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FRAME DESIGN INCLUDING JOINT BEHAVIOUR

VOLUME 1

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FOREWORD

This book, or manual, is the result of the work of eight individuals from four research centres and university departments. This work has been three years long, all the way enjoyable if difficult.

The design of steel frames has long been considered as being composed of two different stages, the design of members being quite a separate task from the design of joints.

The original idea of this manual occurred soon after the moment when the developments of knowledge resulted into the introduction in the European design code (Eurocode 3) of the so-called concept of *semi-rigid design* or the *semi-continuous design*. But the book is more general and refers to any kind of joints in building frames.

As is often the case with progress, those *semi-rigid* developments, while recognising the true behaviour of joints in steel frames, were at first sight seen as a nest of complications for any designer.

When using the wording « progress », it is of course not only for the sake of science, far from that ! One main motivation for this work was to show how the semi-continuous concept may help the designer to achieve a better *global economy* in the project, through proper and « custom-made » balancing between the material costs (members weight) and the fabrication costs (joints).

These ways for better economy have been suggested in several publications already. It is expected that this manual will provide the reader with the tools for mastering the ways towards cheaper structures which means a larger market-share for steel construction. That definitely meant encompassing the complete progress of designing, including global analysis and design checks. But this is not the end.

As was amply demonstrated through the discussions within the drafting panel (and believe me it is quite difficult to get eight people to agree on any practical issue), there seems to exist quite a world between the *frame designers* and the *joint designers*. No doubt that this state of facts is due to the traditional habit of considering joints as pinned or rigid - thus simplifying the assumptions for frame design -, but also to the fact that different people (either within the same company, or in different companies for several European countries) deal with the design of members and joints respectively. Hence this book boldly tries at considering the design of members and joints as a whole, whenever the design organisation allows for it, and please take that statement in a broad sense !

The authors have made their best efforts to include those different design situations in this work, and I truly believe that this is rather unusual in that kind of manual. So praise to them if they have succeeded !

The reader will find in this manual some theoretical background, application rules (compatible with Eurocode 3) and worked examples . Also a software has been developed, and this fits with the present economical way to deal with calculations within design offices.

Regarding joints, it is to be noted that, instead of the *original Annex J* to E*urocode 3*, the *revised Annex J* was considered. This *revised Annex* was adopted in 1994 and its publication as an ENV is expected in 1997.

Within our drafting panel, I was personally the lowest ranking expert (you may understand not a real expert!) either in frame design or in joints behaviour knowledge, the reason, why, supposedly, I was kindly asked by Professor René Maquoi, Project Manager, to produce this foreword, thus acting in the process as a kind of Candid. I am glad, and not the least ashamed, to say that, despite the occasional fences and difficulties in the work, I took profit of it by learning quite a lot, and I feel sure that my fellow writers learned some things also during the process. So what is only left to be expected is that you, our reader, will also gather a lot and take a very good profit from this book. So be it, and long live our well designed steel structures!

But a work is never quite finished. Eurocode 3 will come, slowly or quickly, into common use. Practical experience will for sure suggest a lot of improvements or afterthoughts. Our interest and please will be to know of your comments and criticisms.

Bruno Chabrolin,
CTICM, Head of R&D Department

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VOLUME 1

USER’S MANUAL

SYMBOLS

b_{eff}	effective width
d_n	nominal diameter of the bolt shank
e_o	magnitude of initial out-of-straightness
f	fixity factor
f_u	material ultimate tensile strength
f_y	material yield strength
h_b	depth of beam cross-section
h_c	depth of column cross-section
h_t	distance between the centroïds of the beam flanges
l_{eff}	effective length
r_c	fillet radius of the structural shape used as column
$t_{f,c}$	thickness of the column flange
$t_{f,b}$	thickness of the beam flange
t_p	thickness of the end-plate
$t_{w,c}$	thickness of the column web
z	lever arm of the resultant tensile and compressive forces in the connection
xx	longitudinal axis of a member
yy	major axis of a cross-section

zz	minor axis of a cross-section
A_o	original cross-sectional area
A_s	shear area of the bolt shank
$A_{v,c}$	shear area of the column web
E	Young modulus for steel material
F_b	tensile and compressive forces in the connection, statically equivalent to the beam end moment
$F_{c,Sd}$	design compressive force
$F_{t,Sd}$	design tensile force
$F_{v,Sd}$	design shear force (bolts)
$F_{b,Sd}$	design bearing force (bolt hole)
H	horizontal load
I	second moment of area
I_b	second moment of area of the beam section (major axis bending)
I_c	second moment of area in the column section (major axis bending)
L	member length
L_b	beam span (system length)
L_c	column height (system length measured between two consecutive storeys)
M	bending moment
M_b	bending moment at the beam end (at the location of the joint)
M_c	bending moment in the column (at the location of the joint)
M_j	bending moment experienced by the joint (usually $M_j = M_b$)
$M_{j,u}$	ultimate bending moment resistance of the joint
$M_{j,Rd}$	design bending moment resistance of the joint
$M_{j,Sd}$	design bending moment experienced by the joint

M_{pb}	plastic bending resistance of the beam cross-section
M_{pc}	plastic bending resistance of the column cross-section
$M_{pl,Rd}$	design plastic bending resistance of a cross-section
M_u	ultimate bending moment
M_{Sd}	design bending moment
N	axial force
N_b	axial force in the beam (at the location of the joint)
N_c	axial force in the column (at the location of the joint)
P	axial compressive load
S	rotational joint stiffness
S_j	nominal rotational joint stiffness
$S_{j,app}$	approximate rotational joint stiffness (estimate of the initial one)
$S_{j,ini}$	initial rotational joint stiffness
$S_{j,\ post\text{-}limit}$	post-limit rotational joint stiffness
V	shear force or gravity load
V_b	shear force at the beam end
V_c	shear force in the column
V_{cr}	critical value of the resultant gravity load
V_n	shear force experienced by the column web panel
V_{Sd}	design shear force in the connection
V_{wp}	shear force in the column web panel (sheared panel)
W	gravity load
β	transformation parameter
ε	$= (L/\pi)\ \sqrt{N / EI}$
ε_y	material yield strain**Erreur ! Les arguments du commutateur ne sont pas spécifiés.**

ε_u	material ultimate strain**Erreur ! Les arguments du commutateur ne sont pas spécifiés.**
δ	magnitude of the member deflection or local second-order effect
ϕ	relative rotation between the axes of the connected members or sum of the rotations at the beam ends
ϕ_b	rotation of the beam end
ϕ_t	rotation of the (beam + joint) end
γ	shear deformation
γ_F	partial safety factor for the loads (actions)
γ_M	partial safety factor for the resistance (strength function)
Δ	sway displacement or global second-order effect
η	stiffness reduction factor (S_j / $S_{j,ini}$)
λ	slenderness
λ_L	load parameter
λ_{cr}	elastic critical load parameter (gravity loads)
λ_p	plastic load parameter
$\bar{\lambda}$ **Erreur ! Les arguments du commutateur ne sont pas spécifiés.**	reduced slenderness
θ_b	absolute rotation of the beam end
θ_c	absolute rotation of the column axis
σ_u	average ultimate stress
ψ	load combination factor
Φ	reference rotation of a plastic hinge in a plastic mechanism

ABBREVIATIONS

EC1 Eurocode 1: *Basis for design*

EC3 Eurocode 3: *Design of steel structures.*

EN 10025 Euronorm 10025: *Hot rolled products of non-alloy structural steels - Technical delivery conditions.*

EN 10113 Euronorm 10113: *Hot rolled products in weldable fine grain structural steels.*

SLS Service limit state(s).

ULS Ultimate limit state(s).

CHAPTER 1

INTRODUCTION

1.1 Aims of this manual

The aim of this manual is threefold:

- To present, in simple terms, the behaviour of steel structural frames and the techniques to model this behaviour;
- To provide the designer with practical guidance and tools for the design of the structure, incorporating a treatment of the joints at all design stages;
- To illustrate the possibilities of producing more economical structures using the new approaches offered in *Eurocode* 3.

The organisation of the manual reflects the belief that, in addition to the sizing of the members (beams and columns), consideration should also be given to the joint characteristics throughout the design process. This approach, despite the novelty it may present to many designers, is shown to be relatively easy to integrate into everyday practice using present day design tools.

Hence the present manual addresses design methodology, structural analysis, joint behaviour and design checks, at three different levels:

- Presentation and discussion of concepts;
- Practical guidance and design tools;
- Worked examples.

1.1.1 The present common way in which joints are modelled for the design of a frame

Generally speaking, the process of designing building structures has been up to now made up of the following successive steps:

- Frame modelling (including the choice of rigid or pinned joints);
- Initial sizing of beams and columns;
- Evaluation of internal forces and moments (load effects) for *each ultimate limit state* (*ULS*) and *serviceability limit state* (*SLS*) load combination;
- Design checks of *ULS* and *SLS* criteria;
- Iteration on member sizes until all design checks are satisfactory;
- Design of joints to resist the relevant members end forces (either those calculated or the maximum ones able to be transmitted by the actual

members); the design is carried out in accordance with the prior assumptions (frame modelling) on joint stiffness.

This approach was possible since designers were accustomed to considering the joints to be either pinned or rigid only. In this way, the design of the joints became a separate task from the design of the members. Indeed, joint design was often performed at a later stage, either by other personnel or by another company.

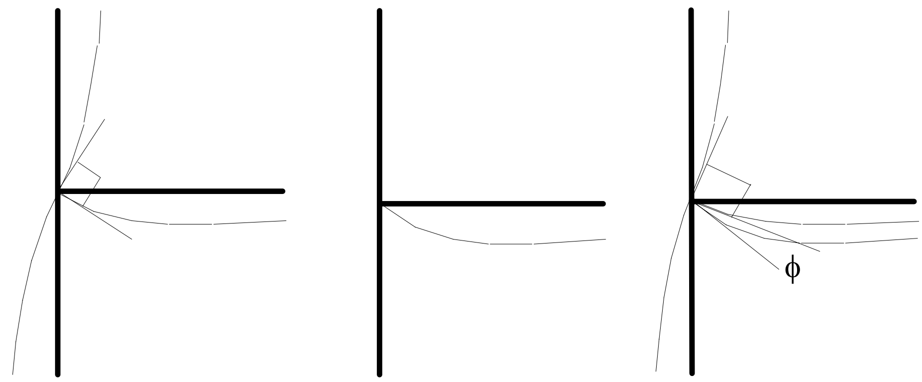
Recognising that most joints have a real behaviour which is intermediate between that of pinned and rigid joints, *Eurocode 3* offers the possibility to account for this behaviour by opening up the way to what is presently known as the *semi-rigid approach*. This approach to design offers the potential for achieving better and more economical structures.

1.1.2 The semi-rigid approach

The rotational behaviour of actual joints is well recognised as being often intermediate between the two extreme situations, i.e. rigid or pinned.

In chapter 4, the difference between *joints* and *connections* will be introduced. For the time being, we will use examples of joints between one beam and one column only.

Let us now consider the bending moments and the related rotations at a joint (Figure 1.1):



(a) Rigid joint (b) Pinned joint (c) Semi-rigid joint

Figure 1.1 Classification of joints according to stiffness

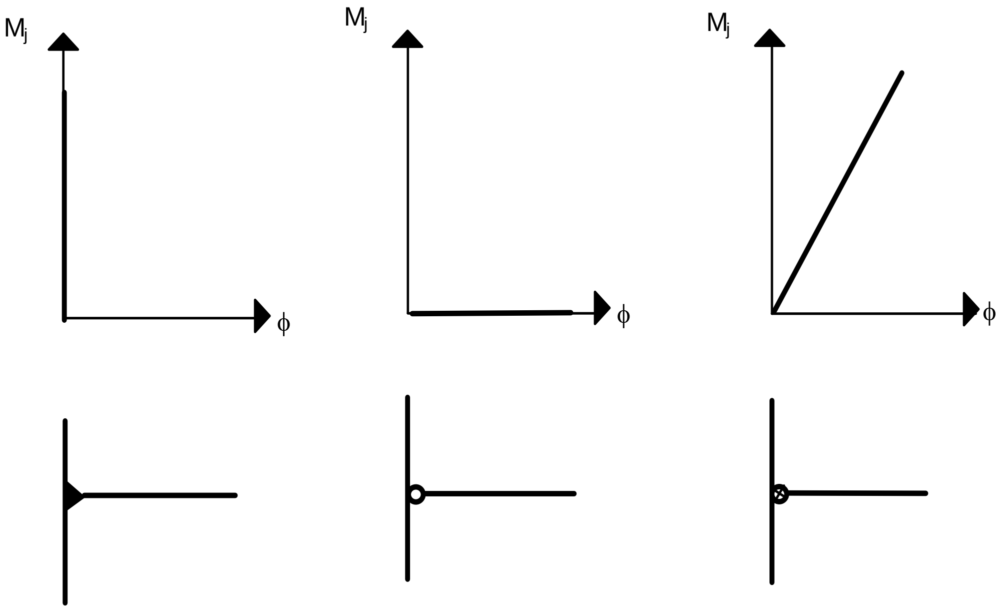
When all the different parts in the joint are sufficiently stiff (i.e. ideally infinitely stiff), the joint is *rigid*, and there is no difference between the respective rotations at the ends of the members connected at this joint (Figure 1.1.a). The joint experiences a single global rigid-body rotation which is the nodal rotation in the commonly used analysis methods for framed structures.

Should the joint be without any stiffness, then the beam will behave just as a simply supported beam, whatever the behaviour of the other connected member(s) (Figure 1.1.b). This is a *pinned* joint.

For intermediate cases (non zero and non infinite stiffness), the transmitted moment will result in there being a difference ϕ between the absolute rotations of the two connected members (Figure 1.1.c). The joint is *semi-rigid* in these cases.

The simplest means for representing the concept is a rotational (spiral) spring between the ends of the two connected members. The rotational stiffness S of this spring is the parameter that links the transmitted moment M_j to the relative rotation ϕ, which is the difference between the absolute rotations of the two connected members.

When this rotational stiffness S is zero, or when it is relatively small, the joint falls back into the pinned joint class. In contrast, when the rotational stiffness S is infinite, or when it is relatively high, the joint falls into the rigid joint class. In all the intermediate cases, the joint belongs to the semi-rigid joint class.



(a) Rigid joint ($\phi = 0$)

(b) Pinned joint ($M_j = 0$)

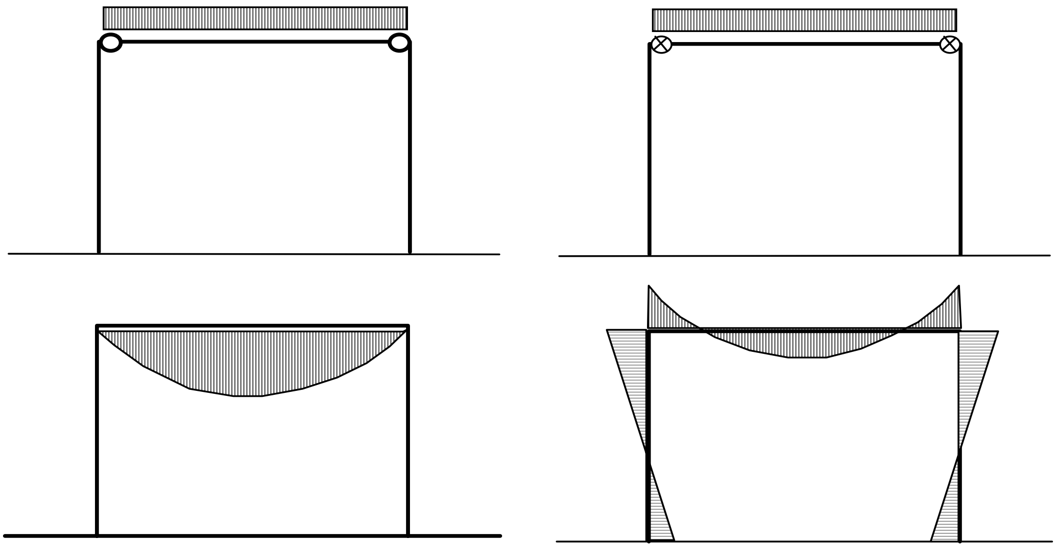
(c) Semi-rigid joint (M_j and $\phi \neq 0$)

Figure 1.2 Modelling of joints (case of elastic global analysis)

For semi-rigid joints the loads will result in both a bending moment M_j and a relative rotation ϕ between the connected members. The moment and the relative rotation are related through a constitutive law depending on the joint properties. This is illustrated in Figure 1.2, where, for the sake of simplicity, the global analysis is assumed to be performed with linear elastic assumptions (how to deal with non-linear behaviour situations will be addressed later on, especially in chapters 3, 5, 7, 9).

It shall be understood that the effect, at the global analysis stage, of having semi-rigid joints instead of rigid or pinned joints is to modify not only the displacements, but also the distribution and magnitude of the internal forces throughout the structure.

As an example, the bending moment diagrams in a fixed-base simple portal frame subjected to a uniformly distributed load are given in Figure 1.3 for two situations, where the beam-to-column joints are respectively either pinned or semi-rigid. The same kind of consideration holds for deflections.



(a) Pinned joints (b) Semi-rigid joints

Figure 1.3 Elastic distribution of bending moments in a simple portal frame

1.1.3 The merits of the semi-rigid approach

Both the *Eurocode 3* requirements and the desire to model the behaviour of the structure in a more realistic way leads to the consideration of the semi-rigid behaviour when necessary.

Many designers would stop at that basic interpretation of *Eurocode 3* and hence would be reluctant to confront the implied additional computational effort involved. Obviously a crude way to deal with this new burden will be for them to design joints that will actually continue to be classified as being either pinned or fully rigid. However such properties will have to be proven at the end of the design process; in addition, such joints will certainly be found to be uneconomical in a number of situations.

It shall be noted that the concept of rigid and pinned joints still exists in *Eurocode 3*. It is accepted that a joint which is *almost rigid* or, on the contrary, *almost pinned*) may still be considered as being *truly rigid* or *truly pinned* in the design process. How to judge whether a joint can be considered as rigid, semi-rigid or pinned depends on the comparison between the joint stiffness and the beam stiffness, which latter depends on the second moment of area and length of the beam.

The designer is strongly encouraged to go beyond this "all or nothing" attitude. Actually it is possible, and therefore of interest, to consider the benefits to be gained from the semi-rigid behaviour of joints. Those benefits can be brought in two ways :

1. The designer decides to continue with the practice of assuming -sometimes erroneously- that joints are either pinned or fully rigid. However, *Eurocode 3* requires that proper consideration be given to the influence that the actual behaviour of the joints has on the global behaviour of the structure, i.e. on the precision with which the distribution of forces and moments and the displacements have been determined. This may not prove to be easy when the joints are designed at a late stage in the design process since some iterations between global analysis and design checking may be required. Nevertheless, the following situations can be foreseen:

    - So that a joint can be assumed to be rigid, it is common practice to introduce web stiffeners in the column. *Eurocode 3* now provides the means to check whether such stiffeners are really necessary for the joint to be both rigid and have sufficient resistance. There are practical cases where they are not needed, thus permitting the adoption of a more economical joint design.

    - When joints assumed to be pinned are later found to have fairly significant stiffness (i.e. to be semi-rigid), the designer may be in a position to reduce beam sizes. This is simply because the moments carried by the joints reduce the span moments in the beams.

2. The designer decides to give consideration, at the preliminary design stage, not only to the properties of the members but also to those of the joints. It will be shown that this new approach is not at all incompatible with the sometimes customary separation of the design tasks between those who have the responsibility for conceiving the structure and carrying out the global analysis and those who have the responsibility for designing the joints. Indeed, both tasks are very often performed by different people, indeed, or by different companies, depending on national or local industrial habits. Adopting this novel manner towards design requires a good understanding of the balance between, on the one hand, the costs and the complexity of joints and, on the other hand, the optimisation of the structural behaviour and

performance through the more accurate consideration of joint behaviour for the design as a whole. Two examples are given to illustrate this:

- It was mentioned previously that it is possible in some situations to eliminate column web stiffeners, therefore reduce costs. Despite the reduction in its stiffness and, possibly, in its strength, the joint can still be considered to be rigid and be found to have sufficient strength. This is shown to be possible for industrial portal frames with rafter-to-column haunch joints, in particular, but other cases can be envisaged.

- In a more general way, it is worthwhile to consider the effect of adjusting the joint stiffness so as to strike the best balance between the cost of the joints and the costs of the beams and the columns. For instance, for braced frames, the use of semi-rigid joints, which are probably more costly than the pinned joints, leads to reducing the beam sizes. For unbraced frames, the use of less costly semi-rigid joints, instead of the rigid joints, leads to increased beam sizes and possibly column sizes.

Of course the task may seem a difficult one, and this is why this manual is aimed at providing the designer with a set of useful design tools. The whole philosophy could be termed as *"As you must do it, so better make the best of it".*

Thus *Eurocode 3* now presents the designer with a choice between a *traditionalist attitude*, where however something may often be gained, and an *innovative attitude*, where the best economical result may best be sought.

It is important to stress the high level of similarity that exists between the member classification and the joint classification. This topic is addressed in the Section 1.1.4.

1.1.4 A parallel between member sections and joints in the semi-rigid approach

Member cross-section behaviour may be considered through an M-ϕ curve for a simply supported beam loaded at mid span (M : bending-moment at mid-span ; ϕ : sum of rotations at the span ends). Joint behaviour will be considered through a similar relationship, but with $M = M_j$ being the bending moment transmitted by the joint and ϕ being the relative rotation between the connected member and the rest of the joint. Those relationships have a similar shape as illustrated in Figure 1.4.

According to *Eurocode 3* member cross-sections are divided into four classes according to their varying ability to resist local instability, when partially or totally subject to compression, and the consequences this may have on the possibility for plastic redistribution. Therefore their resistance ranges from the full plastic resistance (*class 1* and *2*) to the elastic resistance (*class 3*) or the sub-elastic resistance (*class 4*).

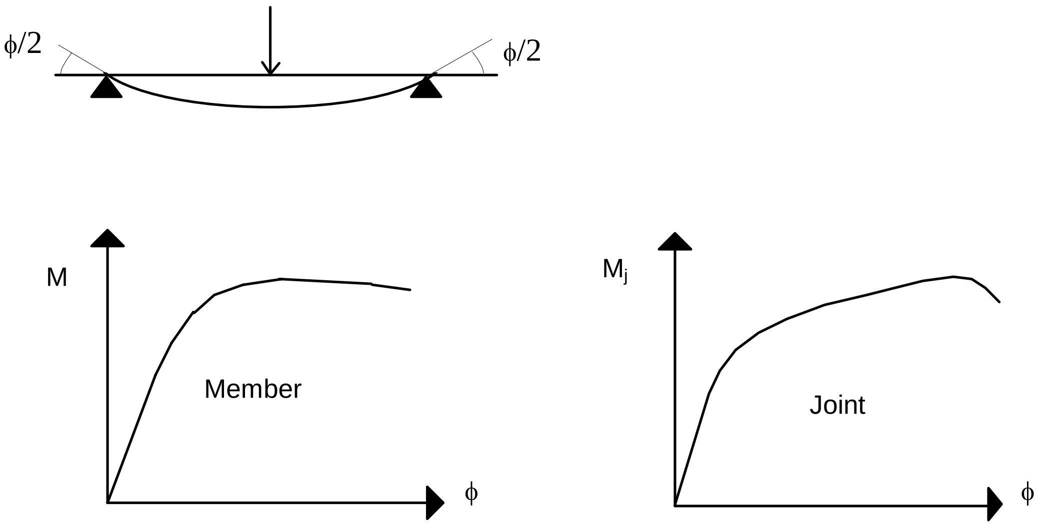


Figure 1.4 *M-ϕ* characteristics for member cross-section and joint

The belonging of a cross-section to a specific class governs the assumptions on:

- The behaviour to be idealised for global analysis (i.e. *class 1* will allow the formation of a plastic hinge and permit the redistribution of internal forces in the frame as loads are increased up to or beyond the design loads);
- The behaviour to be taken into account for local design checks (i.e. *class 4* will imply that the resistance of the cross-section is based on the properties of a relevant effective cross-section rather than of the gross cross-section).

In *Eurocode 3*, the classification of a cross-section is based on the width-to-thickness ratio of the component walls of the section. Ductility is directly related to the amount of rotation during which the design bending resistance will be sustained. The also used *rotation capacity* concept is equivalent to the *ductility* concept.

In a manner similar to that for member cross-sections, joints are classified in terms of *ductility* or *rotation capacity*, joints are classified. This classification is a measure of their ability to resist premature local instability and, even more likely, premature brittle failure (especially due to bolt failure) with due consequences on the type of global analysis allowed.

The practical interest of such a classification for joints is to check whether an elastoplastic global analysis may be conducted up to the formation of a plastic collapse mechanism in the structure, which implies such hinges in at least some of the joints.

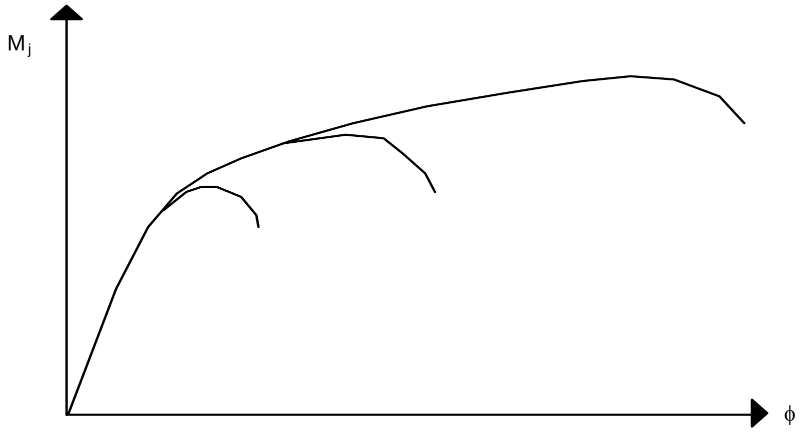


Figure 1.5 Ductility or rotation capacity in joints

As will be shown, this classification of joints by ductility, while not explicitly stated in *Eurocode 3*, may be defined from the geometric and mechanical properties of the joint components (bolts, welds, plate thickness, etc.).

Joints may therefore be classified according to both their *stiffness* and their *ductility*. Moreover, joints may be classified according to their *strength.*

In terms of their strength, joints are classified as *full-strength* or *partial-strength* according to their resistance compared to the resistance of the connected members. For elastic design, the use of partial-strength joints is well understood. When plastic design is used, the main use of this classification is to foresee the possible need to allow a plastic hinge to form in the joint during the global analysis. In order to permit the increase of loads, a partial-strength joint may be required to act as a hinge from the moment when its plastic bending resistance is reached. in that case, the joint must also have sufficient ductility.

The final parallel to be stressed between joints and members is that the same kind of link exists between the global analysis stage and the ultimate limit states design checking stage. The latter has to extend to all aspects that were not implicitly or explicitly taken into account at the global analysis stage. Generally speaking, one can state that the more sophisticated the global analysis is, the simpler are the ultimate limit states design checks required.

The choice of global analysis will thus depend not only of what is required by *Eurocode 3* but also on personal choices, depending on specific situations, available software's, etc. A particular choice means striking a balance between the amount of effort devoted to global analysis and the amount of effort required for the check of remaining *ULS* (Figure 1.6).

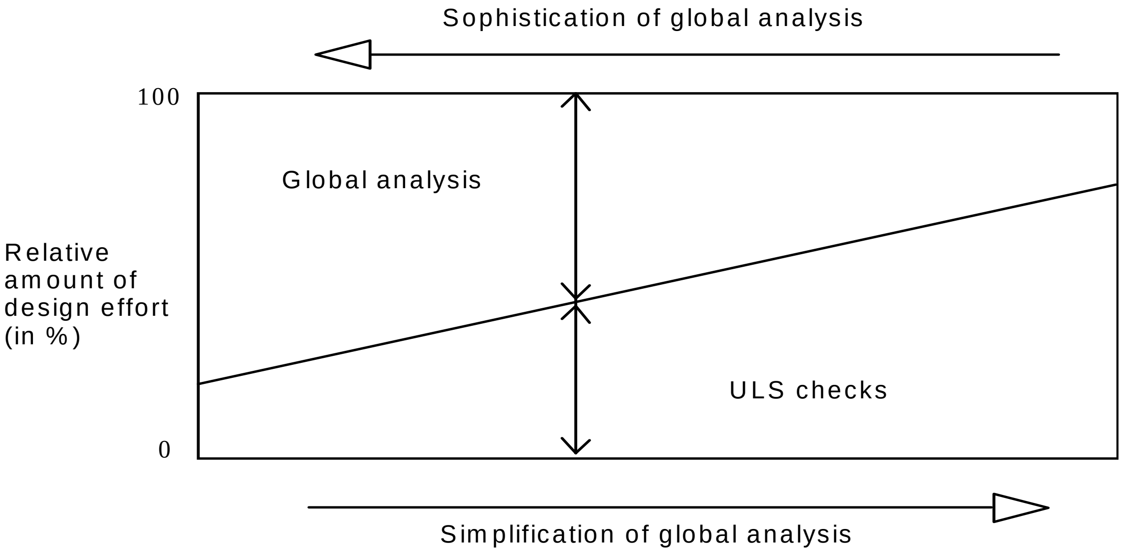


Figure 1.6 Schematic of the proportion of effort for global analysis and for *ULS* checks

1.2 Brief description of the contents of the manual

This manual is divided into three main parts:

- *Part 1* deals with technical background, and it is primarily an explanation of the new concepts as well as a simple presentation of the contents and consequences of the novel approach to design offered by *Eurocode 3*.
- *Part 2* presents application rules and it is aimed at daily use by the designer. It is the translation of *Part 1* in terms of formulae, by-hand methods and software presentation. Guidelines, data and hints are given for those who wish to develop their own software aids on common PC hardware. For example, in addition to the design tables that are given for a variety of joints, the design sheets are also provided. These sheets should allow users having a basic programming knowledge to develop their own programme.
- *Part 3* consists in a set of worked examples.

In *Part 1*, three chapters address in succession the design, the characterisation of joints, the available methods of global analysis and, finally, the decision on the type of global analysis and the related checks of limit states. For *Part 2*, the order is different than in *Part 1*, with Chapters 8 and 9 describing design tools the general use of which is outlined in Chapter 7. *Part 3* is truly the place where the designer will find clues to decide which overall strategy is the best for his purposes.

The contents of the present manual is in accordance with *Eurocode 3-Part 1* and, as such, makes the many references to design values. These design values are related to structural safety as defined in *Eurocode 1* and *Eurocode 3* and they include relevant partial safety factors for resistance (γ_M), as opposed to either nominal or characteristic values. They are generally indicated by the subscript d ; for instance $M_{pl.Rd}$ is the design plastic bending resistance of a member cross-section.

Concerning the loads, only one load case is generally considered in the manual, it being understood that the described operations shall be performed for each load combination case.

1.3 Division of the manual and directions to use it

For a first reading of this manual, it is strongly advised to give a thorough consideration to the present introduction and to *Part 1*, in order to grasp the basic concepts once and for all.

For practical daily use, *Part 2* (design rules) is of the main interest, it being supplemented by the worked examples presented in *Part 3*. *Part 1* may always be reconsulted for clarification of problems related to general fundamentals.

It should be noted that this manual has been written according to the most recent version of *Eurocode 3*, and especially to the *revised Annex J* to *Eurocode 3*, dealing with the design of joints. However, some evolution in *Eurocode 3* is to be expected in the near future, either at the European level or at each national level (on this latter respect, the values of partial safety factors are a particular case of point).

This manual will, as far as possible, be maintained up-to-date. However the reader is strongly advised to check on the current state of *the Eurocode 3-Part 1* Standard and to proceed with the possible modifications that may be required.

Similarly, the software tools that may be developed by third parties, based on the contents of this manual, should be checked to be in accordance with the latest version of the design rules. The same remark obviously holds for any software developed by the reader using the information given in this manual.

To that effect, the date of issue is clearly indicated on the front page of the manual. Any possible revised version of the latter will be supplied with a revision index.

1.4 Types of joints covered

Strictly speaking, the joints covered are those directly covered by *Eurocode 3-(revised) Annex J*, as listed hereafter; however, the same principles are also valid for many other types of joints and cross-sections. The manual addresses:

- Members cross sections : H or I rolled or welded sections.
- Beam-to-column joints : Welded joints (with or without haunch).
  Bolted end-plate connections (with or without haunch).

Bolted top and seat angle connections (with or without web angles).

- Beam splices : Bolted end-plate connections.

When relevant, both major axis (moment acting in the plane of greatest inertia of the column cross-section) and minor axis (moment acting in the plane of lowest inertia of the column cross-section) joints are covered, as regard principles. Application rules deal only with major axis joints.

1.5 Domain of validity

The use of information given in this manual when designing structures and joints shall be restricted to the following:

- Static loading only.
- Steel grades for members and joints:
  S235 to *S355* according to *EN 10025*;
  S355 to *S460* according to *EN 10113*.
- For welded sections and for some rules:
  depth-to-thickness ratio of the web $d/t_w < 69\varepsilon$.

PART 1: TECHNICAL BACKGROUND

CHAPTER 2

DESIGN METHODOLOGY

2.1 Introduction

This chapter describes the various approaches which can be used to design steel frames with due attention being paid to the behaviour of the joints. In practice, this design activity is normally performed by one or two parties, according to one of the following ways:

- An engineering office (in short *engineer*) and a steel fabricator (in short *fabricator*), referred as *Case A*;
- An engineering office (*engineer*) alone, referred as *Case B1*;
- A steel fabricator (*fabricator*) alone, referred as *Case B2*.

At the end of this design phase, fabrication by the steel fabricator takes place.

The share of responsibilities for design and fabrication respectively is given in Table 2.1 for these three cases.

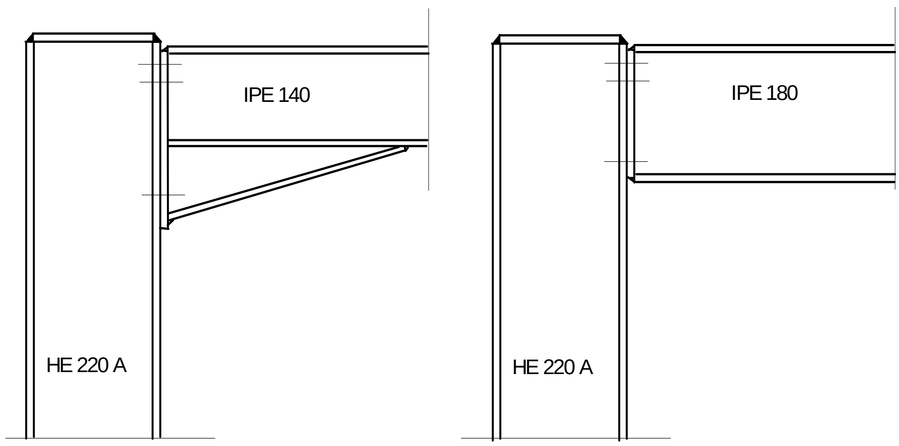
Role	Case A	Case B1	Case B2
Design of members	Engineer	Engineer	Fabricator
Design of joints	Fabricator	Engineer	Fabricator
Fabrication	Fabricator	Fabricator	Fabricator

Table 2.1 Parties and their roles in the design/fabrication process of a steel structure

The design process is ideally aimed at ascertaining that a given structure fulfils architectural requirements, on the one hand, and is safe, serviceable and durable for a minimum of global cost, on the other hand. The parties involved in the design activities also care about the cost of these latter, with a view to optimising their respective profits.

In *Case A*, the engineer designs the members while the steel fabricator designs the joints. It is up to the engineer to specify the mechanical requirements to be fulfilled by the joints. The fabricator has then to design the joints accordingly, keeping in mind the manufacturing aspects also. Due to the disparity in the respective involvements of both parties, the constructional solution adopted by the fabricator for the joints may reveal to be sub-optimal; indeed it is dependent on the beam and column sizing that is made previously by the engineer. The latter may for instance aim at minimum shape sizes, with the consequence that the joints then need stiffeners in order to achieve safety and serviceability requirements. If he chooses larger shapes, then joints may prove to be less elaborated and result in a better economy of the structure as a whole (Figure 2.1).

In *Case B1*, the engineer designs both the members and the joints. He is thus able to account for mechanical joint properties when designing members. He can search for global cost optimisation too. It may happen however that the engineer has only a limited knowledge of the manufacturing requisites (machinery used, available materials, bolt grades and spacing, accessibility for welding, ...); then this approach may contribute some increase in the fabrication cost.



a. Bolted end-plate with haunch
b. Bolted flush end-plate

Figure 2.1 Two solutions: different economy

Case B2 is ideal with regard to global economy. Indeed the design of both members and joints are in the hands of the fabricator who is presumably well aware of all the manufacturing aspects.

Before commenting on these various approaches, it is necessary to introduce some wording regarding joints.

A joint is termed *simple*, *semi-continuous* or *continuous*. This wording is general; it is concerned with resistance, with stiffness or with both. Being a novelty for most readers, some detailed explanations are given in Chapter 4 on joints. In two circumstances only - that are related to the methods of global frame analysis (see Chapter 5) -, this wording leads to more commonly used terms:

1. In an elastic global frame analysis, only stiffness of joints is involved. Then, a simple joint is a *pinned* joint, a continuous joint is a *rigid* joint while a semi-continuous is a *semi-rigid* joint;
2. In a rigid-plastic analysis, only resistance of joints is involved. Then, a simple joint is a *pinned* joint, a continuous joint is a *full-strength* joint while a semi-continuous joint is *a partial-strength* joint.

The various cases described above are commented on in the present chapter. For the sake of simplicity, it is assumed that global frame analysis is conducted based on an elastic method of analysis. This assumption is however not at all a restriction; should another kind of analysis be performed, similar conclusions would indeed be drawn.

For the design of steel frames, the designer can follow one of the following design approaches:

- *Traditional design approach* :

The joints are presumably either simple or continuous. The members are designed first; then the joints are. Such an approach may be used in any Case A, B1 or B2; it is of common practice in almost all the European countries.

- *Consistent design approach* :

Both member and joint properties are accounted for when starting the global frame analysis. This approach is normally used in Cases B1 and B2, and possibly in Case A.

- *Intermediate design approach* :

  Members and joints are preferably designed by a single party (Case B1 or B2).

These approaches are described more in detail in the following paragraphs. They are illustrated by some case studies in Chapter 6.

2.2 Traditional design approach

In the traditional design approach, any joint is assumed to be either simple or continuous. A simple joint is capable of transmitting the internal forces got from the global frame analysis but does not develop a significant moment resistance which might affect adversely the beam and/or column structural behaviour. A continuous joint exhibits only a limited relative rotation between the members connected as long as the applied bending moment does not exceed the bending resistance of the joint.

The assumption of simple and/or continuous joints results in the share of design activities into two more or less independent tasks, with limited data flow in between. The traditional design approach of any steel frame consists of eight steps (Figure 2.2).

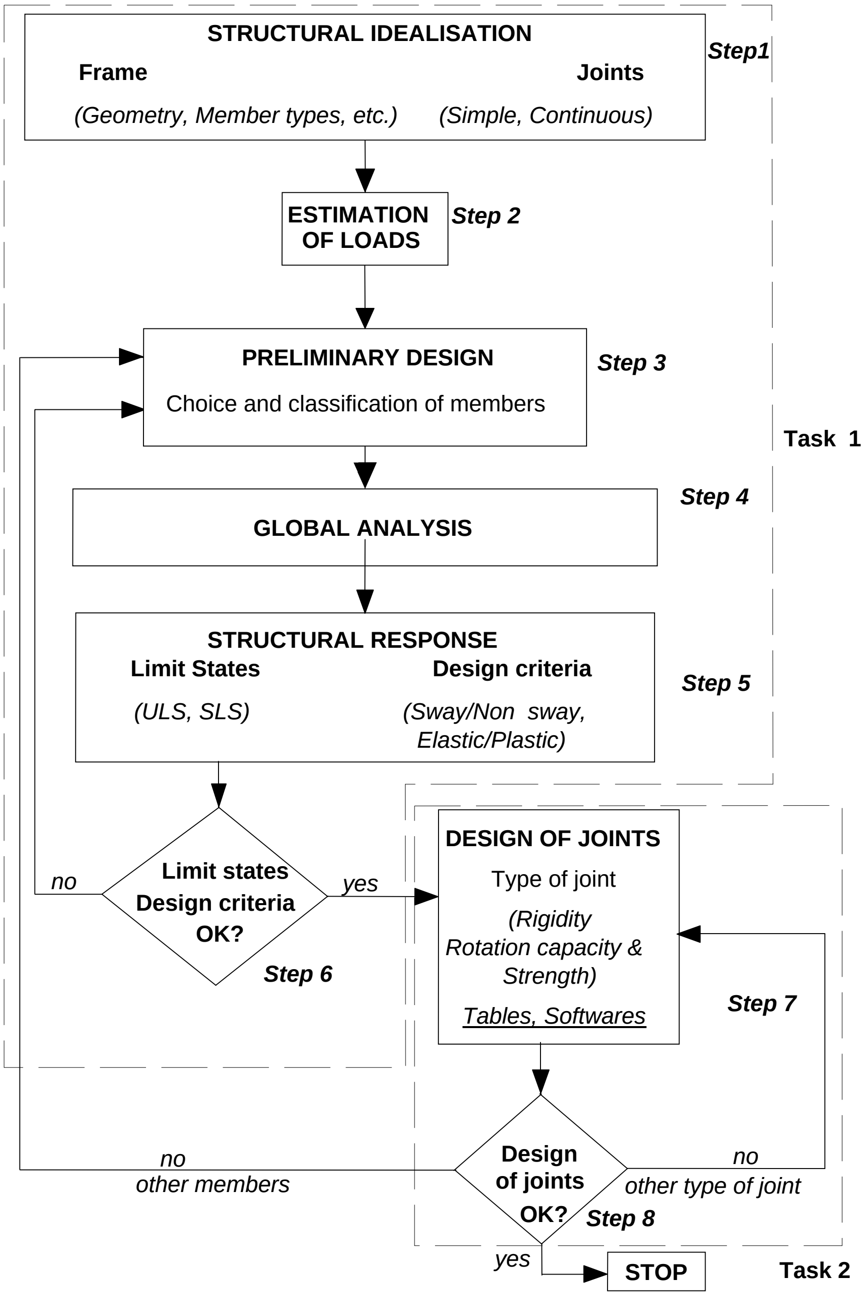


Figure 2.2 Traditional design approach (simple/continuous joints)

Step 1 : The structural idealisation is a conversion of the real properties of the frame into the properties required for frame analysis. Beams and columns are normally modelled as bars. Dependent on the type of frame analysis which will be applied, properties need to be assigned to these bars. For example, if an elastic analysis is used, only the stiffness properties of the members are relevant; the joints are pinned or rigid and are modelled accordingly (Figure 2.3).

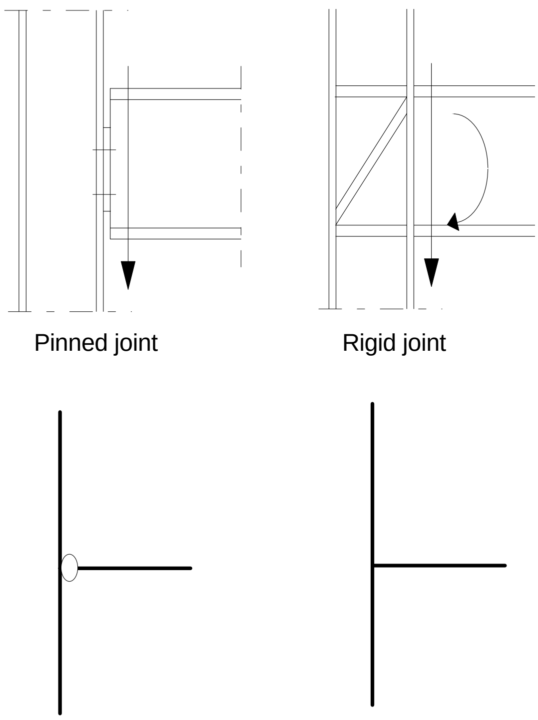


Figure 2.3 Modelling of pinned and rigid joints (elastic global analysis)

Step 2 : Loads are determined based on national or European standards.

Step 3 : The designer normally performs the preliminary design - termed pre-design in short - of beams and columns by taking advantage of his own design experience from previous projects. Should he have a limited experience only, then simple design rules can help for a rough sizing of the members. In the pre-design, an assumption shall be made concerning the stress distribution within the sections (elastic, plastic), on the one hand, and, possibly, on the allowance for plastic redistribution between sections, on the other hand. Therefore classes need to be assumed for the structural shapes composing the frame (see Chapter 3); the validity of this assumption shall be verified later in Step 5.

Step 4 : The input for global frame analysis is dependent on the type of analysis. In an elastic analysis, the input is the geometry of the frame, the loads and the flexural stiffness of the members. In a rigid-plastic analysis, the input is the geometry of the frame, the loads and the resistance of the members. An elastic-plastic analysis requires both resistance and flexural stiffness of the members. Whatever the type of global frame analysis, the distribution and the magnitude of the internal forces and displacements are the output (however rigid-plastic analysis does not allow for any information regarding displacements).

Step 5 : Limit state verifications consist normally in checking the displacements of the frame and of the members under service loading conditions (Serviceability Limit States, in short *SLS*), the resistance of the member sections (Ultimate Limit States, in short *ULS*), as well as the frame and member stability (*ULS*). The assumptions made regarding the section classes (see Step 3) are checked also.

Step 6 : The adjustment of member sizing is to be carried out when the limit state verifications fail, or when undue under-loading occurs in a part of the structure. Member sizing is adjusted by choosing larger shapes in the first case, smaller ones in the second case. Normally the designer's experience and know-how form the basis for the decisions made in this respect.

Steps 7/8: The member sizes and the magnitude of the internal forces that are experienced by the joints are the starting point for the design of joints. The purpose of any joint design task is to find a conception which allows for a safe and sound transmission of the internal forces from the beam into the column. Additionally, when a simple joint is adopted, the fabricator shall verify that no significant bending moment develops in the joint. For a continuous joint, the fabricator shall check whether the joint satisfies the assumptions made in Step 1 (for instance, whether the joint stiffness is sufficiently large when an elastic global frame analysis is performed). In addition, the rotation capacity shall be appropriate when necessary.

The determination of the mechanical properties of a joint is called *joint characterisation* (see Section 4.5). To check whether a joint may be considered as simple or continuous, reference will be made to *joint classification* (see Section 4.6).

Guidance provided in this manual is implemented by design tables for joints. These tables are in compliance with *Eurocode 3*. They can be very helpful during the design process because they enlighten drastically the design tasks. The designer selects the joints out of these tables which provide the strength and stiffness properties of the relevant joints, as well as, when necessary, the rotation capacity. A dedicated software, called *DESIMAN*, developed as a supplement to these tables, may be an alternative to the latter; it requires the whole layout of the joint as input and provides strength, stiffness and rotational capacity as output. Computer based design proceeds interactively by trial and error; for instance, the designer first tries a simple solution and improves it by adjusting the joint layout until the strength and stiffness criteria are fulfilled. Such a convivial software was developed in the frame of the preparation of present manual.

The mechanical properties of a joint shall be consistent with those required by the modelling of this joint in view of global frame analysis. Either the design of the joint and/or that of the members may need to be adjusted. In any case, some steps of the design approach need to be repeated.

2.3 Consistent design approach

In the consistent design approach, the global analysis is carried out in full consistency with the presumed real joint response (Figure 2.4).
It is therefore different from the traditional design approach described in Section 2.2 in several respects:

- *Structural conception* :
  In the structural conception phase, the real mechanical behaviour of the joints is modelled;
- *Preliminary design* :
  In the pre-design phase, joints are selected by the practitioner based on his experience. Proportions for the joint components are determined : end-plate or cleat dimensions, location of bolts, number and diameter of bolts, sizes of column and beam flanges, thickness and depth of column web, etc.;
- *Determination of the mechanical properties* :
  In Step 4, the structural response of both the selected members and joints is determined. First the joints are characterised (see Section 4.5) with the possible consequence of having a non-linear behaviour. This characterisation is followed by an idealisation, for instance according to a linear or bi-linear joint response curve (see Section 4.4), which becomes a part of the input for global frame analysis;
- *Global frame analysis* :
  For the purpose of global frame analysis, any joint structural response is assigned to a relevant spring in the frame model. This activity is called *modelling* (see Section 4.3).

Of course the consistent design approach is only possible when both members and joints are designed by a single party, because the mechanical properties of the joints must be accounted for when starting the global frame analysis. In other words, this approach suits both Case B1 and Case B2. In Chapter 6, information is given on intermediate forms.

As it accounts for any kind of behaviour, the consistent design approach is especially applicable to frames with so-called semi-continuous joints. The modelling of semi-continuous joints is presented in Section 4.3 in relation with the global frame analysis. It may also be applied when designing frames with simple or continuous joints.

