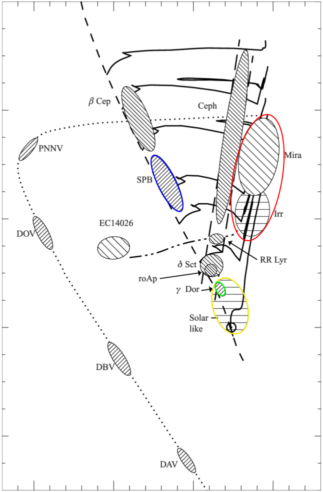
$\alpha_{\mathbf{ov}} = 0.20$ $\alpha_{\mathbf{ov}} = 0.25$ $\alpha_{\mathbf{ov}} = 0.30$

Pros and Cons

- Derive independent coefficients $\Rightarrow$ as uncorrelated as possible indicators;
  - $\rightarrow$ Smaller standard deviations;
- Linear formulation
  - $\rightarrow$ Fast computation;
  - $\rightarrow$ No regularisation needed;
  - $\rightarrow$ Compatible with any minimisation scheme;
- Currently only suited for solar-like;
- Currently no acoustic depth determination.

Future Perspectives

- Kepler Legacy;
- Red giants : see Miglio et al. (2010);
- Red subgiants $\rightarrow$ mixed modes formulation;
- Analysis of g-pulsators : γ Dor and SPB;
- PLATO (Rauer et al. 2014).



Credits: Christensen-Dalsgaard J.

Basis Elements

We selected the basis functions:

- Smooth

  ① $p_0(n) = 1$

  ② $p_1(n) = n$

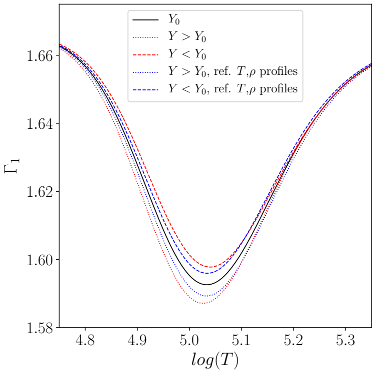
  ③ $p_2(n) = n^2$

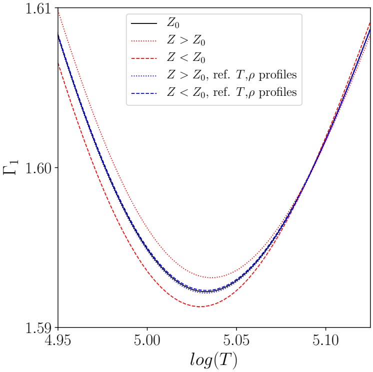
- Glitch

  He
  $$p_{\mathrm{He}Ck}(\tilde{n}) = \cos\left(4\pi T_{\mathrm{He}}\tilde{n}\right)\tilde{n}^{-k}$$
  $$p_{\mathrm{He}Sk}(\tilde{n}) = \sin\left(4\pi T_{\mathrm{He}}\tilde{n}\right)\tilde{n}^{-k}$$
  with $k = 5, 4$, $\tilde{n} = n + l/2$

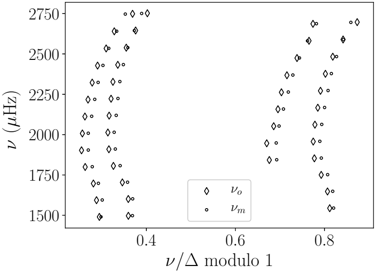
  CZ
  $$p_{\mathrm{C}C}(\tilde{n}) = \cos\left(4\pi T_{\mathrm{CZ}}\tilde{n}\right)\tilde{n}^{-2}$$
  $$p_{\mathrm{C}S}(\tilde{n}) = \sin\left(4\pi T_{\mathrm{CZ}}\tilde{n}\right)\tilde{n}^{-2}$$

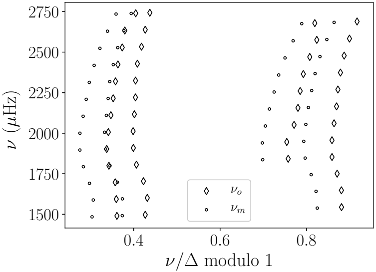
Γ_1 Toy Model



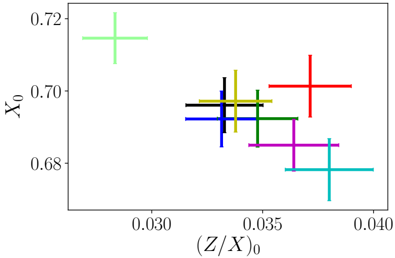


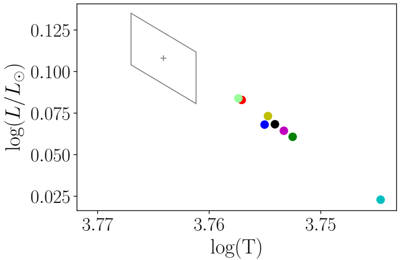
Échelle


2750
2500
2250
2000
1750
1500
ν (μHz)
ν_o
ν_m
0.4
0.6
0.8
ν/Δ modulo 1


2750
2500
2250
2000
1750
1500
ν (μHz)
ν_o
ν_m
0.4
0.6
0.8
ν/Δ modulo 1

16 Cygni B Best Fit Models



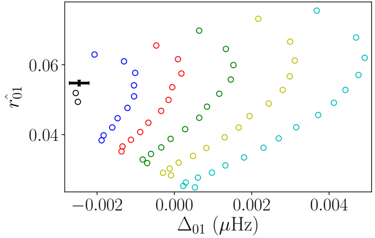


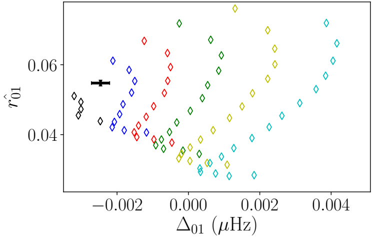
Ref: AGSS09, OPAL, FREE, $\alpha_{\mathrm{MLT}} = 1.82$

Legend: AGSS09 GN93 OP LANL

CEFF OPAL05 Dturb α_{conv}

Composition and Overshoot





$Y_0 = 0.25 \quad (Z/X)_0 = 0.0218$ $\qquad$ $Y_0 = 0.24 \quad (Z/X)_0 = 0.017$