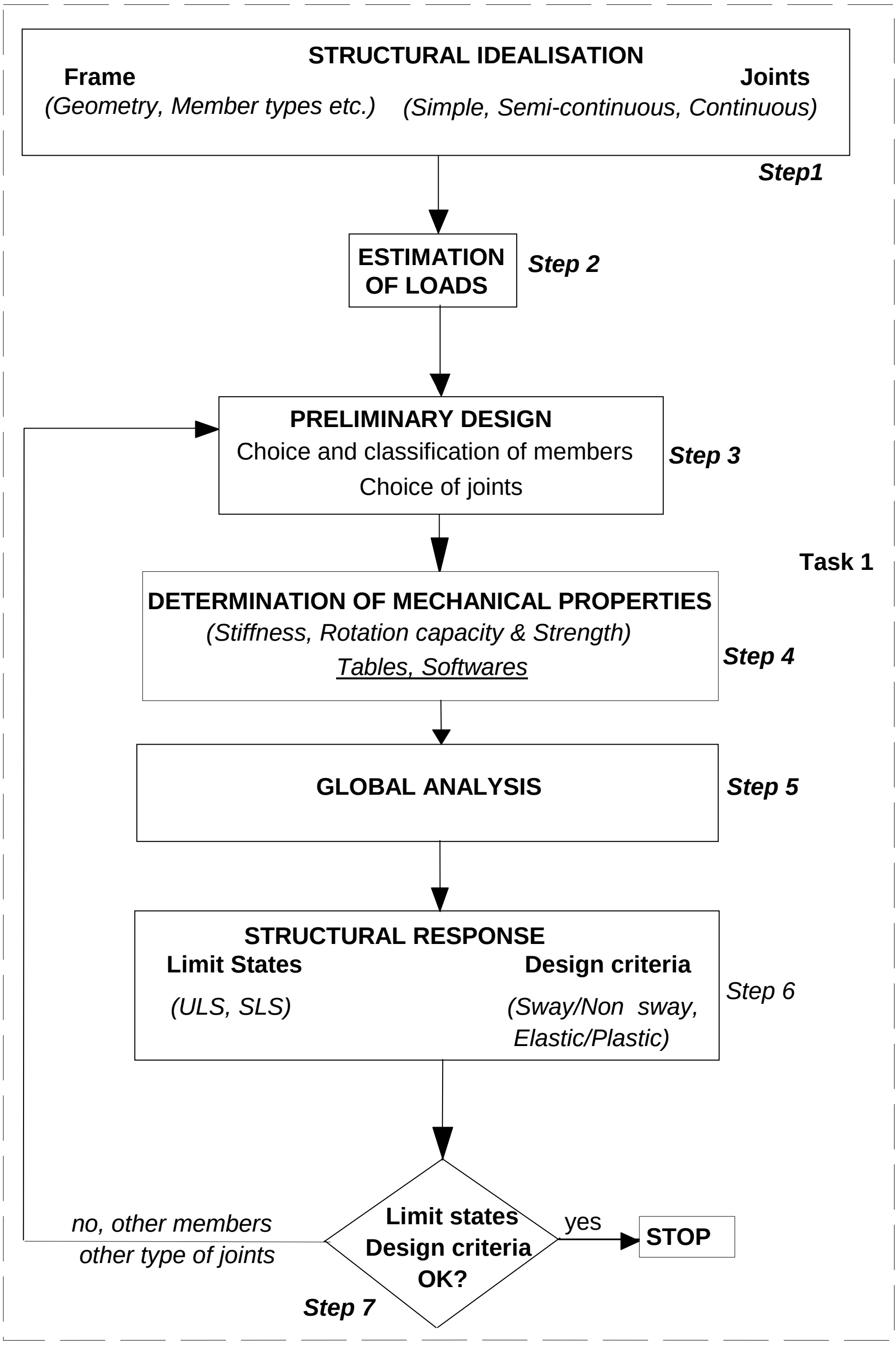


Figure 2.4 Consistent design approach

2.4 Intermediate design approaches

The two design approaches described in Section 2.2 (for frames with simple or continuous joints) and in Section 2.3 (for frames with semi-continuous joints) correspond to extreme situations. Intermediate approaches can be used. For example, the procedure given in Figure 2.1 can also be applied for semi-continuous joints. In that case, during the first pass through the design process, the joints are assumed to behave as simple or continuous joints. Joints are then chosen, the real properties of which are then accounted for in a second pass of the global analysis (i.e. after Step 8). The design process is then pursued similarly to the one described in Figure 2.4.

Some applications of intermediate design approaches are commented on in Chapter 6.

CHAPTER 3

GLOBAL FRAME ANALYSIS

3.1 Introduction

3.1.1 Scope

Global analysis aims at determining the distribution of the internal forces and moments and the corresponding displacements in a structure subjected to a specified loading.

Achieving this purpose requires the adoption of adequate models which incorporate assumptions about the behaviour of the structure and in particular of its component members and joints. Only the essential aspects are reviewed here. For example, as the basic concepts of structural analysis are considered to be largely known and furthermore are covered by many textbooks, they will not be elaborated on here.

3.1.2 Load-displacement relationship of frames

The relationship between the load parameter and a significant displacement parameter characterises the frame behaviour (Figure 3.1). The *load parameter* λ is most often a multiplier applied to all the load components so as to produce monotonous and proportional increase in the loading, while the displacement parameter may be taken as the lateral displacement at the top level. The slope of the curve is a measure of the stiffness of the structure.

One observes that the response of the structure is quasi-linear up to a certain point (the *linear limit*). Once the linear limit is reached, the positive slope of the rising part of the curve gradually reduces due to a combination of three kinds of *non-linearities*: geometrical non-linearity, joint non-linearity and material non-linearity. Joint non-linearity usually manifests itself at relatively low levels of load. Geometrical non-linearity expresses the influence of the actual deformed shape of the structure on the distribution of the internal forces and moments. Typically it becomes evident well before the onset of material yielding. Beyond the latter, the response becomes progressively non-linear as the load increases up to a maximum. Once the maximum load is reached, equilibrium would require a decrease in the magnitude of the loads.

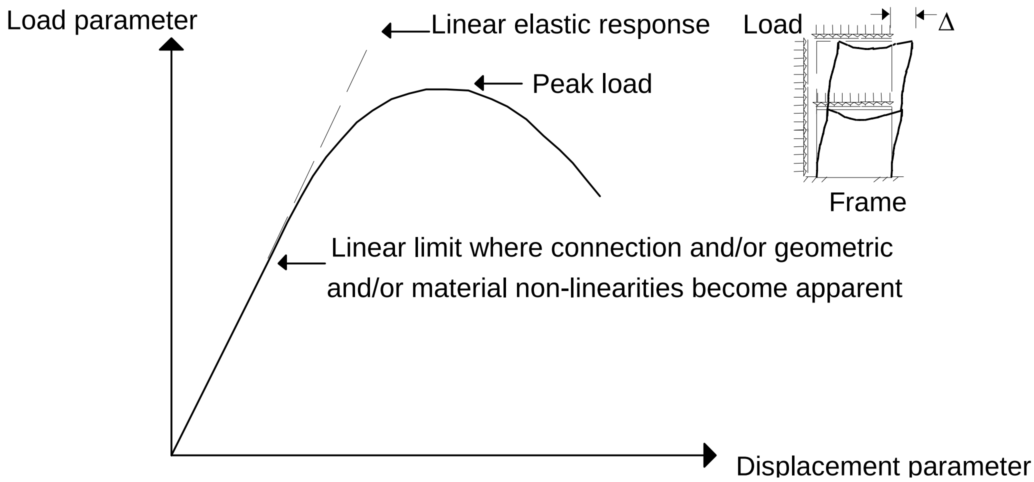


Figure 3.1 Load-displacement response of a framed structure

The slope of the curve (i.e. the frame sway stiffness) is zero at the *peak load*; and then it becomes negative indicating that the structure is henceforward unstable. The peak load, often termed the *ultimate load*, is the point of imminent structural collapse in the absence of the possibility of load shedding.

3.2 Methods of global analysis

3.2.1 General

The determination of the actual load-displacement response generally requires the use of a sophisticated analysis method. For practical purposes, assumptions for the frame and its component members and joints models are made that permit obtaining a safe bound for the ultimate load. Hence models for global analysis range from the simple *elastic analysis* to the most complex *elastoplastic analysis* (in short *plastic analysis*) which can provide the real inelastic behaviour of the structure. The first important distinction that can be made between the methods of analysis is the one that separates elastic and plastic methods. Whilst elastic analysis can be used in all cases, plastic analysis is subjected to some restrictions (see Section 3.4). Another important distinction is between the methods which make allowance for and those which neglect the effects of the actual displaced configuration of the structure. They are referred to respectively as *second-order theory* and *first-order theory* based methods (see Section 3.2.2). The second-order theory can be adopted in all cases, while first-order theory may be used only when the displacement effects on the structural behaviour are negligible.

In the following discussion of the methods of analysis, for clarity, reference will be restricted to two dimensional frames subject to in-plane loading and displacements.

Although only one load combination case is considered, it is understood that a number of load combination cases must be analysed.

3.2.2 Second-order effects

As the displacements due to the external loads may modify the structural response and therefore the distribution of the internal forces and moments, it is necessary to evaluate the degree to which they are significant enough to merit allowance in the design.

For frames, the external loads that cause the largest modifications to the linear response are the axial loads. Shown in Figure 3.2 is a fixed-base cantilever column subject to combined axial and horizontal loads applied at the top. The horizontal displacement at the top of the column as well as the column curvature lead to secondary moments.

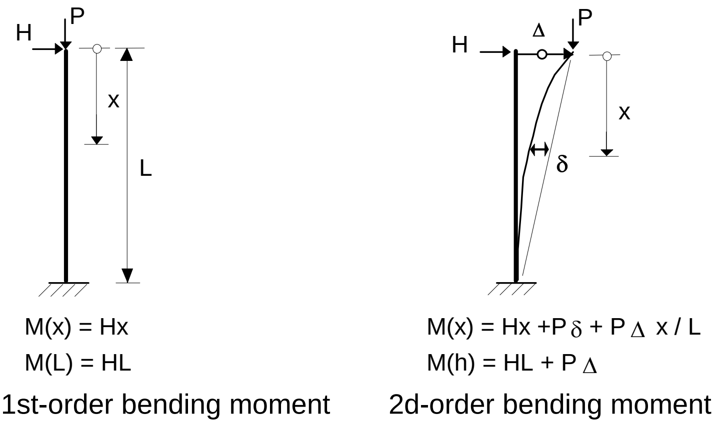


Figure 3.2 First-order and second-order theory

These moment modifications are made up of a *local* or *member second-order effect*, referred to as the *P-δ* effect, and a *global second-order effect*, referred to as the *P-Δ* effect.

Local second-order effects arise in each element subject to axial force due to the member deflections relative to the chord line connecting the member ends. Global second-order effects arise throughout the frame due to relative horizontal displacements between the floors. In general, the *P-Δ* effects, which only occur when sway is permitted, are greater than the *P-δ* effects, which are significant for slender columns only.

When the *P-Δ* and the *P-δ* effects are ignored, each structural element can be characterised by a linear stiffness matrix.

Account for the *P-δ* effects is achieved by modifying the terms of the linear stiffness matrix so that they become functions of the factor $\varepsilon = L\sqrt{P/EI}$, where *P* is the axial load in the member of length *L* and second moment of area *I*.

The *P-Δ* effects result in modifications of the end shear forces and moments; this fact is reflected by additions, corresponding to each of these components, to the member stiffness matrix.

3.3 Elastic global analysis

3.3.1 First-order theory

3.3.1.1 Assumptions, limitations, section/joint requirements

Elastic analysis implies an indefinite linear response of sections and joints (Figure 3.3). In a first-order analysis, equilibrium is expressed with reference to the non-deformed configuration of the structure.

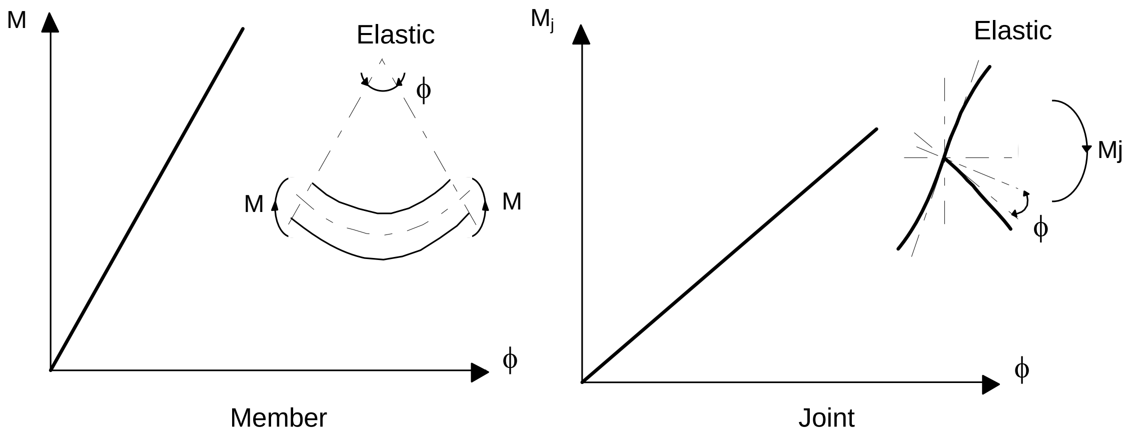


Figure 3.3 Moment-rotation characteristics of member and joint

Sections and joints are not subordinated to any requirement related to their ability to exhibit ductile behaviour (class of member, ductility class of joint).

3.3.1.2 Frame analysis

First-order elastic global analysis with linear member and joint behaviour results in a linear load-displacement curve (Figure 3.4).

Designers are quite familiar with first-order elastic analysis which is the simplest of all possible types of analysis. Over the years a variety of methods have been developed, which are suited to hand calculations (such as *the slope-deflection method* and the *moment-distribution method*). They can be generalised so as to include the joint behaviour. The same applies to procedures based on matrix formulation, which have now almost entirely supplanted the hand calculation methods, ever since computers became commonplace in design offices.

A significant advantage of the first-order elastic analysis is that it permits one to apply the principle of superposition of loading and load effects (i.e. internal forces and moments and displacements).

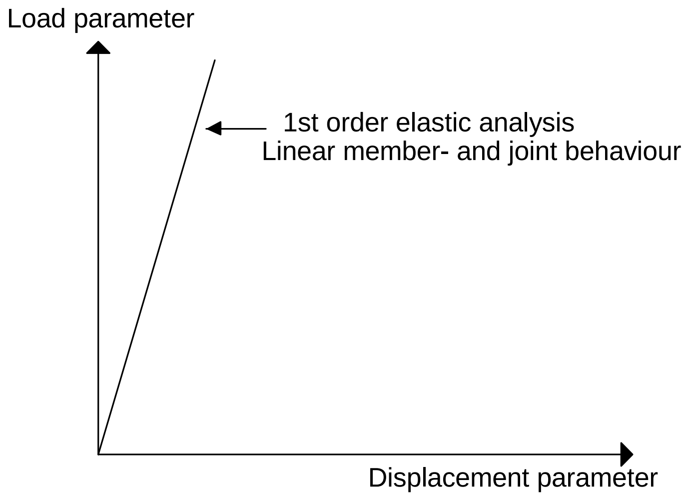


Figure 3.4 Load-displacement response: first-order elastic analysis

3.3.1.3 Frame design

Once the design forces (axial force, bending moment and shear force) have been determined throughout the structure, the resistance (ultimate limit state) check of the cross-section members consists in verifying that the stress (force) in the critical section does not exceed the design stress (resistance). Since elastic analysis has been used, it would seem logical to consider the yield stress in the extreme fibres as the design stress for a member (Figure 3.5). It is however generally accepted that first-order elastic analysis can be used to determine the value λ_{L1} of the load multiplier corresponding to when the first plastic event takes place (onset of the *first* plastic hinge). Therefore, provided the sections meet the requirements for ductile behaviour (class 1 or 2), the section resistance may be checked using the plastic interaction formula. In the same way, the resistance (ultimate limit state) check of the joints consists in verifying that the design forces (bending moment and shear force) in the joint do not exceed the design resistance (moment resistance and shear resistance) of the joint.

However, this assumes that the structure and its members remain stable. Therefore it is of major importance to investigate additionally the instability phenomena (in-plane and/or out-of-plane frame and/or member instability). Instability may indeed reduce the value of λ_{L1}. For the design to be adequate, the value of λ_{L1} must be at least unity (i.e. the structure can withstand at least the applied design loads).

First-order elastic analysis provides a safe basis for design as long as the predicted response of the structure deviates only slightly from the actual response over a considerable range of loading (i.e. structures in which axial loads are low). For structures with heavier axial loads, λ_{L1} is not a lower bound of the maximum load because the second-order effects have been neglected.

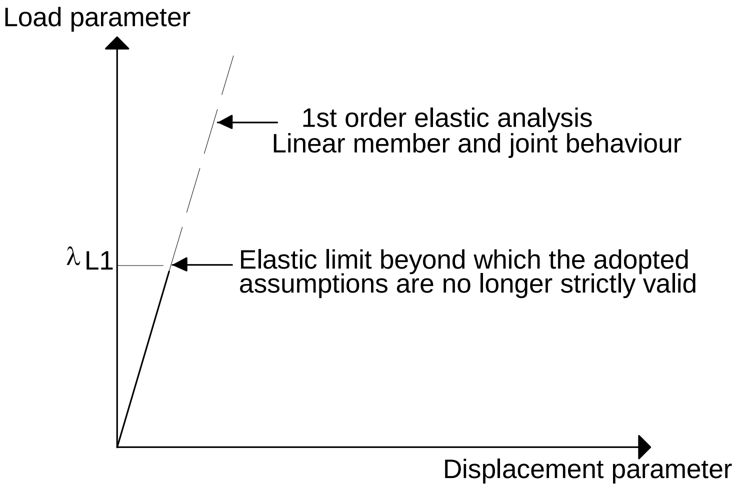


Figure 3.5 Range of validity of first-order elastic analysis

Resistance to concentrated loads may have to be checked for some members.

As regards the serviceability limit state (permissible displacements and/or deflections), a first-order elastic analysis generally provides a good tool for predicting the response of the structure and of its elements.

3.3.2 Second-order theory

3.3.2.1 Assumptions, limitations, section/joint requirements

In this type of elastic analysis, the indefinitely linear elastic response of sections and joints is still implied (Figure 3.3). The distribution of the internal forces and moments is now computed on the basis of a second-order theory of the kind outlined in 3.2.2. Equilibrium equations are formulated with reference to the deformed structure (P-Δ effects) and the decrease in member stiffness due to axial force (P-δ effects) may be included when necessary. As is the case of the first-order elastic analysis, sections and joints are not subordinated to any requirement related to their ability to exhibit ductile behaviour (class member, ductility class of joint).

However one can no longer superimpose the results from the analysis of individual loading cases, as was possible for the first-order elastic analysis.

3.3.2.2 Frame analysis

Figure 3.6 shows the load-displacement response that results from a second-order elastic analysis in which all the loads are increased monotonously by the same load multiplier (proportional loading).

The load-displacement curve, which now includes geometric non-linearity, approaches asymptotically a horizontal corresponding to a peak value of λ_{cr} (critical multiplier). This value of λ_{cr} corresponds to the elastic critical buckling load of the frame, relevant to the specified load case combination.

If the *P*-δ effects are neglected, then the computed peak load may be higher than the actual one. The more slender are the compression members the more significant the *P*-δ effects become. The elastic critical buckling load is an important reference, as it is the highest theoretical load that the frame can experience in the absence of any material yielding. Yielding will reduce the actual maximum load that can be attained to a value often appreciably lower than the elastic critical buckling load.

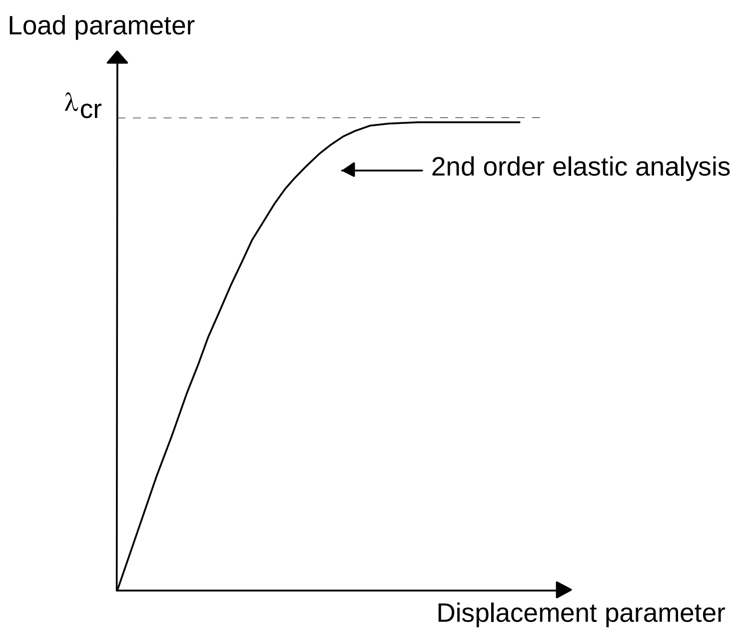


Figure 3.6 Load-displacement response: second-order elastic analysis

3.3.2.3 Frame design

In contrast to first-order elastic analysis, second-order analysis provides internal forces and moments at the ends of the members that include the second-order effects.

The elastic critical buckling load of the frame may also be provided by a second-order elastic analysis but only if the load multiplier is increased sufficiently. In practice the elastic critical buckling load is evaluated using an approximate procedure referred to in Section 7.1.

With due attention being paid to these differences, the frame design may proceed in the same manner as for the first-order elastic analysis. The critically loaded section or joint permits one to obtain the upper limit value of the load multiplier value λ_{L2} (Figure 3.7) for which the elastic analysis is valid.

When second-order elastic analysis is used in calculating frames, the in-plane frame stability in terms of sway buckling is covered by the structural analysis. In most practical cases, the local member imperfections are not introduced and only planar behaviour is considered. Therefore the check against instability of members (in-plane and/or out-of-plane) and of the frame (out-of-plane) may indeed reduce the value of λ_{L2}. For the design to be adequate, the value of λ_{L2} must be at least unity.

Resistance to concentrated loads may have to be checked for some members.

As regards the serviceability limit state (permissible displacements and/or deflections), a first-order elastic analysis generally provides a good tool for predicting the response of the structure and of its elements.

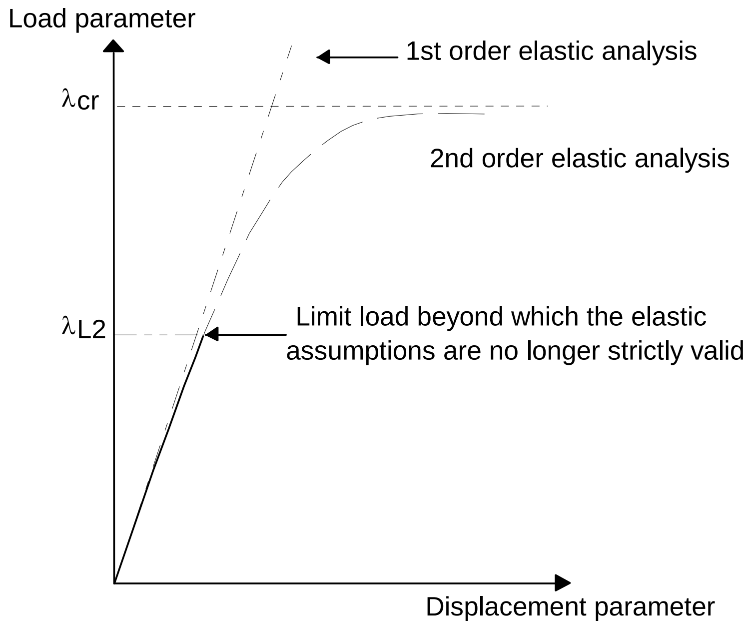


Figure 3.7 Range of validity second-order elastic analysis

3.4 Plastic global analysis

The plastic methods are applicable within the following main restrictions :

1. The steel complies with the following specified requirements :

   - The ratio of the specified minimum tensile strength f_u to the specified minimum yield strength f_y satisfies :

     $$f_u/f_y \geq 1.2$$

   - The elongation at failure on a gauge length of $5.65\sqrt{A_0}$ (where A_0 is the original cross-section area) is not less than *15%*;

   - In a stress-strain diagram, the ultimate strain ε_u corresponding to the ultimate strength f_u is at least *20* times the yield strain ε_y corresponding to the yield strength f_y.

2. Lateral restraint shall be provided at all plastic hinge locations at which plastic hinge rotation may occur under any load case. The restraint should be provided within a distance along the member from the theoretical plastic hinge location not exceeding half the depth of the member.

3. Sections and/or joints where plastic hinges are likely to occur have sufficient rotation capacity and are therefore of class 1 (see Section 5.2.2 of Chapter 5).

3.4.1 Elastic-perfectly plastic analysis (Second-order theory)

3.4.1.1 Assumptions, limitations, section/joint requirements

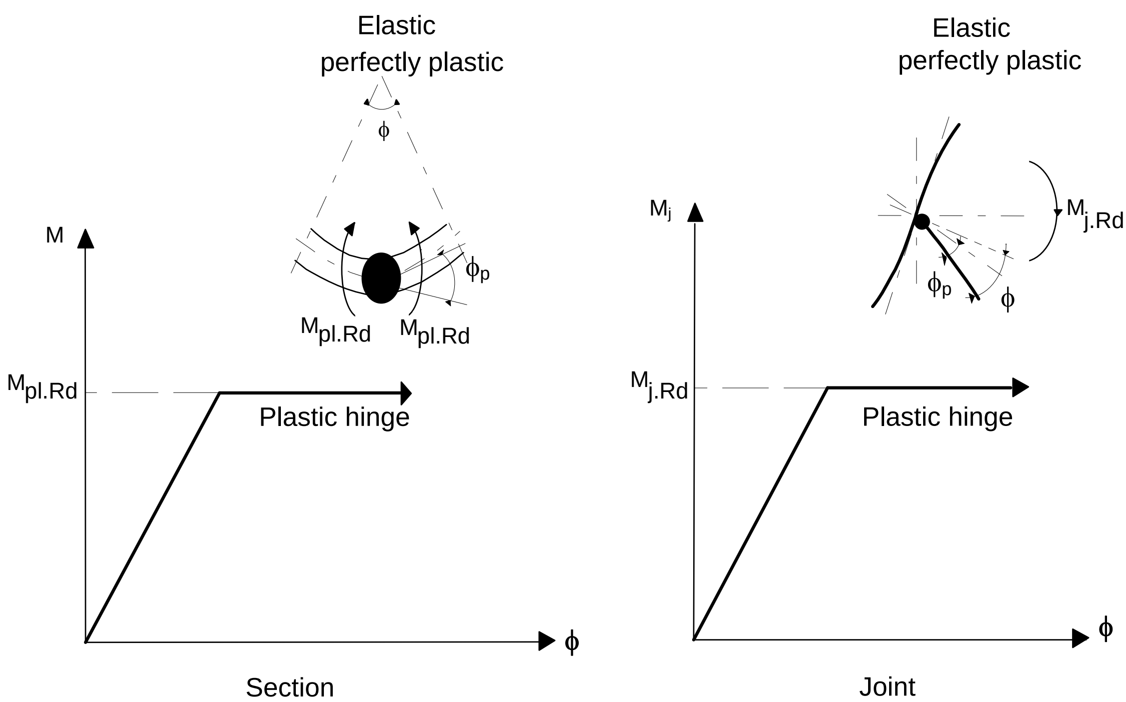


Figure 3.8 Moment-rotation characteristic of section and joint

In the elastic-perfectly plastic analysis, it is assumed that any section and/or joint remains elastic till the plastic moment resistance is reached at which point it becomes ideally plastic. Plastic deformations are assumed to be concentrated at the plastic hinge locations which are assumed to have an infinite rotational capacity. That the actual rotation capacity is sufficient to meet what is required is checked at a later stage. Figure 3.8 shows the elastic-perfectly plastic behaviour of a section and a joint. The influence of the axial force and/or the shear force on the plastic moment resistance of the sections may either be accounted for directly or be checked later at the design verification stage.

The use of elastic-perfectly plastic analysis implies that some requirements concerning the structure and its component members and joints are met. These are needed in order to guarantee that sections and joints, at least at the locations at which the plastic hinges may form, have sufficient rotation capacity to permit all the plastic hinges to develop throughout the structure.

The load-displacement curve of the frame can be determined. Computation of the plastic rotations of the plastic hinges may also be carried out so as to permit the check that the required rotation capacity is available.

3.4.1.2 Frame analysis and design

The load is usually applied by increments. The following is a typical description of how the various steps of the analysis is performed. It assumes that a second-order analysis has been used. For clarity, it is also assumed that the plastic hinges form successively (one by one) without any reversal of the rotation in them, although it is recognised that two (or more) hinges can develop at the same time and that reversal does sometimes occur.

An elastic second-order analysis is first performed and from it, the load at which the first plastic hinge forms in a section and/or in a joint is determined. The next analysis is made for further incremental loads for which the frame behaves differently due to the introduction of a hinge at the first plastic hinge (*modified frame*). It is recalled that a plastic hinge continues to permit rotation without any increase or reduction in its moment and, furthermore, it is assumed to have sufficient ductility to undergo the necessary rotation. The modified frame is said to be the *deteriorated frame*. The next plastic hinge is formed after further increase of the load level and the process is repeated until the failure mechanism is developed.

A typical second-order elastic-perfectly plastic analysis is shown by the solid curve in Figure 3.9. Branch *1* corresponds to a fully elastic frame behaviour. This curve becomes asymptotic to the elastic buckling load of the frame only if infinite elastic behaviour is assumed. A first hinge is formed, and the frame now behaves under further load increments as if one hinge exists in it (branch *2*) until the next hinge is developed. If unlimited elastic behaviour is assumed after the first hinge has formed, then branch *2* continues and becomes asymptotic to the buckling load of the deteriorated frame, which is the frame with one hinge. The process is continued with the load being increased and the structure being progressively « deteriorated » up to the moment when it becomes unstable (formation of plastic mechanism or frame instability). The maximum load of the second-order elastic plastic analysis corresponds to this load level which is shown as the load multiplier $\lambda_L = \lambda_{L3}$ in Figure 3.9.

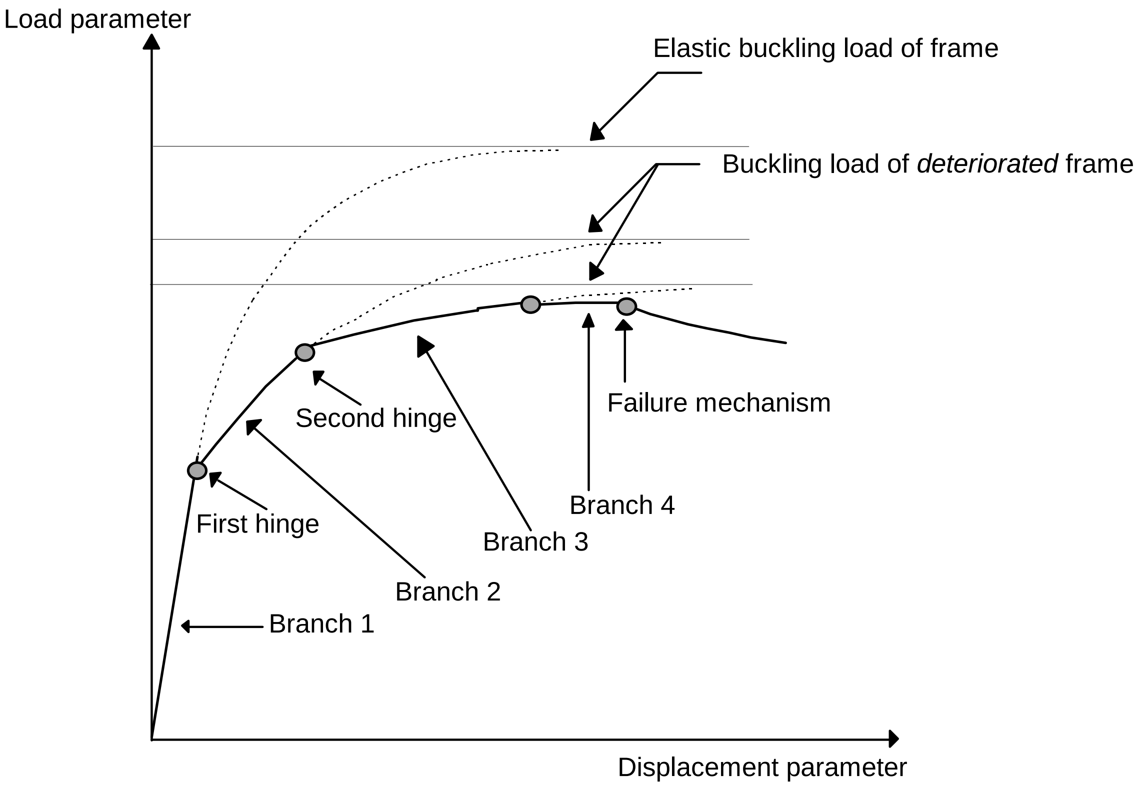


Figure 3.9 Load-displacement response: second-order elastic-perfectly plastic analysis

No additional design checks for the sections and the joints are required when the influence of the axial force and/or the shear force is accounted for in their behavioural model. As the rotations at the plastic hinges have been calculated, this permits one to check that the required rotation capacity is available there.

When second-order theory is used in calculating frames, the in-plane frame stability is covered by the structural analysis. However, the out-of-plane stability of the frame and the members will have to be verified. Except when local member imperfections are taken into account, in-plane stability of the members will also have to be verified These checks may indeed reduce the value of λ_{L3}. For the design to be adequate, the value of λ_{L3} must be at least unity when applied to the factored loads.

Resistance to concentrated loads may have to be checked for some members.

The serviceability limit state checks (permissible displacements and/or deflections) shall be carried out. too.

3.4.2 Elastoplastic analysis (Second-order theory)

3.4.2.1 Assumptions, limitations, section/joint requirements

A better estimation of the maximum load than that provided by the elastic-perfectly plastic analysis can be obtained by carrying out a second-order elastoplastic analysis.

Yielding of members and joints is a progressive process and so the transition from elastic behaviour to plastic one is a gradual phenomenon. As the moment in the member cross-section continues to increase, the plastic zone extends partially along the member. This behaviour is considered by the plastic zone theory. Figure 3.10 shows the moment-rotation characteristics of section and joint which are adopted in this type of analysis.

The ductility requirements for the sections and joints and the procedure for analysis and check of the frame are the same as those outlined in Section 3.4.1.

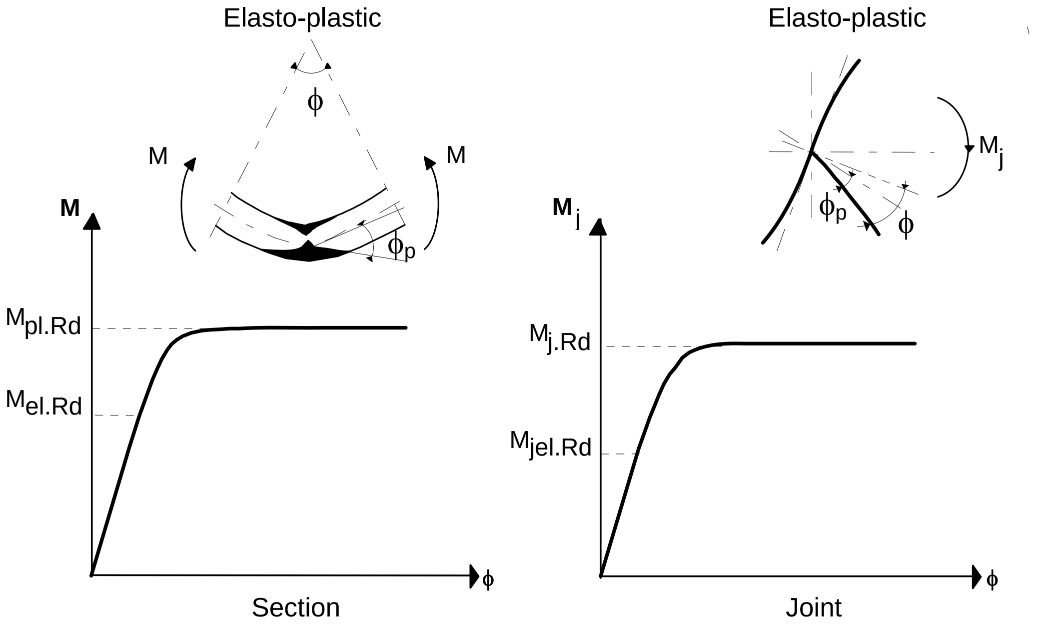


Figure 3.10 Moment-rotation characteristics of section and joint

3.4.2.2 Frame analysis and design

The second-order elastoplastic analysis gives the values of the maximum load that the frame can carry, as well as the magnitude of the displacements corresponding to any load level.

Usually only the in-plane behaviour of the frame is considered. Therefore separate checks on out-of-plane stability shall be carried out.

The elastoplastic method of analysis restricted to computer applications and because of its complexity, it is unlikely to be used for practical design purposes, but more as a tool for research.

3.4.3 Rigid-plastic analysis (First-order theory)

3.4.3.1 Assumptions, limitations, section/joint requirements

In this type of plastic analysis, the elastic strains in members, joints and foundations are disregarded, as they are small compared to plastic strains. Material strain-hardening is also ignored. In addition, plastic deformations are arbitrarily concentrated in sections and joints where plastic hinges are likely to occur. These sections and joints are assumed to have an infinite rotational capacity. Figure 3.11 shows the idealised rigid-plastic response of the sections and the joints which are adopted for the analysis. As a result, the values of the design moment resistance for sections and joints as well as the structural configuration and the loading are the only parameters that affect rigid-plastic analysis.

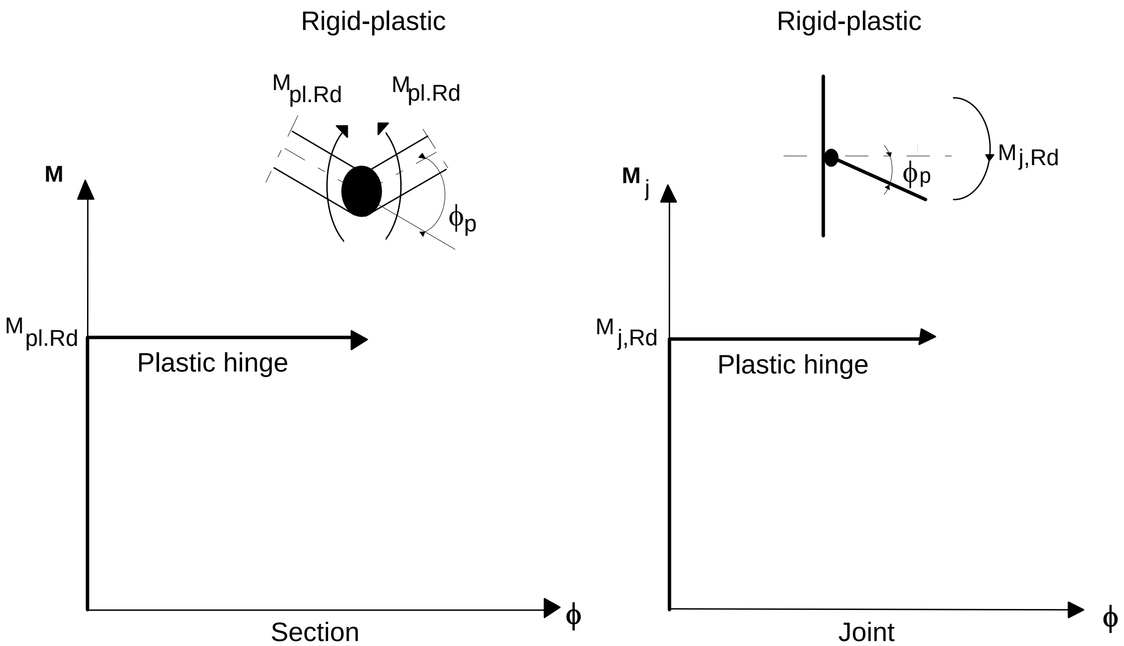


Figure 3.11 Moment-rotation characteristics of section and joint

The ductility requirements for the sections and the joints are the same as those outlined for elastic-perfectly plastic analysis.

3.4.3.2 Frame analysis

On the basis that the maximum loading a given structure can sustain corresponds to that for which collapse occurs (because a realistic plastic mechanism has been created), the analysis consists in identifying the governing plastic mechanism.

The basic principle behind this method is that when the collapse load, i.e. the maximum load that the structure can sustain, is attained, the following three conditions are satisfied simultaneously:

- *Mechanism*: there are a sufficient number of plastic hinges or real hinges (pinned joints) for the structure to form a kinematically admissible mechanism;

- *Equilibrium*: the bending moment distribution throughout the structure is in equilibrium with the external loads and reactions;

- *Plasticity*: the plastic moment resistances of the sections and joints are nowhere exceeded.

The collapse load can be obtained by the direct application of the fundamental theorems of simple plastic design. These fundamental theorems are the lower bound and the upper bound theorems, also known as the static theorem and the kinematic theorem respectively.

An approach suitable for the manual application of the upper bound theorem is summarised here.

According to this theorem, for a given structure and loading, any assumed plastic collapse mechanism occurs at a value of the load multiplier configuration that is greater than or equal to the value of the collapse load multiplier. By examining the various possible mechanisms, one identifies the collapse mechanism for which the value of the load multiplier is least and which is both statically and plastically admissible.

Figure 3.12 shows the elementary mechanisms *1* and *2* as well as the combined mechanism *3* for a simple portal frame. Any load-displacement response is represented by a horizontal line, the ordinate of which is the associated collapse load. In accordance with the upper bound theorem, the lowest load parameter, which is that for mechanism *3*, shall be retained. The collapse load given by the rigid-plastic analysis for this structure and loading corresponds to the load level shown as the reference load multiplier λ_{L4} in the figure.

For most cases of simple rectangular frames, the by hand application of the rigid plastic concept is simple and straightforward. Pitched roof portal frames can also be analysed using this approach. For multi-storey and/or multi-bay frames, the use of computers is usually required.

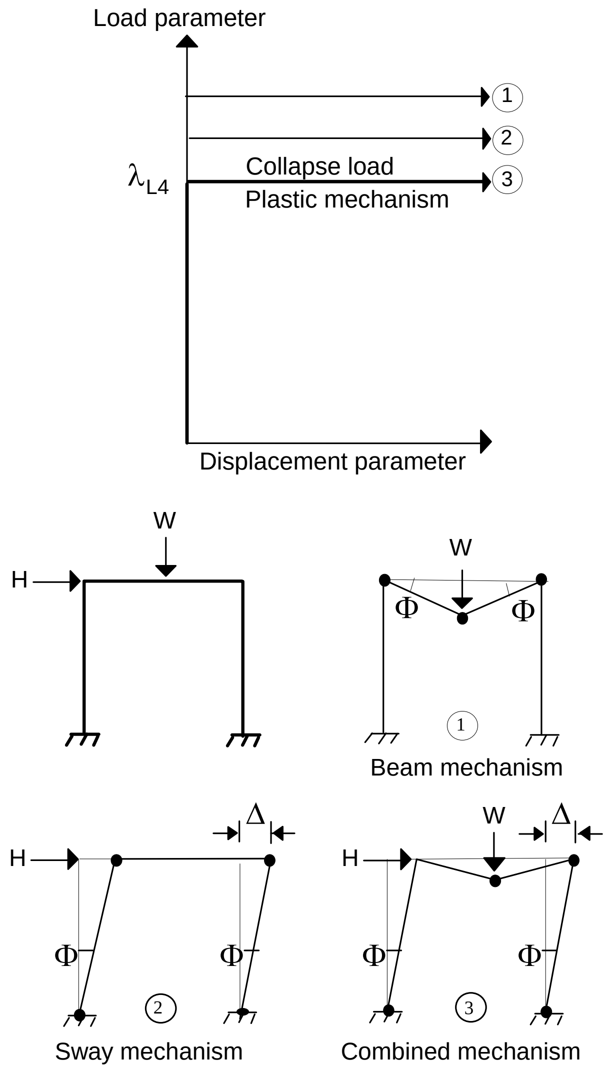


Figure 3.12 Load-displacement response: rigid-plastic analysis

3.4.3.3 Frame design

Additional design checks for sections and joints is required if the influence of the axial force and/or the shear force on the design moment resistance may not be negligible. As the rotations at the plastic hinges have been supposed infinite, these requirements must also be checked.

The in-plane frame instability load should be assessed, with provision for interaction with plasticity (e.g. *Merchant-Rankine* estimate) when necessary. The out-of-plane frame stability will have to be verified as well as the in-plane and/or the out-of-plane member stability. These checks may indeed reduce the value of λ_{L4}. For the design to be adequate, the value of λ_{L4} must be at least unity when applied to the factored loads.

Resistance to concentrated loads may have to be checked for some members.

The rigid-plastic analysis provides direct information in terms of the design frame resistance, but it does not provide any information on the displacements and rotations that have occurred. Therefore, it has usually to be complemented by an elastic analysis of the structure for the serviceability loading conditions.

CHAPTER 4

JOINT PROPERTIES AND MODELLING

4.1 Introduction and definitions

Building frames consist of beams and columns, usually made of H or I shapes, that are assembled together by means of connections. These connections are between two beams, two columns, a beam and a column or a column and the foundation (Figure 4.1).

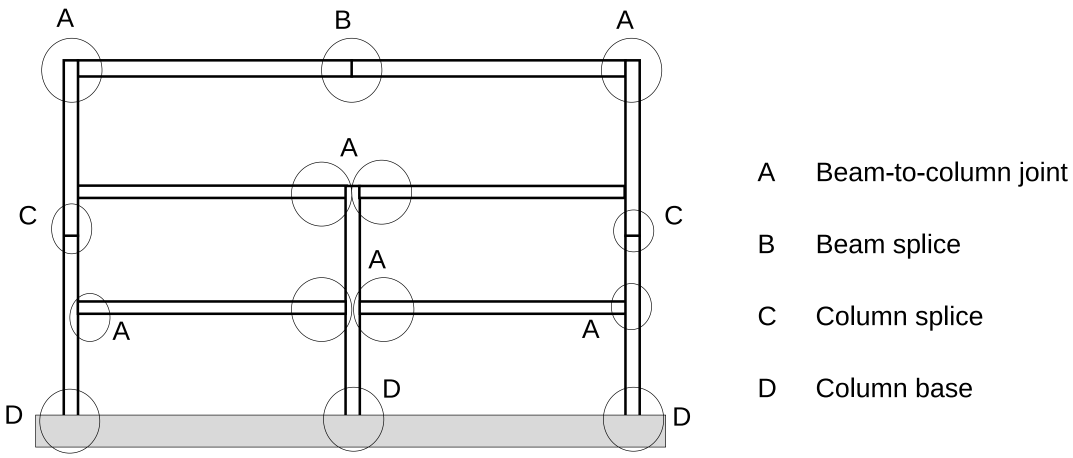
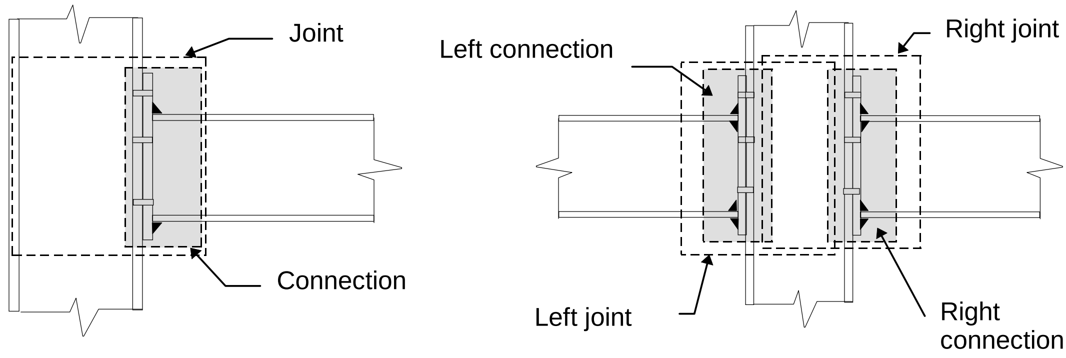


Figure 4.1 Different types of connections in a building frame

A *connection* is defined as the set of the physical components which mechanically fasten the connected elements. One considers the connection to be concentrated at the location where the fastening action occurs, for instance at the beam end/column interface in a major axis beam-to-column joint. When the connection as well as the corresponding zone of interaction between the connected members are considered together, the wording *joint* is then used (Figure 4.2.a).

Depending on the number of in-plane elements connected together, single-sided and double-sided joint configurations are defined (Figure 4.3). In a double-sided configuration (Figure 4.3.b), two joints - left and right - have to be considered (Figure 4.2.b.).

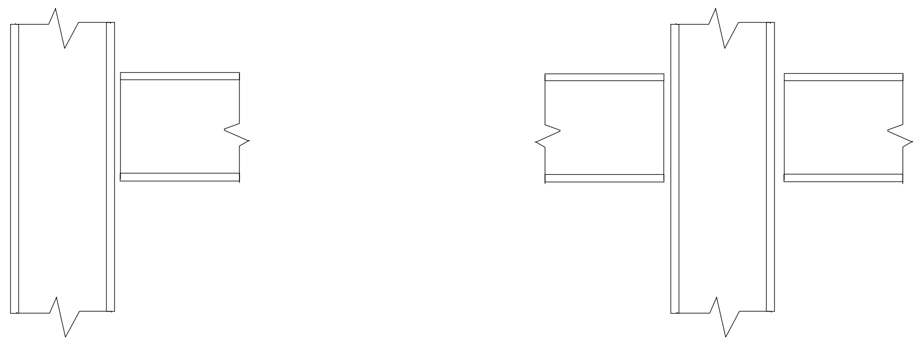
The definitions illustrated in Figure 4.2 are valid for other joint configurations and connection types.



(a) Single sided joint configuration (b) Double sided joint configuration

Figure 4.2 Joints and connections

As explained previously, the joints which are traditionally considered as rigid or pinned and are designed accordingly, possess, in reality, their own degree of flexibility resulting from the deformability of all the constitutive components. Section 4.2 is aimed at describing the main *joint deformability sources*. Section 4.3 provides information on how to *model* the joints in view of the frame analysis; this modelling depends on the level of joint flexibility. In Section 4.4, the way in which the shape of the non-linear joint deformability curves may be *idealised* is given. Section 4.5 refers to the *component method* as a general tool for the prediction of the main joint mechanical properties in bending. The concept of *joint classification* is introduced in Section 4.6. Finally, it is commented on the *ductility classes* of joints in Section 4.7.



(a) Single-sided (b) Double-sided

Figure 4.3 In-plane joint configurations

4.2 Sources of joint deformability

As said in Chapter 1, the rotational behaviour of the joints may affect the local and/or global structural response of the frames. In this section, the sources of rotational deformability are identified for beam-to-column joints, splices and column bases.

It is worthwhile mentioning that the rotational stiffness, the joint resistance and the rotation capacity are likely to be affected by the shear force and/or the axial force acting in the joint.

These shear and axial forces may obviously have contributions to the shear and axial deformability within the connections. However it is known that these contributions do not affect significantly the frame response; therefore, the shear and axial responses of the connection, in terms of rotational deformability, are neglected.

4.2.1 Beam-to-column joints

4.2.1.1 Major axis joints

In a major axis beam-to-column joint, different sources of deformability can be identified. For the particular case of a single-sided joint (Figure 4.4.a and Figure 4.5.a), these are:

- *The deformation of the connection*. It includes the deformation of the connection elements : column flange, bolts, end-plate or angles,... and the load-introduction deformation of the column web resulting from the transverse shortening and elongation of the column web under the compressive and tensile forces F_b acting on the column web. The couple of F_b forces are statically equivalent to the moment M_b at the beam end. These deformations result in a relative rotation ϕ_c between the beam and column axes; this rotation, which is equal to θ_b - θ_c (see Figure 4.4.a) is concentrated mainly along edge BC and provides a flexural deformability curve M_b - ϕ_c.
- *The shear deformation of the column web panel* associated to the shear force V_{wp} acting in this panel. It leads to a relative rotation γ between the beam and column axes; this rotation makes it possible to establish a shear deformability curve V_{wp}-γ.

The deformability curve of a connection may obviously be influenced by the axial and shear forces possibly acting in the connected beam.

Similar definitions apply to double-sided joint configurations (Figure 4.4.b and Figure 4.5.b). For such configurations, two connections and a sheared web panel, forming two joints, must be considered.

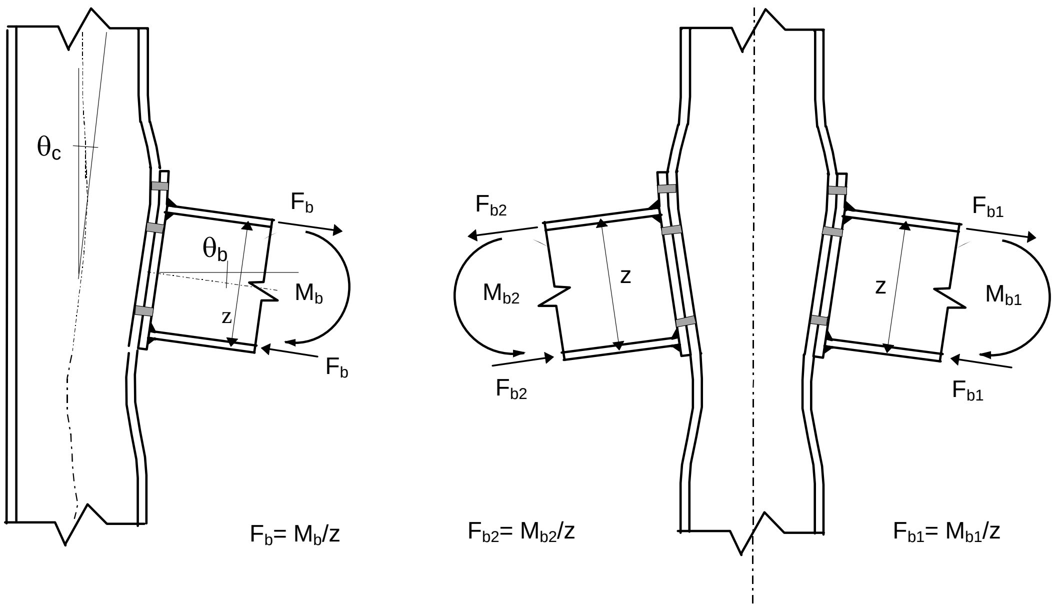
In short, the main sources of deformability which must be contemplated in a beam-to-column major axis joint are :

Single-sided joint configuration :

- the connection deformability M_b-ϕ_c characteristic;
- the column web panel shear deformability V_{wp}-γ characteristic.

Double-sided joint configuration :

- the left hand side connection deformability M_{b1}-ϕ_{c1} characteristic;
- the right hand side connection deformability M_{b2}-ϕ_{c2} characteristic;
- the column web panel shear deformability V_{wp}-γ characteristic.



(a) Single-sided joint configuration (b) Double-sided joint configuration

Figure 4.4 Sources of joint deformability

The deformability of the connection (connection elements + load-introduction) is only due to the couple of forces transferred by the flanges of the beam (equivalent to the beam end moment M_b). The shear deformability of the column web panel results from the combined action of these equal but opposite forces and of the shear forces in the column at the level of the beam flanges. Equilibrium equations of the web panel provide the shear force V_{wp} (see Figure 4.5 for the sign convention) :

$$V_{wp} = \frac{M_{b1} - M_{b2}}{z} - \frac{V_{c1} - V_{c2}}{2} \tag{4.1}$$

Another formula to which it is sometimes referred, i.e. :

$$V_{wp} = \frac{M_{b1} - M_{b2}}{z} \tag{4.2}$$

is only a rough and conservative approximation of (4.1).

In both formulae, *z* is the lever arm of the resultant tensile and compressive forces in the connection(s). How to derive the value of *z* is explained in Chapter 8.

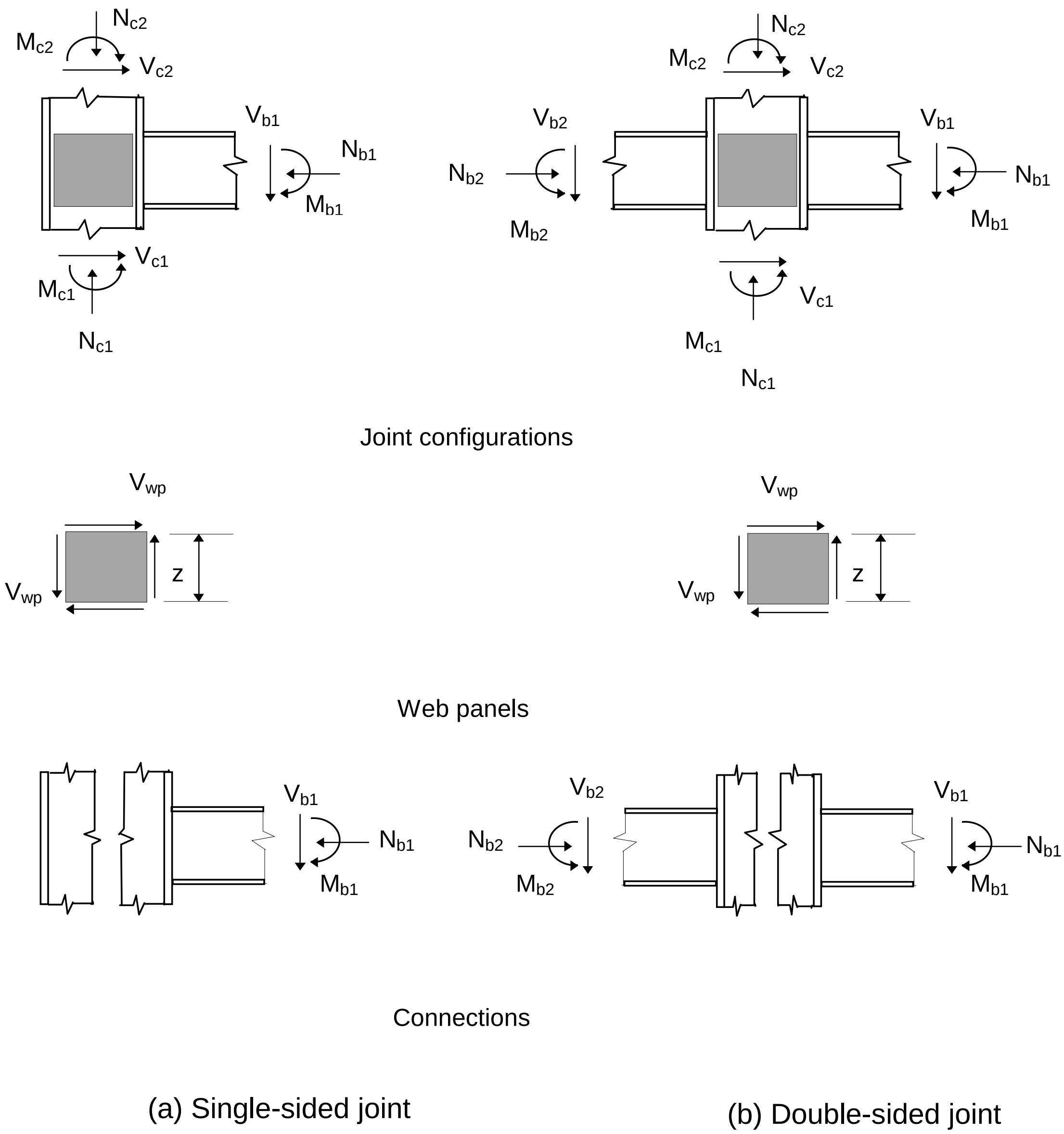


Figure 4.5 Loading of the web panel and the connections

4.2.1.2 Minor axis joints

A similar distinction between *web panel* and *connection* shall also be made for a minor axis joint (Figure 4.6). The column web exhibits a so-called out-of-plane deformability while the connection deforms in bending as it does in a major axis joint. However no load-introduction deformability is involved.

In the double-sided joint configuration, the out-of-plane deformation of the column web depends on the bending moments experienced by the right and left connections (see Figure 4.7) :

$$\Delta M_b = M_{b1} - M_{b2} \tag{4.3.}$$

For a single-sided joint configuration (Figure 4.6), the value of ΔM_b equals that of M_b.

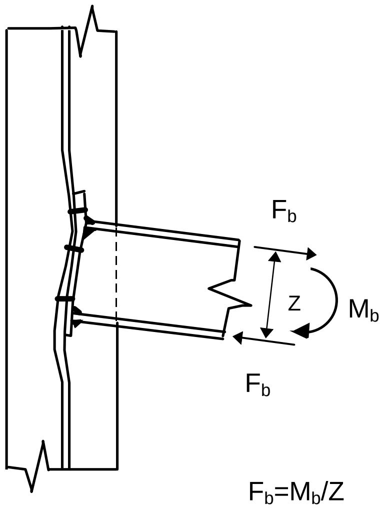


Figure 4.6 Deformability of a minor axis joint

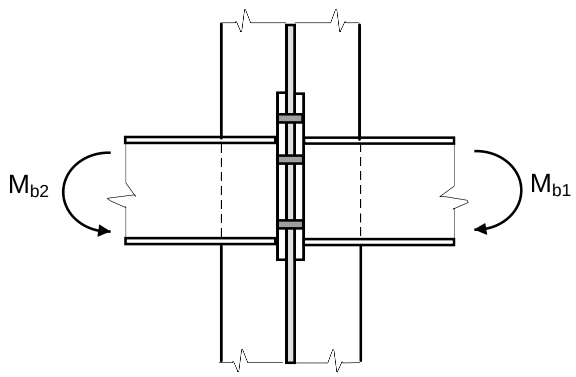


Figure 4.7 Loading of a double-sided minor axis joint

4.2.1.3 Joints with beams on both major and minor column axes

A 3-D joint is (Figure 4.8) characterised by the presence of beams connected to both the column flange(s) and web. In such joints, a shear deformation (see 4.2.1.1) and an out-of-plane deformation (see 4.2.1.2) of the column web develop coincidently.

The loading of the web panel appears therefore as the superimposition of the shear loading given by formulae (4.1) or (4.2) and the out-of-plane loading given by formula (4.3).

The joint configuration of Figure 4.8 involves two beams only; configurations with three or four beams can also be met.

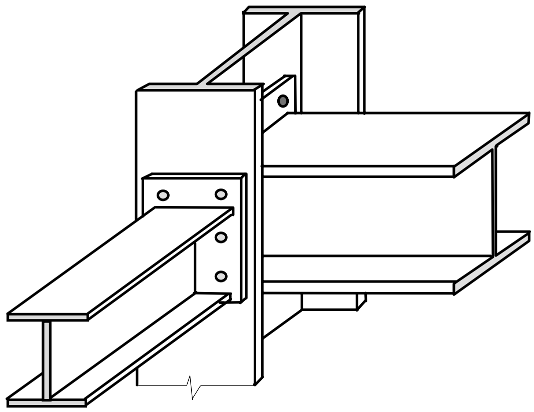


Figure 4.8 Example of a 3-D joint

4.2.2 Beam splices and column splices

The sources of deformability in a beam splice (Figure 4.9) or in a column splice (Figure 4.10) are less than in a beam-to-column joint; indeed they are concerned with connections only. The deformability is depicted by the sole M_b-ϕ curve.

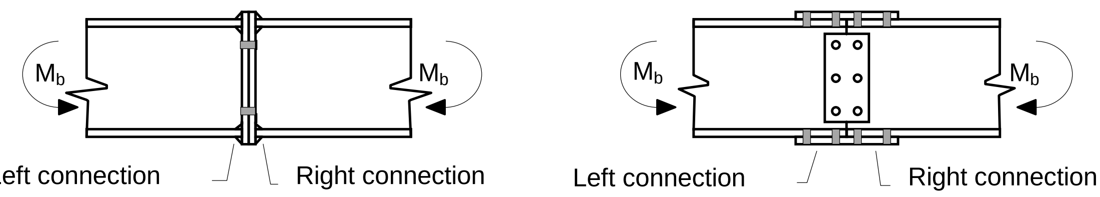


Figure 4.9 Deformation of a beam splice

The single M_b-ϕ curve corresponds to the deformability of the whole joint, i.e. the two constituent connections (left connection and right one in a beam splice, upper connection and lower one in a column splice).

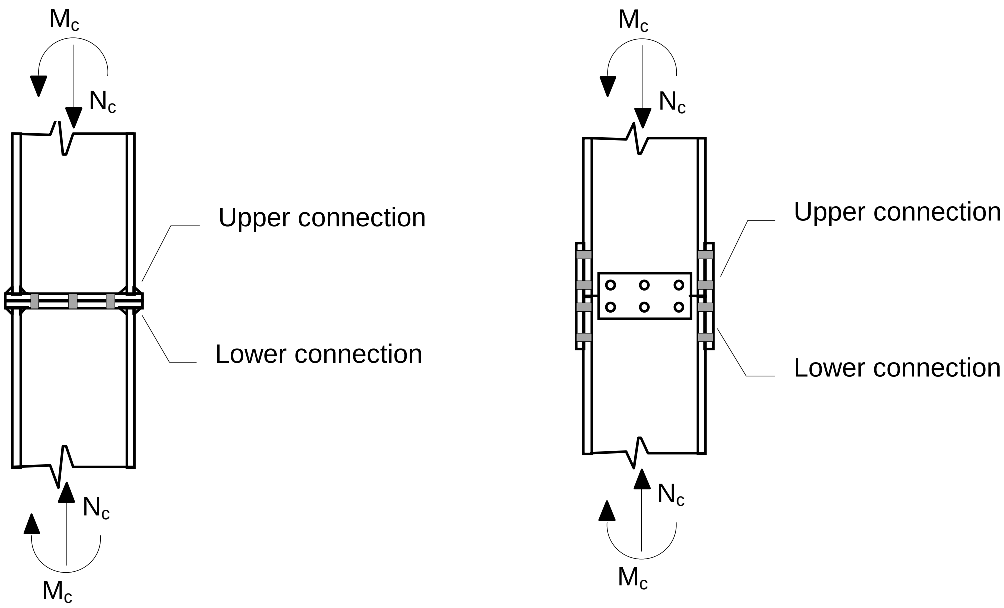


Figure 4.10 Deformation of a column splice

In a column splice where the compressive force is predominant, the axial force affects in a significative way the mechanical properties of the joint, i.e. its rotational stiffness, its strength and its rotation capacity. The influence, on the global frame response, of the axial deformability of splices is however limited; therefore it is neglected.

4.2.3 Beam-to-beam joints

The deformability of a beam-to-beam joint (Figure 4.11) is quite similar to the one of a minor axis beam-to-column joint; the loadings and the sources of deformability are similar to those expressed in 4.2.1.2 and can therefore be identified.

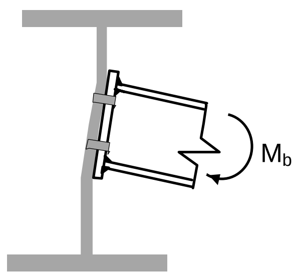


Figure 4.11 Deformation of a beam-to-beam joint

4.2.4 Column bases

In a column base, two connection deformabilities need to be distinguished (Figure 4.12) :

- the deformability of the connection between the column and the concrete foundation (*column-to-concrete connection*);
- the deformability of the connection between the concrete foundation and the soil (*concrete-to-soil connection*).

For the column-to-concrete connection, the bending behaviour is represented by a M_c - ϕ curve, the shape of which is influenced by the ratio of the bending moment to the axial load at the bottom of the column.

For the connection between the concrete foundation and the soil, two basic deformability curves are identified:

- a N_c-u curve which corresponds to the soil settlement due to the axial compressive force in the column; in contrast with the other types of joint, this deformability curve may have a significant effect on the frame behaviour;
- a M_c - ϕ curve characterising the rotation of the concrete block in the soil.

As for all the other joints described above, the deformability of the column base due to the shear force in the column may be neglected.

The column-to-concrete connection and concrete-to-soil connection M_c-ϕ characteristics are combined in order to derive the rotational stiffness at the bottom of the column and conduct the frame analysis and design accordingly.

Similar deformability sources exist in column bases subjected to biaxial bending and axial force. The connection M_c-ϕ characteristics are then defined respectively for both the major and the minor axes.

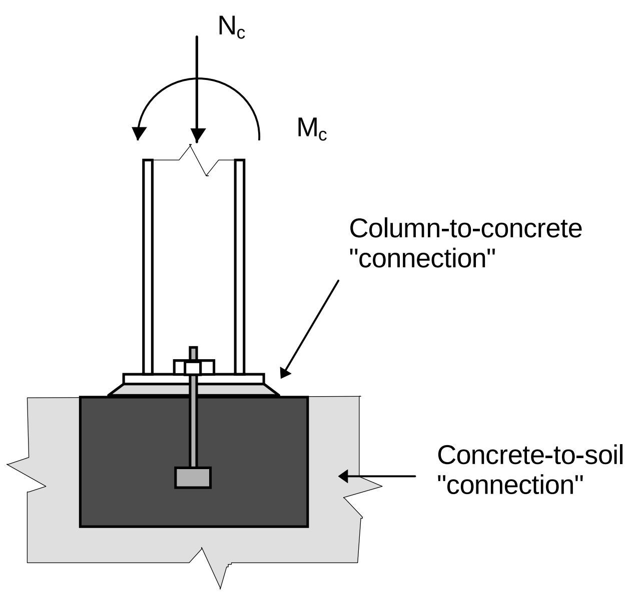


Figure 4.12 The connections in a column base

4.3 Joint modelling

4.3.1 General

Joint behaviour affects the structural frame response and shall therefore be modelled, just like beams and columns are, for the frame analysis and design. Traditionally, the following types of *joint modelling* are considered :

For rotational stiffness :
- rigid
- pinned

For resistance :
- full-strength
- partial-strength
- pinned

When the joint rotational stiffness is of concern, the wording *rigid* means that no relative rotation occurs between the connected members whatever be the applied moment. The wording *pinned* postulates the existence of a perfect (i.e. frictionless) hinge between the members. In fact these definitions may be relaxed, as explained in Section 4.6 devoted to the joint classification. Indeed rather flexible but not fully pinned joints and rather stiff but not fully rigid joints may be considered as fairly pinned and fairly rigid respectively. The stiffness boundaries allowing one to classify joints as rigid or pinned are examined in Section 4.6.

For what regards the joint resistance, a *full-strength joint* is stronger than the weaker of the connected members, what is in contrast with a *partial-strength joint.* In the daily practice, partial-strength joints are used whenever the joints are designed to transfer the internal forces and not to resist the full capacity of the connected members. A *pinned joint* transfers no moment. Related classification criteria are conceptually discussed in Section 4.6.

Consideration of rotational stiffness and resistance joint properties leads to three significant joint modellings:

- rigid/full-strength;
- rigid/partial-strength;
- pinned.

However, as far as the joint rotational stiffness is considered, joints designed for economy may be neither rigid nor pinned but semi-rigid. There are thus new possibilities for joint modelling :

- semi-rigid/full-strength;
- semi-rigid/partial-strength.

With a view to simplification, *Eurocode 3 - Chapter 6* and *Annex J* account for these possibilities by introducing three joint models (Table 4.1) :

- *continuous* : covering the rigid/full-strength case only;
- *semi-continuous* : covering the rigid/partial-strength, the semi-rigid/full-strength and the semi-rigid/partial-strength cases;
- *simple* : covering the pinned case only.

STIFFNESS	RESISTANCE		
	Full-strength	Partial-strength	Pinned
Rigid	Continuous	Semi-continuous	*
Semi-rigid	Semi-continuous	Semi-continuous	*
Pinned	*	*	Simple
* : Without meaning			

Table 4.1 Types of joint modelling

The following meanings are given to these terms :

- *continuous:* the joint ensures a full rotational continuity between the connected members;
- *semi-continuous:* the joint ensures only a partial rotational continuity between the connected members;
- *simple:* the joint prevents from any rotational continuity between the connected members.

The interpretation to be given to these wordings depends on the type of frame analysis to be performed. In the case of an elastic global frame analysis, only the stiffness properties of the joint are relevant for the joint modelling. In the case of a rigid-plastic analysis, the main joint feature is the resistance. In all the other cases, both the stiffness and resistance properties govern the manner the joints shall be modelled. These possibilities are illustrated in Table 4.2.

MODELLING	TYPE OF FRAME ANALYSIS		
	Elastic analysis	Rigid-plastic analysis	Elastic-perfectly plastic and elastoplastic analysis
Continuous	Rigid	Full-strength	Rigid/full-strength
Semi-continuous	Semi-rigid	Partial-strength	Rigid/partial-strength Semi-rigid/full-strength Semi-rigid/partial-strength
Simple	Pinned	Pinned	Pinned

Table 4.2 Joint modelling and frame analysis

4.3.2 Modelling and sources of joint deformability

The difference between the loading of the connection and that of the column web in a beam-to-column joint (see Section 4.2) requires, from a theoretical point of view, that account be taken separately of both deformability sources when designing a building frame.

However doing so is only feasible when the frame is analysed by means of a sophisticated computer program which enables a separate modelling of both deformability sources. For most available softwares, the modelling of the joints has to be simplified by concentrating the sources of deformability into a single rotational spring located at the intersection of the axes of the connected members.

4.3.3 Simplified modelling according to *Eurocode 3*

For most applications, the separate modelling of the connection and of the web panel behaviour is neither useful nor feasible; therefore only the simplified modelling of the joint behaviour (see Section 4.3.2) will be considered in the present document. This idea is the one followed in the *ENV 1993-1-1* experimental standard (*Chapter 6* and *Annex J*). Table 4.3, excerpted from the revised *Annex J*, shows how to relate the simplified modelling of typical joints to the basic wordings used for the joint modelling: simple, semi-continuous and continuous.

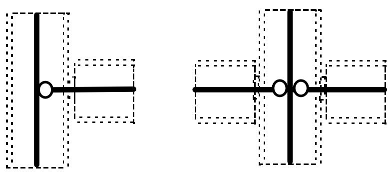
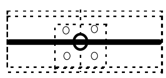
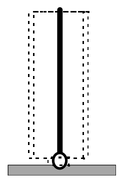
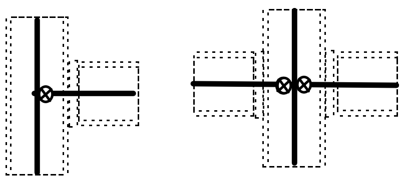
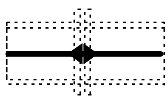
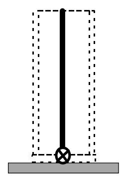
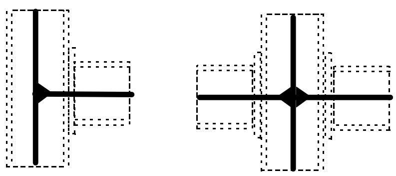
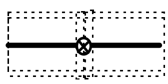
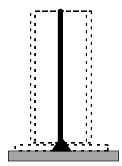
JOINT MODELLING	BEAM-TO-COLUMN JOINTS MAJOR AXIS BENDING	BEAM SPLICES	COLUMN BASES
SIMPLE			
SEMI-CONTINUOUS			
CONTINUOUS			

Table 4.3 Simplified modelling for joints according to EC3

4.3.4 Concentration of the joint deformability

For the daily practice a separate account of both the flexural behaviour of the connection and the shear (major axis beam-to-column joint) or out-of-plane behaviour of the column web panel (minor axis beam-to-column joint configurations or beam-to-beam configurations) is not feasible. This section is aimed at explaining how to concentrate the two deformabilities into a single flexural spring located at the intersection of the axes of the connected members.

4.3.4.1 Major axis beam-to-column joint configurations

In a single-sided configuration, only one joint is concerned. The characteristic shear-rotation deformability curve of the column web panel (see Figure 4.4 and

Figure 4.13.b) is first transformed into a M_b-γ curve through the use of the *transformation parameter* β. This parameter, defined in

Figure 4.14.a, relates the web panel shear force to the (load-introduction) compressive and tensile forces connection (see also formulae 4.1 and 4.2).

The M_b-ϕ spring characteristic which represents the joint behaviour is shown in

Figure 4.13.c; it is obtained by summing the contributions of rotation, from the connection (ϕ_c) and from the shear panel (γ). The M_j-ϕ characteristic of the joint rotational spring located at the beam-to-column interaction is assumed to identify itself to the M_b-ϕ characteristic obtained as indicated in Figure 4.13.c.

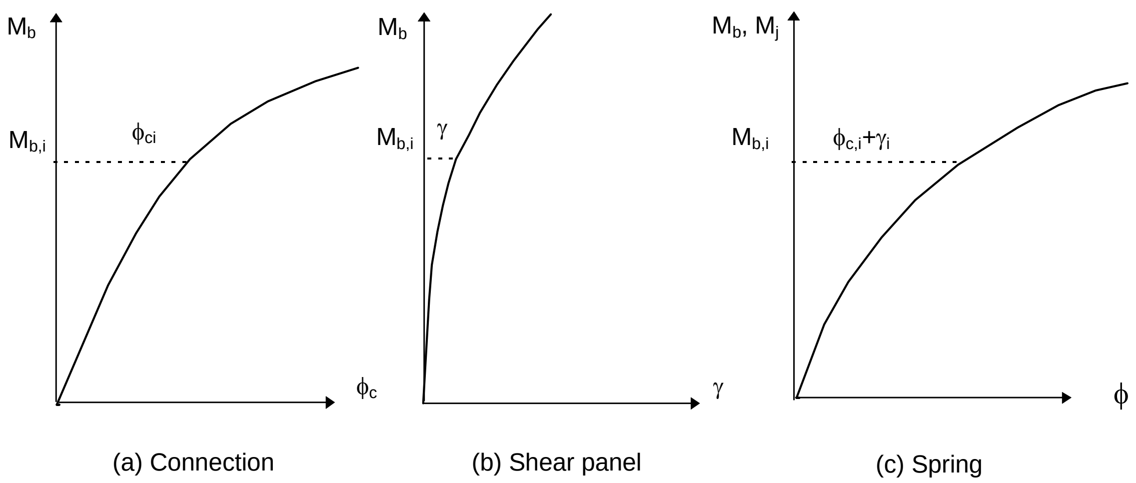
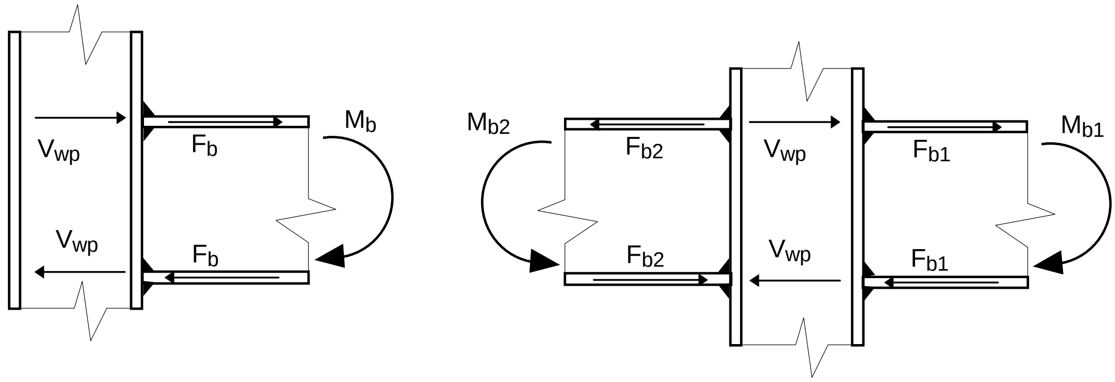


Figure 4.13 Flexural characteristic of the spring



$V_{wp} = \beta\, F_b$

where $F_b = M_b/z$

(a) Single-sided joint configuration

$V_{wp} = \beta_1\, F_{b1}$
$= \beta_2\, F_{b2}$
where $F_{b1} = M_{b1}/z$
$F_{b2} = M_{b2}/z$

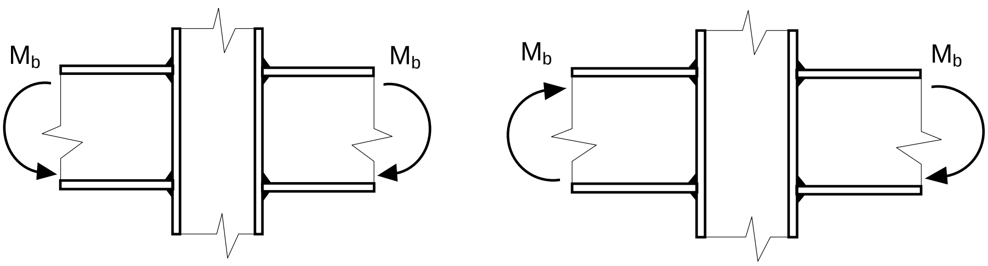
(b) Double-sided joint configuration

Figure 4.14 Definition of the transformation parameter β

In a double-sided configuration, two joints - the left one and the right one - are concerned. The derivation of their corresponding deformability curves is conducted similarly as in a single-sided configuration by using transformation parameters β_1 and β_2 (Figure 4.14.b).

Because the values of the β parameters can only be determined once the internal forces are known, their accurate determination requires an iterative process in the global analysis. For practical applications, such an iterative process is hardly acceptable provided safe β values be available. These values may be used a priori to model the joints and, on the basis of such joint modelling, the frame analysis may be performed safely in a non-iterative way.

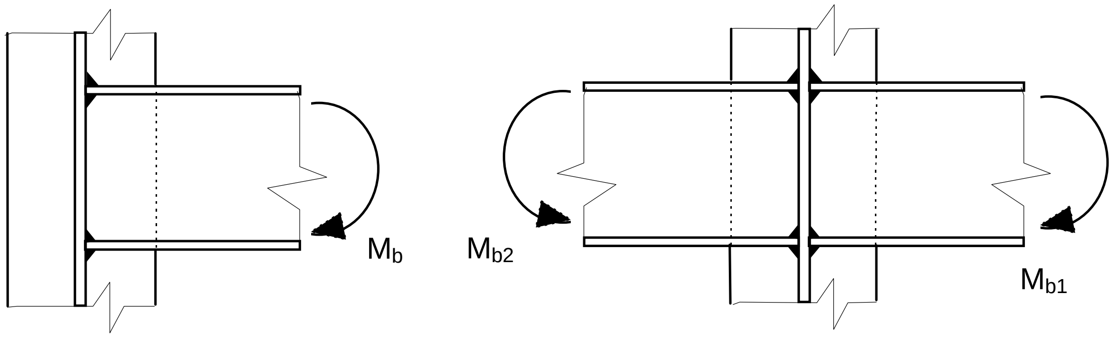
The recommended but approximate values of β, where β_1 is taken as equal to β_2 for double-sided configurations, are given in Chapter 8. They vary from $\beta = 0$ (double-sided joint configuration with balanced moments in the beams) to $\beta = 2$ (double-sided joint configuration with equal but unbalanced moments in the beams). These two extreme cases are illustrated in Figure 4.15.



(a) Balanced beam moments

(b) Equal but unbalanced beam moments

Figure 4.15 Extreme cases for β values



$\Delta M_b = \beta\ M_b$
$= M_b$
$\Rightarrow \beta = 1$

(a) Single-sided joint configuration

$\Delta M_b = \beta_1\ M_{b1}$
$= \beta_2\ M_{b2}$

(b) Double-sided joint configuration

Figure 4.16 Definition of the transformation parameter β

4.3.4.2 Minor axis beam-to-column joint configurations and beam-to-beam configurations

Similar concepts as those developed in Section 4.3.4.1 are referred to for minor axis beam-to-column joint configurations and beam-to-beam configurations. The definition of the transformation parameter is somewhat different (see Figure 4.16).

Approximate values of β (assuming $\beta_1 = \beta_2$) for these joint configurations are also given in Chapter 8.

4.4 Joint idealisation

The non-linear behaviour of the isolated flexural spring which characterises the actual joint response does not lend itself towards everyday design practice. However the moment-rotation characteristic curve may be idealised without significant loss of accuracy. One of the most simple idealisations possible is the elastic-perfectly plastic one (Figure 4.17.a). This modelling has the advantage of being quite similar to that used traditionally for the modelling of member cross-sections subject to bending (Figure 4.17.b).

The moment $M_{j,Rd}$ that corresponds to the yield plateau is termed *design moment resistance* in *Eurocode 3*. It may be understood as the *pseudo-plastic moment resistance* of the joint. Strain-hardening effects and possible membrane effects are henceforth neglected; that explains the difference in Figure 4.17 between the actual M-ϕ characteristic and the *yield plateau* of the idealised one.

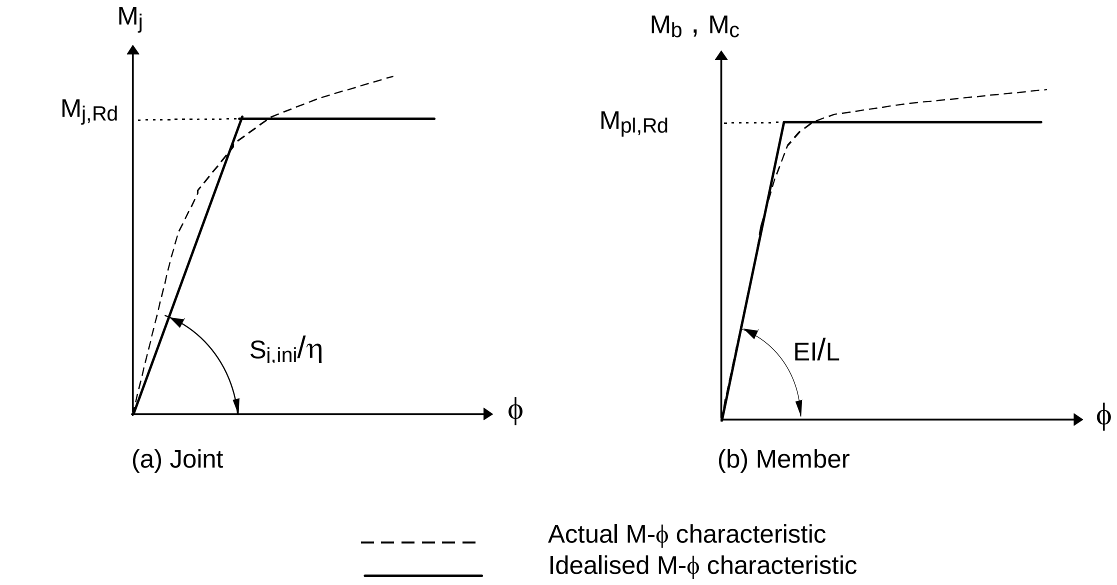


Figure 4.17 Bi-linearisation of moment-rotation curves

The value of the constant stiffness is discussed below.

In fact there are different possible ways to idealise a joint $M\text{-}\phi$ characteristic. The choice of one of them is subordinated to the type of frame analysis which is contemplated:

- *Elastic idealisation for an elastic analysis* (Figure 4.18) :

  The main joint characteristic is the constant rotational stiffness.

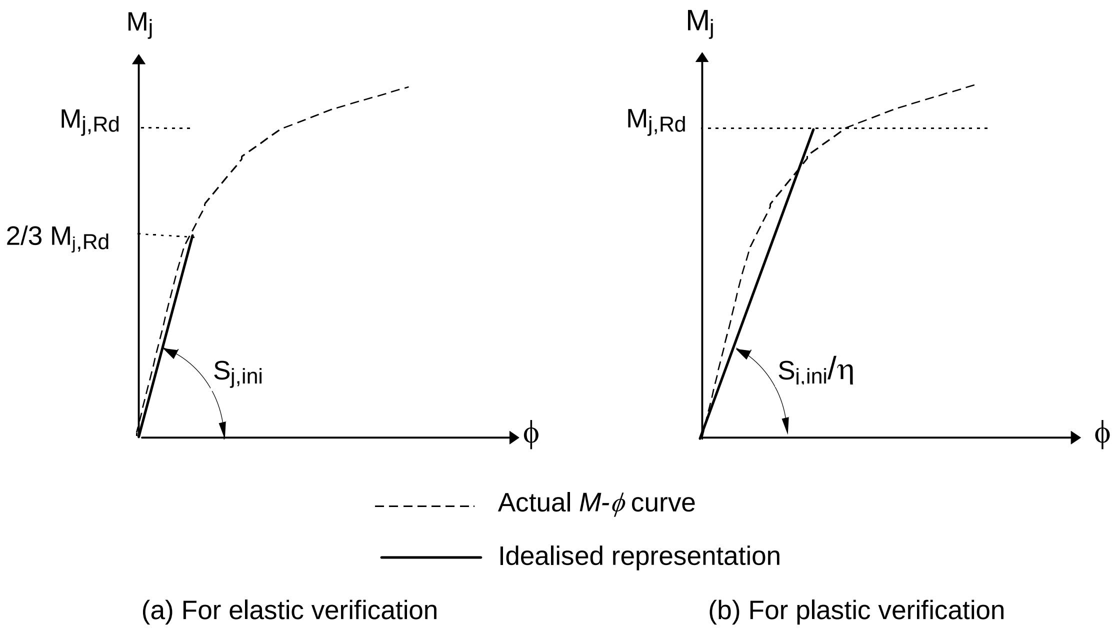


Figure 4.18 Linear representation of a $M\text{-}\phi$ curve

Two possibilities[*] are offered in *Eurocode 3-(revised) Annex J* :

- *Elastic verification of the joint resistance* (Figure 4.18.a) : the constant stiffness is taken equal to the initial stiffness $S_{j.ini}$; at the end of the frame analysis, it shall be checked that the design moment M_{Sd} experienced by the joint is less than the maximum elastic joint moment resistance defined as 2/3 $M_{j,Rd}$;
- *Plastic verification of the joint resistance* (Figure 4.18.b) : the constant stiffness is taken equal to a fictitious stiffness, the value of which is intermediate between the initial stiffness and the secant stiffness relative to $M_{j,Rd}$; it is defined as $S_{j.ini}/\eta$ (values of η are given in Chapter 8). This idealisation is valid for M_{Sd} values less than or equal to $M_{j,Rd}$.

- *Rigid-plastic idealisation for a rigid-plastic analysis* (Figure 4.19).

Only the design resistance $M_{j,Rd}$ is needed. In order to allow the possible plastic hinges to form and rotate in the joint locations, it shall be checked that the joint has a sufficient rotation capacity.

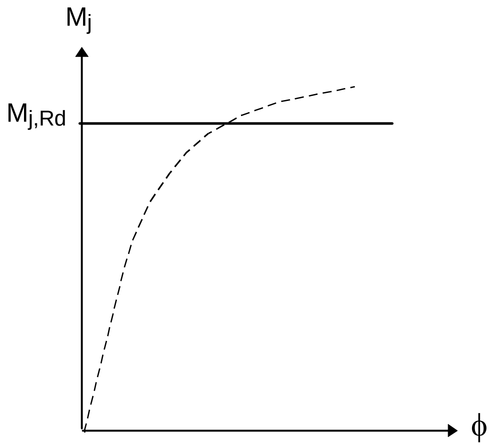


Figure 4.19 Rigid-plastic representation of a M-ϕ curve

- *Non-linear idealisation for an elastic-plastic analysis* (Figure 4.20).

The stiffness and resistance properties are of equal importance in this case. The possible idealisations range from bi-linear, tri-linear representations, ... to the fully non-linear curve. Again rotation capacity is required in joints where plastic hinges are likely to form and rotate.

[*] A third possibility is expressed in Annex J. It leads to an iterative analysis procedure and is not of practical interest. It is therefore not presented here.

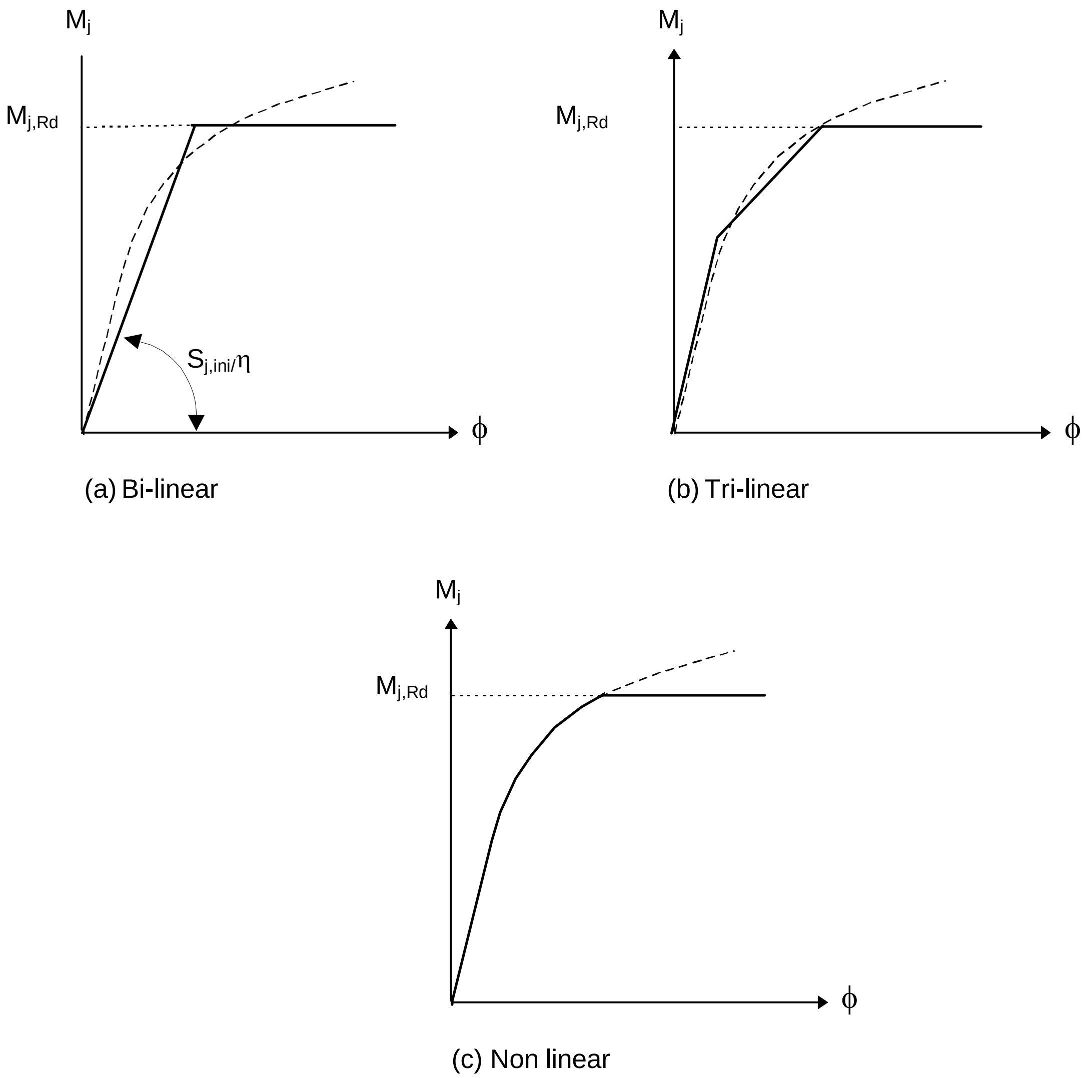


Figure 4.20 Non-linear representations of a *M*-ϕ curve

4.5 Joint characterisation

4.5.1 General

An important step when designing a frame consists in the characterisation of the rotational response of the joints.

Three main approaches may be followed :

- experimental;
- numerical;
- analytical.

The only practical one for the designer is the analytical approach. Analytical procedures enable a prediction of the joint response based on the knowledge of the mechanical and geometrical properties of the joint components.

In this section a general analytical procedure, termed *component method*, is introduced. It applies to any type of steel or composite joints, whatever the geometrical configuration, the type of loading (axial force and/or bending moment, ...) and the type of member sections.

The method is used in Chapter 8 where the mechanical properties of joints subjected to bending moment and shear force are computed.

4.5.2 Introduction to the component method

A joint is generally considered as a whole and studied accordingly ; the originality of the component method is to consider any joint as a set of *individual basic components*. For the particular joint shown in Figure 4.4.a (joint with an extended end-plate connection subject to bending), the relevant components are the following :

- column web in compression;
- beam flange and web in compression;
- column web in tension;
- column flange in bending;
- bolts in tension;
- end-plate in bending;
- beam web in tension;
- column web panel in shear.

Each of these basic components possesses its own strength and stiffness either in tension or in compression or in shear. The column web is subject to coincident compression, tension and shear. This coexistence of several components within the same joint element can obviously lead to stress interactions that are likely to decrease the resistance of the individual basic components.

The application of the component method requires the following steps :
a) *identification* of the active components in the joint being considered;
b) *evaluation of the stiffness and/or resistance characteristics* for each individual basic component (specific characteristics - initial stiffness, design resistance, ... - or whole deformability curve) ;
c) *assembly* of all the constituent components and evaluation of the stiffness and/or resistance characteristics of the whole joint (specific characteristics - initial stiffness, design resistance, ... - or whole deformability curve).

The assembly procedure consists in deriving the mechanical properties of the whole joint from those of all the individual constituent components. That requires a preliminary distribution of the forces acting on the joint into internal forces acting on the components in a way that satisfies equilibrium and respects the behaviour of the components.

In Eurocode 3 Annex J, the analytical assembly procedures are described for the evaluation of the initial stiffness and the design moment resistance of the joint; these two properties enable to build design joint moment-rotation characteristic whatever the type of analysis (Figure 4.18 to Figure 4.20).

The application of the component method requires a sufficient knowledge of the behaviour of the basic components. Those covered by *Eurocode 3* are listed in Table 4.4. The combination of these components allows one to cover a wide range of joint configurations, which should be largely sufficient to satisfy the needs of practitioners as far as beam-to-column joints and beam splices in bending are concerned. Examples of such joints are given in Figure 4.21.

Some fields of application can also be contemplated :

- Joints subject to bending moment (and shear) and axial force;
- Column bases subject to coincident bending moment, shear force and axial force where the components such as :
    - concrete foundation in compression;
    - end-plates with specific geometries;
    - anchorages in tension;
    - contact between soil and foundation,

  will be activated.

These situations are however not yet covered, or only partially covered, by *Eurocode 3*.

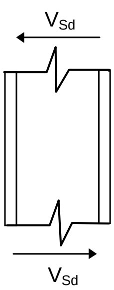
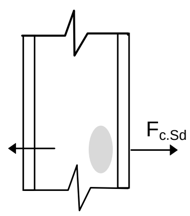
N°	Component	
1	Column web panel in shear	
2	Column web in compression	

Table 4.4 List of components covered by *Eurocode 3* (continued)

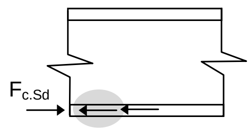
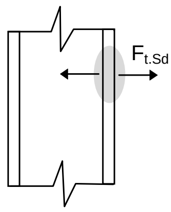
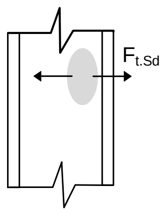
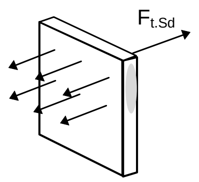
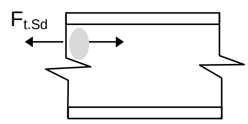
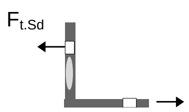
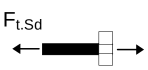
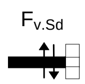
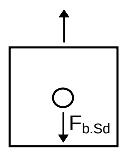
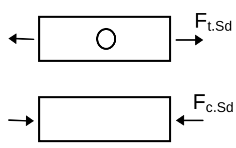
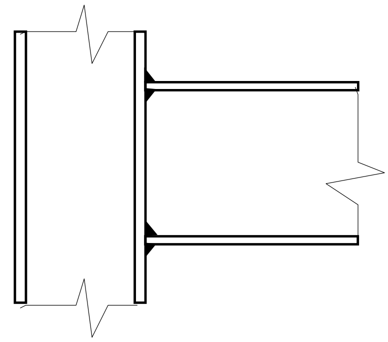
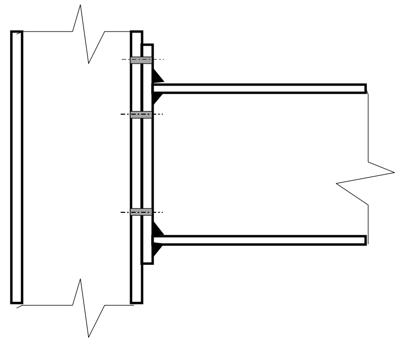
3	Beam flange and web in compression	
4	Column flange in bending	
5	Column web in tension	
6	End-plate in bending	
7	Beam web in tension	
8	Flange cleat in bending	
9	Bolts in tension	
10	Bolts in shear	
11	Bolts in bearing (on beam flange, column flange, end-plate or cleat)	
12	Plate in tension or compression	

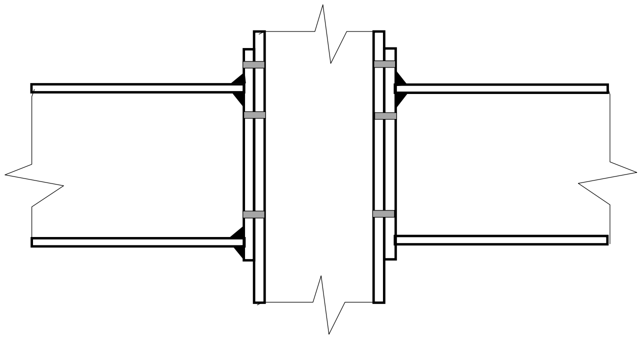
Table 4.4 List of components covered by *Eurocode 3*



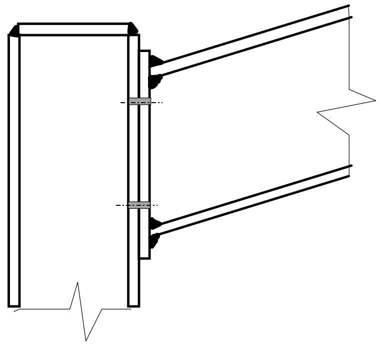
(a) Welded joint



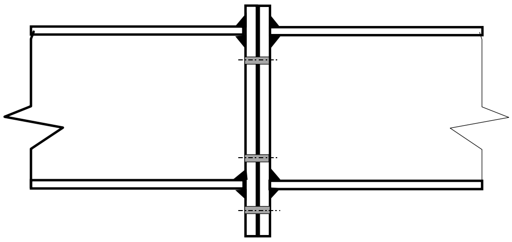
(b) Bolted joint with extended end-plate



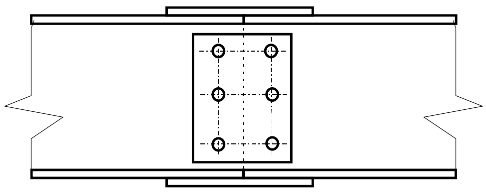
(c) Two joints with flush end-plates
(Double-sided configuration)



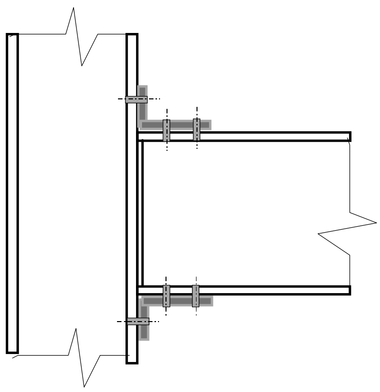
(d) Joint with flush end-plate



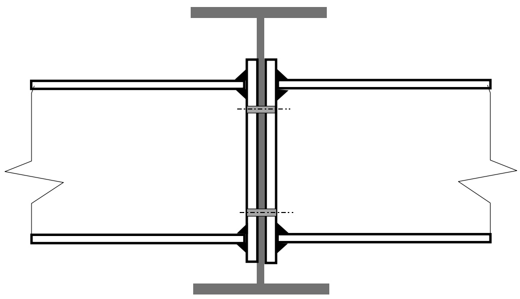
(e) End-plate type beam splice



(f) Cover-joint type beam splice



(g) Bolted joint with angle flange cleats



(h) Two beam-to-beam joints
(Double-sided configuration)

Figure 4.21 Examples of joints covered by *Eurocode 3*

4.6 Joint classification

4.6.1 General

In Section 4.3, it is shown that the joints need to be modelled for the global frame analysis and that three different types of joint modelling are introduced : simple, semi-continuous and continuous.

It has also been explained that the type of joint modelling to which it shall be referred is dependent both on the type of frame analysis and on the class of the joint in terms of stiffness and/or strength (Table 4.2).

Classification criteria are used to define the stiffness class and the strength class to which the joint belongs and also to determine the type of joint modelling which shall be adopted for analysis. They are described in Section 4.6.2.

4.6.2 Classification based on mechanical joint properties[1]

The *stiffness classification* is performed by comparing simply the design joint stiffness to two stiffness boundaries (Figure 4.22). For sake of simplicity, the stiffness boundaries have been derived so as to allow a direct comparison with the *initial design* joint stiffness, whatever the type of joint idealisation that is used afterwards in the analysis (Figure 4.18 and Figure 4.20).

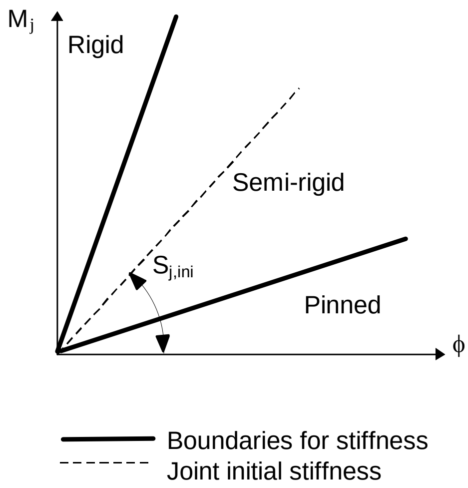


Figure 4.22 Stiffness classification boundaries

The *strength classification* simply consists in comparing the joint *design* moment resistance to "full-strength" and "pinned" boundaries (Figure 4.23).

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[1] This classification is that given in *Eurocode 3-(revised) Annex J*. As this annex is not fully compatible with the questionable classification diagram proposed in *Eurocode 3-Chapter 6.9*, no information on the latter is given in the present manual.

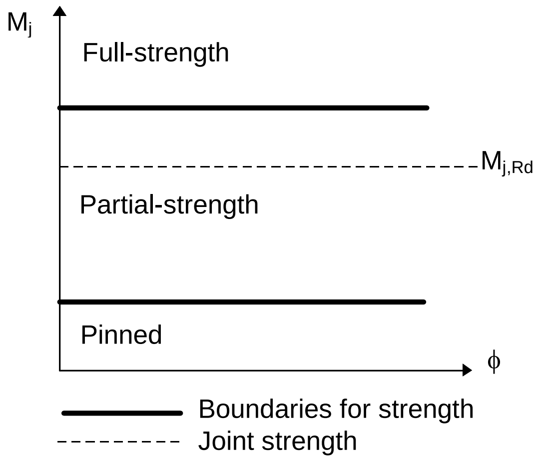


Figure 4.23 Strength classification boundaries

It is while stressing that a classification based on the *experimental* joint *M-ϕ* characteristics is not allowed, as *design* properties only are of concern.

The stiffness and strength boundaries for the joint classification are given in Chapter 8.

4.7 Ductility classes

4.7.1 General concept

Experience and proper detailing result in so-called *pinned* joints which exhibit a sufficient rotation capacity to sustain the rotations imposed on them.

For moment resistant joints the concept of ductility classes is introduced to deal with the question of rotation capacity.

For most of these structural joints, the shape of the *M-ϕ* characteristic is rather bi-linear (Figure 4.24.a). The initial slope $S_{j,ini}$ corresponds to the elastic deformation of the joint. It is followed by a progressive yielding of the joint (of one or some of the constituent components) until the design moment resistance $M_{j,Rd}$ is reached. Then a post-limit behaviour ($S_{j,post\text{-}lim}$) develops which corresponds to the onset of strain-hardening and possibly of membrane effects. The latter are especially important in components when rather thin plates are subject to transverse tensile forces as, for instance, in minor axis joints and in joints with columns made of rectangular hollow sections.

In many experimental tests (Figure 4.24.a) the collapse of the joints at a peak moment $M_{j,u}$ has practically never been reached because of high local deformations in the joints involving extremely high relative rotations. In the others (Figure 4.24.b) the collapse has involved an excessive yielding (rupture of the material) or, more often, the instability of one of the constituent components (ex : column web panel in compression or buckling of the beam flange and web in compression) or the brittle failure in the welds or in the bolts.

In some joints, the premature collapse of one of the components prevents the development of a high moment resistance and high rotation. The post-limit range is rather limited and the bi-linear character of the M_j-ϕ response is less obvious to detect (Figure 4.24.c).

As explained in Section 4.4, the actual M_j-ϕ curves are idealised before performing the global analysis. As for beam and column cross-sections, the usual concept of plastic hinge can be referred to for plastic global analysis.

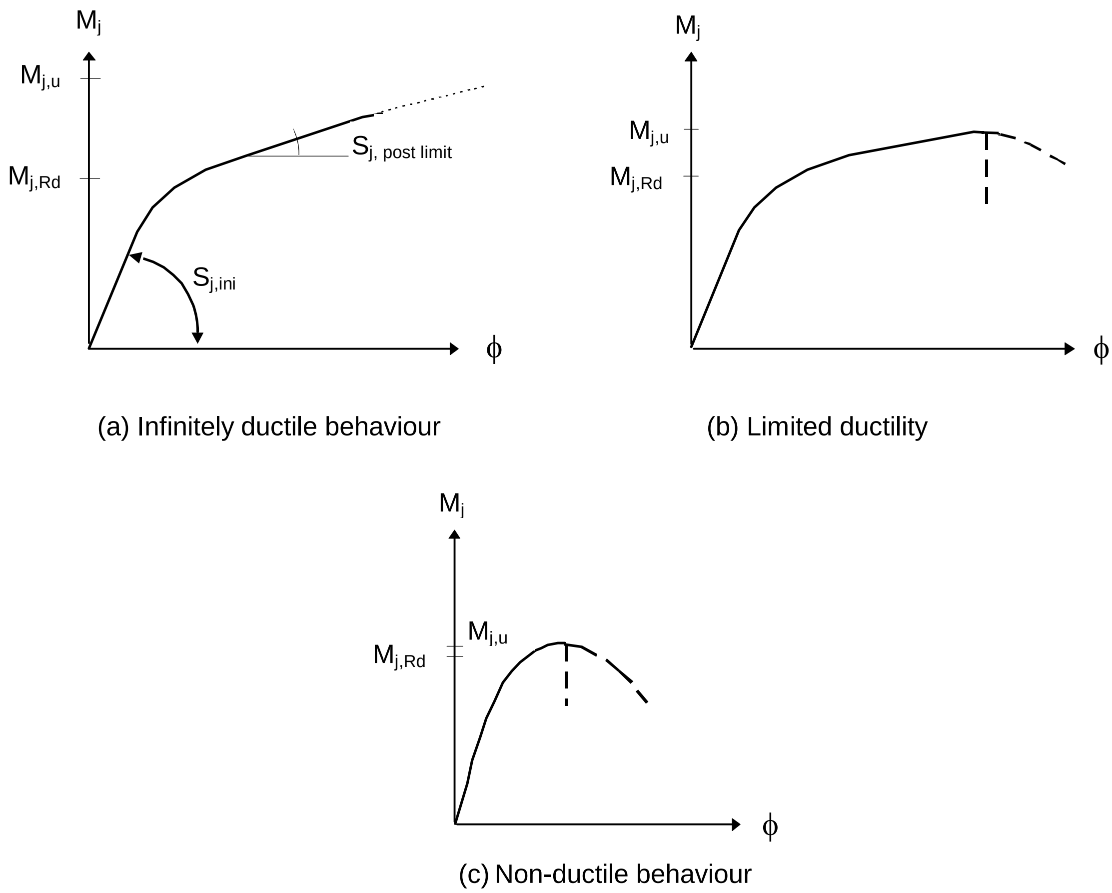


Figure 4.24 Shape of joint M-ϕ characteristics

The development of plastic hinges during the loading of the frame and the corresponding redistribution of internal forces in the frame require, from the joints where hinges are likely to occur, a sufficient rotation capacity. In other words, there must be a sufficiently long yield plateau ϕ_{pl} (Figure 4.25) to allow the redistribution of internal forces to take place.

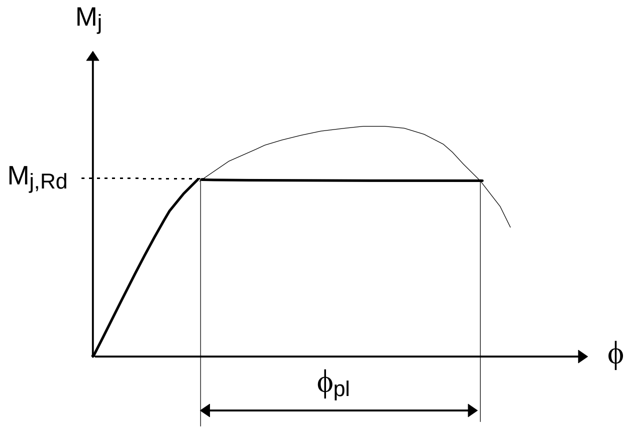


Figure 4.25 Plastic rotation capacity

For beam and column sections, deemed-to-satisfy criteria allow one to determine the class of the sections and therefore the type of global frame analysis which can be contemplated (see Chapter 3).

A strong similarity exists for what regards structural joints; moreover a similar classification may be referred to :

- *Class 1 joints* : $M_{j,Rd}$ is reached by full plastic redistribution of the internal forces within the joints and a sufficiently good rotation capacity is available to allow, without specific restrictions, a plastic frame analysis and design to be performed if required;
- *Class 2 joints* : $M_{j,Rd}$ is reached by full plastic redistribution of the internal forces within the joints but the rotation capacity is limited. An elastic frame analysis possibly combined with a plastic verification of the joints has to be performed. A plastic frame analysis is also allowed as long as it does not result in a too high required rotation capacity in the joints where hinges are likely to occur. The available and required rotation capacities have therefore to be compared before validating the analysis;
- *Class 3 joints* : brittle failure (or instability) limits the moment resistance and does not allow a full redistribution of the internal forces in the joints. It is compulsory to perform an elastic verification of the joints unless it is shown than no hinge occurs in the joint locations.

As the moment design resistance $M_{j,Rd}$ is known whatever the collapse mode and the resistance level, no Class 4 has to be defined as for member sections.

4.7.2 Requirements for classes of joints

In *Eurocode 3*, the procedure given for the evaluation of the design moment resistance of any joint provides the designer with other information such as :

- the collapse mode;
- the state of stresses inside the joint at collapse.

Through this procedure, the designer knows directly whether the full plastic redistribution of the forces within the joint has been reached - the joint is then Class 1 or 2 - or not - the joint is then classified as Class 3.

For Class 1 or 2 joints, the knowledge of the collapse mode, and more especially of the component leading to collapse, gives an indication about whether there is adequate rotation capacity for a global plastic analysis to be permitted. The related criteria are expressed in Chapter 8.

CHAPTER 5

FRAME AND ELEMENT DESIGN

5.1 Frames and their components

5.1.1 Introduction

Global frame analysis is conducted based on assumptions made regarding the section and joint behaviour (elastic/plastic) and the geometric response (first-order/second-order theory). Once the analysis is achieved, the design checks of all these frame components shall be performed.

5.1.2 Frame components

A frame is composed of *members* and *joints* (Figure 5.1).

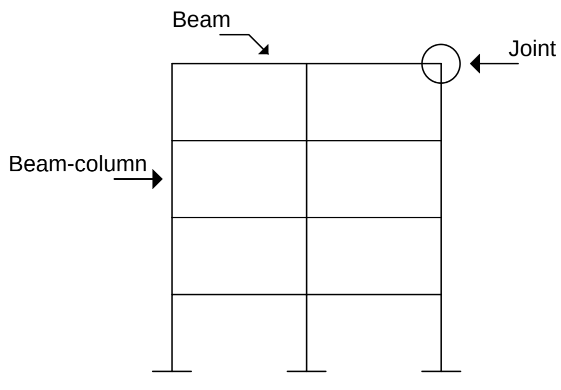


Figure 5.1 Frame and their components

A member is a structural element which is much longer than it is deep; a joint is an assembly of basic components which enable the sound transfer of internal forces from one member to another one. Member is a general wording; very often a member is designated by an appellation which reflects the kind of its loading : *beam* if bending predominates, *beam-column* if significant amounts of both bending and axial force are coincident and *column* if compressive force is dominant.

5.1.2.1 Beams

Most beams are designed to carry gravity loads which produce bending about the major principle axis of the cross-section only; this mono-axial bending is commonly called in-plane bending. Beams are typically the horizontal or slightly sloped members of a framed structure.

Because of the unavoidable initial geometrical imperfections in the beam geometry and in the load transfer, some torsion will most often be present in the case of in-plane bending. If the beam is laterally supported in an efficient way, the torsional effects may usually be disregarded and in-plane bending is governing the design. In contrast, for a laterally unsupported beam, the out-of-plane deformations are magnified by an increase of the in-plane loading and the ultimate limit state refers then to lateral-torsional buckling when this occurs.

Beams are made of rolled or of welded (built-up) sections. In the latter case, the wall components may be prone to local plate buckling because of their large slenderness. Local plate buckling may possibly interact with lateral-torsional buckling and precipitate the collapse of welded beams.

When bending develops simultaneously about both principal axes of the beam cross-section, the beam is said subject to bi-axial bending. As for the in-plane bending case, some torsion is generally present.

In addition, the effects of material yielding within the cross-section can accelerate the onset of the ultimate limit state.

5.1.2.2 Columns

Columns are also termed compression members. A short column, post or pedestal reaches its ultimate limit state when the resistance in compression of its cross-section is exhausted. A long or slender column fails by buckling in the elastic or elasto-plastic range. The ultimate buckling load of any column depends basically on the column slenderness. It is less than the squash load of the cross-section and than the elastic critical buckling load because of unavoidable member out-of-straightness, load eccentricities and residual stresses present in the cross-section, on the one hand, and of the elasto-plastic behaviour of the material, on the other hand.

5.1.2.3 Beam-columns

Members which support loads causing both significative bending and axial compression are called beam-columns. Such members are typically the vertical members of a framed structure. (In the current language, the vertical members of a frame are usually termed columns despite the fact they are most often subject to combined axial force and bending). Indeed they transfer gravity loads from the beams to the foundation and are subject to bending moments because of the full or partial continuity of the beam-to-column joints. Strictly speaking most members of a frame are beam-columns; a beam represents the limit case where the axial force can be disregarded and a column the limit case where the bending moments are not significant.

5.1.2.4 Joints

Because described extensively in Chapter 4, there is no need to comment anymore on the joints.

5.2 Classification of frames and their components

5.2.1 Classification of frames

5.2.1.1 Braced and unbraced frames

For a frame to be classified as a *braced* frame, it must possess a bracing system which is adequately stiff (Figure 5.3.a).

Common bracing systems are trusses or shear walls (Figure 5.2) or even a concrete central core where stairs and lifts take place.

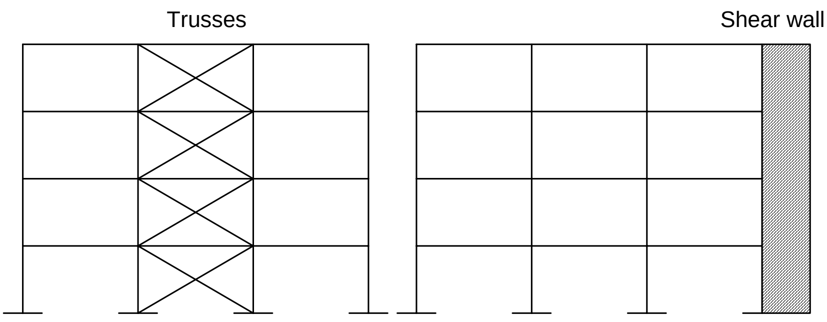


Figure 5.2 Common bracing systems

When it is justified to classify the frame as braced, it is possible to analyse the frame and the bracing system separately as follows:
- The frame without its bracing system resists all the vertical loads;
- The bracing system resists all the horizontal loads.

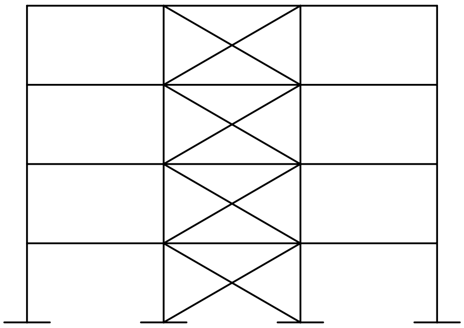
In the cases where there is either no bracing system or no stiff enough bracing system, the frame is said *unbraced* (Figure 5.3.b). The frame with possibly its bracing system is then the single structure to be analysed.

In chapter 9, specific application rules are provided which enable to classify frames as braced or unbraced.

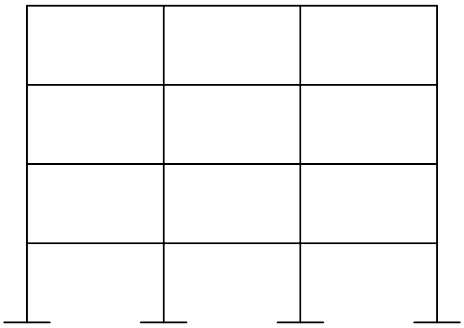
5.2.1.2 Sway and non-sway frames

The wording *non-sway* frame applies to a frame when its response to in-plane horizontal forces is sufficiently stiff for it to be acceptable to neglect any additional forces or moments arising from horizontal displacements of its storeys; that means that the global second-order effects may be neglected. When the later is not negligible, the frame is said *sway*.

In Chapter 9, criteria are given which enable to classify frames as sway or non-sway.



a. Braced frame
(may be sway if bracing is unduly flexible)



b. Unbraced frame
(may be non-sway if it is insensitive to horizontal loads)

Figure 5.3 Braced and unbraced frame

5.2.2 Classification of frame components

5.2.2.1 Classification of member cross-sections

Local buckling can be prevented by limiting appropriately the width-to-thickness ratio of the wall elements. These limits are given in *Eurocode 3*. The class of a cross-section is determined by the worst class of its wall components. Cross-sections are classified as follows according to their behaviour in bending and/or compression.

For member cross-sections subject to bending, there are 4 classes (Figure 5.4):

- *Class 1* : *Plastic* cross-sections.

A plastic cross-section is able to develop the plastic moment resistance of the gross section (plastic hinge) and to exhibit a sufficiently large rotation capacity.

- *Class 2* : *Compact* cross-sections.

A compact cross-section is able to develop the plastic moment resistance of the gross section but local buckling in the plastic range limits significantly the rotation capacity once the plastic moment resistance is reached.

- *Class 3* : *Semi-compact* cross-section.

  A semi-compact cross-section is only able to develop the elastic moment resistance of the gross section because local buckling in the elasto-plastic range occurs before the plastic moment resistance is reached.

- *Class 4* : *Slender* cross-section.

  Premature local plate buckling occurs before the elastic moment resistance is reached. The bending resistance of an appropriate effective cross-section shall be considered.

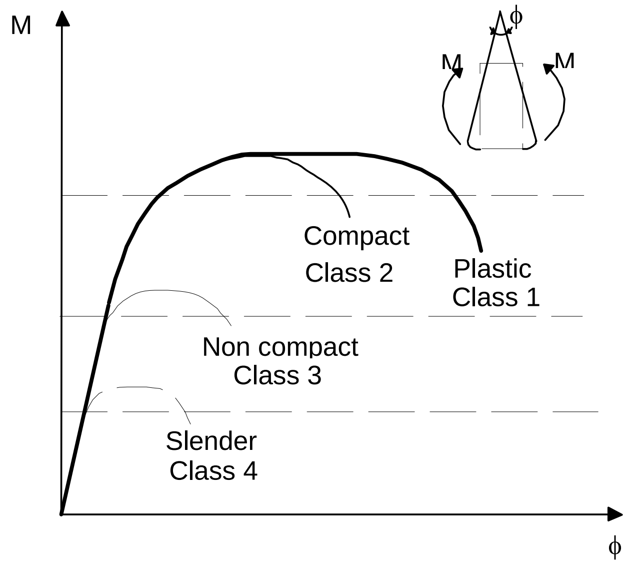


Figure 5.4 Classification of the cross-section in bending

Above concept of classes applies also to cross-sections subject to combined dominant bending and axial force.

For member cross-sections subject to axial compression only, there are only two classes, according to the ability of the cross-section to reach or not its squash load. There is indeed no further need for some available rotation capacity.

In general, the classification will depend on both the loading case - i.e. on the distribution of the direct stresses within the cross-section - and the loading plane.

5.2.2.2 Classification of joints according to ductility

For joints a classification similar to the one used for member cross-sections may be introduced (Figure 5.5).

There are three classes of joints:

- *Class 1* : *Ductile* joints.

A ductile joint is able to develop its plastic moment resistance and to exhibit a sufficiently large rotation capacity.

- *Class 2* : Joints of *intermediate ductility*.

A joint of intermediate ductility is able to develop its plastic moment resistance but exhibits only a limited rotation capacity once this resistance is reached.

- *Class 3* : *Non ductile* joints.

Premature failure (due to instability or to brittle failure of one of the joint components) occurs within the joint before the moment resistance based on a full plastic redistribution of the internal forces is reached.

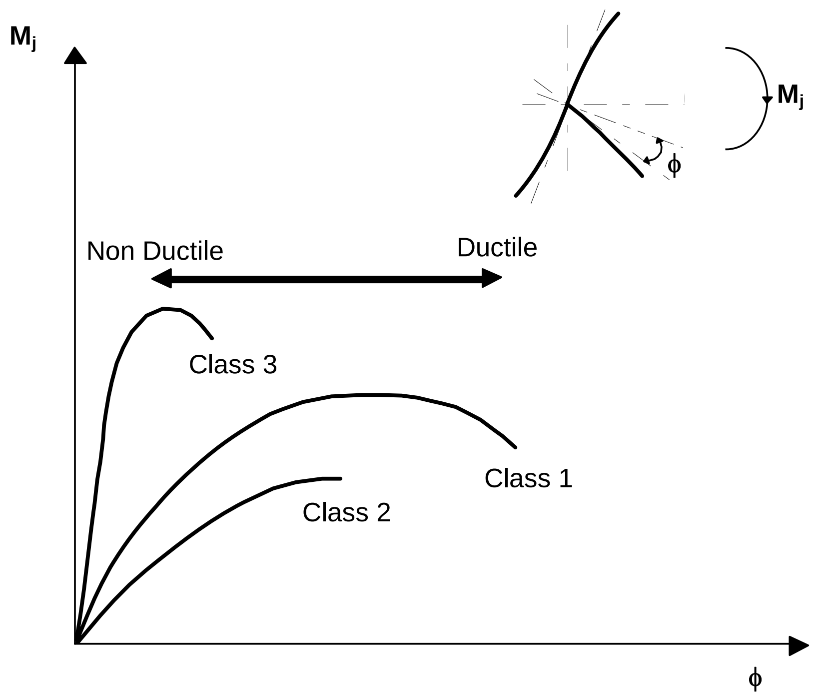


Figure 5.5 Classification of the joint by ductility

5.3 Checking frame components

5.3.1 Resistance check of member cross-sections

To which extent the check of the resistance of the member cross-sections is concerned depends upon the type of global frame analysis that has been performed.

5.3.1.1 Elastic global analysis

A strictly elastic design would limit the carrying capacity of a frame to the onset of the very first yielding in the outer fibre(s) of any cross-section.

That leads to an elastic resistance design check of the member cross-sections.

However a less severe attitude is acceptable. While an elastic global analysis is performed, the internal forces and moments that are obtained accordingly can be checked against the resistance of the cross-sections by using the relevant plastic interaction formulae for section resistance. Doing so allows only for one plastic hinge (or more than one but occurring simultaneously) to form in the structure. That leads to a plastic resistance design check of the member cross-sections.

- *Elastic resistance check*

The elastic distribution of the direct stresses σ in an I-section and a T-section subject to mono-axial bending is shown in Figure 5.6.

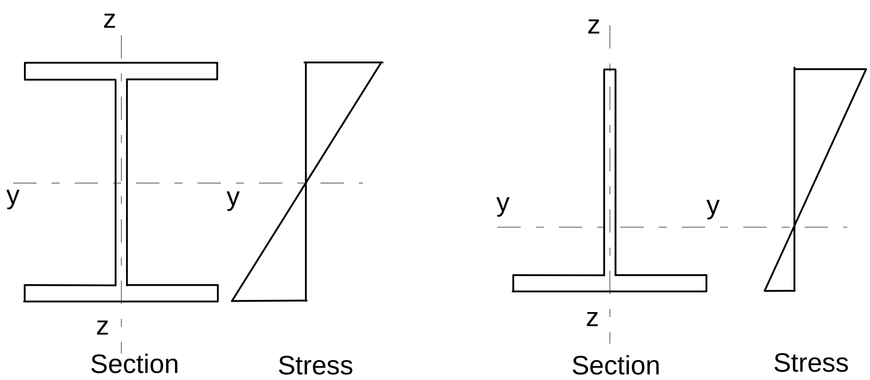


Figure 5.6 Beams subject to elastic mono-axial bending

The design bending moment M_{Sd} in any cross-section, in the absence of the shear and axial force, shall be less than or equal to the design elastic moment resistance $M_{el,Rd}$ of the cross-section. The width-to-thickness ratio limits for class 3 shall be met by all the wall components of these cross-sections. If the width-to-thickness ratio of any wall element is larger than the relevant limit for class 3, then an elastic moment resistance based on the effective cross-section characteristics $M_{el,eff,Rd}$ shall be used.

Similar principles apply when bi-axial bending.

Interaction with shear and/or axial forces shall be accounted for by referring to the equivalent stress deduced from the *von Mises* criterion.

- *Plastic resistance check*

The plastic distribution of the direct stresses in an I-section and a T-section subject to mono-axial bending is shown in Figure 5.7. Such a distribution is permitted when the section belongs either to class 1 or to class 2.

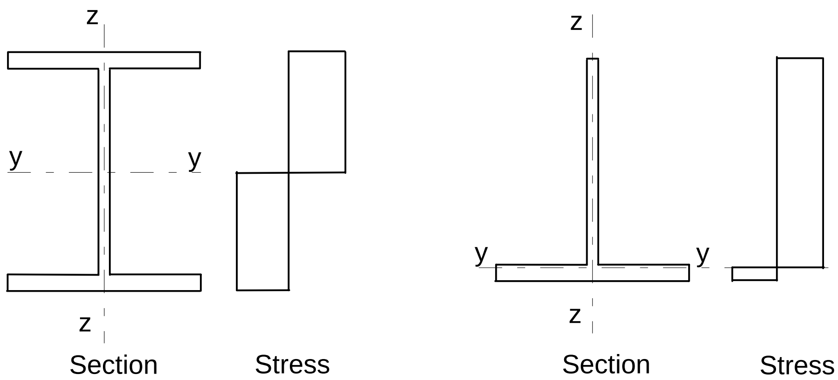


Figure 5.7 Beams subject to plastic mono-axial bending

As far as the interaction between bending moment and shear and/or axial force(s) is sufficiently small for it to be regarded as negligible, the design value of the bending moment M_{Sd} in each cross-section shall be less than or equal to the plastic moment resistance $M_{pl,Rd}$ of the cross-section.

When the interaction between bending and shear and/or axial force(s) can no more be disregarded, a plastic interaction formula shall be used when checking the resistance of the cross-section. It will be done similarly when bi-axial bending.

5.3.1.2 Plastic global analysis

When an elastic-perfectly plastic or a rigid-plastic global frame analysis is performed, the behaviour of the cross-sections is accounted for in the global analysis with the result of a redistribution of internal forces.

For this purpose, the bending response of a cross-section is usually simplified as shown in Figure 5.8. Any cross-section where a plastic hinge is likely to occur shall be either a class 1 or a class 2 section. However when a class 2 section is used, the available rotation capacity must be checked to ensure that it is sufficient to meet what is required by the formation of the relevant plastic mechanism.

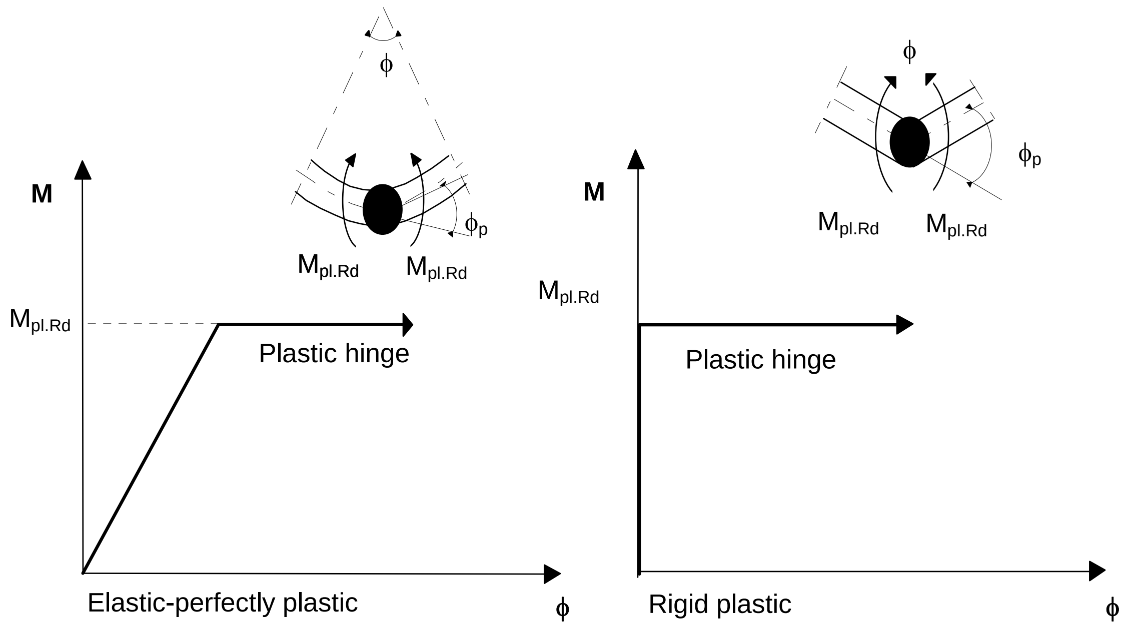


Figure 5.8 Moment-rotation characteristics of a cross-section : plastic global analysis

5.3.2 Check of member instability

5.3.2.1 Beam in bending and column in compression

Any member which is subject to direct compressive stresses over the whole or a part only of its cross-section may become unstable because of a buckling phenomenon: column buckling for an axially loaded column and lateral-torsional buckling for a beam subject to predominant bending (Figure 5.9).

An axially loaded column the cross-section of which is doubly symmetrical may collapse by in-plane buckling either by bending about the major axis of its cross-section or, more commonly, about the minor axis. When the cross-section is not doubly symmetrical, instability may occur by torsional-flexural buckling; this phenomenon is of major importance in thin-walled cold-formed compression members.

Lateral-torsional buckling of a beam has much in common with the minor axis column buckling phenomenon due to the buckling of the compression part by bending about the minor axis of the section. This column buckling is however accompanied by torsional effects that are induced by the necessary continuity between both tensile and compressive parts of the cross-section. The greater the slenderness of the member about the minor axis, the smaller the lateral-torsional buckling resistance of the member.

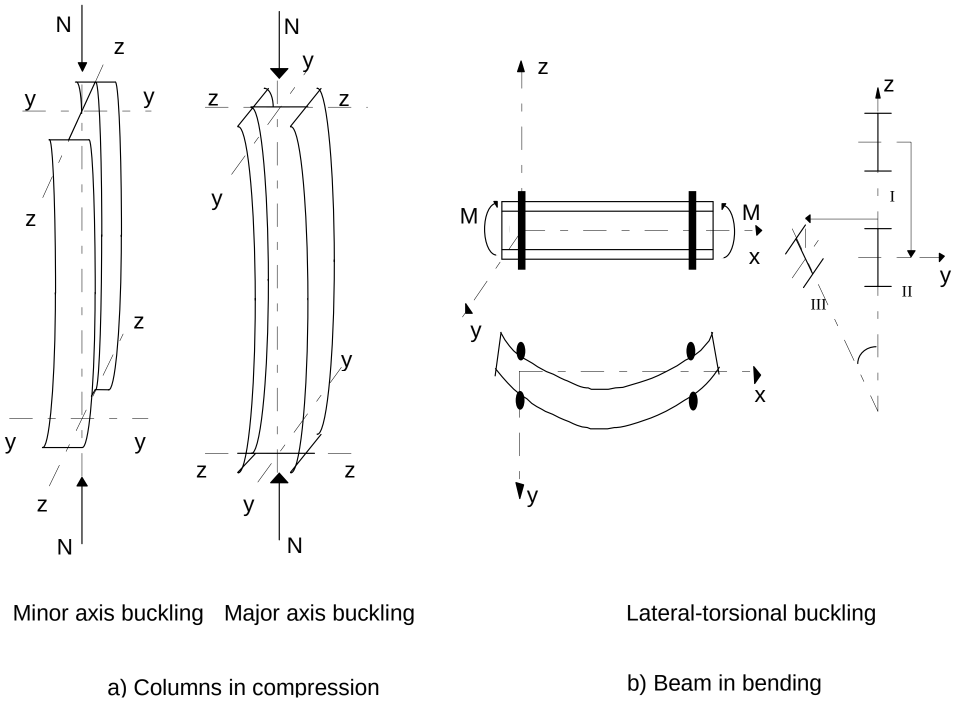


Figure 5.9 Buckled position of columns and beam

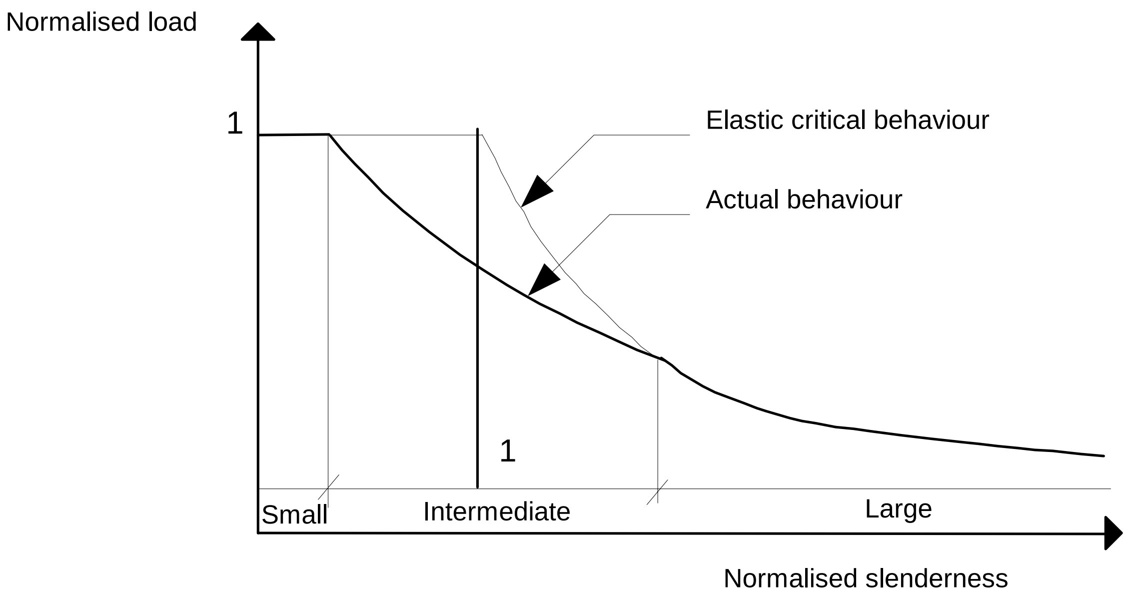


Figure 5.10 Buckling behaviour curve of columns and beams

Column buckling resistance (respect. lateral-torsional buckling resistance) depends on the member slenderness. It can be described schematically by the solid line in Figure 5.10, where use is made of normalized co-ordinates. The dotted lines correspond to an ideal behaviour : plastic resistance of the cross-section in compression (respect. in bending) and elastic critical column buckling load (respect. lateral-torsional buckling load of the beam). The loss in resistance with regard to such an ideal behaviour is due to geometrical imperfections (initial out-of-straightness or initial twist), structural imperfections (residual stresses) and effects of material yielding. It is usual to distinguish three slenderness ranges :

- *Small slenderness* range, where the member slenderness is so small that the detrimental effects of imperfections is more than compensated by the strain-hardening effects (the latter are generally not accounted for in the design rules), so that only yielding governs the resistance;

- *Large slenderness* range, where the member slenderness is sufficiently large for the instability to occur in the elastic range with the result that only the imperfections affect the resistance;

- *Intermediate slenderness* range, when the interaction between the detrimental effects of material yielding and member imperfections is the most pronounced, with the result of the largest drop in resistance compared to the ideal behaviour.

Guidance on the design for buckling resistance of columns and beams is given in *Eurocode 3*.

5.3.2.2 Beam-column

The behaviour of a beam-column combines the respective behaviours of both a column and a beam. That results in a complex structural response for what regards instability. One can identify five cases according to the member unbraced length and the type of loading (see Table 5.1).

5.3.3 Additional resistance checks

In addition to cross-section checks (see Section 5.3.1) and member instability checks (see Section 5.3.2), the following checks shall be considered:
- resistance of the web to shear buckling;
- resistance of the web to transverse concentrated loads (patch loading).

Eurocode 3 provides the information required for those additional checks.

5.3.4 Resistance check of joints

The type of resistance checks to be carried out depends on the type of global analysis that has been performed.

Case	Type member / Type of loading / Restraints	Failure mode
1	Short member subject to combined axial compressive force and either mono-axial bending or bi-axial bending.	Exhaustion of the elastic or plastic resistance of a cross-section.
2	Slender member subject to combined axial compressive force and mono-axial bending about the major axis *(yy)* and supported laterally so that buckling about the minor axis *(zz)* is prevented.	Buckling about the major axis *(yy)*.
3	Slender member with no lateral support, subject to combined axial compressive force and mono-axial bending about the minor axis *(zz)* and where no lateral buckling out of the plane of bending is prevented.	Buckling about the minor axis *(zz).*
4	Slender member with no lateral support, subject to combined axial compressive force and mono-axial bending about the major axis (*yy*).	Combination of buckling and lateral-torsional buckling. The member section twists as well as deflects in both major and minor axes.
5	Slender member with no lateral support, subject to combined axial compressive force and bi-axial bending.	Similar to Case 4 but accentuated by the minor axis bending.

Table 5.1 Failure modes of beam-columns

5.3.4.1 Elastic global analysis

When an elastic global frame analysis is performed, the behaviour of the joints is assumed to be indefinitely linear. Any joint shall be verified on the basis of its so-called design elastic or plastic resistance. The *design elastic bending resistance* of the joint is taken nominally equal to *2/3* of the *design plastic bending resistance* $M_{j,Rd}$, when the analysis has been carried out based on the initial stiffness $S_{j,ini}$ of the joint. To be able to use the design plastic resistance $M_{j,Rd}$, a reduced stiffness $S_{j,ini}/\eta$ of the joint shall be adopted in the global analysis (see Figure 5.11).

5.3.4.2 Plastic global analysis

When either an elastic-perfectly plastic or a rigid-plastic global frame analysis is performed, the joint behaviour is idealised as shown in Figure 5.12; a quite similar idealisation is currently adopted for the member cross-sections. Any joint where a plastic hinge is likely to occur shall be either a class 1 or a class 2 joint. Shall this joint belong to class 2, then it shall be checked whether the available rotation capacity of the joint is at least equal to the rotation capacity required by the relevant plastic mechanism.

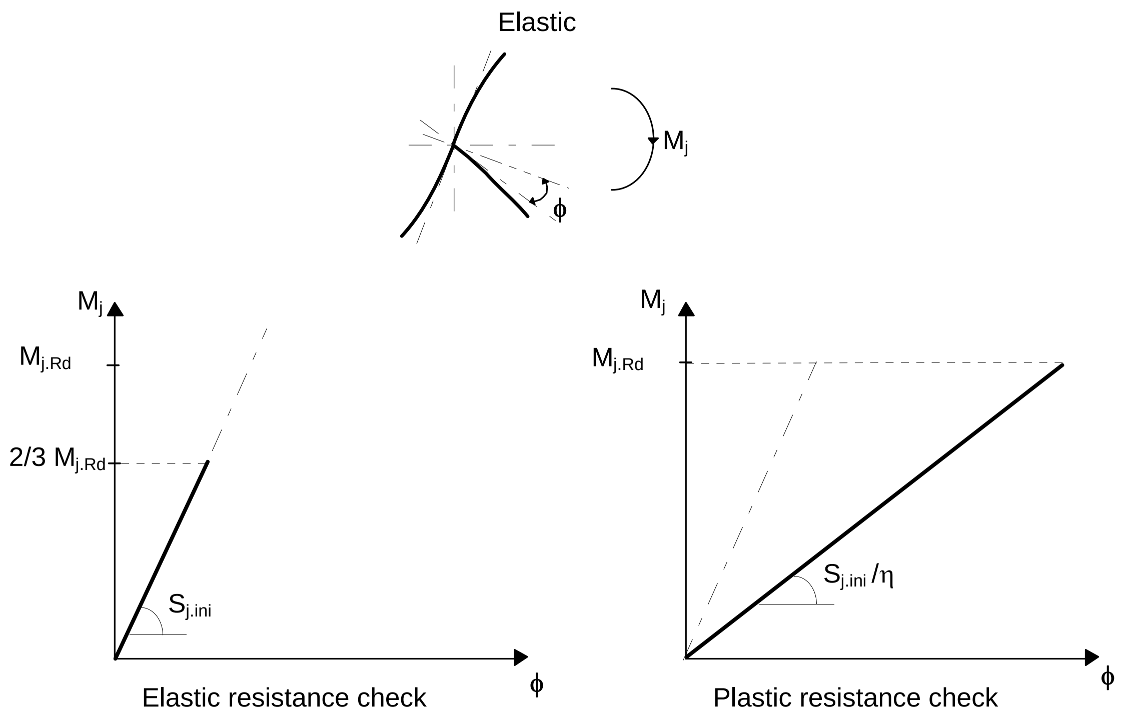


Figure 5.11 Elastic idealisation of the joint behaviour

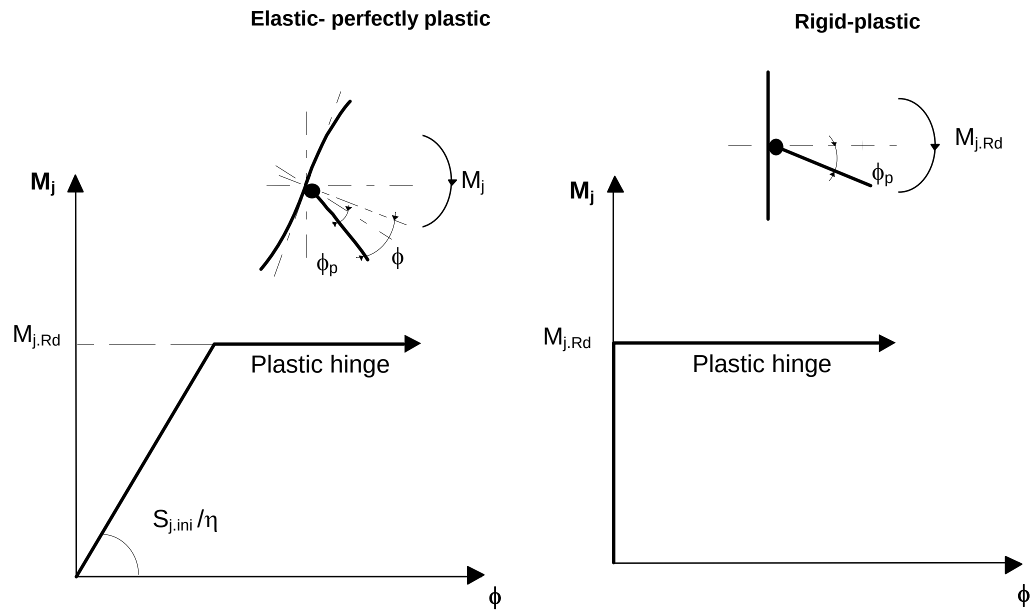


Figure 5.12 Plastic idealisation of the joint behaviour

5.4 Modelling of joints

5.4.1 Design assumptions

Joint *modelling* requires a particular attention as joints are classified by strength, stiffness and ductility. The following table (Table 5.2) shows the different cases of joint modelling (simple, continuous and semi-continuous) as related to both the type of global analysis and the joint classification.

5.4.2 Simple joints

In a simple joint, the connection between the members may be assumed to be unable to develop any bending moment. The in-plane sway stability of a frame with simple joints only must be provided by the bracing system which shall resist all the horizontal loads. Vertical loads are resisted by both the frame and the bracing system. Such a frame is invariably designed as a non-sway frame, with the result that the distribution of internal forces and moments is not significantly affected by the sway displacements.

GLOBAL ANALYSIS	JOINT CLASSIFICATION		
Elastic	Nominally pinned	Rigid	Semi-rigid
Rigid-Plastic	Nominally pinned	Full-strength	Partial-strength
Elastic-Plastic	Nominally pinned	Rigid and full-strength	Semi-rigid and partial-strength Semi-rigid and full-strength Rigid and partial-strength
JOINT MODELLING	Simple	Continuous	Semi-continuous

Table 5.2 Type of joint model

5.4.3 Continuous joints

In a frame with continuous joints, the joints ensure a full continuity between the connected members. The internal forces and moments can be determined either by an elastic or a plastic global analysis, with account being taken for second-order effects when necessary.

5.4.4 Semi-continuous joints

In a frame with semi-continuous joints, only a partial continuity exists between the connected members. The internal forces and moments can be determined by either an elastic or a plastic global analysis, with account being taken for second-order effects when necessary. As partial-strength and/or semi-rigid joints usually influence significantly the frame behaviour, the analysis of the frame shall account for the joint structural response (strength, stiffness).

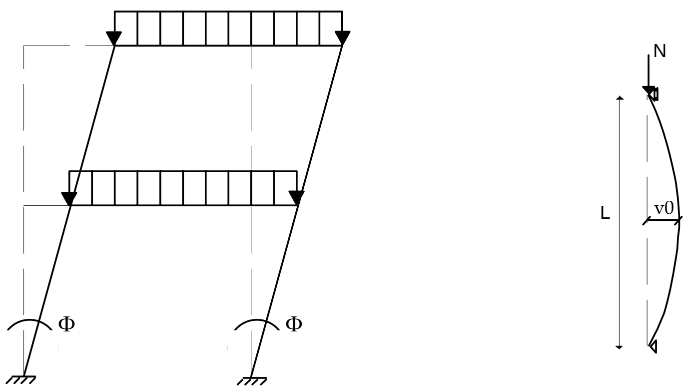
5.5 Frame design procedure tasks

At the preliminary design stage, the frame must be first defined. In particular, in addition to the structure layout and the member and joint sizing, the braced or unbraced character of the overall frame must be determined.

The *frame imperfections* are taken as an initial out-of-plumb of the vertical members, i.e. by an initial sway of the storeys (Figure 5.13.a); how to assess these imperfections is described in *Eurocode 3*.

For sake of easiness equivalent horizontal loads at the floor levels are generally substituted for the global frame imperfections; then the global analysis may still be conducted on the ideal frame, i.e. without out-of-plumb, but these equivalent loads must of course be superimposed to the other actions.

The *member imperfections* are taken as geometrical imperfections (out-of-straightness) and residual stresses. Usually residual stresses are not accounted for explicitly but converted into a magnification of the initial out-of-straightness. Thus the member imperfection is normally represented by an equivalent initial out-of-straightness. These imperfections have sometimes, but rarely, to be taken into account by fitting the members with an appropriate initial camber (Figure 5.13.b) and conducting the global analysis accordingly.



(a) Frame imperfections (b) Member imperfections

Figure 5.13 Imperfections

At the next stage, an appropriate method of global analysis is chosen. Whereas a second-order analysis is always practicable, there are cases where a less sophisticated analysis provides the designer with a satisfactorily accurate distribution of internal forces :

- A first-order analysis may be sufficient, without there being any need to account for second-order effects;

- A first-order analysis may be performed, but its results need for some corrections because of significant second-order effects.

To make this selection possible, the designer needs a mean to evaluate to which extent the sway displacements can modify the distribution of internal forces. That is measured by the *elastic critical load parameter* λ_{cr}, i.e. the ratio between the vertical load resultant V_{cr} producing sway instability and the resultant V_{Sd} of the design vertical loads applied onto the frame. If the value of this ratio is sufficiently large (non-sway frame), first-order analysis is adequate. If it is sufficiently small (sway frame), second-order analysis is required. However many structures are characterised by an intermediate value of the critical load parameter; then it is acceptable to conduct a first-order analysis provided that the first-order internal forces and moments are amplified appropriately to account for the second-order effects that have been fully disregarded by the global analysis.

The designer will also have to opt for the use either of an elastic method of analysis or, when appropriate, of a plastic method of analysis. While an elastic global analysis is always permitted, the use of a plastic global analysis is subordinated to some requirements (see Section 5.5.2).

When the design internal forces have been determined accordingly (first-order, second-order of first-order with amplification), the resistance and the stability of both the frame and its components have to be checked.

The aim of Section 5.5.1 and Section 5.5.2 is to give guidelines for global elastic analysis and plastic analysis respectively.

The question of the overall equilibrium of the frame in terms of uplifting, overturning and sliding is not addressed here. It can usually be verified without additional modelling of the structure. The support reactions provided by the global frame analysis shall be compared to the corresponding foundation support resistances.

5.5.1 Elastic global analysis and relevant design checks

To enable an elastic global analysis, the frame components (sections and joints) are not subordinated to any requirement related to their ability to exhibit ductile behaviour. Figure 5.14 summarizes the different possibilities of the elastic global analysis and the relevant checks with reference to *Eurocode 3*.

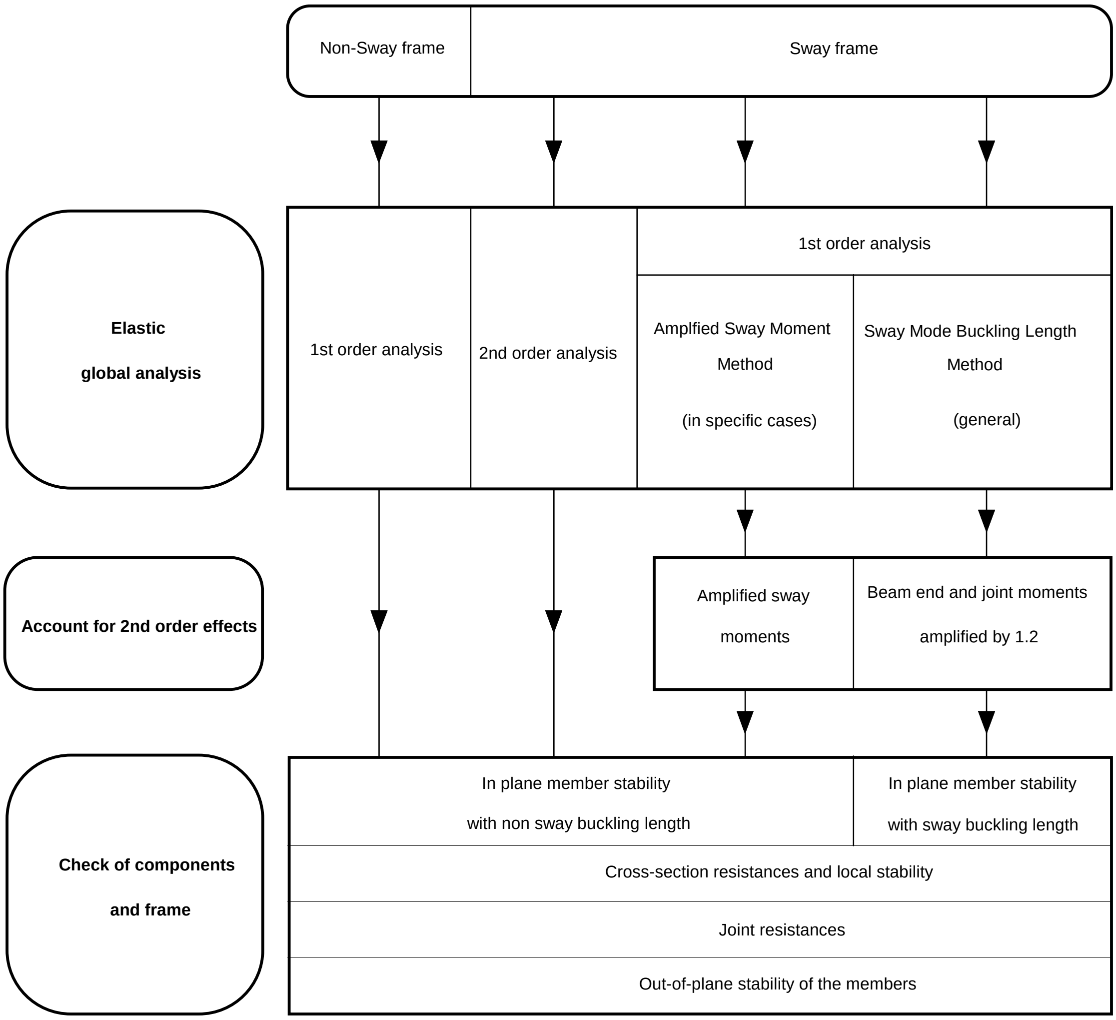


Figure 5.14 Elastic global analysis and relevant design checks (according to *Eurocode 3*)

In Section 5.5.1.1 and Section 5.5.1.2 details are given on how to proceed when an elastic global analysis is used.

Once the designer has opted for an elastic global analysis, he shall question about a first-order or a second-order theory for this analysis.

5.5.1.1 First-order analysis

First-order elastic analysis with appropriate allowance for frame imperfections may be used to calculate the distribution of internal forces in a non-sway frame.

The frame shall be analysed for various load combinations. A maximum error of 10% is expected on the values of both first-order sway and bending moments compared to those got from a second-order analysis.

The resistance of the frame components (member sections and joints) and of the in-plane member stability, using the buckling lengths for the non-sway mode, are checked. Separate design checks of the local resistance and of the out-of-plane stability of the members shall also be carried out.

No further check of the in-plane global frame stability for sway buckling mode is required because of the non-sway character of the frame.

A first-order analysis may also be used for sway frames provided some corrections be made to allow for the second-order effects (see Section 5.5.1.2).

5.5.1.2 Second-order analysis

Second-order analysis with due allowance made for frame imperfections may be used in all cases. It is required when the frame is classified as sway. *Eurocode 3* allows the procedures described hereafter.

- *General Method*

Second-order effects due to frame imperfections and sway displacements are explicitly accounted for in the method of global analysis. Second-order effects due to member imperfections and in-plane member deflections can also be taken into account when necessary.

The resistance of the frame components (member sections and joints) and of the in-plane stability of each member, using buckling lengths for the non-sway buckling mode, are checked. The latter check is not necessary when member imperfections are considered prior to the global analysis. Separate design checks of the local resistance to concentrated forces and of the out-of-plane stability of the members shall also be carried out.

No further check of the in-plane global frame stability for the sway buckling mode is required because it is already implicitly covered by the type of theory used for the global analysis.

Alternatively, the approximate method known as the *Equivalent Lateral Load* procedure can be used. It consists in an iterative procedure, using the results of a standard first-order analysis, to include the sway frame effects. The same considerations as above still hold for the design checks of the frame.

What is especially of concern in a sway frame is the determination of internal forces and moments which include second-order effects. It is thus more a problem of an appropriate assessment of these internal forces and moments than, strictly speaking, a question of theory used for the analysis. In this respect, the heading of present section could appear a bit ambiguous to the attentive reader.

Therefore tricks have been imagined which enable the designer to make a realistic assessment of the design internal forces and moments without conducting a so-called second-order analysis properly. Of course the range of application of such tricks is, as said previously, subordinated to some conditions. The basic philosophy of these alternative approaches is first to conduct a first-order analysis and second to introduce an estimate of the second-order effects. In this respect, *Eurocode 3* allows for two simplified procedures :

a) the so-called *amplified sway moment method* where second-order effects are accounted for by amplifying the loading side in the check expressions to be fulfilled;

b) the so-called *sway mode buckling length method* where second-order effects are introduced both as a magnification of the loading side and as a penalty on the resistance side in the check expressions to be fulfilled.

Let us comment a bit more on both procedures.

- *Amplified Sway Moment Method*

As an alternative to a second-order elastic analysis, the simplified method known as the *Amplified Sway Moment Method* may be adopted, provided that the distribution of the second-order bending moments shows approximate affinity to the one of the first-order bending moments; that is acceptably the case in structures exhibiting a low to moderate sway. The *sway moments* result from the horizontal applied loads and, if the frame and/or the vertical loading are asymmetrical, from the effects of the sway displacements.

They can be obtained by proceeding as follows :

1. Perform a first-order elastic analysis of the frame under the whole loading with, in addition to the real supports, the floor levels restrained against horizontal displacement;

2. Determine the horizontal reactions at those horizontal restraints;

3. Perform a first-order elastic analysis of the real frame, i.e. where the horizontal restraints are removed, under the sole horizontal forces equal but opposite to the horizontal reactions found in above Step 1;

4. The sway moments are the moments obtained at the end of above Step 3.

The moments to be used for further design checks are equal to the sum of :

a) the moments obtained at Step 1;

b) the sway moments obtained at Step 4 but magnified by an appropriate amplification factor.

Of course, the amplification process described above for bending moments applies as well to shear and axial forces.

The resistance of the frame components (sections and joints) and the out-of-plane stability of the beams, using the amplified moments, are then checked.

The in-plane and the out-of-plane column stability, using the non-amplified moments (i.e. the sum of those obtained respectively at Steps 1 and 3), and the in-plane buckling length for the sway mode are then checked.

Separate design checks of the local resistance of the members are carried out.

According to *Eurocode 3*, the fulfilment of these checks is sufficient to guarantee the overall sway stability of the frame.

- *Sway Mode Buckling Length Method*

The simplified method known as the *Sway Mode Buckling Length Method* shall be adopted for structures either which exhibit a large sway or the sway sensitivity of which is unknown and uneasily predictable. The internal forces and moments are computed based on a first-order analysis; then the internal moments at the beam ends and joints are amplified by a nominal factor *1,2*.

Besides the resistance of the frame components (member sections and joints) and of the in-plane member stability, using the *buckling lengths for the sway mode*, are then checked. Separate design checks of the local resistance to concentrated loads and of the out-of-plane stability of the members shall also be carried out.

According to *Eurocode 3*, the fulfilment of these checks is sufficient to guarantee the overall sway stability of the frame.

5.5.2 Plastic global analysis and relevant design checks

The use of a plastic global analysis is subordinated to the fulfilment of some requirements regarding the steel grades and the classes of cross-sections and joints. It is indeed necessary to ensure that the full plastic resistance of sections and joints can be reached and that sufficient rotations can develop at the locations where plastic hinges are likely to form. Figure 5.15 summarizes the different possibilities for performing a plastic global analysis and the relevant checks with reference to *Eurocode 3*.

In Section 5.5.2.1 and Section 5.5.2.2 details are given on how to proceed when plastic global analysis is used.

Once the designer is allowed to perform a plastic global analysis and has decided to do accordingly, he shall question about a first-order or a second-order theory for this analysis.

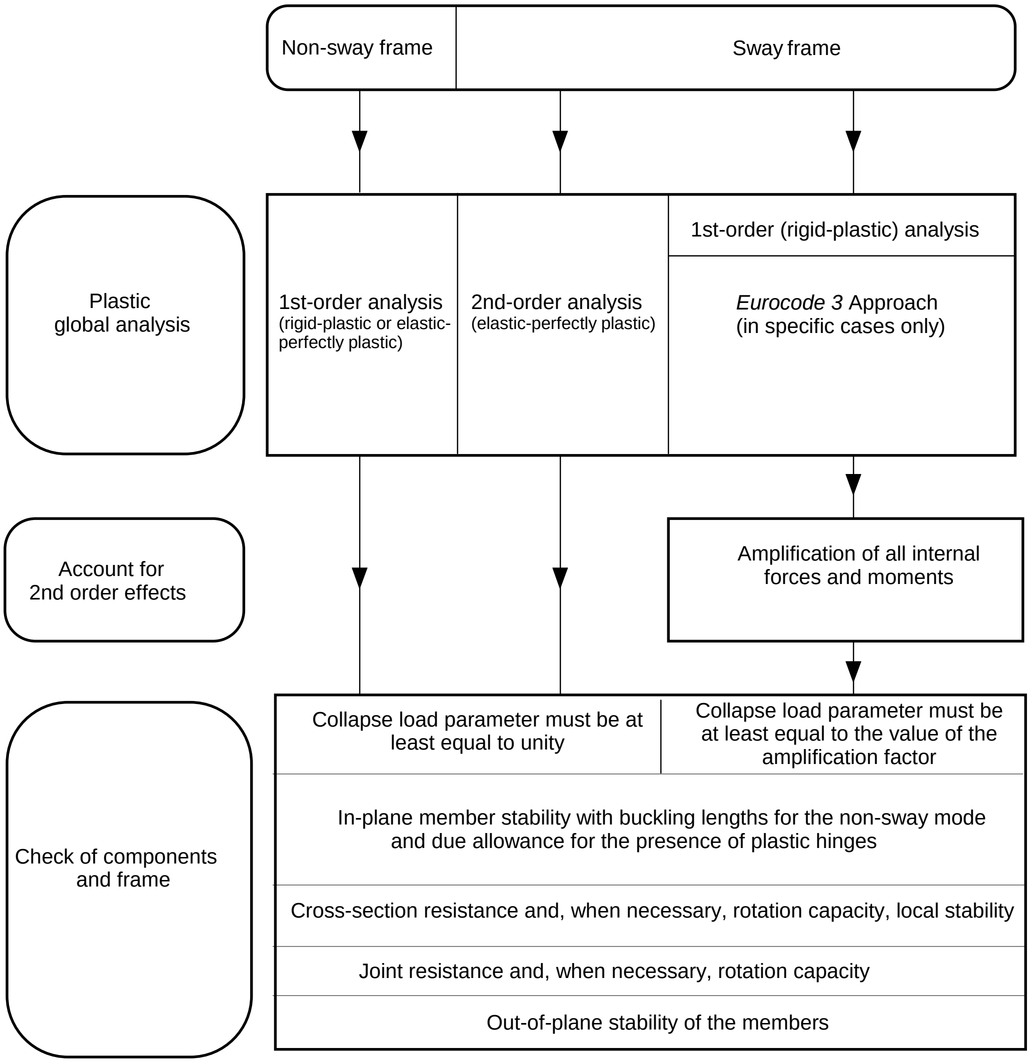


Figure 5.15 Plastic analysis and relevant design checks (according to *Eurocode 3*)

5.5.2.1 First-order analysis

First-order analysis (rigid-plastic analysis or elastic-perfectly plastic analysis) is especially appropriate for non-sway frames. Rigid-plastic analysis is also permitted for sway frames in specific cases only. The loads are assumed to increase in a proportional and monotonic manner. The plastic load parameter λ_p, at which collapse occurs by the onset of a plastic hinge mechanism, must at least amounts unity.

Checks of the resistance of the cross-sections and joints are required to account for the influence of axial and/or shear forces. As the first-order plastic method does not make allowance for any buckling in-plane and, out-of-plane phenomena in the members, these checks shall be carried out with due account taken of the presence of plastic hinges and of their effects on buckling lengths. Separate design checks of local resistance to concentrated forces shall also be performed.

No further check of the in-plane global frame stability for the sway buckling mode is required because of the non-sway character of the frame.

First-order plastic analysis may also be applicable to sway frames under the reservation that some corrections be brought to the results to allow for second-order effects (see the alternative methods to second-order plastic analysis described in Section 5.5.2.2.).

5.5.2.2 Second-order analysis

Second-order analysis, with due allowance being made for frame imperfections is applicable in any circumstance. Second-order theory is especially required when the frame is classified as sway.

Second-order effects due to global frame imperfections and sway displacements are necessarily included in the results of the global analysis. Second-order effects due to local member imperfections and in-plane member deflections can also be accounted for provided that the global analysis is conducted on the structure with initially deflected members. The influence of the axial and/or shear forces on the plastic moment resistance of the sections may also be reflected if the software proceeds appropriately.

As the loads are assumed to be incremented proportionally, the plastic collapse load parameter λ_u, which is the one at which collapse develops in the structure, due either to a plastic mechanism or to frame instability, must at least amount unity.

Additional design checks for the cross-sections and joints are required only when the influence of the axial and/or shear forces has not yet been accounted for in the global analysis. As the rotations in the plastic hinges are results of the global analysis, a direct check of the rotation capacity is possible when necessary. The design checks of the members against in-plane buckling phenomena, using the *buckling lengths for the non-sway mode* and with due allowance for the presence of plastic hinges, shall be carried out except when the member imperfections have yet been introduced in the structure prior to its global analysis. Local resistance to concentrated forces may have to be checked in some members. In most cases where elastic-perfectly plastic global analysis is used, only the in-plane behaviour of members is considered. Therefore separate checks of the out-of-plane member stability shall be needed.

No further check of the in-plane frame stability for the sway buckling mode is required, because it has been covered by the structural analysis.

Similarly to what was said about second-order elastic global analysis (see Section 5.5.1.2), it is still possible to account for second-order effects by means of corrections to be brought to the results of a first-order plastic analysis. In this respect, two simplified approaches are available :

a) the *Eurocode 3* approach (Simplified a[1]pproach) ;

b) the so-called *Merchant-Rankine* approach.

- *Simplified second-order elastic-perfectly plastic analysis - Eurocode 3 Approach*[1]

As an alternative to the general second-order elastic-perfectly plastic analysis, *Eurocode 3* offers a simplified second-order method, which may be adopted for sway frames in specific cases (frames with low to moderate sway).

In this approach, the collapse load parameter λ_p is first computed based on a first-order rigid-plastic analysis. It is then reduced to take account of second-order effects and must then have a value which is at least equal to unity.

The internal forces and moments resulting from the rigid-plastic analysis are amplified to generate a consistent set of internal forces and moments which then include presumably second-order effects.

Safety checks of the cross-section and joint resistance are required which account for the influence of axial and/or shear forces. Checks of the in-plane member stability, using the buckling lengths for the non-sway mode are carried out with allowance being made for the presence of plastic hinges. Separate design checks of the local resistance to concentrated loads and the out-of-plane stability member shall also be performed.

According to *Eurocode 3*, these checks, when fulfilled, guarantee the overall sway stability of the frame.

- *Merchant-Rankine approach*

The *Merchant-Rankine* approach can only be used for sway frames which meet the following requirement :

$$4 \leq \frac{\lambda_{cr}}{\lambda_p} \leq 10$$

---

[1] The practical application of this simplified approach raises several questions to which no clear answer is provided by Eurocode 3. In the present manual, as in some *Eurocode 3 National Application Documents*, this approach is not recommended for application as far as all the clarifications have not been brought. For these reasons, no guidelines for application are given in Chapter 7.

where λ_{cr} is the elastic critical load parameter and λ_p, the first-order plastic collapse load parameter. When evaluating λ_p, due allowance has to be made for possible interactions between bending moment and shear and/or axial forces in cross-sections and joints where hinges form.

The safety check of the whole frame consists in ensuring that the ultimate load λ_u, which is calculated from the *Merchant-Rankine* formula, amounts at least unity.

This value of the ultimate load parameter λ_u may be calculated by the following formula :

$$\frac{1}{\lambda_u} = \frac{1}{\lambda_{cr}} + \frac{0.9}{\lambda_p}$$

This criterion is very simple to apply for checking frames.

When the frame is designed using the *Merchant-Rankine* criterion, no further verification for cross-section and joint resistance is required; only the member stability needs to be checked. Local resistance to concentrated forces may have to be checked for some members.

PART 2 : APPLICATION RULES

CHAPTER 6

GUIDELINES FOR DESIGN METHODOLOGY

This chapter treats the strategy for an appropriate application of *the design methodology*, i.e. of the various *design strategies*. The design strategy followed by the designer is of vital importance for an efficient design process. A good strategy leads normally to economical solutions for both members and joints.

In Chapter 2, several approaches for the design process have been given. The traditional design approach reflects current practice where joints are modelled as either simple or continuous. In the consistent design approach, global frame analysis is started with due account taken of the joint response.

When the joints are semi-continuous, the traditional design approach may be adopted too but in an iterative way (see section 2.4). Then it is of course far better that only a single party be in charge of the whole design process; indeed the actual joint properties, that depend on the joint sizing and detailing -i.e. on the fabricator's task- need to be included in the global frame analysis -i.e in the engineer's task-. Such a use of the traditional design approach is neither very efficient nor suits well for a rather common practice where the respective designs of members and joints are carried out by different parties. For these reasons this chapter gives design strategies which focus on frames with semi-continuous joints. The purpose of these three strategies is to allow for a two task process; the design of the frame is separated from the design of the joints, as in the traditional design approach, with the aim of efficiency and allowance for semi-continuity.

These strategies are:

- Use of a *good guess* for joint stiffness with a view to elastic global frame analysis;
- Use of the fixity factor in the traditional design approach;
- Design of braced frames with rigid-plastic analysis.

The first two strategies are especially applicable when elastic frame design, possibly when elasto-plastic one; they address mainly unbraced frames but also braced frames. The third strategy focuses on one type of plastic frame design; its use may be recommended for braced frames only.

6.1 Use of a *good guess* for joint stiffness

This strategy refers to the traditional design approach (see Figure 2.2). However, two changes are needed to make it suitable for semi-continuous design (Figure 6.1):

1) *Account for joint stiffness in the elastic global frame analysis*:

In Step 3, use is made of a *good guess* of the initial joint stiffness. In the till now common practice, a rough guess is to assume that the joint is rigid. As joints behave normally as semi-rigid ones, it is not really a *good guess*. Therefore a table is given to make a better *good guess* of the actual initial joint stiffness than assuming rigid. This good guess is based on the beam and column properties and on the type of the joints; that is explained more in detail in Chapter 6.1.1. This stiffness of the joint is the one to be used for the elastic global frame analysis (Step 4) when the designer intends to perform elastic design checks ; it shall be divided by an appropriate η factor (see Section 4.4) when plastic design checks are carried out.

2) *Verification of stiffness in the design of joints*:

In Step 8 of Figure 6.1, it shall be verified whether the actual stiffness of any of the joints is in reasonable agreement with the relevant approximate stiffness that is accounted for in the elastic global frame analysis. This replaces the verifications for rigid and/or pinned joints in the traditional design. Chapter 6.1.2 gives some rules aimed at enabling this verification; their philosophy is quite similar to the classification diagrams of *Eurocode 3*. These rules can be applied easily in combination with a global analysis software.

6.1.1 Simple prediction of the initial joint stiffness

In the preliminary frame design phase, it is difficult to assess the stiffness of the (semi-rigid) joints; indeed the joints have not been designed yet. To overcome this problem, some simplified formulae have been derived based on *Eurocode 3-(revised) Annex J*. By means of these formulae, the designer can determine the stiffness of a joint by selecting the joint configuration only.

Of course these formulae are based on some fixed choices regarding the connection detailing. These are:

For *end-plated connections*:
- The connection has two bolt rows only in the tension zone;
- The bolt diameter is approximately *1.5* times the column flange thickness;
- The location of the bolt is as close as possible to the root radius of the column flange, the beam web and flange (about *1.5* times the thickness of the column flange);
- The end-plate thickness is similar to the column flange thickness.

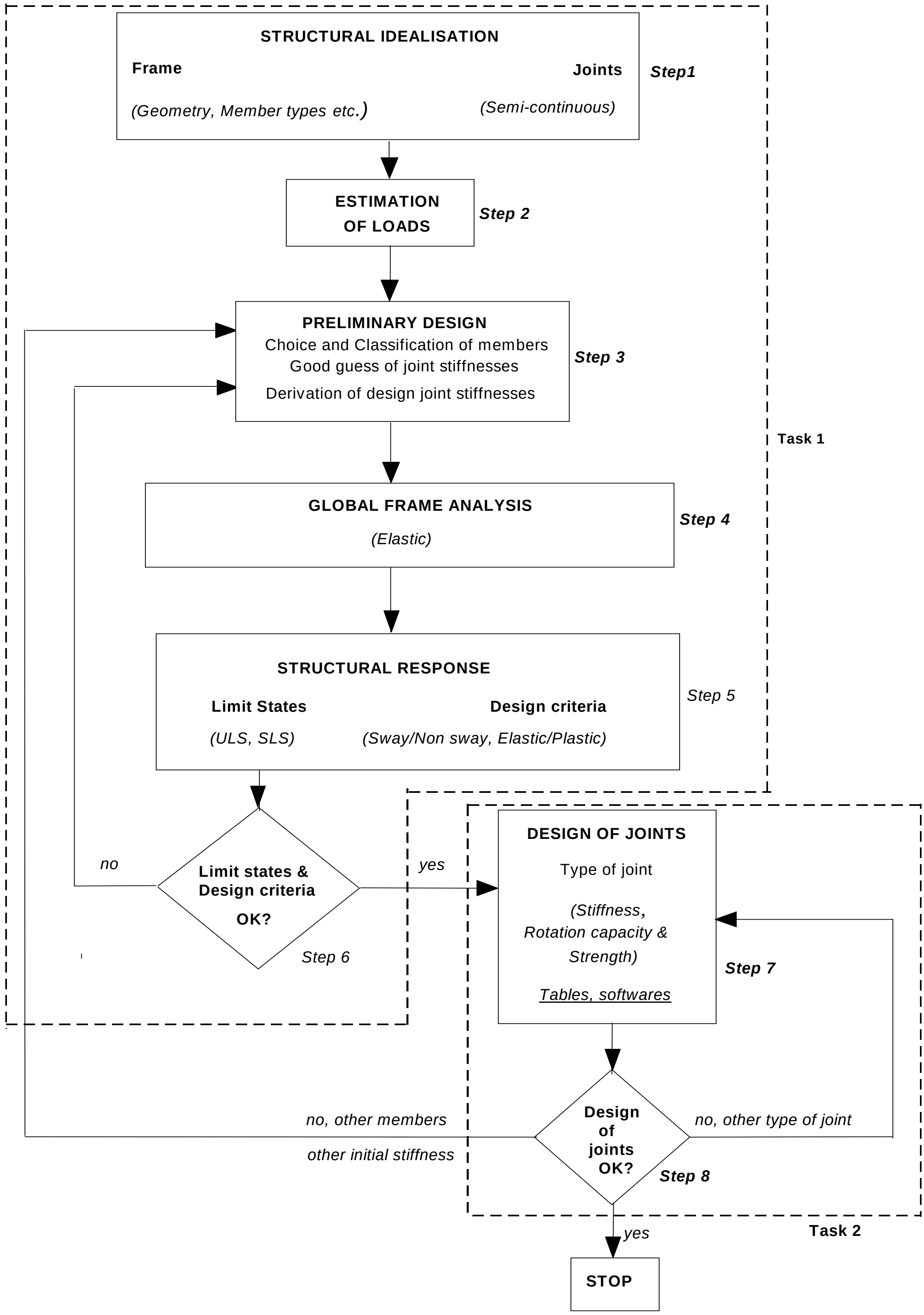


Figure 6.1 Design strategy when semi-continuous joints (elastic global analysis)

For *cleated connections*:

- The connection has one bolt row in the tension zone;
- The bolt diameter is approximately *1.5* times the cleat thickness;
- The location of the bolt is as close as possible to the root radius of the column flange and the cleat (about *1.5* times the thickness of the column flange);
- The cleat thickness is whenever possible similar to the column flange thickness.

The approximate value $S_{j,app}$ of the initial joint stiffness is expressed as :

$$S_{j,app} = \frac{Ez^2 t_{fc}}{C}$$

where the values of the factor C are given in Table 6.1 for different joint configurations and loadings. These values of the joint stiffness involve two parameters only: z and $t_{f,c}$; z is the distance between the compression and tensile resultants and $t_{f,c}$ is the thickness of the column flange. In an extended end-plate connection with two bolt rows, the distance z is approximately equal to the beam depth. For the same joint with a haunch, z is equal to the sum of the beam depth and the haunch depth.

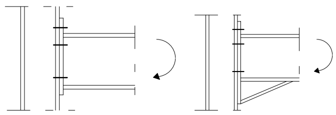
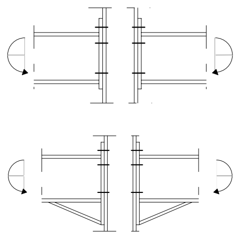
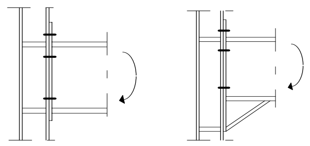
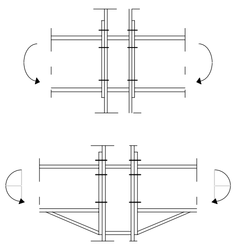
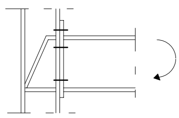
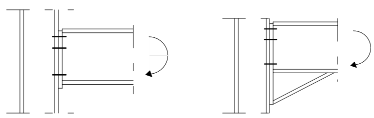
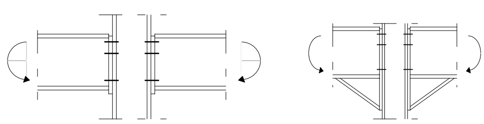
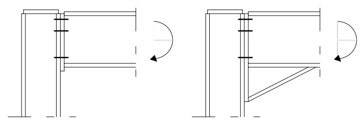
6.1.2 Required joint stiffness

Eurocode 3 provides the designer with two diagrams which enable the classification of joints according to their stiffness (pinned, semi-rigid, rigid): one for braced and one for unbraced frames.

Accordingly, for braced frames, a joint may be regarded as rigid if the actual initial joint stiffness fulfils :

$$S_{j.ini} \geq \frac{8EI_b}{L_b}$$

That condition ensures that the real flexibility of the joints does not result in a more than 5% drop in bearing capacity of the frame. The acceptance of this drop magnitude permits not to restart the global frame analysis with a finite value of the joint stiffness. Here, the wording *actual initial stiffness* is the best value a designer can obtain for the initial stiffness of a particular joint. This is, for example, the value obtained from experiments, from numerical simulations or computed based on *Eurocode 3-(revised) Annex J*.

Joint configuration		*C*
Extended end-plate, single sided, unstiffened (β=1)		13
Extended end-plates, double sided, unstiffened, symmetrical (β=0)		7,5
Extended end-plate, single sided, stiffened in tension and compression (β=1)		8,5
Extended end-plates, double sided stiffened in tension and compression symmetrical (β=0)		3
Extended end-plate, single sided, *Morris* stiffener (β=1)		3
Flush end-plate, single sided (β=1)		14
Flush end-plates, double sided, symmetrical (β=0)		9,5
Flush end-plate, single sided, cover plate at column top (β=1)		11,5

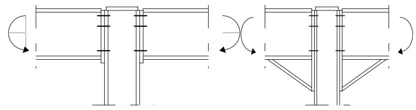
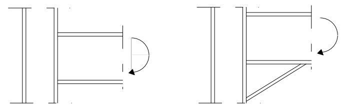
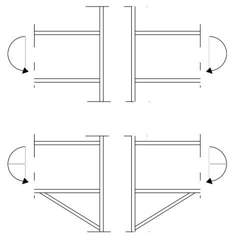
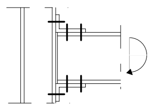
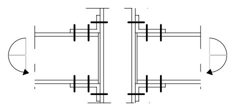
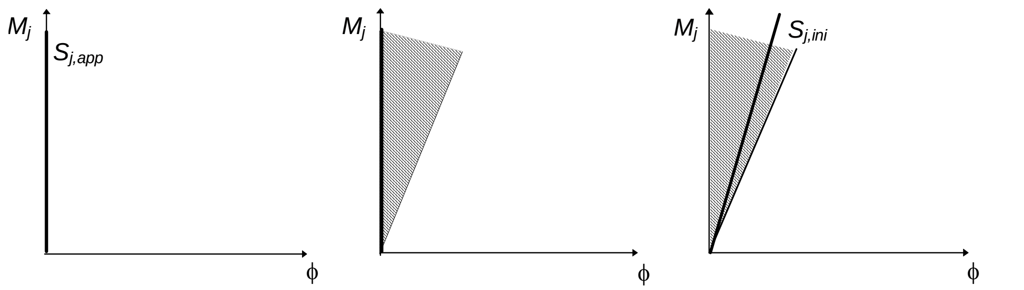
Flush end-plates, double sided, cover plate at column top, symmetrical (β=0)		6
Welded joint, single sided, unstiffened (β=1)		11,5
Welded joints, double sided, unstiffened, symmetrical (β=0)		6
Cleated, single sided (β=1)		70
Cleated, double sided, symmetrical (β=0)		65
<u>Note:</u> For the rare cases of double-sided joint configurations where β=*2* (*unbalanced* moments), the value of the *C* factor is obtained by adding *11* to the relevant value for symmetrical conditions (*balanced* moments).		

Table 6.1 Good guess of the initial stiffness for typical beam-to-column joint configurations

To check whether a joint is rigid needs a three steps procedure (see Figure 6.2) :

- *Step a* : frame analysis conducted with the assumption of rigid joints (Step *3* of Figure 6.1).
- *Step b* : range in which the actual initial stiffness should be (in Step 8 of Figure 6.1).

- *Step c* : check whether the actual initial stiffness is in this range (in Step 8 of Figure 6.1).



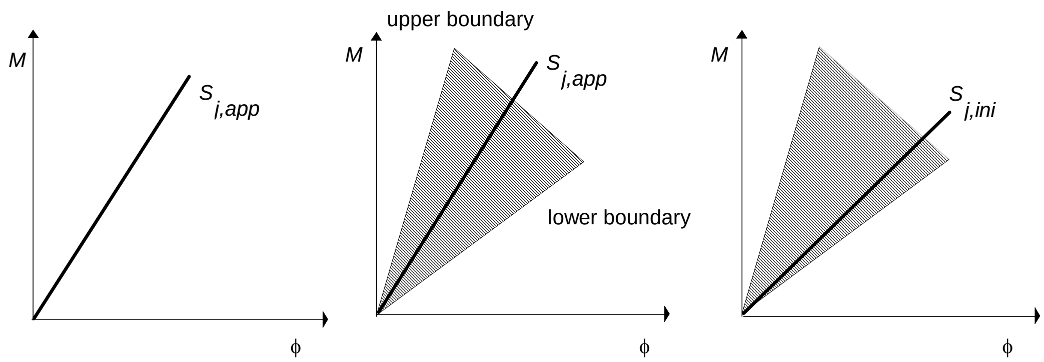
Step a) :
Frame analysis based on the assumption $S_{j,app} = \infty$ (joint *presumably* rigid)

Step b) :
Stiffness range for rigid joints based on $S_{j,app} = \infty$ see *Eurocode 3*

Step c) :
Check if actual initial stiffness $S_{j,ini}$ is in range

Figure 6.2 Check of the stiffness requirement for a rigid joint

This concept can be generalised to check whether a difference between the approximate joint stiffness and the actual joint stiffness of semi-rigid joints has a significant influence on the frame behaviour (Figure 6.3). The corresponding formulae for the variance between the approximate joint stiffness and the actual initial one are given in Table 6.2. These criteria may be used to check whether a difference between these stiffnesses has a no more than *5%* effect on the frame carrying capacity.



Step a): engineer
Frame analysis based on a design joint stiffness derived from$S_{j,app}$

Step b): fabricator
Stiffness range for semi-rigid joints based on $S_{,app}$

Step c): fabricator
Check if actual initial stiffness $S_{j,ini}$ is in range

Figure 6.3 Check of stiffness requirement of a semi-rigid joint

Frame	Lower boundary	Upper boundary
Braced	$S_{j,ini} \geq \dfrac{8S_{j,app}EI_b}{10EI_b + S_{j,app}L_b}$	If $S_{j,ini} \leq \dfrac{8EI_b}{L_b}$ then $S_{j,ini} \leq \dfrac{10S_{j,app}EI_b}{8EI_b - S_{j,app}L_b}$ else $S_{j,ini} \leq \infty$
Unbraced	$S_{j,ini} \geq \dfrac{24S_{j,app}EI_b}{30EI_b + S_{j,app}L_b}$	If $S_{j,ini} \leq \dfrac{24EI_b}{L_b}$ then* $S_{j,ini} \leq \dfrac{30S_{j,app}EI_b}{24EI_b - S_{j,app}L_b}$ else $S_{j,ini} \leq \infty$

in which:
$S_{j,app}$ = approximate joint stiffness (estimate of the initial one)
$S_{j,ini}$ = actual initial joint stiffness
E = Young modulus
L_b = beam length
I_b = second moment of area of the beam cross-section

*For sake of simplicity, the stiffness boundary of *Eurocode 3* is modified from

$S_j \geq \dfrac{25EI_b}{L_b}$ to $S_j \geq \dfrac{24EI_b}{L_b}$ for *unbraced* frames.

Table 6.2 Boundaries for variance between *actual* and *approximate* initial stiffnesses

6.2 Use of the fixity factor concept (traditional design approach)

Another strategy for a preliminary design is the use of the so-called *fixity factor f*. The fixity factor *f* is defined as the rotation ϕ_b of the beam end, due to a unit end moment applied at the same end, divided by the corresponding rotation ϕ_t of the beam plus the joint.

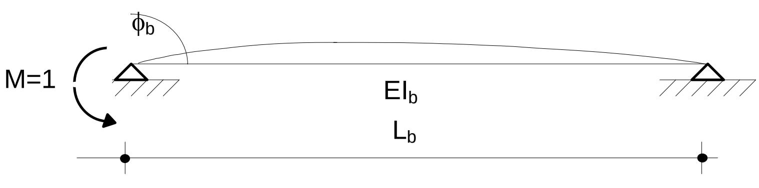


Figure 6.4 Beam end rotation

The end rotation of the beam for a unit moment is (Figure 6.4):

$$\phi_b = \frac{L_b}{3EI_b}$$

The rotation of the beam plus the joint for the same moment is:

$$\phi_t = \frac{L_b}{3EI_b} + \frac{1}{S_j}$$

wherefrom the fixity factor:

$$f = \frac{\phi_b}{\phi_t} = \frac{1}{1+1.5\alpha}$$

where the symbols E, L_b, I_b and S_j are defined in Table 6.2 and :

$$\alpha = \frac{2EI_b}{L_b S_j}$$

For a *truly pinned* joint, f is equal to 0 while for a *truly rigid* joint, f amounts 1.

To speed up the process of converging to a solution, the designer can decide to adopt a fixity factor between 0 and 1 and start the global frame analysis accordingly. Recommended values are $0.1 \leq f \leq 0.6$ for braced frames and $0.7 \leq f \leq 0.9$ for unbraced frames. Let us assume $f = 0.5$ for braced frames; then a joint stiffness of $3\ EI_b/L_b$ should be adopted in the global frame analysis. If $f = 0.8$ is used for unbraced frames, the corresponding value of the joint stiffness is 12 EI_b/L_b.

These respective values can be considered as a good guess in the design procedure of Figure 6.1. It shall be verified that this good guess is in reasonable agreement with the actual initial stiffness of the joints. For this verification, Table 6.2 can be used, but it merges to Table 6.3 if a fixity factor $f = 0.5$ for braced frames or $f = 0.8$ for unbraced frames is adopted.

6.3 Design of non-sway frames with rigid-plastic global frame analysis

The strategy described above focuses on the joint stiffness only and therefore suits especially for elastic global frame analysis. When plastic design is used - the latter is especially appropriate to braced non-sway frames-, the resistance of the joints is governing the global frame analysis too.

Frame	Lower boundary	Upper boundary
Braced (f = 0.5) $S_{j,app} = \dfrac{3EI_b}{L_b}$	$S_{j,ini} \geq \dfrac{24EI_b}{13L_b}$	$S_{j,ini} \leq \dfrac{6EI_b}{L_b}$
Unbraced (f= 0.8) $S_{j,app} = \dfrac{12EI_b}{L_b}$	$S_{j,ini} \geq \dfrac{48EI_b}{7L_b}$	$S_{j,ini} \leq \dfrac{30EI_b}{L_b}$

in which:
$S_{j,app}$ = approximate joint stiffness used for the global frame analysis
$S_{j,ini}$ = actual initial joint stiffness
E = Young modulus
L_b = beam length
I_b = second moment of area of the beam cross-section

Table 6.3 Boundaries for actual initial stiffness (given fixity factors)

For this reason, an alternative strategy can be contemplated; it is illustrated in Figure 6.5. In a first step, the frame is designed assuming simple joints. In a second step, the beam section depth is reduced by one size. So any joint has to transfer some bending moment. If this moment is small enough, simple partial-strength joints are sufficient. This strategy focuses very much on the economy of the frame: it is presumed that savings in material make more than compensate the additional cost due to the use of partial-strength joints. Should these partial-strength joints need to be stiffened, then increase the beam size reveals often a better solution.

The strategy is conducted as follows:

Steps 1/2: Quite similar to those of the strategy depicted in Figure 6.1.

Step 3: First design of the beams as fitted with presumably simple joints at the ends; for instance a beam subject to a uniformly distributed load is first sized with regard to the field moment only. The beam size is however chosen one size smaller then the one resisting just the maximum field moment (As a consequence, the joints will have to transfer some bending moment).

Step 4: Design of columns as if they were pinned connected to the beams.

Step 5: Determination of the bending moments to be transferred through the joints.

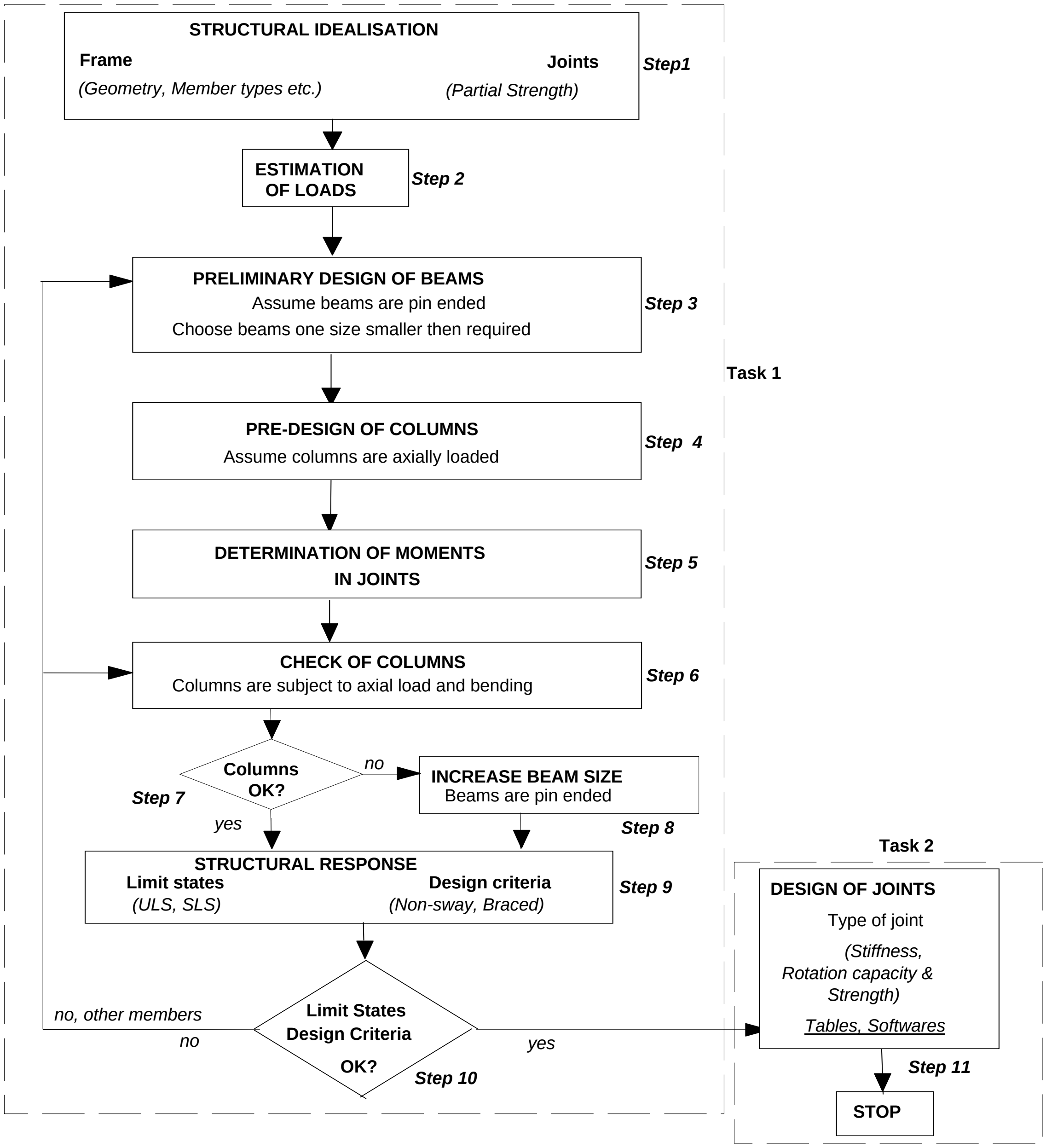


Figure 6.5 Design strategy for partial-strength joints in non-sway frames

Step 6: Columns are checked for coincident axial force and moments (section resistance and stability).

Steps 7/8: If any check of step 6 fails, economy commands probably to increase the beam size and to have simple joints rather than increase the column size and adopt partial-strength joints.

Steps 9/10: The checks concerned with the limit states are carried out. If the beams do not fulfil the serviceability limit states, they can be chambered. The stiffness of the joints can also be taken into account for this purpose; an estimate can be found using Table 6.1.

Alternatively the beam sizes can be increased. These options are a matter of economy.

Step 11: If the checks of the ultimate limit states is successful, the joints are designed in such a way that they are able to resist the relevant bending moments. If a design joint stiffness derived from an approximate one is used for the check of the serviceability limit states, it shall be checked whether the actual joint stiffness is in compliance with this estimate (see also Section 6.1.2).

In the conceptual stage of the design process of Figure 6.5, it is useful for the designer to have a rapid indication of the type of joint he will end-up with the last step (Step 11) . It is important to have this indication as early as possible, preferably before the check of beams and columns. If it appears that the joint needs stiffeners, it may be more economical to adopt simple joints combined with an increased beam size.

To obtain such a rapid indication, Table 6.4 can be used.

Joint detailing	Single-sided	Double-sided
	$M_{j,\mathrm{Rd}}$	$M_{j,\mathrm{Rd}}$
Simple	0	0
Intermediate	$\leq 5\, f_y\, z\, t_{f.c}^2 / \gamma_{M0}$	$\leq 7\, f_y\, z\, t_{f.c}^2 / \gamma_{M0}$
Complex	$> 5\, f_y\, z\, t_{f.c}^2 / \gamma_{M0}$	$> 7\, f_y\, z\, t_{f.c}^2 / \gamma_{M0}$

z distance between centres of compression and tension;

f_y yield strength of the column flange;

$t_{f.c}$ column flange thickness;

γ_{M0} partial safety factor for resistance of members.

Table 6.4 Strength recommendations for joints during preliminary design

Simply detailed joints are those joints which traditionally are considered as nominally pinned. Joints with complex detailing are able to transfer bending moments but require stiffening. Joints which are able to transfer bending moments but do not require stiffening are described as intermediate. In general, stiffening is labour expensive and thus “complex” joints may not lead to economical solutions. Partial-strength joints in general fall in the “intermediate” or “complex” categories.

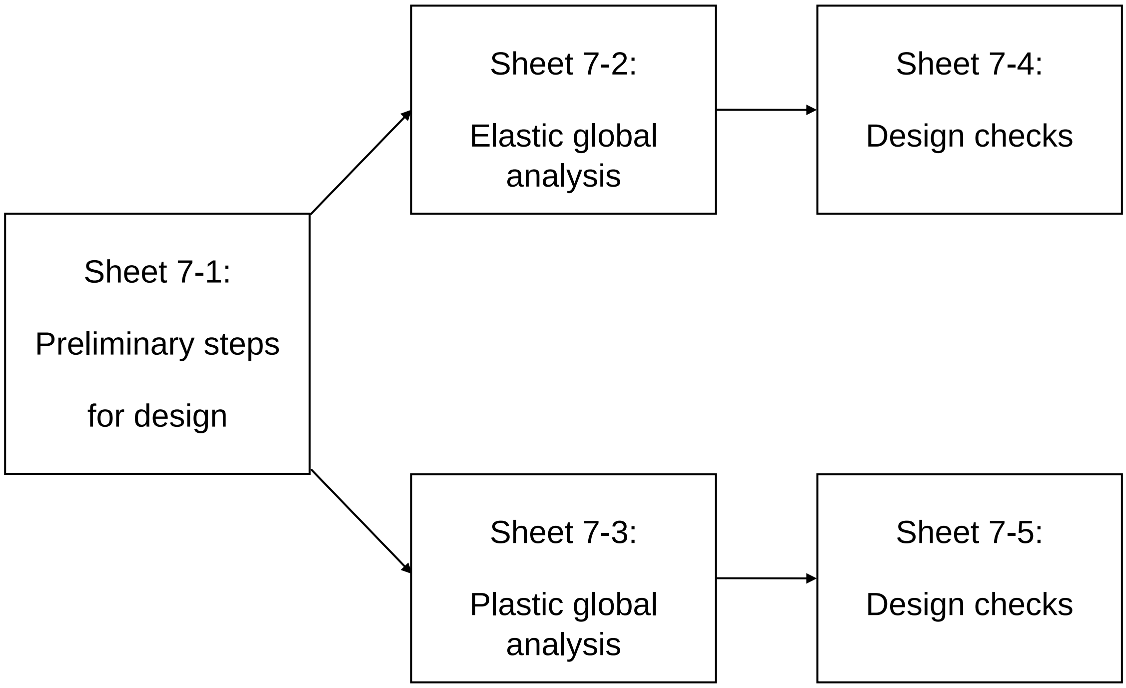
With help of this table a designer can check whether he may expect to end up with “moderate” joints during the design process of Figure 6.5. This can be done directly after Step 5. In this step, the moments M_{Sd} which shall be transmitted from beam to column have been determined. In general, the design moment resistance M_{Rd} of the joint shall be greater than or equal to this M_{Sd}. In other words, for example in the case of a single-sided joint, whenever the design moment at the joint M_{Sd} is such that $M_{Sd} < 5\ f_y\ z\ t_{f.c}^2\ /\ \gamma_{M0}$, it may be expected that the final solution for the joint can be without stiffeners. Otherwise, stiffening is likely to be required and it may be better to increase the beam depth and to choose simple connections in that case.

Table 6.4 is especially useful for the design of non-sway frames by rigid-plastic global frame analysis. However, it may also be helpful in the design of braced or unbraced sway frames using either plastic global frame analysis : elastic or plastic.

CHAPTER 7

GUIDELINES FOR FRAME AND ELEMENT DESIGN

General organisation



<table>
<tr><td colspan="3">Subject : Preliminary steps for design</td></tr>
<tr><td colspan="2">Design Code : Eurocode 3</td><td>Sheet n° : 7-1</td></tr>
<tr><th>Reference</th><th colspan="2">Purpose</th></tr>
<tr><td>Annex 7-A</td><td colspan="2">Step 1 Determination of the structural system</td></tr>
<tr><td>Eurocode 1</td><td colspan="2">Step 2 Load cases</td></tr>
<tr><td>Eurocode 1</td><td colspan="2">Step 3 Load combination cases</td></tr>
<tr><td>Annex 7-B</td><td colspan="2">Step 4 Global imperfections or equivalent loads ΦV</td></tr>
<tr><td>Eurocode 3
5.3</td><td colspan="2">Step 5 Class of beam and column cross-sections
Ductility class of joints</td></tr>
<tr><td>Sheet 7-2

Sheet 7-3</td><td>Step 6

Elastic analysis

Range of application :

• No limitations

•</td><td>Plastic analysis
Range of application (ductility requirements) :

• Class 1 cross-sections and joints are required where plastic hinges form(*).
• Class 2 cross-sections and/or joints are allowed if rotation capacity is sufficient (to be checked at the end of the design process)(*)
• Deemed-to-satisfy criteria for steel: § 3.2.2.2 of EC3

(*) Any full-strength joint with a resistance less than 1.2 times the full-strength resistance (see Chapter 8) which is adjacent to a section where a plastic hinge forms shall meet this requirement.</td></tr>
<tr><td>Sheet 7-4
Sheet 7-5</td><td colspan="2">Step 7 Design checks</td></tr>
</table>

Subject : Elastic global analysis (to be carried out for each load case)

Design code: *Eurocode 3*

First order theory	**Second-** Eurocode 3
	Amplified sway moment method
Range of application : • $V_{Sd} / V_{cr} \leq 0{,}1$ (non-sway frame)	Range of application : • $V_{Sd} / V_{cr} < 0{,}25$

First order

For design checks of the sections and joints and of the stability of beams and columns : • No amplification of moments and internal forces	For design checks of the sections and joints and of the stability of beams and columns and internal forces: • Amplification of sway moments and internal forces by factor: $1 / (1 - V_{Sd} / V_{cr})$

In-plane buckling lengths for the non-sway mode

Annex 7-C

	Annex 7-D
	Sheet n° : 7-2
order theory	
- 5.2.6.2	
Sway mode buckling length method	**General method**
Range of application: • No limitations	Range of application: • No limitations
elastic analysis *Chapter 9*	**Second order elastic analysis** *Chapter 9*
For design checks of the sections and joints and of the stability of beams : • Amplification of sway moments and internal forces by factor 1,2	
For design checks of the column stability : • No amplification factor • In-plane buckling lengths for the sway mode *Annex 7-C*	• In-plane buckling lengths for non-sway mode *Annex 7-C*

Subject : Plastic global analysis (to be carried out for each load case)	
Design code: ***Eurocode 3***	
First order theory	**Second-** *Eurocode 3*
	Simplified method **(NOT RECOMMENDED)**
Range of application : • $V_{Sd} / V_{cr} \leq 0{,}1$ (non-sway frames)	Range of application : * $V_{Sd} / V_{cr} < 0{,}2$ • Specific limitations (see *Eurocode-5.2.6.3)*
First-order plastic analysis Chapter 9	**First-order (rigid-)plastic analysis** λ_p : plastic load parameter *Chapter 9*
For design checks of the sections and joints and of the stability of beams and columns : • No amplification factor *($\lambda_u = \lambda_p$)*	For design checks of the sections and joints, and of the stability of beams and columns : • Amplification of all the internal forces and moments by factor: *$(1 - V_{Sd} / V_{cr}\ )$*
Conditions on the location of plastic hinges	
In-plane buckling lengths for the non-sway mode, with due allowance for the presence of plastic hinges *Annex 7-C*	

	Chapter 5
	Sheet n° : 7-3
order theory *5.2.6.3*	
Merchant-Rankine approach	**General method**
Range of application : • $4 \leq \lambda_{cr} / \lambda_p \leq 10$ λ_{cr} : critical load parameter (V_{cr} / V_{Sd}) λ_p : plastic load parameter	Range of application : • No additional limitations
First-order plastic analysis λ_p : plastic load parameter *Chapter 9*	**Second-order elastic-perfectly plastic analysis** λ_u : collapse load parameter *Chapter 9*
Estimate of the collapse load parameter λ_u obtained through a reduction of the plastic load parameter λ_p by factor : $1/(0,9 + \lambda_p / \lambda_{cr})$	
: *Eurocode 3 - 5.2.1.4(3) and (4)*	
In-plane buckling lengths = system lengths Axial forces only	In-plane buckling lengths for the non-sway mode, with due allowance for the presence of plastic hinges *Annex 7-C*

Subject : Design checks following an elastic global analysis	
Design Code : *Eurocode 3*	***Sheet n° : 7-4***

Code ref.	**Beams**
5.3.2 to 5.3.6	• Class of the cross-sections (if not already done)
4.2 to 4.3	• Serviceability
5.4.5 to 5.4.7+ 5.6	• Resistance of the cross-sections to bending and/or shear (including shear buckling)
5.4.10	• Resistance of the cross-sections to transverse forces on web
5.7	• Resistance of the cross-sections to crushing, crippling, buckling
5.5.2	• Stability of the members : lateral-torsional buckling (Note : For all types of joints, it is recommended to assume no fixity for end rotations due to free and/or restrained warping)
	Columns and Beam/columns
5.3.2 to 5.3.6	• Class of the cross-sections (if not already done)
5.4.3 to 5.4.9+5.6	• Resistance of the section to bending, compression, tension, shear (including shear buckling)
5.4.10	• Resistance of the sections to transverse forces on web
5.7	• Resistance of the sections to crushing, crippling, buckling
	• Stability of the members :
5.5.1	- Buckling in compression
5.5.2	- Lateral torsional buckling (see beam-checks)
5.5.3	- Lateral torsional buckling in bending and tension
5.5.4	- Buckling in compression and bending
	Joints
See Chapter 8	• Stiffness • Strength

Subject : Design checks following a plastic global analysis	
Design Code : *Eurocode 3*	**Sheet n° : 7-5**

Code ref.	**General check : $\lambda_u \geq 1$**
	Beams
5.3.2 to 5.3.6 *4.2 to 4.3* *5.4.7 and 5.6* *5.4.10* *5.7* *5.5.2*	• Class of the cross-sections (if not already done) and rotation capacity • Serviceability • Interaction of stress resultants in the cross-sections (if not already done) • Resistance of the cross-sections to transverse forces • Resistance of the cross-sections to crushing, crippling, buckling • Stability of the members : lateral-torsional buckling (UU<u>Note</u> : For all types of joints, it is recommended to assume no fixity for end rotations due to free and/or restrained warping)
	Columns and Beam-columns
5.3.2 to 5.3.6 *5.4.3 to 5.4.9+ 5.6* *5.4.10* *5.7* *5.5.1* *5.5.2* *5.5.3* *5.5.4*	• Class of the cross-sections (if not already determined) • Interaction of stress resultants in the cross-sections (if not already done) • Resistance of the section to transverse forces on web • Resistance of the section to crushing, crippling, buckling • Stability of the member : - Buckling in compression - Lateral torsional buckling (see beam-checks) - Lateral torsional buckling in bending and tension - Buckling in compression and bending
	Joints
See Chapter 8	• Stiffness • Strength • Rotation capacity

ANNEX 7-A DETERMINATION OF THE STRUCTURAL SYSTEM

- No bracing system : the frame is *unbraced*.
- Bracing system :

  If $\Psi_{br} > 0.2\ \Psi_{unbr}$: the frame is *unbraced.*

  If $\Psi_{br} \leq 0.2\ \Psi_{unbr}$: the frame is *braced.*

  Ψ_{br} is the lateral flexibility of the structure with bracing system

  Ψ_{unbr} is the lateral flexibility of the structure without the bracing system

In the case of a braced frame, and if the bracing system considered alone can be analysed as a non-sway system ($V_{Sd} / V_{cr} \leq 0.1$), the frame can be considered as fully supported, and both systems (frame and bracing) can be analysed separately. Each system is then analysed under its own vertical loads, and all the horizontal loads are applied on the bracing system.

In all the other cases, both systems shall be analysed as a single structural system.

Note : to evaluate V_{cr} , see Sheet 9-2 in Chapter 9.

ANNEX 7-B ASSESSMENT OF IMPERFECTIONS

In *Eurocode 3*, the effects of frame imperfections must be included in the global analysis of any frame. The frame imperfections are treated as a load case to be used in conjunction with all the critical load combinations acting on the frame. They are quantified in terms of an initial sway rotation at the base of the columns. The initial sway imperfections are determined directly from the following formula given in *Eurocode 3-Section 5.2.4.3(1)* :

$$\Phi = k_c\, k_s\, \Phi_0$$

with :

$$\Phi_0 = 1/200$$

$$k_c = (0.5 + \frac{1}{n_c})^{0.5} \quad \text{but } k_c \leq 1$$

$$k_s = (0.2 + \frac{1}{n_s})^{0.5} \quad \text{but } k_s \leq 1$$

n_c is the number of full height columns per plane;

n_s is the number of storeys.

Eurocode 3 proposes an alternative method to introduce the global imperfection of the frame, which may be more practical than the introduction of the out of plumb of the frame.

The initial sway imperfection may be replaced by a closed system of equivalent horizontal forces. The equivalent horizontal forces at each roof and floor level are calculated by multiplying the proportion of the vertical load applied at the level by the initial sway imperfection. They may be applied in any horizontal direction, but only in one direction at time.

At the supports, the equivalent horizontal forces obtained by multiplying the vertical reactions by the initial sway imperfections are applied so that the equivalent horizontal forces on the entire frame form a closed system, which results in a net horizontal reaction of zero in the absence of actual horizontal loads.

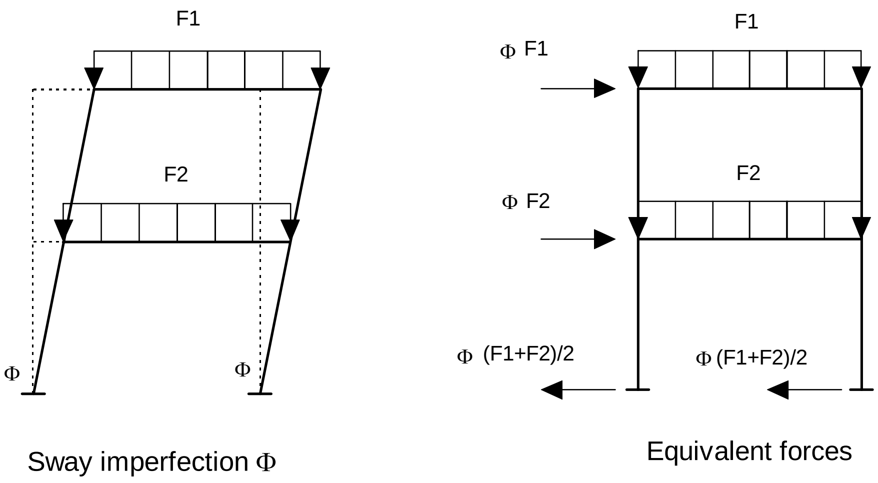


Figure 7-B.1 Global frame imperfections

According to *Eurocode 3*, the effects of member imperfections may be neglected when carrying out the global analysis of frames, except in some specific cases, as described in *Eurocode 3 - 5.2.4.2(4).*

ANNEX 7-C EFFECTIVE BUCKLING LENGTH FOR COLUMNS WITH END RESTRAINTS

The effective length of columns with semi-rigid beam-to-columns joints can be computed as described in the *Eurocode 3 - Annex E*.

The effective length for the *non-sway mode* can be obtained either by Figure 7-C.3, or by :

- $\frac{l}{L} = 0{,}5 + 0{,}14(\eta_1 + \eta_2) + 0{,}055(\eta_1 + \eta_2)^2$

or alternatively :

- $\frac{l}{L} = [\frac{1 + 0{,}145(\eta_1 + \eta_2) - 0{,}265\eta_1\eta_2}{2 - 0{,}364(\eta_1 + \eta_2) - 0{,}247\eta_1\eta_2}]$

The effective length for the *sway mode* can be obtained either by Figure 7-C.4, or by the following expression :

- $\frac{l}{L} = [\frac{1 - 0{,}2(\eta_1 + \eta_2) - 0{,}12\eta_1\eta_2}{1 - 0{,}8(\eta_1 + \eta_2) + 0{,}6\eta_1\eta_2}]^{0{,}5}$

In these figures and expressions, η_1 and η_2 coefficients are given by the following formulae :

$$\eta_1 = \frac{K_C + K_1}{K_C + K_1 + K_{11}^* + K_{12}^*}$$

$$\eta_2 = \frac{K_C + K_2}{K_C + K_2 + K_{21}^* + K_{22}^*}$$

- K_C column stiffness coefficient I_c/L_c
- K_1 and K_2 stiffness coefficients for the adjacent lengths of columns, if any.
- $K_{11}^*, K_{12}^*, K_{21}^*$ and K_{22}^* are effective (beam + joint) stiffness coefficients defined as :

$$K_{mn}^* = K_{mn}\left(\frac{1}{1 + \frac{4K_{mn}}{S_{j,mn}}}\right)$$

where $m,n = 1,2$ and $S_{j,mn}$ is the design stiffness of the beam-to-column joint connected to the beam *mn* (see Figure 7-C.2) and K_{mn} is the effective beam stiffness coefficient for this beam, given in Table 7-C.1.

Conditions of rotational restraint at far end of beam	Effective beam stiffness coefficient K_{mn} (provided that beam remains elastic)
Fixed at far end	EI_b / L_b
Pinned at far end	$0.75\ EI_b / L_b$
Rotation as at near end (double curvature)	$1.5\ EI_b / L_b$
Rotation equal and opposite to that at near end (single curvature)	$0.5\ EI_b / L_b$

Table 7-C.1 Effective beam stiffness coefficients

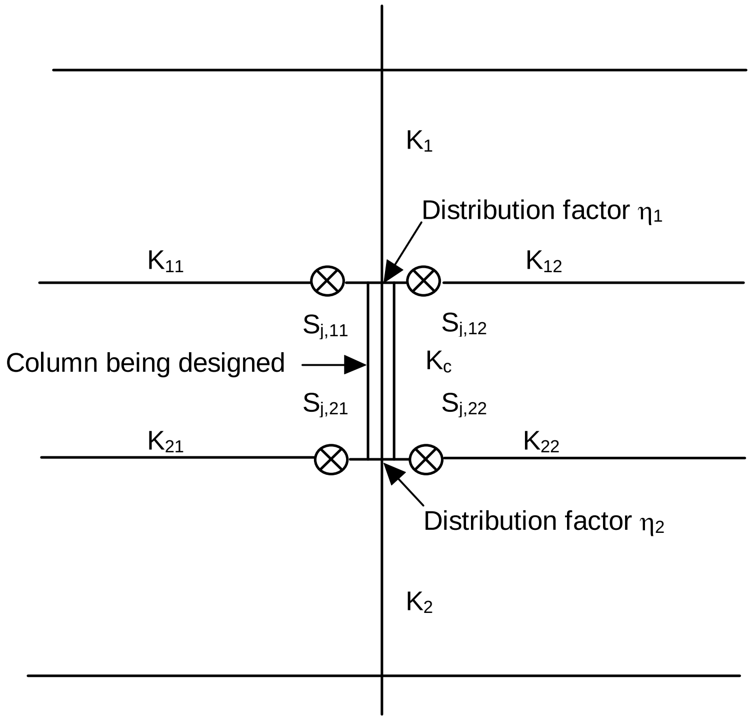
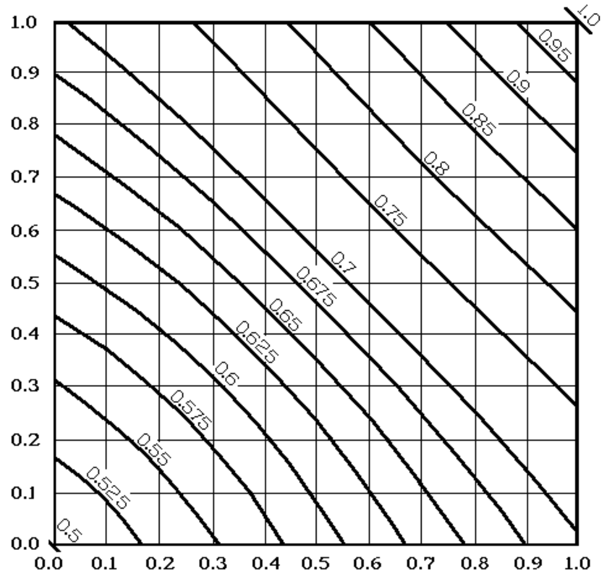


Figure 7-C.2 Distribution factors for continuous columns

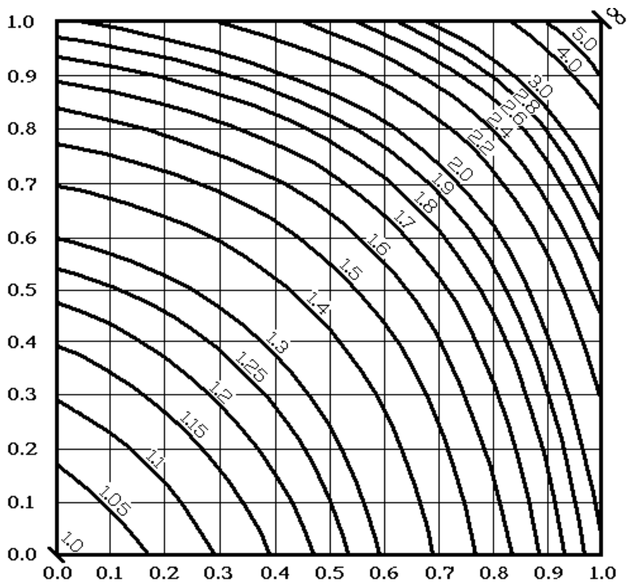
Pïnned



Fixed Pinned

Figure 7-C.3 Buckling length ratio l_c/L_c for a column in a non-sway mode

Pinned



Fixed Pinned

Figure 7-C.4 Buckling length ratio l_c/L_c for a column in a sway mode

CHAPTER 8

GUIDELINES FOR JOINT PROPERTIES AND MODELLING

Subject: Joint characterisation

Design code : *Eurocode 3-(revised) Annex J*	**Sheet n°: 8-1**

Manual ref.	**Purpose and means**
	Evaluation of the stiffness and resistance properties
	Four available design tools :
Sheet 8-1.A	• ***Eurocode 3-(revised) Annex J***
	Accurate procedures for beam-to-column joints and beam splices with bolted and welded connections.
Sheet 8-1.B	• ***Design sheets***
	Simplified calculation procedures for beam-to-column joints with end-plate or cleated connections and beam splices with flush end-plates.
Sheet 8-1.C	• ***Design tables***
	Tables providing stiffness and moment and shear design resistances for joints covered by design sheets(presently limited to standardised configurations between IPE beams and HEB columns.
Sheet 8-1.D	• ***DESIMAN Software***
	For all types of beam-to-column joints and beam splices.
	Ranges of application specified in 8-1.A to D sheets.
Sheet 8-1.E	**Evaluation of the rotation capacity**
	Based on the nature of the design failure mode identified by the resistance calculation.
	Possible use of deemed-to-satisfy criteria in a preliminary design step.

Subject: Joint characterisation	
Design code : *Eurocode 3-(revised) Annex J*	**Sheet n°: 8-1.A**
Manual ref.	**Purpose and means**
	Evaluation of the stiffness and resistance properties using *Eurocode 3-(revised) Annex J*
Annex 8-A	Principles expressed in an annex to the present chapter. Range of application : • Connection types allowed by the list of available components. • H or I hot-rolled profiles or built-up profiles with similar dimensions. • Column webs where $d_c / t_{wc} \leq 69\ \varepsilon$. d_c is the clear depth of the web and t_{wc} its thickness; $\varepsilon = (235/f_{ywc})^{0,5}$ where f_{ywc} is the yield strength of the web in Mpa. • In double-sided beam-to-column joint configurations without diagonal stiffeners on the column web, the two beams are assumed to have similar depths. • Joints with preloaded and non-preloaded bolts, but in both cases joints properties are evaluated by assuming no preloading in the bolts. • Joints under static loading. • Limited axial force N_{Sd} in the connected beam : $N_{Sd}/N_{Rd} < 0,1$ (N_{Rd} is the design resistance of the connected beam in tension or compression). • Steel grades up to *S460*.

Subject: Joint characterisation	
Design code : *Eurocode 3-(revised) Annex J*	**Sheet n°: 8-1.B**

Manual ref.	**Purpose and means**
	Evaluation of the stiffness and resistance properties using design sheets
Volume 2	Design sheets and guidelines for use available in Volume 2.
	Range of application:
	• Same as for *(revised) Annex J* (see Sheet 8-1.A).
	• Limited scope in terms of joint configurations :
	♦ Single-sided and double-sided joints configurations with extended end-plates (4 bolts in the tension zone).
	♦ Single-sided and double-sided joints configurations with flush end-plates (2 bolts in the tension zone).
	♦ Beam splices with flush end-plates (same end-plates each side and 2 bolts in the tension zone).
	♦ Single-sided and double-sided joints configurations with flange cleats.
Annex 8-B	Extension to haunched beams given.
	Possible extension to further joint configurations may be easily contemplated.

Subject: Joint characterisation	
Design code : *Eurocode 3-(revised) Annex J*	**Sheet n°: 8-1.C**

Manual ref.	**Purpose and means**
	Evaluation of the stiffness and resistance properties using design tables
Volume 2	Design tables available and guidelines for use in Volume 2.
	Range of application:
Sheet 8-1.A	• Same as for *(revised) Annex J* (see Sheet 8-1.A).
	• Limited scope in terms of joint configurations:
Sheet 8-1.B	♦ Configurations covered by design sheets.
	♦ Tables of standardised joints between:
	* IPE profiles for beam splices;
	* HEB columns and IPE beams for beam-to-column joints.
Annex 8-B	Extension to haunched beams given.
Sheet 8-1.D	Possible extension to further joint configurations and other profiles may be easily contemplated through the use of the DESIMAN software.

Subject: Joint characterisation	
Design code : *Eurocode 3-(revised) Annex J*	**Sheet n°: 8-1.D**

Manual ref.	**Purpose and means**
	Evaluation of the stiffness and resistance properties using the DESIMAN software
Sheet 8-1.A Sheet 8-1.B	Calculations according to *Eurocode 3-(revised) Annex J* or simplified design sheets presented in Sheet 8-1.B.
Sheet 8-1.A	Range of application same as for *(revised) Annex J* (see Sheet 8-1.A).
Volume 3	Guidelines for use and installation given.

Subject: Joint characterisation

Design code : *Eurocode 3-(revised) Annex J*	**Sheet n°: 8-1.E**

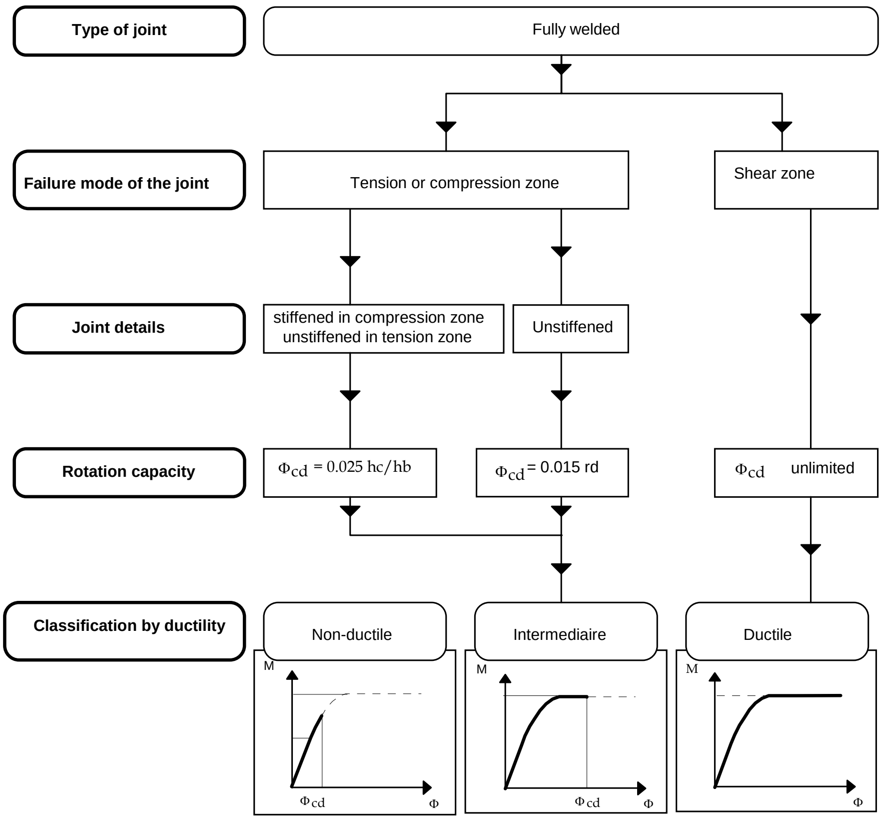
Purpose and means

Evaluation of the rotation capacity

The rotation capacity depends on the design failure mode identified by the resistance calculation.

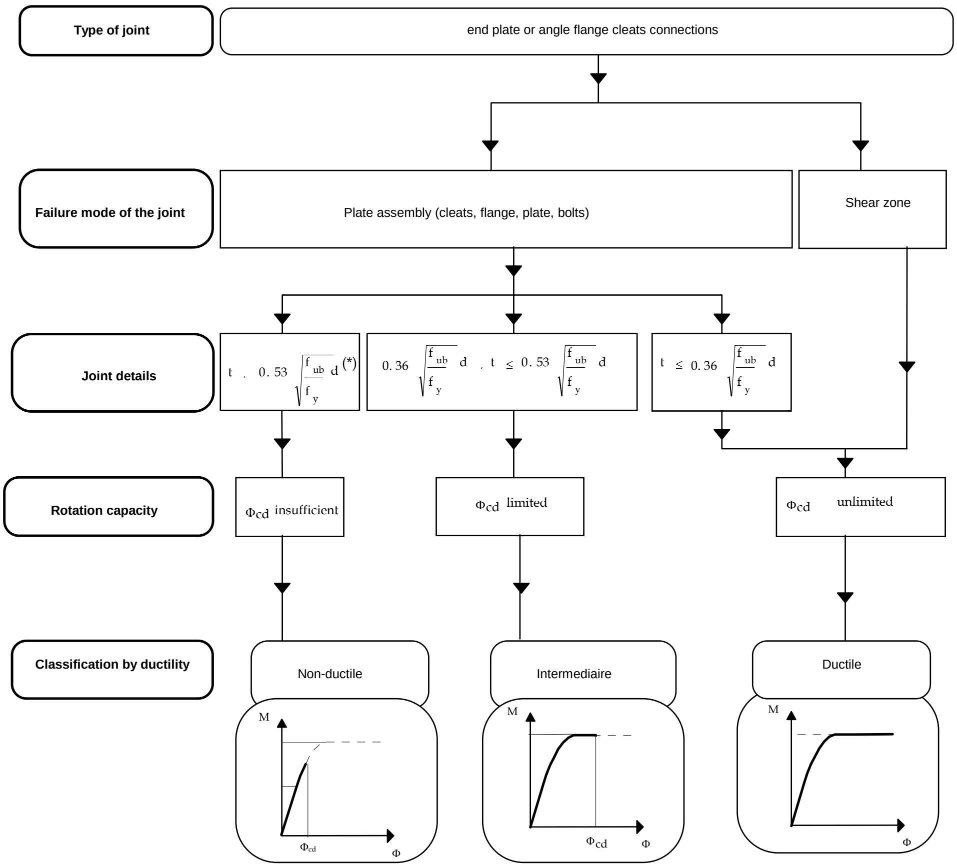
Following flow charts indicate how to proceed for welded joints and bolted joints with end-plates or flange cleats :

- Welded joints :



where : h_c is the height of the column
h_b is the height of the beam

- Bolted joints :



where : *t* is the thickness of either the column flange or the end-plate or the tension flange cleat;
d is the nominal diameter of the bolts;
f_{ub} is the ultimate tensile strength of the bolts;
f_y is the yield strength of the relevant basic components.

(*) Simple criterion in accordance with that expressed in *Eurocode 3-(revised) Annex J* (see Volume 2 - Chapter 1)

Above flow charts cover only some failure modes; for those not covered, the rotation capacity may be determined by testing or through models validated by testing.

For practical applications, it may be recommended to select a ductile failure mode in design when rotation capacity is required.

Limitations of the plate thickness with regards to bolt diameter expressed in the second flow chart may be used as deemed-to-satisfy criteria for the preliminary design of joints.

Subject: Joint classification

Design code : *Eurocode 3-(revised) Annex J*	**Sheet n°: 8-2**

Manual ref.	**Purpose and means**

Classification by stiffness

Beam-to-column joint in an unbraced frame[(*)/(**)] :

Pinned joint : $S_{j,ini} \leq 0{,}5\ EI_b / L_b$
Semi-rigid joint : $0{,}5\ EI_b / L_b < S_{j,ini} < 25\ EI_b / L_b$
Rigid joint : $S_{j,ini} \geq 25\ EI_b / L_b$

Beam-to-column joint in a braced frame[(*)] :

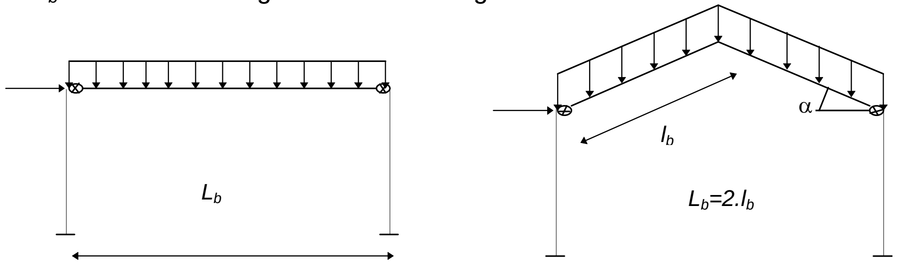
Pinned joint : $S_{j,ini} \leq 0{,}5\ EI_b / L_b$
Semi-rigid joint : $0{,}5\ EI_b / L_b < S_{j,ini} < 8\ EI_b / L_b$
Rigid joint : $S_{j,ini} \geq 8\ EI_b / L_b$

Beam splices[(***)] *:*

Pinned joint : $S_{j,ini} \leq 0{,}5 EI_b / L_b$
Semi-rigid joint : $0{,}5\, EI_b / L_b < S_{j,ini} < 25\, EI_b / L_b$
Rigid joint : $S_{j,ini} \geq 25\ EI_b / L_b$

Important remark : The validity of the stiffness classification for beam-to-column joints is restricted to structures where possible beam splices are classified as rigid. Fortunately, this limitation is not very restrictive for practical applications.

(*) $S_{j,ini}$ initial joint stiffness;
E Young modulus for steel;
I_b second moment of area of the beam;
L_b beam length as defined in figures herebelow:



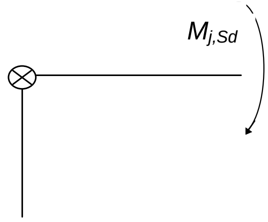
(**) Provided that in every storey $K_b/K_c \geq 0{,}1$ where:
K_b is the mean value of I_b/L_b for all the beams at the top of that storey;

	K_c is the mean value of I_c/L_c for all the columns in that storey. (***) Limit not specified in *Eurocode 3-(revised) Annex*

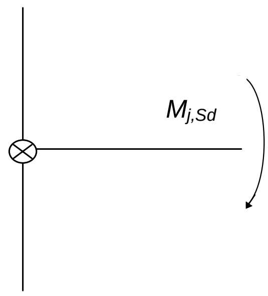
Classification by resistance

Full strength joint :

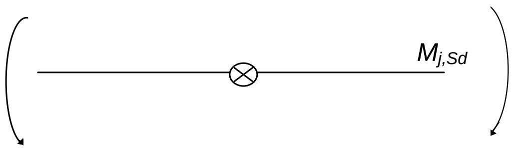
- Beam-to-column joint (top column)



- Beam-to-column joint (within column height)



- Beam splice



- Pinned joint :

  Joint with design resistance lower than 25 % of the full-strength resistance.

- Partial-strength joint :

  Joint with design resistance lower than the full-strength resistance and which has not been classified as pinned.

	Classification by ductility
	Class 1 joint :
Sheet 8-1.E	Is recognised as *ductile* in Sheet 8-1.E.
	Class 2 joint :
Sheet 8-1.E	Is recognised as having an *intermediate* ductility in Sheet 8-1.E.
	Class 3 joint :
Sheet 8-1.E	Is recognised as *non-ductile* in Sheet 8-1.E.

Subject: Joint modelling

Design code : *Eurocode 3-(revised) Annex J*	**Sheet n°: 8-3**

Manual ref.	**Purpose and means**

Joint modelling for the frame analysis

Three types of joint modelling:

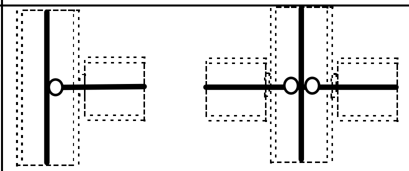
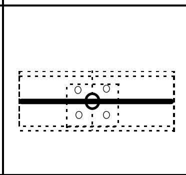
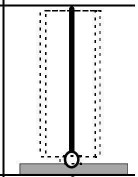
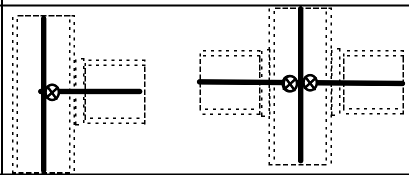
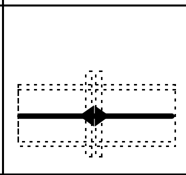
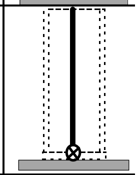
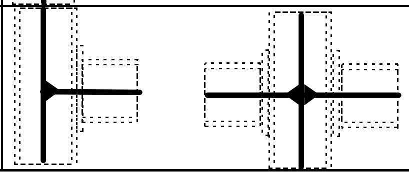
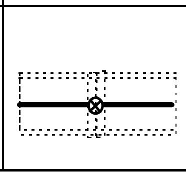
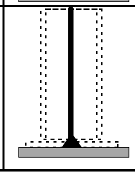
- Continuous
- Semi-continuous
- Simple

Sheet 8-2

The appropriate joint modelling depends on the type of frame analysis and on the stiffness and/or strength class of the joint; it is determined according to the following table :

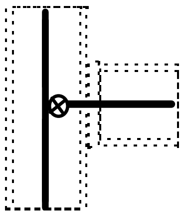
MODELLING	TYPE OF FRAME		
	Elastic analysis	Rigid-plastic analysis	Elastic-perfectly plastic and elastoplastic analysis
Continuous	Rigid	Full-strength	Rigid/full-strength
Semi-continuous	Semi-rigid	Partial-strength	Rigid/partial-strength Semi-rigid/full-strength Semi-rigid/partial-strength
Simple	Pinned	Pinned	Pinned

The local behaviour of the joint is schematically represented as follows :

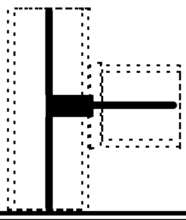
JOINT MODELLING	BEAM-TO-COLUMN JOINTS MAJOR AXIS BENDING	BEAM SPLICES	COLUMN BASES
SIMPLE			
SEMI-CONTINUOUS			
CONTINUOUS			

The physical representation of a semi-continuous joint is usually achieved through one of the following means:

- A <u>rotational spring</u> characterised by the stiffness $S_j^{(*)}$ and/or resistance $M_{j,Rd}$ properties of the joint



- A <u>short beam element</u> (with a length equal to half of the column depth for a major axis beam-to-column joint) with:
  - Flexural resistance equal to that of the joint (for instance: $M_{b,Rd} = M_{j,Rd}$)
  - Flexural stiffness equal to that of the joint (for instance, $EI_b/(h_c/2) = S_j$ where I_b and h_c are the second moment of area of the short beam element and the column depth respectively)



(*) This stiffness may be infinite in some cases.

Subject: Joint idealisation

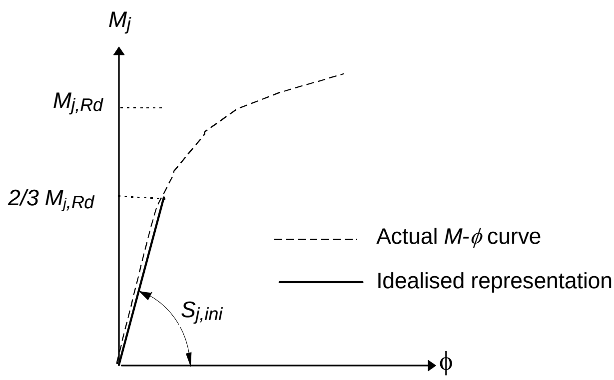
Design code : *Eurocode 3-(revised) Annex J*	Sheet n°: 8-4

Manual ref.	Purpose

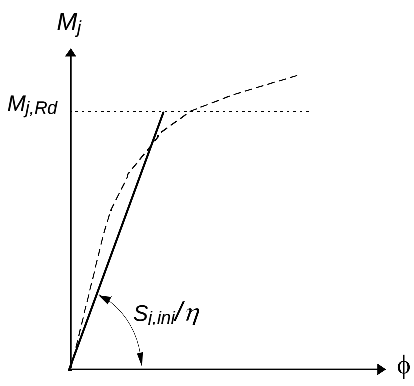
Idealisation of the joint response

Elastic frame analysis

- Elastic verification of the resistance of the joint

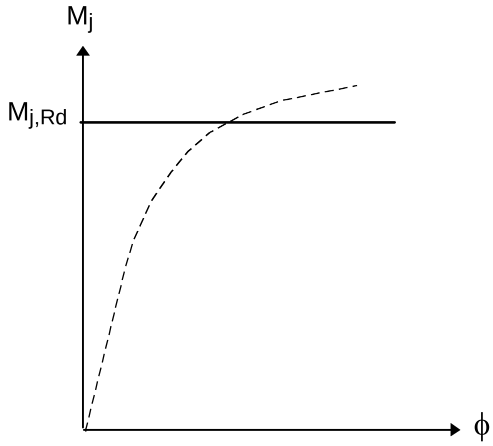


- Plastic verification of the resistance of the joint

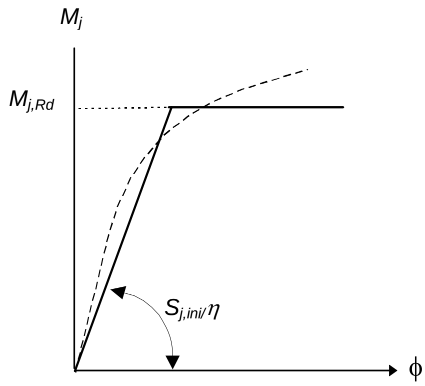


Type of connection	Beams-to-column joints	Other types of joints
Welded	2	3
Bolted end-plate	2	3
Bolted flange cleats	2	3,5

<table>
<tr><td></td><td>Values of the η coefficient</td></tr>
<tr><td>Annex 8-A</td><td>
Rigid-plastic frame analysis

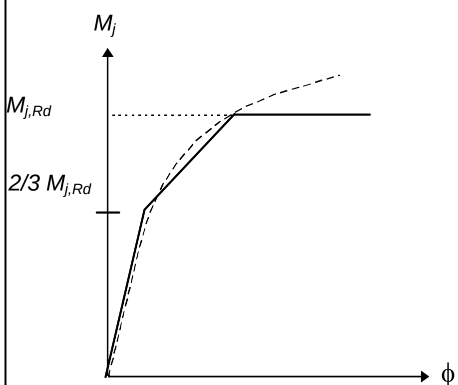


Elastic-perfectly plastic frame analysis



Elastoplastic frame analysis

- Tri-linear idealisation



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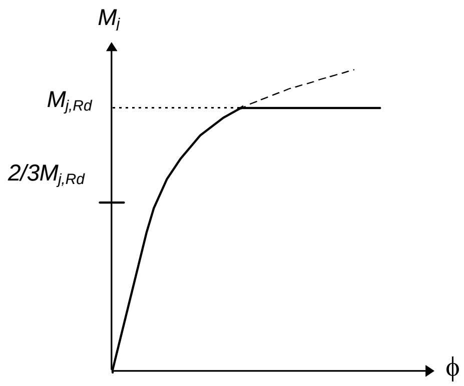
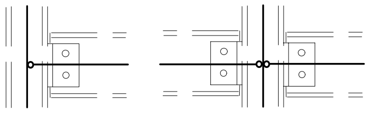
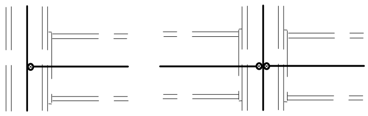
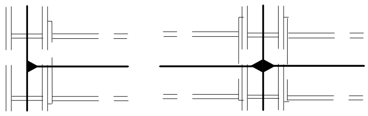
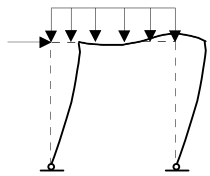
Annex 8-A	• Non-linear idealisation 

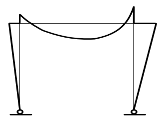
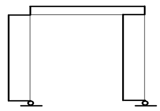
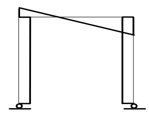
CHAPTER 9

GUIDELINES FOR GLOBAL FRAME ANALYSIS

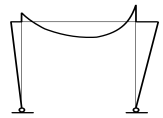
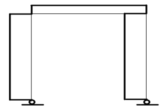
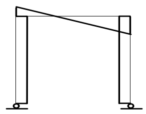
Subject : Joints		
Design Code : Eurocode 3		***Sheet n° : 9-1***
Manual ref.	***Purpose***	***Output***
Chapter 8	Joint modelling • Simple • Semi-continuous • Continuous	 Simple  Semi-continuous  Continuous
Manual ref.	***Analysis Routine***	***Device***
Chapter 4	Joint idealisation: • Idealize $M - \phi$ curve in accordance with the chosen method of analysis.	SPRING ELEMENT
Chapter 8	Joint characterisation: • Stiffness and/or strength.	
Chapter 4	Joint idealisation: • Idealize $M - \phi$ curve in accordance with the chosen method of analysis.	EQUIVALENT BEAM ELEMENT
Chapter 8	Joint characterisation: • Length, second moment of area and/or strength.	

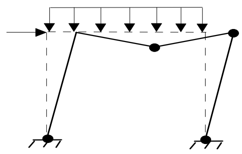
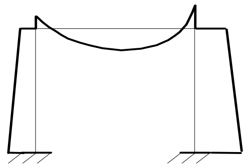
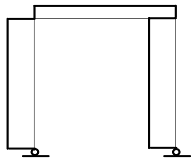
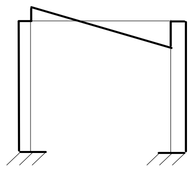
Subject : Joints		
Design Code : Eurocode 3		***Sheet n° : 9-1***
Manual ref.	***Purpose***	***Output***
	Before frame analysis, four separate actions have to be carried out :	
Chapter 8	• Joint characterization Evaluation of the properties of the joint : stiffness, resistance and/or rotation capacity	
Chapter 8	• Joint classification Definition of the class in terms of stiffness, resistance and/or ductility	
Chapter 8	• Joint modelling How to include joint response in global frame analysis	
Chapter 8	• Joint idealisation Idealisation of the M-ϕ joint response according to the method of analysis (elastic, rigid-plastic, elastic-plastic or elastoplastic)	

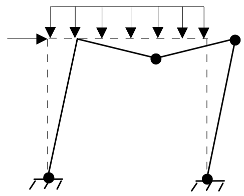
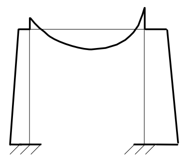
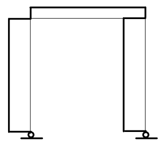
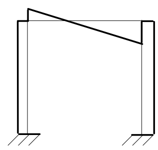
Subject : Elastic critical load in the sway mode		
Design Code : Eurocode 3		**Sheet n° : 9-2**
Code ref.	**Purpose**	**Output**
5.2.3.1.1	Assess the sensitivity of the structure to the *(P-Δ)* effects.	
5.2.5.2	Assessed by the ratio between the vertical load resultant producing sway instability (V_{cr}) and the actual design vertical load resultant (V_{Sd}).	Sway frame or Non sway frame for a given load case
	Values of this ratio shall be obtained for each load case.	
Manual ref.	***Analysis Routine for Calculation of V_{cr}***	***Tools***
	Specialized softwares: • Bifurcation analysis. • Second order step by step elastic analysis.	COMPUTER PROGRAM
	Joints shall normally be characterised by their initial design stiffness $(S_{j,ini})$.	
Annex 9-A	An approximate procedure is given in *Eurocode 3*.	EUROCODE 3 PROCEDURE
Annex 9-A	An approximate procedure is available.	APPROXIMATE PROCEDURE
	Charts are available for specific frame configurations (see for instance *Petersen*).	CHARTS OR FORMULAE

Subject : First-order elastic analysis at the ultimate limit state		
Design Code : Eurocode 3		**Sheet n° : 9-3**
Code ref.	***Purpose***	***Output***
5.2.1.3 *5.2.1.2.1*	May be used to calculate the distribution of design internal forces and moments in non sway frames.	 Bending moments
5.2.1.2.2 *5.2.6.2*	May also be used for sway frames provided allowance be made for the second-order effects (amplified moment method and/or sway buckling length method).	 Normal forces
5.2.4.1.3 *5.2.4.3.1*	Global frame imperfections shall be considered in the analysis of each load case or load combination case.	 Shear forces
Manual ref.	***Analysis Routine***	***Tools***
	Computer programs are widely available: • Computer programs shall include joint behaviour; otherwise proceed with the equivalent beam procedure (see also *Sheet 9-1*). Joints shall be characterised by their initial or nominal design stiffnesses, as appropriate.	COMPUTER PROGRAM
Annex 9-B	• Slope deflection method. • Moment distribution method. Both methods can be generalised so as to include the joint behaviour. Joints shall be characterised by their initial or nominal design stiffnesses, as appropriate.	HAND CALCULATION

Subject : Second-order elastic analysis at the ultimate limit state

Design Code : Eurocode 3		***Sheet n° : 9-4***
Code ref.	***Purpose***	***Output***
5.2.1.3	May be used in all cases.	
5.2.1.2.3	Required when the frame is classified as sway (see also *Sheet n°:9-3*).	Bending moment
5.2.4.1.3	Global frame imperfections shall be considered in the analysis of each load combination case.	 Normal forces
5.2.4.2.4 *5.2.4.3.1*	Local member imperfections shall be considered for very slender compression members/ may always be considered in order to avoid any further in-plane buckling check.	 Shear forces Include *P-Δ* effects and, when considered, *P-δ* effects.
Manual ref.	***Analysis Routine***	***Tools***
	Computer programs are widely available. Joints shall be characterised by their initial or nominal design stiffnesses, as appropriate.	COMPUTER PROGRAM
Annex 9-B	Iterative procedure. Joints shall be characterised by their initial or nominal design stiffnesses, as appropriate.	EQUIVALENT LATERAL LOAD
Annex 9-B	Non iterative procedure. Joints shall be characterised by their initial or nominal design stiffnesses, as appropriate.	SLOPE DEFLECTION METHOD

Subject : First-order plastic analysis at the ultimate limit state		
Design Code : Eurocode 3		***Sheet n° : 9-5***
Code ref.	***Purpose***	***Output***
5.2.1.1.4 *5.2.7* *5.3.3* *3.2.2.2* *5.2.1.2.1* *5.2.1.2.2* *5.2.1.4.1* *5.2.1.4.3* *5.2.1.4.4* *5.2.1.4.5* *5.2.4.1.3* *5.2.4.2.4* *5.2.4.3.1*	Especially appropriate for non sway frames May also be used for sway frames provided allowance be made for the second order effects (*Eurocode 3* method and *Merchant-Rankine* method). Requirements are imposed on the steel grades and on the cross-section and joint classes. Global frame imperfections shall be considered in the analysis of each load combination case.	 Collapse load multiplier.  Bending moment (ULS)  Normal forces (ULS)  Shear forces (ULS)
Manual ref.	***Analysis Routine***	***Tools***
	Computer programs available. Joints shall be characterised by their design moment resistances and, when the elastic behaviour is included, by their nominal stiffnesses.	COMPUTER PROGRAM
	• Virtual work method for rigid-plastic mechanisms. • Graphical method. Joints shall be characterised by their design moment resistances.	HAND CALCULATION

Subject : Second-order elastic perfectly-plastic analysis at the ultimate limit statel		
Design Code : Eurocode 3		***Sheet n° : 9-6***
Code ref.	***Purpose***	***Output***
5.2.1.1.4 *5.2.7* *5.3.3* *3.2.2.2* *5.2.1.2.3* *5.2.4.1.3* *5.2.4.2.4* *5.2.4.3.1*	May be used in all cases. It is required when the frame is classified as sway. Requirements are imposed on the steel grades, cross-section and joint classes. Global frame imperfections must be considered in the analysis of each load case. Local member imperfections shall be considered for very slender compression members, and may be considered in order to avoid a future in-plane buckling check.	 Collapse load multiplier.  Bending moment (ULS)  Normal forces (ULS)  Shear forces (ULS) Include the non linear behaviour of joints and sections, *P*-Δ effects and, when considered, *P*-δ effects.
Manual ref.	***Analysis Routine***	***Tools***
	Computer programs are widely available. Joints shall be characterised by their nominal design stiffnesses and design moment resistances.	COMPUTER PROGRAM

Annex 9-A : ASSESSMENT OF THE ELASTIC BUCKLING CRITICAL LOAD IN THE SWAY MODE

9-A.1 Eurocode 3 procedure

For plane frames in used in building structures, with beams connecting the columns at each storey level, the elastic critical buckling load for the sway buckling mode may be calculated according to the following procedure :

A first order elastic analysis is conducted for the given load combination case. The horizontal displacements due to the design loads (both horizontal and vertical ones) is determined at each storey. The elastic critical load of the frame for the sway buckling mode may be estimated by :

$$\frac{V_{Sd}}{V_{cr}} = Max\left[\frac{\delta}{h}\frac{V}{H}\right]_i$$

where :

i designation of the i[th] storey;

V_{Sd} design value of the resultant of the vertical loads;

V_{cr} elastic critical load for the sway buckling mode;

δ horizontal displacement at the top of the storey, relative to the bottom of the same storey;

h storey height;

H total horizontal reaction at the bottom of the storey;

V total vertical reaction at the bottom of storey.

9-A.2 Approximate procedure

A multi-storey multi-bay frame with semi-rigid joints can be replaced by an equivalent substitute single bay frame having rigid joints and columns and beams fitted with appropriate equivalent stiffnesses (Figure 9-A.1 (a)-(b).

It is assumed that the columns behave elastically and are continuous over their whole height. Accordingly the stiffness of the column at each storey is obtained as follows :

$$K_c = \frac{1}{2}\sum_j K_{c,j}$$

where $K_{c,j}$ is the stiffness coefficient of the column *j*, i.e. $I_{c,j}/L_{c,j}$.

The equivalent stiffness coefficient of the beam with linear end restraints at each storey is obtained as follows :

$$K_b = \sum_i K_{b,equi,i}$$

where :

$$K_{b,equi,i} = I_{b,equi,i}/l_{b,i}$$

in which :

$$I_{b,equ,i} = I_{b,i}/(1+3\alpha_i) \quad \text{and} \quad \alpha_i = 2EI_{b,i}/S_{j,ini,i}l_{b,i}$$

$EI_{b,i}/l_{b,i}$ flexural stiffness of the beam *i*;

$S_{j,ini,i}$ initial joint stiffness at the end of the beam *i* in the actual structure.

For a beam in which the joint stiffnesses are not the same at both ends, either the lowest joint stiffness is used (conservative) or an assessment has to be made in order to get an appropriate single value for the individual equivalent beam stiffness.

Since the so-called substitute frame has rigid joints, the associated *Grinter* frame can then be derived (Figure 9-A.1(b)-(c)). The stiffness of the members in the *Grinter* frame are :

$$K_b^* = 3\sum_i K_{b,equi,i} \quad \text{and} \quad K_c^* = \sum_j K_{c,j}$$

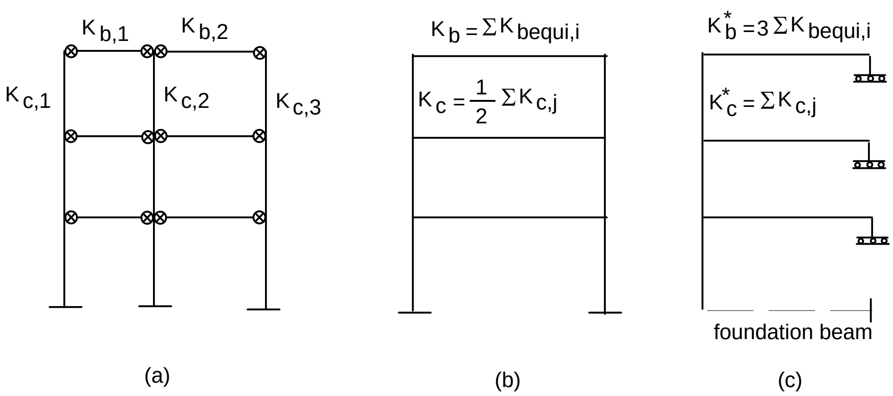


Figure 9-A.1(a) Actual frame (semi-rigid joints), (b) Substitute frame (rigid joints),(c) *Grinter* frame

The elastic critical load of the actual frame with semi-rigid joints can be computed by referring to the associated *Grinter* frame. The computation steps are as follows :

1. The elastic critical load of each column, V_{cr}^* ,is computed on the basis of the buckling length for the non sway buckling mode but taking the end restraints into account (see *Eurocode 3- Annex E*).

Each column of the *Grinter* frame is thus characterised by a value of V_{cr}^*. The lowest of all these values, i.e. $V_{cr,min}^*$, is selected as being a safe lower bound for the elastic critical load of the whole *Grinter* frame and thus of the whole actual frame.

Annex 9-B : METHODS FOR ELASTIC GLOBAL ANALYSIS

9-B.1 First order theory

9-B.1.1 Displacement method

This method, which has been the subject of many publications, is explained here very concisely.

The displacement method states that the forces $\{\overline{Q}\}$ applied to the nodes of the structure consist of those associated with the nodal displacement $\{D\}$, i.e. $[K]\{D\}$, and those due to the loads acting on the members fitted with fixed ends, i.e. $\{Q\}$. This is expressed by the following matrix equilibrium equation:

$$\{\overline{Q}\} = [K]\{D\} + \{Q\}$$

The structure stiffness matrix $[K]$, which is a square matrix of an order equal to the number of degrees of freedom of the structure, is assembled from the stiffness matrix $[k^i]$ of the individual members.

For a first order elastic analysis, this matrix does not include coefficients which would account for relative rotations at the beam ends and for changes in the column flexural stiffness due to axial loads.

Above matrix equation is solved for the unknown displacements $\{D\}$, which are then used to determine the forces acting on each individual member.

Softwares based on the displacement method constitute the mainstay of structural analysis in modern design office practice. Many of these programs are suited for rigid frame analysis only, since they do generally not permit joint flexibility as far as appropriate modifications are not introduced in the terms of the stiffness matrix. In the following section, some straightforward modifications are outlined which would enhance considerably the analytical capabilities of many existing tools for global plane frame analysis.

9-B.1.2 Element stiffness

The stiffness of a beam element is obtained as:

$$K_{ij}^{SR} = \varepsilon . S_{ij} . K_{ij}^{R} \qquad \varepsilon = \frac{1}{1 + 4\alpha + 3\alpha^2}$$

where :

$$K_{ij} = \text{ element stiffness } \begin{cases} K^{R} & \text{rigid joint} \\ K^{SR} & \text{semi-rigid joint} \end{cases}$$

$$S_{11} = S_{33} = (1+\alpha) \qquad \alpha = \frac{2EI_b}{S_j L}$$

$$S_{22} = S_{44} = \left(1+\frac{3}{2}\alpha\right) \qquad \frac{2EI_b}{L} \text{ is the beam flexural stiffness}$$

$$S_{22} = 1 \qquad S_j \text{ is the joint design flexural stiffness}$$

$$\left.\begin{array}{l} S_{12} = S_{34} \\ S_{31} = S_{41} \\ S_{32} \end{array}\right\} = (1+\alpha)$$

$$S_{ij} = S_{ji}$$

9-B.1.3 Fixed end moments

The fixed end forces on the member ends due to gravity loads are :

$$\begin{aligned} Q_1^{SR} &= Q_1^{R} \\ Q_2^{SR} &= -Q_2^{R}\left(\frac{1}{1+\alpha}\right) \\ Q_3^{SR} &= Q_3^{R} \\ Q_4^{SR} &= -Q_4^{R}\left(\frac{1}{1+\alpha}\right) \end{aligned} \qquad Q_i^{SR} = \text{fixed end forces } \begin{cases} \{Q\}^{R} & \text{rigid restraint} \\ \{Q\}^{SR} & \text{semi-rigid restraint} \end{cases}$$

9-B.1.4 Slope deflection method

The basic equations of the slope-deflection method give the end moments in a member as the superimposition of the end moments due to external loads on the member with its ends restrained, on the one hand, and of the moments caused by the actual end displacements and rotations, on the other hand.

A set of simultaneous equations is written, that express the equilibrium of the joints, in which the end moments are expressed in terms of the joint displacements and rotations.

The solution of these equations gives the unknown joint displacements and rotations. Substituting the latter into the original slope-deflection equations provides the designer with the end moments in the members.

The slope deflection method is in fact an application of the more general stiffness method, in which the effects of axial and shear strain energy are neglected compared to the bending strain energy. This simplification is usually justified when analysing frames.

Figure 9-B.1 shows the symbols adopted as well as the assumptions for the positive sign of moments and shear forces.

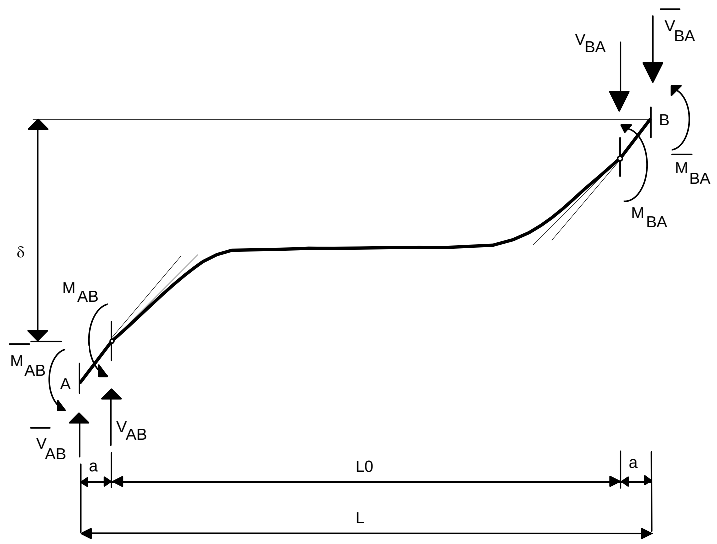


Figure 9-B.1 Deformation of beam with flexible end joints

The basic slope deflection equations for a member with identical joints at both ends may then be written as follows :

$$\overline{M}_{AB} = \varepsilon\left[R^*_{AB}\left(S_{AA}\theta_A + S_{AB}\theta_B - S_{AC}\psi_{AB}\right) - F_{AA}M_{AB} - F_{AB}M_{BA}\right] - V_{AB}a$$

$$\overline{M}_{BA} = \varepsilon\left[R^*_{AB}\left(S_{BA}\theta_A + S_{BB}\theta_B - S_{BC}\psi_{AB}\right) - F_{BA}M_{AB} - F_{BB}M_{BA}\right] - V_{BB}a$$

with, in addition to the symbols defined in the Figure 9-B.1 :

$\overline{M}_{AB}$ and $\overline{M}_{BA}$	moments at the center of nodes A and B;
V_{AB} and V_{BA}	shear forces at the ends of the simply supported member of length L_0;
M_{AB} and M_{BA}	fixed end moments due to transverse loads applied between the beam ends;
$R^*_{AB} = \dfrac{2EI_b}{L_0}$	beam flexural stiffness;

$\varepsilon = \dfrac{1}{1 + 4\alpha + 3\alpha^2}$ with $\alpha = \dfrac{2EI_b}{L_0 S_j}$ where S_j is the joint design flexural stiffness;

$$\psi_{AB} = \frac{\delta}{L_0}$$

The values of the coefficients S_{AA}, S_{AB}, S_{BA}, S_{BB}, S_{AC}, S_{BC}, F_{AA}, F_{AB}, F_{BA} and F_{BB} are given in Table 9-B.1.

Symbol	Expression when beam+ rigid stub of length *a*	Expression when zero stub length
$S_{AA} = S_{BB}$	$2+3\alpha+6(1+\alpha)\dfrac{a}{L_0}\left(1+\dfrac{a}{L_0}\right)$	$2+3\alpha$
$S_{AB} = S_{BA}$	$1+6(1+\alpha)\dfrac{a}{L_0}\left(1+\dfrac{a}{L_0}\right)$	1
$S_{AC} = S_{BC}$	$3(1+\alpha)\left(1+2\dfrac{a}{L_0}\right)$	$3(1+\alpha)$
$F_{AA} = F_{BB}$	$1+2\alpha+(1+\alpha)\dfrac{a}{L_0}$	$1+2\alpha$
$F_{AB} = F_{BA}$	$\alpha-(1+\alpha)\dfrac{a}{L_0}$	α

Table 9-B.1 Semi-rigid joints - Slope-deflection method

The expressions given for the M_{AB} and M_{BA} are introduced in the usual node and storey equilibrium equations of the slope-deflection method. The solution of these latter gives the values of the moments at the nodes; the corresponding shear forces can be computed by the following relationships :

$$\overline{V}_{AB} = -\frac{(M_{AB}+M_{BA})}{L} + V_{AB}$$

$$\overline{V}_{BA} = -\frac{(M_{AB}+M_{BA})}{L} + V_{BA}$$

where :

V_{AB} and V_{BA} shear forces at the ends of the simply supported beam of span L.

$\overline{V}_{AB}$ and $\overline{V}_{BA}$ which are constant in the zone of the joints, represent the values of the shear forces at the joint. The values of the moments at the joints are derived from the equilibrium equations of the rigid beam end stubs of length *a*:

$$\overline{M}_{AB} = M_{AB} + \overline{V}_{AB}.a$$
$$\overline{M}_{BA} = M_{BA} + \overline{V}_{BA}.a$$

It is usually sufficiently accurate to disregard the joint size compared to the beam length and thus to neglect the stub length *a.*

The basic equations then become:

$$\overline{M}_{AB} = \varepsilon\left[R^*_{AB}\left((2+3\alpha)\theta_A + \theta_B - 3(1+\alpha)\psi_{AB}\right) - (1+2\alpha)M_{AB} - \alpha M_{BA}\right]$$
$$\overline{M}_{BA} = \varepsilon\left[R^*_{AB}\left(\theta_A + (2+3\alpha)\theta_B - 3(1+\alpha)\psi_{AB}\right) + \alpha M_{AB} + (1+2\alpha)M_{BA}\right]$$

The effective stiffness $K_{b,eff}$ of a member with linear elastic restraints, for which the loading conditions produce a single bending curvature, is obtained by introducing $\theta_A = -\theta_B$ and $\delta = 0$. One obtains :

$$K_{b,eff} = \frac{R^*_{AB}}{1+\alpha}$$

The effective stiffness $K_{b,eff}$ of a member with linear elastic restraints, for which the loading conditions produce a double bending curvature, is obtained by introducing $\theta_A = \theta_B$ and $\delta = 0$. One obtains:

$$K_{b,eff} = \frac{R^*_{AB}}{1+3\alpha}$$

9-B.1.5 Moment distribution method

The moment distribution method is a particular application of the slope-deflection method. The sole difference lies in the way the equations of equilibrium are solved. In the slope-deflection method, use is made of any of available analytical tools for solving sets of equations, while the moment distribution method proceeds by a relaxation procedure .

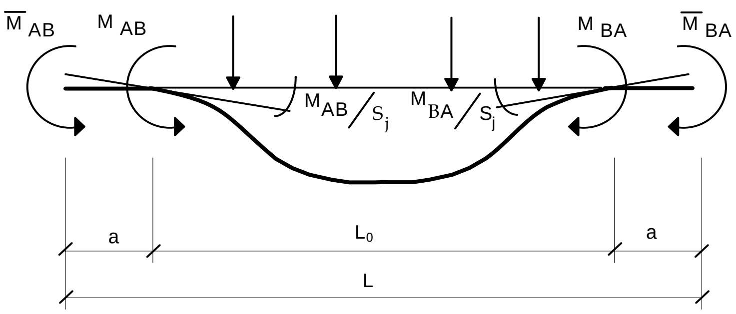


Figure 9-B.3 Semi-fixed end moment

$$V_A = 1 \qquad V_B = 0 \qquad \delta = 0$$

Symbol	Expression when beam + rigid stub of length *a*	Expression when zero stub length
$\overline{M}_{SAB}$ semi-fixed end moments at the joint centre	$\dfrac{\left[1+2\alpha+\dfrac{a}{L_0}(1+\alpha)\right]M_{AB}+\left[\alpha+\dfrac{a}{L_0}(1+\alpha)\right]M_{BA}}{1+4\alpha+3\alpha^2}+V_{AB}.a$	$\dfrac{(1+2\alpha)M_{AB}+\alpha.M_{BA}}{1+4\alpha+3\alpha^2}$
$\bar{r}_{AB}$ carry over factor between the joint centre	$\dfrac{1+6(1+\alpha)\dfrac{a}{L_0}\left(1+\dfrac{a}{L_0}\right)}{2+3\alpha+6(1+\alpha)\dfrac{a}{L_0}\left(1+\dfrac{a}{L_0}\right)}$	$\dfrac{1}{2+3\alpha}$
$\overline{K}_M$ end rotation stiffness	$\dfrac{2EK_b\left[2+3\alpha+6(1+\alpha)\dfrac{a}{L_0}\left(1+\dfrac{a}{L_0}\right)\right]}{1+4\alpha+3\alpha^2}$	$\dfrac{2EK_b(2+3\alpha)}{1+4\alpha+3\alpha^2}$
$\overline{K}_S$ side-sway stiffness	$\dfrac{12EK_b}{L_0^2}\left[\dfrac{1+\alpha}{1+4\alpha+3\alpha^2}\right]$	$\dfrac{12EK_b}{L_0^2}\left[\dfrac{1+\alpha}{1+4\alpha+3\alpha^2}\right]$
$\overline{M}_{VAB}$ side-sway end moment at node *A*	$-\dfrac{6EK_b}{L_0}\left[\dfrac{1+\alpha+2\dfrac{a}{L_0}(1+\alpha)}{1+4\alpha+3\alpha^2}\right]$	$-\dfrac{6EK_b}{L_0}\left[\dfrac{1+\alpha}{1+4\alpha+3\alpha^2}\right]$

Table 9-B.2 Semi-rigid joints - Moment distribution method

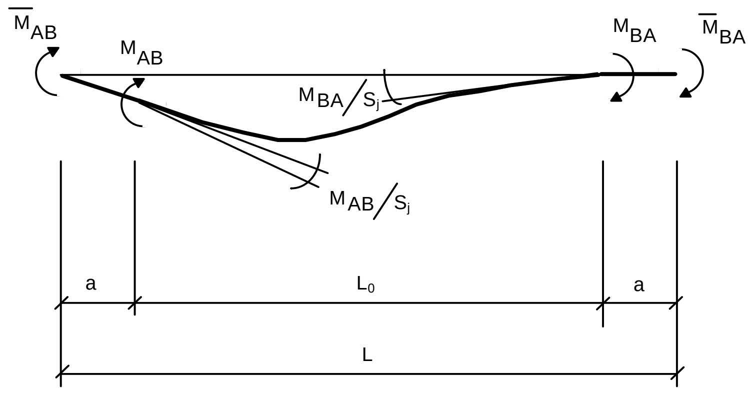


Figure 9-B.4 Moment carry over and rotation stiffness

When the moment distribution method is used, one must determine the carry-over factors, the rotation and side-sway stiffness factors and the side-sway end moments. For frames with flexible joints, the moment distribution method requires the definition of the semi-fixed-end moments (Figure 9-B.3, Figure 9-B.4 and Table 9-B.2).

The relaxation procedure is identical to that for frames with rigid joints.

9-B.2 Second order theory

9-B.2.1 Equivalent lateral load procedure

The *Equivalent lateral load* procedure consists in an iterative procedure, using the results of a standard first order analysis, to include the sway displacement effects.

The initial step in the modification is to compute the sway forces. The lateral and vertical loads are applied to the system, and the lateral displacements, denoted as Δ_i in Figure 9-B.5 (a), are computed by first order theory. The storey level is denoted by *i*. The additional storey shears due to the vertical loads are computed by :

$$V_i^{'} = \frac{\sum P_i}{h_i}\left(\Delta_{i+1} - \Delta_i\right)$$

in which :

$V_i^{'}$	additional shear in storey *i* due the sway forces;
$\sum P_i$	sum of the column axial loads in storey;
h_i	height of storey *i*;
Δ_{i+1}, Δ_i	displacements of levels *i+1* and *i* respectively.

The sway forces due to the vertical loads $H_i^{'}$ are then computed as the difference between the additional storey shears at each level, i.e.:

$$\mathrm{H}_\mathrm{i}^{'} = \mathrm{V}_{\mathrm{i}-1}^{'} - \mathrm{V}_\mathrm{i}^{'}$$

The sway forces $H_i^{'}$ are added to the applied lateral loads, and the structure is re-analysed using the first order theory. When the Δ_i values at the end of cycle are nearly equal to the previous cycle, the method has sufficiently converged. If it does not converge within five or six trials, it can be concluded that the structure is unstable. Once convergence is established, the resulting forces or moments in every member now include the *P-Δ* effects.

The method is summarised in Figure 9-B.5.(b).

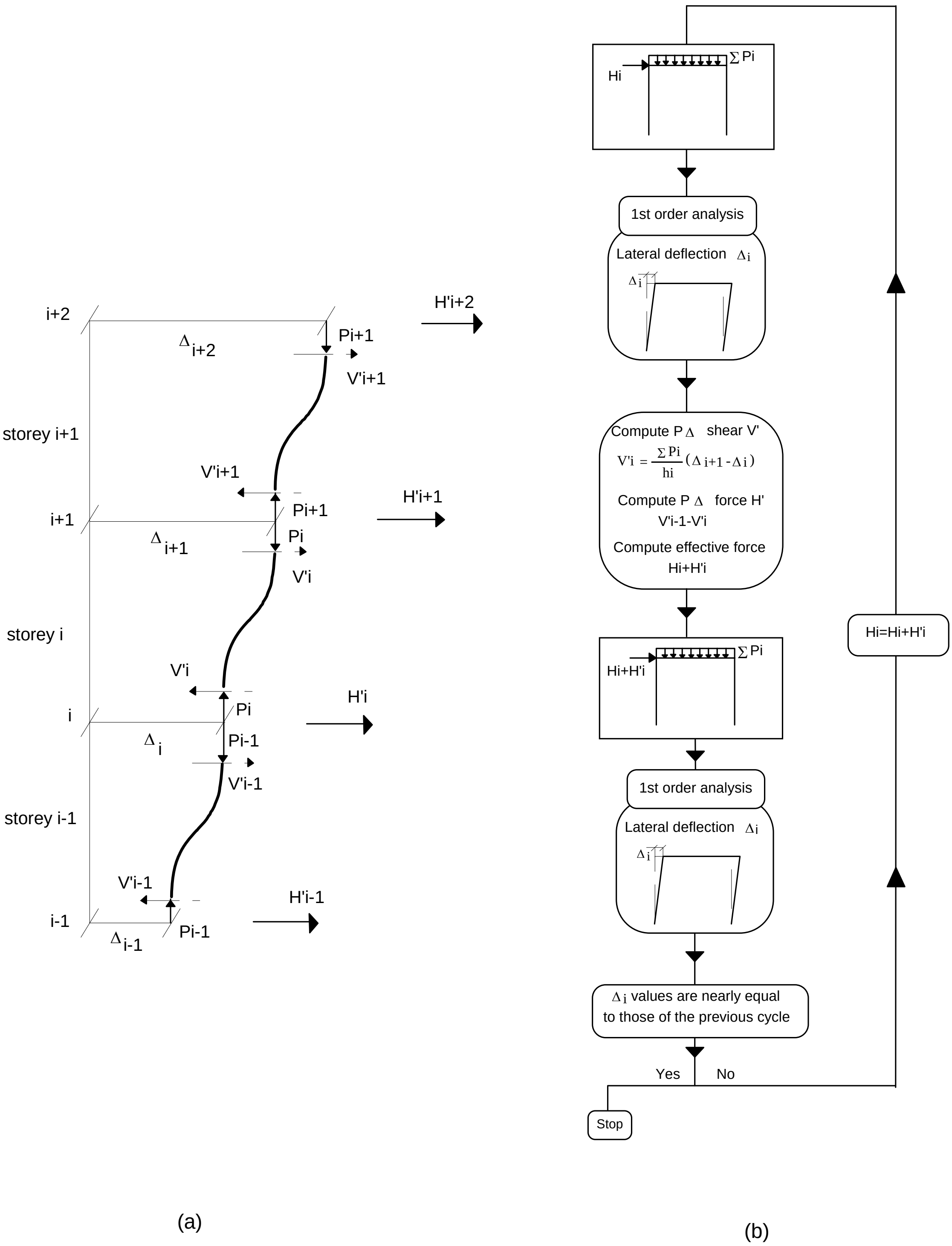


Figure 9-B.5 Equivalent lateral force procedure

9-B.2.2 Slope-deflection method

Only the equation for the equilibrium of translation at each storey need to be modified in order to account for global second order effects. This equation is transformed by means of the following one:

$$\frac{1}{h_i}\sum_i\left(\overline{M}_{AB}+\overline{M}_{BA}\right)-\psi_i\sum_i V=\sum_i H-\sum_i R_H$$

where $\sum_i V$ is the sum of the vertical components of all the loads acting on the frame above the plane (aa) as shown in Figure 9-B.6.

Setting up the equation is not any more difficult than for the usual application of the slope deflection equations, and the solving procedure is precisely the same. One obtains the system of equations which is linear in terms of the joint rotations θ and in terms of sway angles ψ.

Solving the former system immediately provides the values of the joint rotations θ, the sway angles ψ, and the bending moments, including the global second order effects.

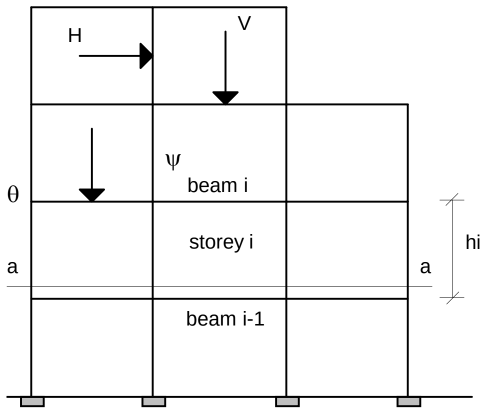


Figure 9-B.6

PART 3 : WORKED EXAMPLES

CHAPTER 10

WORKED EXAMPLE 1

DESIGN OF A BRACED FRAME

10.1 Frame geometry and loading

Figure 10.1 shows a non-sway braced frame. The frame consists of two storeys and two bays. The beam span is 7,2 m. The height from column foot to the beam at floor level is 4,5 m, the height from floor to roof is 4,2 m. It is assumed that the column foot is pinned to the foundation.

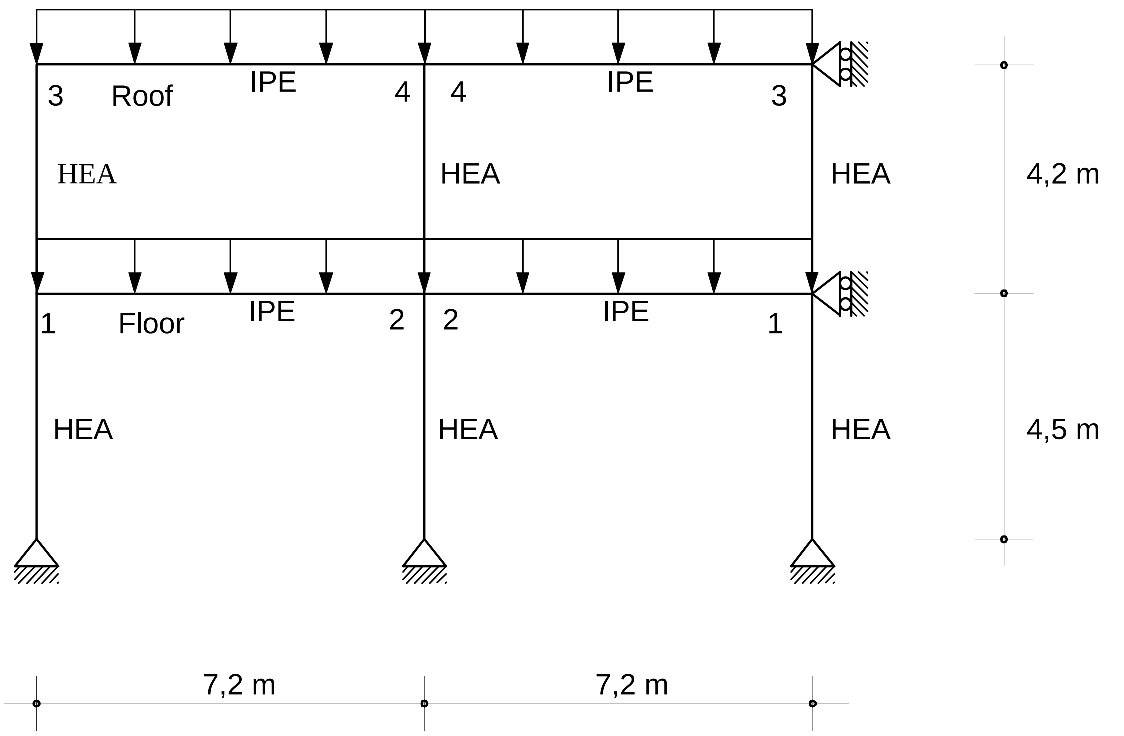


Figure 10.1 Geometry of the braced frame

The following load case, corresponding to dead and life load (no horizontal loads) is governing; the design loads include the partial safety factors for actions.

Design loads	Roof	Floor
Serviceability limit state	40 kN/m	59 kN/m
Ultimate limit state	54 kN/m	81 kN/m

Table 10.1 Governing load case

The steel grade chosen for beams, columns and joints is *S235*, with f_y = 235 N/ mm^2. The following partial safety factors for strength have been adopted during the design :

$$\gamma_{M0} = 1,1;$$

$$\gamma_{M1} = 1,1;$$

$$\gamma_{Mb} = 1,25.$$

The columns are HEA sections. They are fully supported against lateral torsional buckling. The columns are continuous from column foot to roof. They are supported against out-of-plane movement at foot, floor and roof levels.

Beams are IPE sections. The beams are supported against lateral torsional buckling; they have no initial camber.

All the sections are fulfilling the requirements for Class 1 sections.

10.2 Objectives and design steps

In this example, the beams, columns and joints will be designed. For this, three solutions will be studied :

- Frame design with pinned joints;
- Frame design with semi-rigid joints;
- Frame design with partial-strength joints.

One starts with the design of a simple frame with pinned joints (Section 10.3). Then, one introduces joints having a certain rotational stiffness and moment resistance; this allows probably lower beam sizes. To see whether this expectation is confirmed, an elastic analysis (Section 10.4) and a plastic analysis (Section 10.5) have been respectively carried out.

10.3 Frame design with pinned joints

10.3.1 Introduction

Frame design with pinned joints is typical when the members are designed by an engineer and the joints in a subsequent step by the steel fabricator. The steps described in Sections 10.3.2 to 10.3.4.2 are normally performed by the engineer; the step described in Section 10.3.5 is the task of the steel fabricator (refer to Chapters 2 and 6 for more explanation in this respect).

10.3.2 Preliminary design of beams and columns

The following sizes for beams and columns are chosen :

- Floor beams : IPE 550
- Roof beams : IPE 450
- Outer columns : HE 200 A
- Inner columns : HE 240 A.

10.3.3 Frame analysis

10.3.3.1 Serviceability limit state

The maximum deflection amounts :

$$u = \frac{5 q_{Sd} l_b^4}{384 E I_b}$$

where :

u deflection;

E Young modulus equal to 210000 N/mm^2;

I_b second moment of area of the beam;

q_{Sd} design uniformly distributed load at serviceability limit state.

Floor beams : $\dfrac{5 \times 59 \times 7200^4}{384 \times 210000 \times 67120 \times 10^4} = 14{,}6$ mm

Roof beams : $\dfrac{5 \times 40 \times 7200^4}{384 \times 210000 \times 33740 \times 10^4} = 19{,}8$ mm

10.3.3.2 Ultimate limit state

In the case of a frame with pinned joints, the frame analysis is based on simple equilibrium equations.

The maximum moment in the beam is :

$$M_{Sd} = 1/8\ q_{Sd}\ l_b^2$$

where :

M_{Sd} moment in the beam span;

q_{Sd} design uniformly distributed load at ultimate limit state;

l_b beam span.

Floor beams : $M_{Sd} = 1/8 \times 81 \times 7{,}2^2 = 525$ kNm

Roof beams : $M_{Sd} = 1/8 \times 54 \times 7{,}2^2 = 350$ kNm

The maximum axial force in the column is :

$$N_{Sd} = \Sigma\ 1/2\ q_{Sd}\ l_b$$

where :

N_{Sd} axial force in the columns;

Σ summation sign for all the connected beams at all floor and roof levels.

Outer columns : $N_{Sd} = 1/2 \times 81 \times 7{,}2 + ½ \times 54 \times 7{,}2 = 486$ kN

Inner columns : $N_{Sd} = 2 \times (1/2 \times 81 \times 7{,}2 + 1/2 \times 54 \times 7{,}2) = 972$ kN

10.3.4 Design checks

10.3.4.1 Serviceability limit state

To reduce the volume of calculations reported here, only the δ_{max} limit (see *Eurocode 3- Figure 4.1*) has been checked :

$u \leq \delta_{max} = l_b / 250$ (floor beam) and $u \leq \delta_{max} = l_b / 200$ (roof beam)

Floor beam : 14,6 mm $\leq$ 72000 mm / 250 = 28,8 mm

Roof beam : 19,8 mm $\leq$ 72000 mm / 200 = 36 mm

$\Rightarrow$ Satisfactory.

10.3.4.2 Ultimate limit state

To reduce the volume of the worked example, the check of the ultimate limit state is limited to the following aspects :

- Column stability;
- Cross-sectional checks of beams and columns.

10.3.4.2.1 Column stability

In accordance with *Eurocode 3-Clause 5.5.1.1*, the criterion to be fulfilled is :

$$N_{Sd} \leq \chi\, \beta_A\, A\, f_y / \gamma_{M1}$$

where :

N_{Sd} acting compressive force in the column;

χ reduction factor for the relevant buckling mode, i.e. χ_z in this case;

β_A = 1 for Class 1 cross-sections;

γ_{M1} partial safety factor, taken equal to 1,1;

A cross-sectional area of the column.

The value of χ depends on the reduced slenderness of the columns :

$$\bar{\lambda} = \lambda / \lambda_1 (\beta_A)^{1/2}$$

where :

λ $= l / i_z$

λ_1 = 93,9 (steel grade *S235*);

i_z radius of gyration;

l column buckling length (taken here equal to the system length).

Outer columns HE 200 A :

$\lambda = l / i_z$ = 4500 / 49,8 = 90,4

$\bar{\lambda} = \lambda / \lambda_1 (\beta_A)^{1/2}$ = 90,4 / 93,9 = 0,96 hence χ = 0,58 (buckling curve c).

N_{Sd} = 486 kN $\leq \chi\, \beta_A\, A\, f_y / \gamma_{M1}$ = 0,58 · 5380 · 235 / 1,1 = 666 kN.

$\Rightarrow$ Satisfactory.

Inner columns HE 240 A:

$\lambda = l \,/\, i_z = 4500 / 60{,}0 = 75$

$\bar{\lambda} = \lambda \,/\, \lambda_1 (\beta_A)^{1/2} = 75 / 93{,}9 = 0{,}80$ hence $\chi = 0{,}66$ (buckling curve c).

$N_{Sd} = 972$ kN $\leq \chi\, \beta_A\, A\, f_y \,/\, \gamma_{M1} = 0{,}66 \cdot 7680 \cdot 235 / 1{,}1 = 1082$ kN

$\Rightarrow$ Satisfactory.

10.3.4.2.2 Section check of the beams

The check of beam cross-sections is conducted in accordance with *Eurocode 3-Clause 5.4.5.1*. Because the beam sections are Class 1, a plastic verification is permitted :

$$M_{Sd} \leq M_{c.Rd} = W_{pl}\, f_y \,/\, \gamma_{M0}$$

where W_{pl} is the plastic section modulus.

Floor beam : 525.10^6 Nmm $\leq 2780.10^3 \cdot 235 / 1{,}1 = 593.10^6$ Nmm.

Roof beam : 350.10^6 Nmm $\leq 1702.10^3 \cdot 235 / 1{,}1 = 364.10^6$ Nmm,.

$\Rightarrow$ Satisfactory.

10.3.4.2.3 Section check of the columns

The section check of the columns is covered by the buckling check carried out in Section 10.3.4.2.1.

10.3.5 Design of joints

Joints may be designed as simple joints, e.g.

- Web cleated joints;
- Fin plate joints;
- Flexible end-plate joints (thin end-plates only welded to the web of the beam).

These types of connections need to be designed for shear force only. Detailing should be such that the rules of *Eurocode 3-Chapter 6* are satisfied.

10.4 Frame design with semi-rigid joints

10.4.1 Introduction

In this chapter, just like in Section 10.3, the members can be designed by the engineer and the joints by the steel fabricator. During member design (Sections 10.4.2 to 10.4.5), the effect of the joints on the frame behaviour is taken into account by assuming that the engineer makes a good assessment of the mechanical properties of the joints. In the subsequent step of the joint design (Section 10.4.6), the steel fabricator needs to ensure that the mechanical properties of the joints are close enough to the assumptions made by the engineer (See also Chapters 2 and 6 for more explanations).

10.4.2 Preliminary design of beams, columns and joints

Column sizes will be chosen as found in Section 10.3. However, we will try to save on the beam sizes. Therefore, beams are chosen one section size lower than in the case of the frame with pinned joints :

- Floor beams : IPE 500
- Roof beams : IPE 400
- Outer columns : HE 200 A
- Inner column : HE 240 A.

For the joints, extended end-plate connections will be contemplated. A first assessment of the initial stiffness of these joints is made using the following formula :

$$S_{j.app} = \frac{E z^2 t_{f.c}}{k_x}$$

where :

$S_{j.app}$ approximate initial stiffness of the joint;

k_x coefficient taken from Table 10.2;

z distance between the compression and tension resultants. For extended end-plate joints, this distance equals approximately the beam height;

$t_{f.c}$ column flange thickness;

E Young modulus (= 210.000 N/mm^2).

The values of $S_{j.app}$ are listed in Table 10.2.

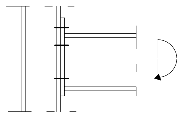
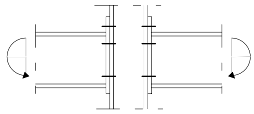
Type of joint		k_x
Extended end-plate, single sided		13
Extended end-plate, double sided symmetrically		7,5

Table 10.2 k_x-factor for different types of joints

	Joint	$S_{j.app}$ (Nmm/rad)
1	IPE 500 floor HE200A column	$\frac{210000 \times 500^2 \times 10}{13} = 40.10^9$
2	IPE 500 floor HE240A column	$\frac{210000 \times 500^2 \times 10}{7,5} = 84.10^9$
3	IPE 400 roof HE200A column	$\frac{210000 \times 400^2 \times 10}{13} = 26.10^9$
4	IPE 400 roof HE240A column	$\frac{210000 \times 400^2 \times 12}{7,5} = 54.10^9$

Table 10.3 Approximate initial joint stiffness

10.4.3 Frame analysis

In this case, a first order linear frame analysis is carried out. In accordance with *Eurocode 3-(revised) Annex J*, half of the initial stiffness of the joint is introduced in the frame analysis at the ultimate limit state. Since serviceability limit state is not governing the design of the frame, the same value of the joint stiffness has been used for this limit state.

In this worked example, it is assumed that the available frame analysis software cannot reflect the stiffness of the joint by means of a spring element. For this reason, the stiffness of the joint is modelled with a short beam element. The second moment of area of this element is (See Figure 10.2) :

$$I_j = S_j \, l \, / \, E$$

where :

S_j half of the initial stiffness ($S_{j.app}/2$);

l short beam element length (about half the column depth, 100 mm);

E Young modulus taken equal to 210000 N/mm^2.

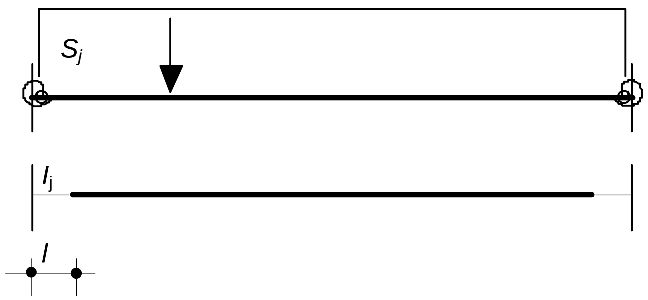


Figure 10.2 Modelling of joint stiffness S_j by an equivalent beam with length l

	Joint	$S_{j.app}$ (Nmm/rad)	I_j (mm^4)
1	IPE 500 floor HE200A column	40.10^9	95.10^4
2	IPE 500 floor HE240A column	84.10^9	2000.10^4
3	IPE 400 roof HE200A column	26.10^9	620.10^4
4	IPE 400 roof HE240A column	54.10^9	1280.10^4

Table 10.4 Calculation of the equivalent beam properties

10.4.3.1 Serviceability limit state

The results of the frame analysis program are as follows :

Floor beam : u = 12,2 mm.

Roof beam : u = 15,9 mm.

10.4.3.2 Ultimate limit state

The results of the frame analysis program are as follows (Figure 10.3) :

Outer columns : N_{Sd} = 444 kN M_{Sd} = 23 kNm

Inner column : N_{Sd} = 1055 kN M_{Sd} = 0 kNm

Floor beam : M_{Sd} = 360 kNm

Roof beam : M_{Sd} = 225 kNm

1 joint IPE 500 floor beam-to-HE200A column : M_{Sd} = 78 kNm

2 joint IPE 500 floor beam-to HE240A column : M_{Sd} = 251 kNm

3 joint IPE 400 floor beam-to HE200A column : M_{Sd} = 60 kNm

4 joint IPE 400 floor beam-to HE240A column : M_{Sd} = 188 kNm

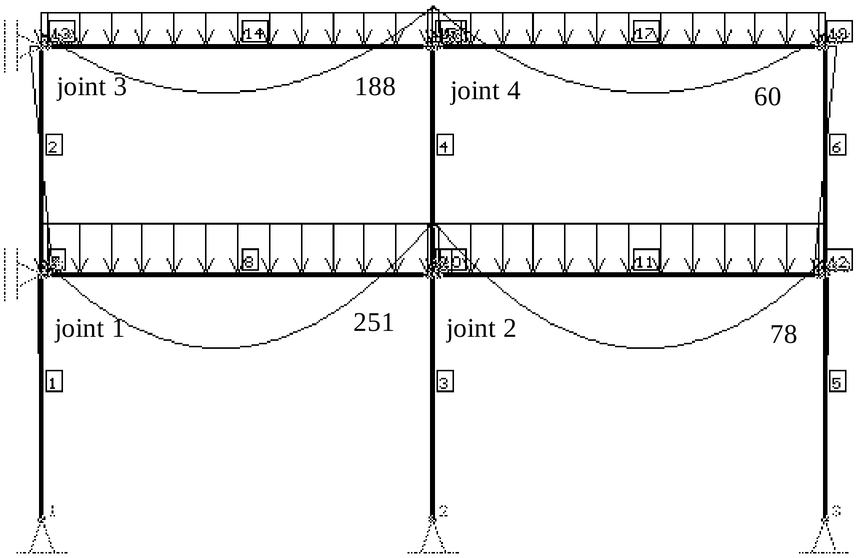


Figure 10.3 Moment distribution (ultimate limit state)

10.4.4 Design checks

10.4.4.1 Serviceability limit state

Only the δ_{max} limit of *Eurocode 3-Figure 4.1* has been checked here :

$u \leq \delta_{max} = l_b$ / 2500 (floor) and $u \leq \delta_{max} = l_b$ / 200 (roof)

Floor beam : 12,2 mm ≤ 72000 mm / 250 = 28,8 mm.

Roof beam : 15,9 mm ≤ 72000 mm / 200 = 36 mm.

⇒ Satisfactory.

10.4.4.2 Ultimate limit state

In this worked example, the check of the ultimate limit state is again limited to the following aspects :

- Column stability;
- Cross-sectional checks of beams and columns.

10.4.4.2.1 Column stability

Inner column HE 240A :

$$N_{Sd} \leq \chi\, \beta_A\, A\, f_y / \gamma_{M1}$$

N_{Sd} = 1055 kN ≤ $\chi\, \beta_A\, A\, f_y / \gamma_{M1}$ = 0,66 · 7680 · 235 / 1,1 = 1082 kN,.

⇒ Satisfactory.

Outer column HE 200 A :

It is assumed that lateral torsional buckling is not a possible failure mode.

(It has to be noted that Eurocode 3 is rather conservative concerning stability checks of I or H columns loaded with axial force and uni-axial bending about the strong-axis compared to tests and other national design standards (for example the German and the Dutch Standards).

According to *Eurocode 3-Clause 5.5.4,* the criterion to be fulfilled is :

$$\frac{N_{sd}}{\chi_{min} A f_y / \gamma_{M1}} + \frac{k_y M_{y.Sd}}{W_{pl.y} f_y / \gamma_{M1}} \leq 1$$

where :

χ_{min} the smaller of the χ values relative respectively to weak-axis and strong-axis buckling, here χ_z = 0,58;

k_y $= 1 - \frac{\mu_y N_{sd}}{\chi_y A f_y} = 1$

μ_y $= \bar{\lambda}_y (2\beta_{My} - 4) + (W_{pl.y} - W_{el.y}) / W_{el.y}$ but $\mu_y \leq 0{,}9$

$= 0{,}27 \times (2 \times 1{,}8 - 4) + (430.10^3 - 389.10^3) / 389.10^3 \quad = 0$

β_{My} determined based on *Eurocode-Figure 5.5.3*, in this case 1,8;

$\bar{\lambda}_y$ $= \lambda / \lambda_1 (\beta_A)^{1/2} = 25{,}6 / 93{,}9 = 0{,}27$

λ $= l / i_y = 4500 / 17{,}6 = 25{,}6$

λ_1 $= 93{,}9$

l column buckling length, i.e. 4500 mm.

$$\frac{444.10^3}{0{,}58 \times 5380 \times 235/11} + \frac{1 \times 23 \times 10^6}{430.10^3 \times 235/11} \leq 1$$

$0{,}67 + 0{,}24 = 0{,}91 \leq 1$.

$\Rightarrow$ Satisfactory.

10.4.4.2.2 Section check of the beams

Check of the beams with *Eurocode 3-Clause 5.4.5.1* :

IPE 500 floor beam :

360.10^6 Nmm $\leq 2200.10^3$ x 235 /1,1 = 517.10^6 Nmm.

$\Rightarrow$ Satisfactory.

IPE 400 roof beam :

225.10^6 Nmm $\leq 1308.10^3$ x 235 /1,1 = 307.10^6 Nmm.

$\Rightarrow$ Satisfactory.

10.4.4.2.3 Section check of the columns

Since weak-axis buckling is the governing failure mode for the inner column, no check of the inner column section needs to carried out.

The section check of the outer column can be carried out using *Eurocode 3 - Clause 5.4.8.1(3)* :

$$N_{Sd} / N_{pl.Rd} = \frac{444.10^3}{5380\, x\, 235/1{,}1} = 0.38 \leq 1$$

$$M_{Sd} / M_{y.pl} = \frac{23.10^6}{430.10^3\, x\, 235/1{,}1} = 0{,}25 \leq 1$$

Since $N_{Sd} / N_{pl.Rd} \leq 0.5$, there is no need to check the interaction between axial force and bending moment.

⇒ Satisfactory.

10.4.5 Design of the joints

Joints have been designed using the DESIMAN software (Table 10.5). Extended end-plates have been contemplated for all the joints. This was unsuccessful because extended end-plate joints possess insufficient strength to transfer the bending moments got from the frame analysis. Therefore, haunched connections have been chosen, see Figure 10.4. This type of connection has the advantage that column web stiffeners can be avoided whilst the web remains free for erection of beams out of the plane of the frame. All the end-plates have been chosen 20 mm thick. Bolts are M20 8.8. All the connections have four bolt-rows.

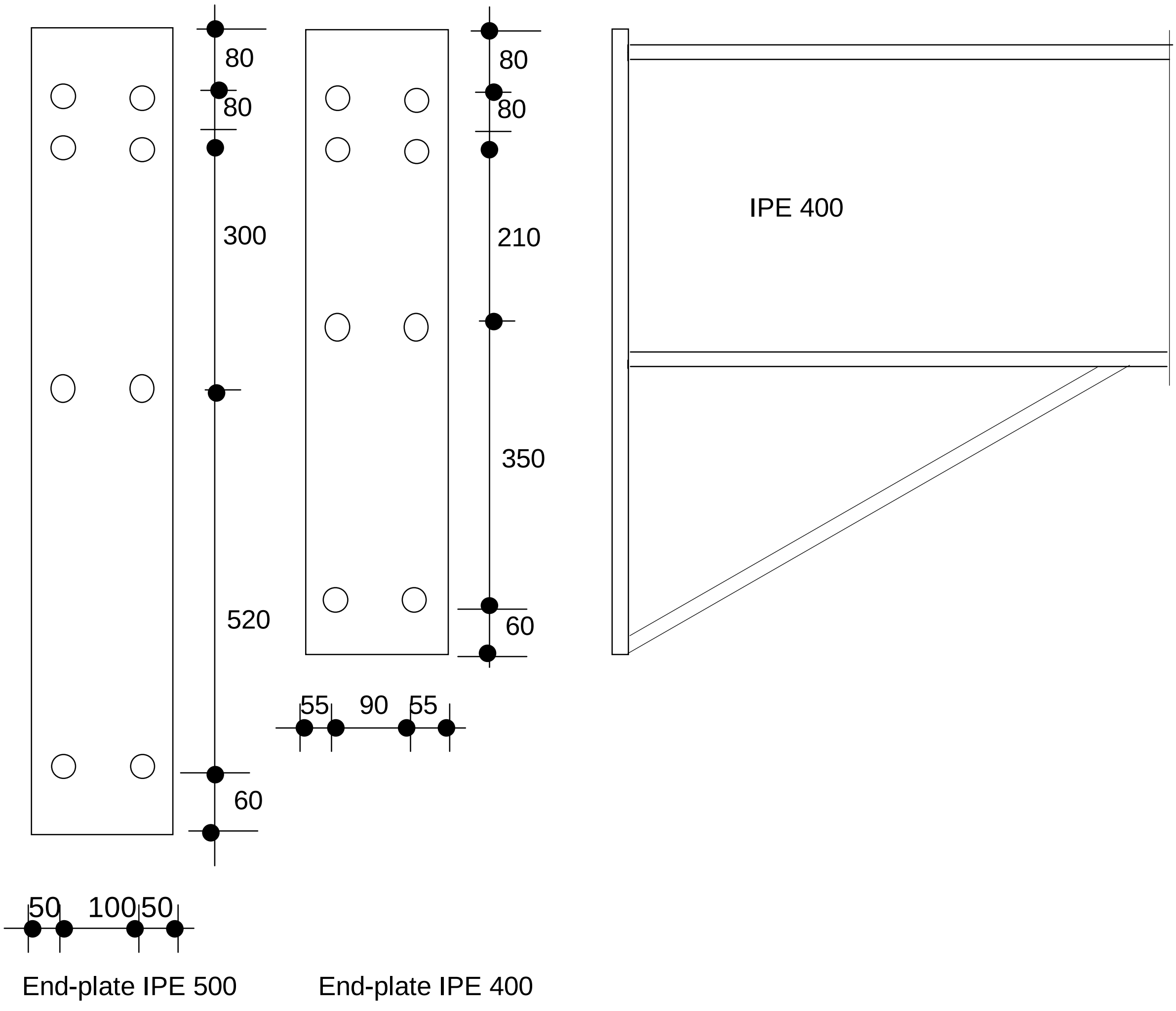


Figure 10.4 Geometry of the haunched connections

	Joint	M_{Rd} (kNm)	$S_{j.ini}$ (Nmm/rad)
1	IPE 500 floor beam HE200A column	160	78.10^9
2	IPE 500 floor beam HE240A column	246	359.10^9
3	IPE 400 roof beam HE200A column	128	58.10^9
4	IPE 400 roof beam HE240A column	140	228.10^9

Table 10.5 Joint properties according to DESIMAN calculations

Whether the stiffness was accurately enough introduced in the frame analysis has to be checked. This can be done with Table 10.6.

Since all the approximate values $S_{j.app}$ are lower, then the actual stiffness $S_{j.act}$ computed based on DESIMAN software, it is only required to check the upper limit.

Verification of the joints between an IPE 400 roof beam and a HE 240A inner column :

$$\begin{aligned} S_{j.act} &\geq \frac{10 S_{j.app} EI_b}{8EI_b - S_{j.app} l_b} \\ &\geq S_{j.app} \frac{10}{8 - \dfrac{S_{j.app} l_b}{EI_b}} \\ &= 54.10^9 \frac{10}{8 - \dfrac{54.10^9 \, x \, 7200}{210000 \, x \, 23131.10^4}} = \infty \text{ Nmm} / rad \end{aligned}$$

The results for the other joints in the frame are summarised in Table 10.7. It appears in each case that the actual stiffness is higher than the approximate stiffness. Therefore, only a check on the upper boundary is required.

This check is satisfactory. The difference between the actual stiffness and the approximate stiffness will not lead to more than a 5% drop in frame bearing resistance.

Frame	Lower boundary	Upper boundary
Braced	$S_{j.act} \geq \frac{8 S_{j.app} E I_b}{10 E I_b + S_{j.app} l_b}$	If $S_{j.act} \geq \frac{8 E I_b}{l_b}$ then: $S_{j.act} \leq \frac{10 S_{j.app} E I_b}{8 E I_b - S_{j.app} l_b}$ else: $S_{j.act} \leq \infty$
where : $S_{j.app}$ assumed stiffness adopted in the frame analysis (this is an approximation of the 'actual' stiffness); $S_{j.act}$ 'actual' stiffness of a joint; E Young modulus; l_b beam length; I_b second moment of area of the beam.		

Table 10.6 Boundaries for variance between actual an approximate stiffnesses

	Joint	$S_{j.app}$ (Nmm/rad)	Upper boundary (Nmm/rad)	$S_{j.act}$ (Nmm/rad)
1	IPE 500 floor beam HE200A column	40.10^9	78.10^9	78.10^9
2	IPE 500 floor beam HE240A column	84.10^9	∞	359.10^9
3	IPE 400 roof beam HE200A column	26.10^9	68.10^9	58.10^9
4	IPE 400 roof beam HE240A column	54.10^9	∞	228.10^9

Table 10.7 Check of joint stiffness

10.5 Frame design with partial-strength joints

10.5.1 Introduction

In this application, the theory of plasticity is used. The latter can lead to economical results, as it is shown in the following.

In contrast to the design procedures given in Sections 10.3 and 10.4, both member and joint designs are preferably performed by one single party, e.g. the steel fabricator. The reason for this is as follows : when using plastic design joints, properties need to be determined at an early stage of the design procedure and be included in the frame analysis. This is in contrast with the previous applications, where it was possible first to design beams and columns and then, in a second step, to design the joints.

10.5.2 Preliminary design of beams and columns

The same beam and column sizes as in Section 10.4 will be used :

- Floor beams : IPE 500
- Roof beams : IPE 400
- Outer columns : HE 200 A
- Inner column : HE 240 A.

10.5.3 Design of the joints

In a first step, flush end-plate joints will be contemplated. All the flush end-plates have been chosen 12 mm thick. Bolts are *M20* grade *8.8*. In all the joints, four bolt rows have been used. Figure 10.5 shows the geometry of the flush end- plates.

The joint properties were computed by means of the DESIMAN software; they are listed in Table 10.8.

The governing failure mode of the joints is related to the bending of the end-plate. So a sufficient rotation capacity is available and a plastic design is permitted.

	Joint	M_{rd} (kNm)	$S_{j.ini}$ (Nmm/rad)	Rotational capacity
1	IPE 500 floor beam HE200A column	70	25.10^9	sufficient
2	IPE 500 floor beam HE240A column	104	60.10^9	sufficient
3	IPE 400 roof beam HE200A column	54	16.10^9	sufficient
4	IPE 400 roof beam HE240A column	80	32.10^9	sufficient

Table 10.8 Joint properties according to *DESIMAN* calculations

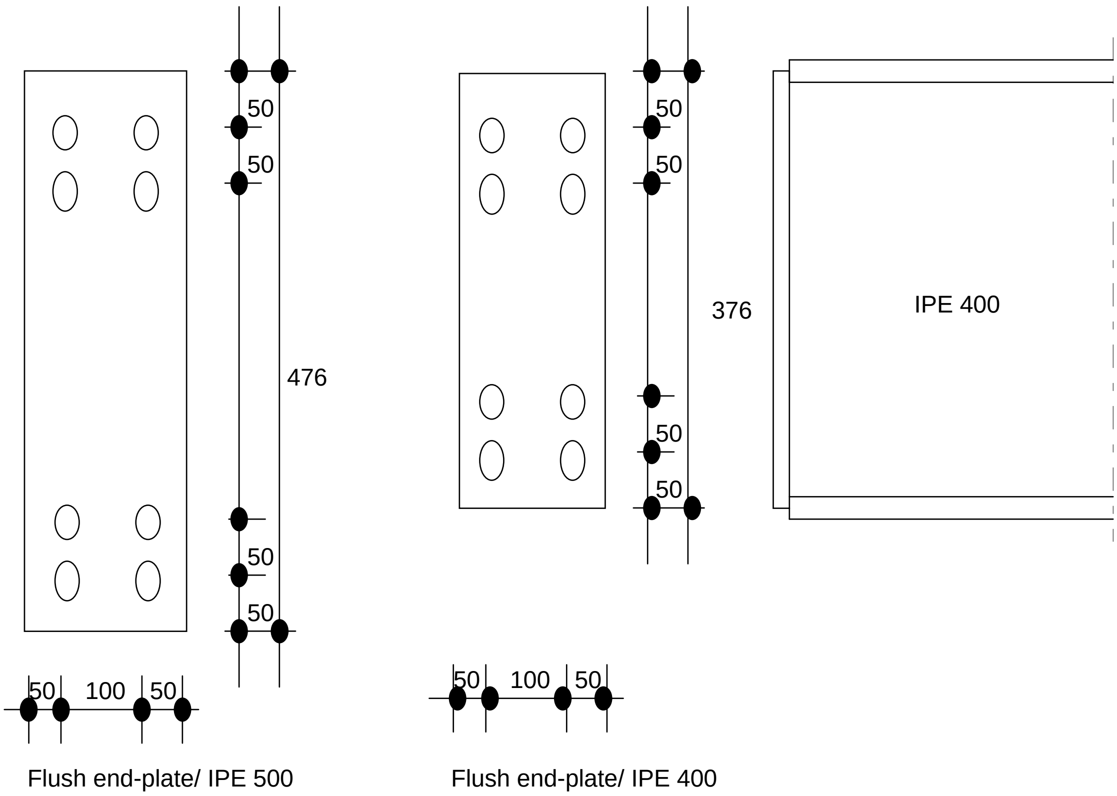


Figure 10.5 Geometry of the flush end plate connections

For the design of the joints connecting a beam to the columns, the following rule should be satisfied (see Figure 10.6) :

$$\frac{1/\,8 q_{Sd} l_b^2}{M_{pl.y} + 0{,}5 M_{Rd.1} + 0{,}5 M_{Rd.2}} \leq 1$$

where :

$M_{pl.y}$ resistance moment of the beam, see Section 10.4.4.2.2;

$M_{Rd.1}$ resistance moment of the joint connecting the beam to the outer column;

$M_{Rd.2}$ resistance moment of the joint connecting the beam to the inner column.

In this case, a bending moment of 1/8 q_{Sd} l_b^2 is taken at mid span, considering that this assumption is sufficiently accurate.

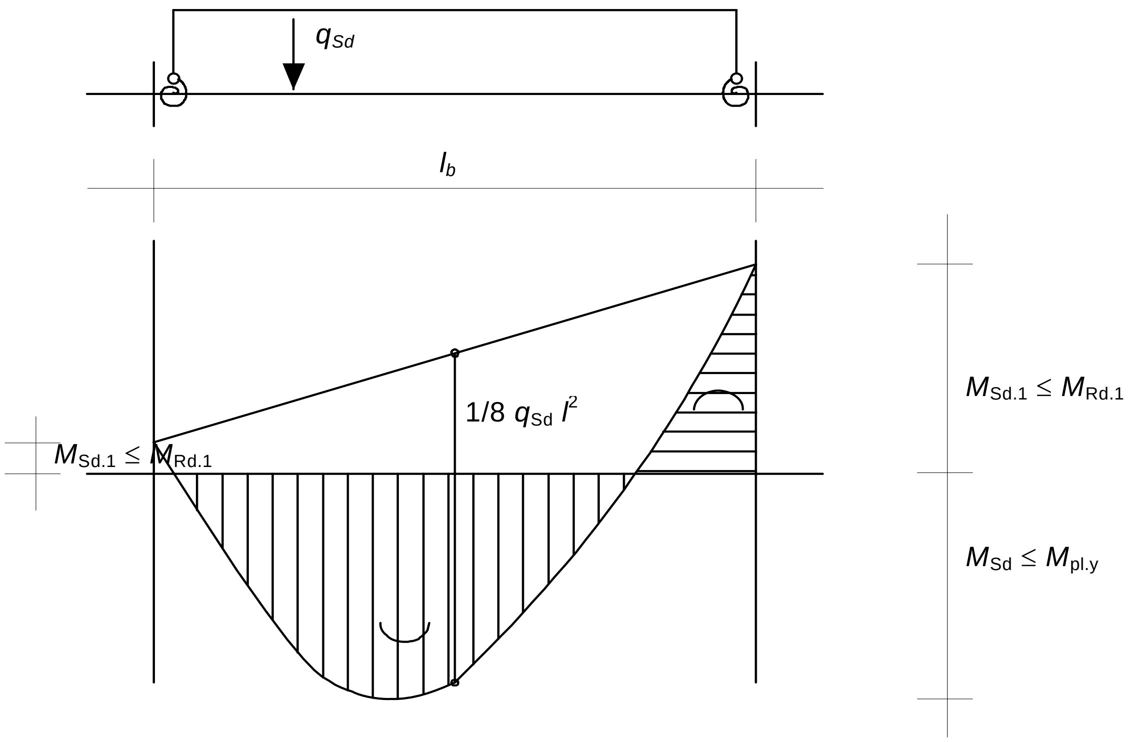


Figure 10.6 Equilibrium in a beam

Check of the floor beam :

$$\frac{525}{517 + 0{,}5 \times 70 + 0{,}5 \times 104} = 0{,}87 \leq 1$$

$\Rightarrow$ Satisfactory.

Check of the roof beam:

$$\frac{350}{307 + 0{,}5 \times 54 + 0{,}5 \times 80} = 0{,}93 \leq 1$$

$\Rightarrow$ Satisfactory.

10.5.4 Frame analysis

10.5.4.1 Serviceability limit state

For sake of simplicity, the entire stiffness of the joints is neglected when determining the deflections at the serviceability limit state. This yields the following results :

Floor beam $: u = \dfrac{5 \times 59 \times 7200^4}{384 \times 210000 \times 48200.10^4} = 20{,}3$ mm

Roof beam : $u = \dfrac{5 \times 40 \times 7200^4}{384 \times 210000 \times 23130.10^4}$ = 28,8 mm

10.5.4.2 Ultimate limit state

It is assumed that, at collapse, the outer column is still continuous, but pinned connected to the beams and loaded with 54 kNm (design moment capacity of the joint) at roof level and 70 kNm (design moment capacity joint) at the floor level (see Figure 10.7). Then, the axial force in the outer column is equal to :

$N_{c.Sd}$ = Σ (1/2 q_{Sd} l_b + ($M_{Rd.1}$ - $M_{Rd.2}$) / l_b))

= 1/2 x 81 x 7,2 + (70 - 104) / 7,2 +

1/2 x 54 x 7,2 + (54 - 80) / 7,2

= 478 kN

The axial force in the inner column is equal to :

$N_{c.Sd}$ = Σ (1/2 q_{Sd} l_b + ($M_{Rd.2}$ - $M_{Rd.1}$) / l_b))

= (1/2 x 81 x 7,2 + (70 - 104) / 7,2 +

1/2 x 54 x 7,2 + (54 - 80) / 7,2) x 2

= 989 kN

With help of first order elastic analysis, the moment in the outer column just below the floor is :

M_{Sd} = 23 kNm.

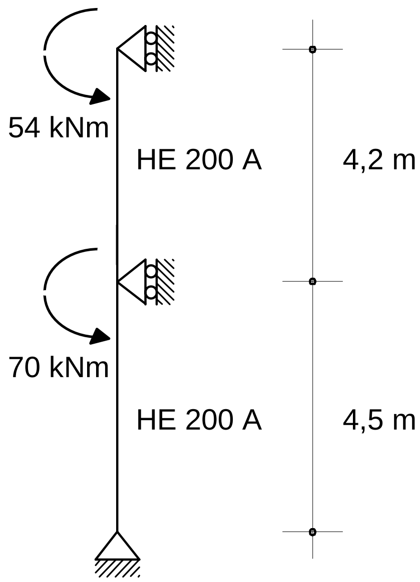


Figure 10.7 Moments acting on the outer column

10.5.5 Design checks

10.5.5.1 Serviceability limit state

Floor : 20,3 mm ≤ 72000 mm / 250 = 28,8 mm.

Roof : 28,8 mm ≤ 72000 mm / 200 = 36 mm.

⇒ Satisfactory.

(Note : if this verification was not satisfactory, then a first order elastic frame analysis could be carried out, taking the stiffness of the joints into consideration).

10.5.5.2 Ultimate limit state

10.5.5.2.1 Column stability

With reference to 10.4.4.2.1, the check of column stability will be satisfactory for the inner column.

For the outer columns, *Eurocode 3-Clause 5.5.4* needs to be checked (see also Section 10.4.4.2.1).

$$\frac{N_{sd}}{\chi_{min} A f_y / \gamma_{M1}} + \frac{k_y M_{y.Sd}}{W_{pl.y} f_y / \gamma_{M1}} \leq 1$$

$$\frac{478.10^3}{0{,}58 \times 5380 \times 235 / 1{,}1} + \frac{123.10^6}{430.10^3 \times 235 / 1{,}1} \leq 1$$

$$0{,}71 + 0{,}25 = 0{,}96$$

⇒ Satisfactory.

10.5.5.2.2 Section checks of the beams

No check is required, since this has been done in Section10.5.2

10.5.5.2.3 Section checks of the columns

Since weak-axis buckling is the governing failure mode for the inner column, no check of the inner column section needs to be carried out.

The section check of the outer columns can be carried out using *Eurocode 3-Clause 5.4.8.1(3)* :

$$N_{Sd} / N_{pl.Rd} = \frac{N_{sd}}{A f_y / \gamma_{M0}} = \frac{478.10^3}{5380 x\, 235 / 1{,}1} = 0{,}41 \leq 1$$

$$M_{Sd} / M_{y.pl} = \frac{M_{y.Sd}}{W_{pl.y} f_y / \gamma_{M0}} = \frac{23.10^6}{430.10^3 \, x \, 235 / 1{,}1} = 0{,}25 \leq 1$$

Since N_{Sd} / $N_{pl.Rd}$ ≤ 0,5, there is no need to check the interaction between axial force and bending moment.

⇒ Satisfactory.

10.6 Conclusions

This worked example showed that semi-rigid / partial-strength concept, as given in Chapter 6.3 can be applied to braced frames in a straightforward manner. Based on the design of a simple frame with pinned joints, the beam and column sizes can be derived. General rules for economic design are :

- The columns in the frame with semi-rigid / partial-strength joints are identical to those used in the frame with simple (pinned) joints;
- The beam sizes are one section lower.

Calculations to the frames with semi rigid / partial strength joints showed that a plastic frame analysis with partial strength joints normally leads to simpler joints then elastic frame analysis.

The three possibilities are summarised in Table 10.9 :

Possibility	Frame analysis	Joint modelling	Type of joint	Beam sizes	Parties
Section 10.3	Elastic	Simple	Angle cleats, fin plate or partial depth end plate	IPE 550 / IPE 450	Engineer <u>and</u> Steel Fabricator
Section 10.4	Elastic	Semi-rigid	Haunched end plates	IPE 500 / IPE 400	Engineer <u>and</u> Steel Fabricator
Section 10.5	Plastic	Partial strength	Flush end plates	IPE 500 / IPE 400	Engineer <u>or</u> Steel Fabricator

Table 10.9 Summary of frame alternatives

Chapter 11

WORKED EXAMPLE 2

DESIGN OF A 3-STOREY UNBRACED FRAME

11.1 Frame geometry and loading

11.1.1 Frame geometry

The skeletal structure of a three bay three storey building is shown in Figure 11.1 (dimensions in mm).

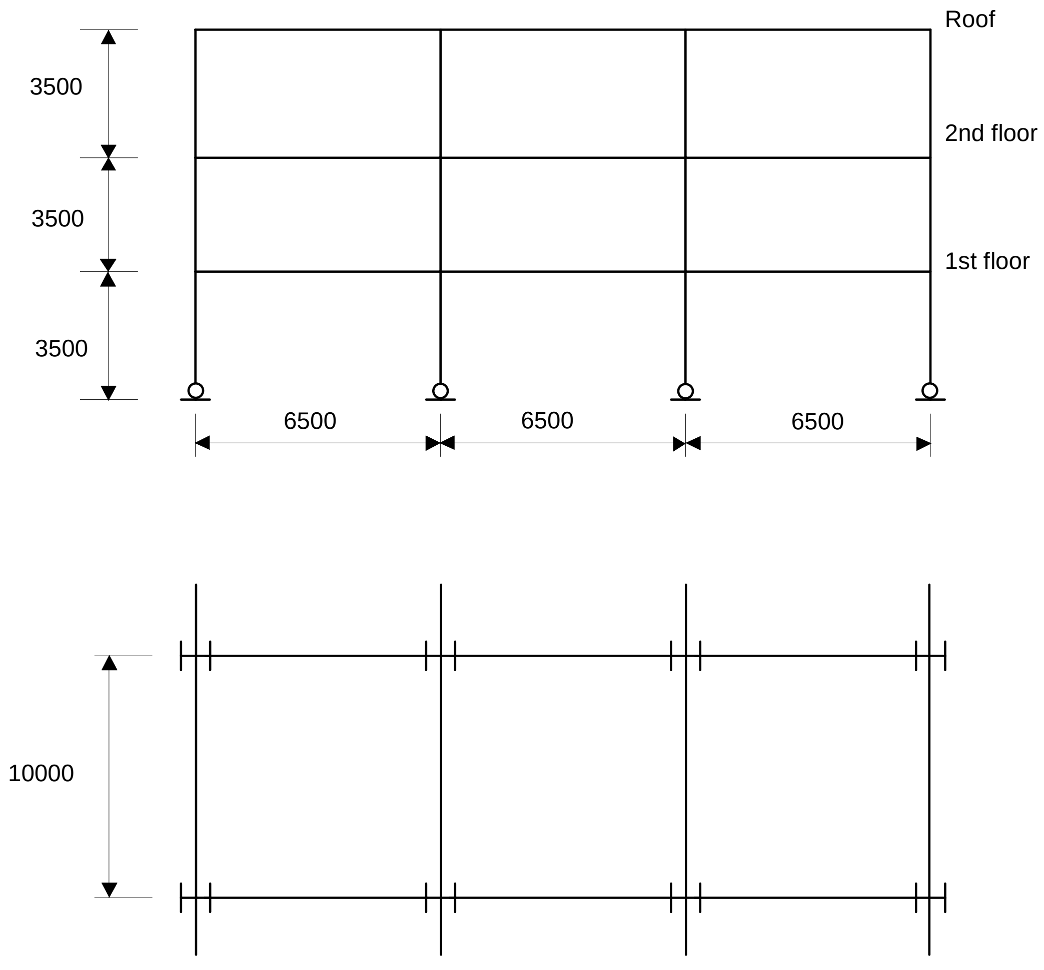


Figure 11.1 Frame geometry

The frame is 19.5 m wide, each span being 6.5 m; its total height is 10.5 m, each storey being 3.5 m high.

The spacing of the frames is 10 m.

The structure is assumed to be braced out of its plane and to be unbraced in its plane. In the longitudinal direction of the building, i.e. in the direction perpendicular to the frame plane, a bracing does exist so that the top of the columns is held in place. The lateral support for the floor beams are provided by the floor slabs. The bases of the columns (foundation level) are assumed to be nominally pinned.

The total height of the building is less than the maximum length allowed for transportation (about 12 m); therefore, it was decided to use continuous columns throughout the total height of the building.

For the members, use is made of standard hot rolled sections.

Members, end-plates and stiffeners are made of *S235* steel, according to *EN 10025*.

Bolts are property *class 10.9*, according to *EN 20898-1* and *EN 20898-2.*

11.1.2 Loading

11.1.2.1 Basic loading

The values for the characteristic permanent and variable actions are :

Roof level

Variable actions (imposed loads) :	6 kN/m.
Permanent actions :	20 kN/m.

Floor level

Variable actions (imposed loads) :	18 kN/m.
Permanent actions :	30 kN/m.

The wind loads were established, according to the French regulations, for a building erected in France, at an altitude of 200 m in wind region II. They are applied as point loads of respectively 10.5 kN at the roof level and 21 kN at the 1st and 2nd floor levels.

The basic loading cases, which are shown schematically in Figure 11.2, have been considered in appropriate combinations.

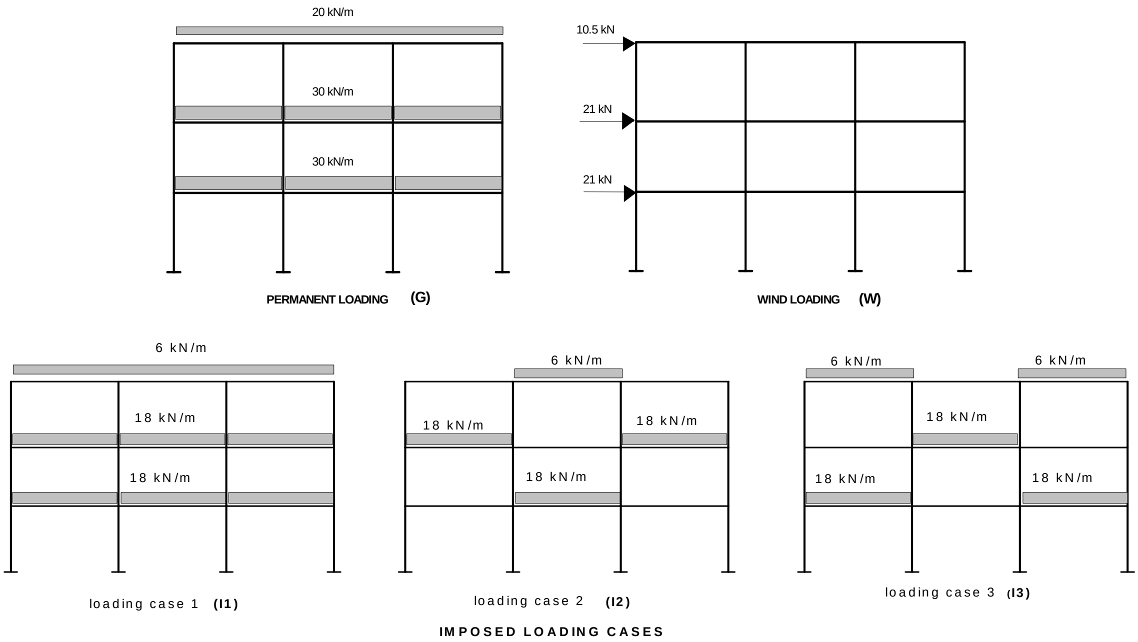


Figure 11.2 Loading cases

11.1.2.2 Frame imperfections

Frame imperfections are considered by means of equivalent horizontal. The initial sway imperfection is given as follows *(Eurocode 3 - 5.2.4.3(1))* :

$$\phi = k_c k_s \phi_0$$

with :

$$k_c = \sqrt{0.5 + \frac{1}{n_c}} \leq 1$$

$$k_s = \sqrt{0.2 + \frac{1}{n_s}} \leq 1$$

$$\phi_0 = \frac{1}{200}$$

Here, one has $n_c = 4$ (number of full height columns per floor) and $n_s = 3$ (number of storeys in the frame), wherefrom :

$$k_c = \sqrt{0.5 + \frac{1}{4}} = 0.866$$

$$k_s = \sqrt{0.2 + \frac{1}{3}} = 0.73$$

$$\phi = (0.866).(0.73).\frac{1}{200} = \frac{1}{315}$$

The equivalent horizontal load $H = V\phi$ at each storey of the frame is derived from the initial sway ϕ and the total design vertical load V in any storey for a given load case *(Eurocode 3 - 5.2.4.3 (7))*. The relevant values are listed in Table 11.1 for all the basic loading cases.

Basic loading case (see Figure 11.2)	Storey	V (kN)	H (kN)
G	Roof	390	1.24
	2nd floor	585	1.86
	1st floor	585	1.86
I_1	Roof	117	0.37
	2nd floor	351	1.11
	1st floor	351	1.11
I_2	Roof	39	0.12
	2nd floor	234	0.74
	1st floor	117	0.37
I_3	Roof	78	0.25
	2nd floor	117	0.37
	1st floor	234	0.74

Table 11.1 Equivalent horizontal forces

They must of course be affected by the appropriate partial safety factors on actions.

11.1.2.3 Load combination cases

It was decided to use the simplified combinations for the ultimate limit state (*Eurocode 3 - 2.3.3.1 (5)*) and the serviceability limit state (*Eurocode 3 - 2.3.4.(5)*).

	Ultimate limit state
Load combination case 1	$1.35\ G + 1.5\ W$
Load combination case 2	$1.35\ G + 1.5\ I_1$
Load combination case 3	$1.35\ G + 1.5\ I_2$
Load combination case 4	$1.35\ G + 1.5\ I_3$
Load combination case 5	$1.35\ G + 1.35\ W + 1.35\ I_1$
Load combination case 6	$1.35\ G + 1.35\ W + 1.35\ I_2$
Load combination case 7	$1.35\ G + 1.35\ W + 1.35\ I_3$

Table 11.2 Load combination cases at ULS

The basic load cases are combined at the ultimate limit state as summarised in Table 11.1.

	Serviceability limit state
Load combination case 1	$G + W$
Load combination case 2	$G + I_1$
Load combination case 3	$G + I_2$
Load combination case 4	$G + I_3$
Load combination case 5	$G + 0.9\ W + 0.9\ I_1$
Load combination case 6	$G + 0.9\ W + 0.9\ I_2$
Load combination case 7	$G + 0.9\ W + 0.9\ I_3$

Table 11.3 Load combination cases at SLS

The basic load cases are combined at the serviceability limit state as summarised in Table 11.3.

The requirements on the frame displacements at the serviceability limit state are as follows for a multi-storey building *(Eurocode 3 - 4.2.2 (1) and (4))* :

The allowable horizontal deflection in each storey is :

h/300 = 3500/300 = 11.7 mm.

The allowable horizontal deflection of the structure as a whole is :

h_0/500 = 10500/500 = 21 mm.

The allowable vertical deflection of the floor beams is :

L/250 = 6500/250 = 26 mm.

The allowable vertical deflection of the roof beam shall is :

L/200 = 6500/200 = 32.5 mm.

11.1.3 Partial safety factors on resistance

For this worked example, the values of the partial safety factors on resistance are adopted as follows :

$\gamma_{M0} = 10$ for the resistance of cross-sections;

$\gamma_{M1} = 110$ for the buckling resistance of members;

$\gamma_{Mb} = 125$ for the resistance of bolts;

$\gamma_{Mw} = 125$ for the resistance of welds.

(The value of $\gamma_{M0} = 10$ is permitted in France provided that the steel material bears the quality mark NF).

11.2 Objectives

The objective is to aim at *joint economy* . It is assumed that there is an interaction between the two respective tasks of frame design and joint design. The use of unstiffened joints, which may consequently become semi-rigid, is a priori considered as the principal means of obtaining this economy.

To provide a basis for evaluating the effect on costs, the frame with rigid joints shall be compared to the frame with semi-rigid joints.

Elastic global analysis is used to compute the internal forces and moments.

For the frame with rigid joints, it is only reported on the column and beam sizes and on the connection detailing.

For the frame with semi-rigid joints, the detailed calculations of the members are given in addition.

11.3 Frame with rigid joints

11.3.1 Assumptions and global analysis

All the beam-to-column joints were assumed to be perfectly rigid. A linear elastic analysis was carried out for each load case.

11.3.2 Member sizes

The member sizes were first determined based on a preliminary design; their validity was confirmed a posteriori on base of detailed calculations conducted at the end of the global analysis for the various load combination cases.

The member sizes obtained accordingly are (Figure 11.3) :

Inner columns :	HEB 260
Outer columns :	HEB 220
Floor beams :	IPE 360
Roof beams :	IPE 450

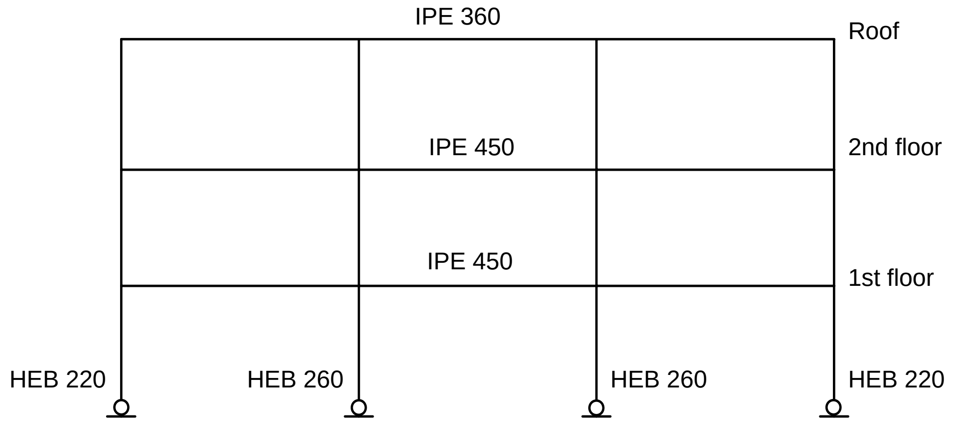


Figure 11.3 Member sizes (rigid joints)

11.3.3 Serviceability limit state requirements

The maximum horizontal deflection of the storeys is 10.1 mm.

(< 11.7 mm)

The maximum horizontal deflection of the structure as a whole is 13.5 mm.

(< 21 mm)

The maximum vertical deflection of the floor beams is 6.9 mm.

(26 mm)

The maximum vertical deflections of the roof beams is 7.8 mm.

(32.5 mm)

All the serviceability limit state requirements on the frame deflections are thus fulfilled.

11.3.4 Joint design

At the ultimate limit state, the joints have to resist the following values of bending moment (Table 11.4) :

	Maximum bending moments (*kN.m*)	
	Inner columns	Outer columns
Roof level	147.2	81.1
Floor level	337.8	192.8

Table 11.4 Moments at joints (rigid joints)

To realize rigid joints with the required resistance, it was decided to use extended end-plate moment connections. The joint detailing is represented in Table 11.5.

11.4 Frame with semi-rigid joints

11.4.1 Design strategy

It was decided to use joints with no shear stiffeners and no horizontal web stiffeners in the columns so as to achieve the best economy in both the fabrication and erection stages by simplifying the joint detailing. As a result, the joints become semi-rigid and there is possibly a need for resizing the members.

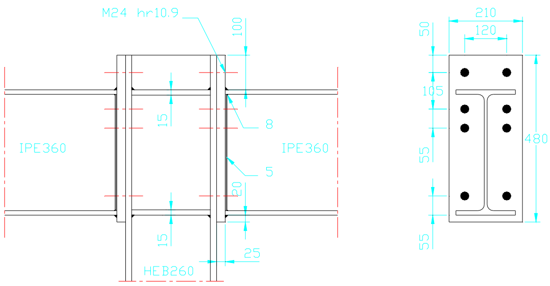
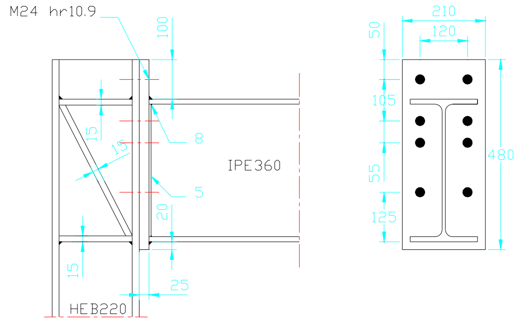
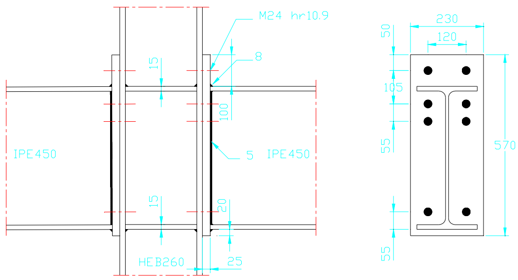
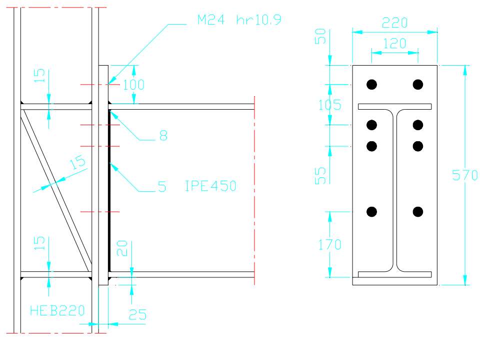
	Interior column / beam	Exterior column / Beam
Roof level	 	
Floor level	 	

Table 11.5 Joint details (rigid joints)

11.4.2 Preliminary design

The member sizes obtained in Section 11.3.2 were used as a matter of preliminary sizing of the members. However, due to the semi-rigidity of the joints, the horizontal deflections will then be larger than those computed in Section 11.3.3 for the frame with rigid joints. In order to fulfil the serviceability limit state requirements, the decrease in joint stiffness was compensated by an increase in column sizes.

It was decided to try , for the columns, one section size more than in the case of rigid joints; thus sections HEB 280 and HEB 240 were adopted for inner and outer columns respectively.

The beam-to-column joint detailing was inspired by a constructive solution listed in the tables produced in Part 3 of the present manual.

A quick check of the serviceability limit state of the frame with semi-rigid joints was carried out with the mechanical joint properties given in the same tables. On this base, the following column and beam sizes were finally selected (Figure 11.4) :

Inner columns : HEB 280

Outer columns : HEB 240

Floor beams : IPE 360

Roof beams : IPE 450

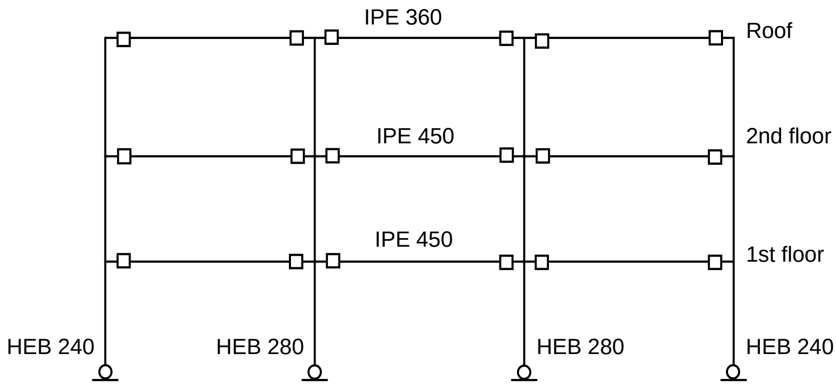


Figure 11.4 Member sizes (semi-rigid joints)

The detailing of the unstiffened end-plate connections which were adopted is given in Table 11.6.

11.4.3 Characteristics and classification of the joints

The mechanical properties of the joints given in the tables are computed based on the simplified method of *Eurocode 3-(revised) Annex J*; therefore the moment resistance of the joints is underestimated.

In order to get more accurate values of the mechanical properties, it was decided to use the DESIMAN software which refers to the general method described in *Eurocode 3-(revised) Annex J* (see Table 11.7).

For the calculation of the joints on the inner columns, the parameter β is taken equal to 1; this assumption allows a non-iterative global analysis (see comments in Section 11.5).

For an unbraced frame, the joint can be classified as rigid if the following criterion is met (*Eurocode 3-(revised) Annex J*) :

$$\frac{S_{j,ini}}{\dfrac{EI_b}{L_b}} \geq 25$$

with : $S_{j,ini}$: initial stiffness of the joint;

EI_b / L_b : rigidity of the beam of span L_b.

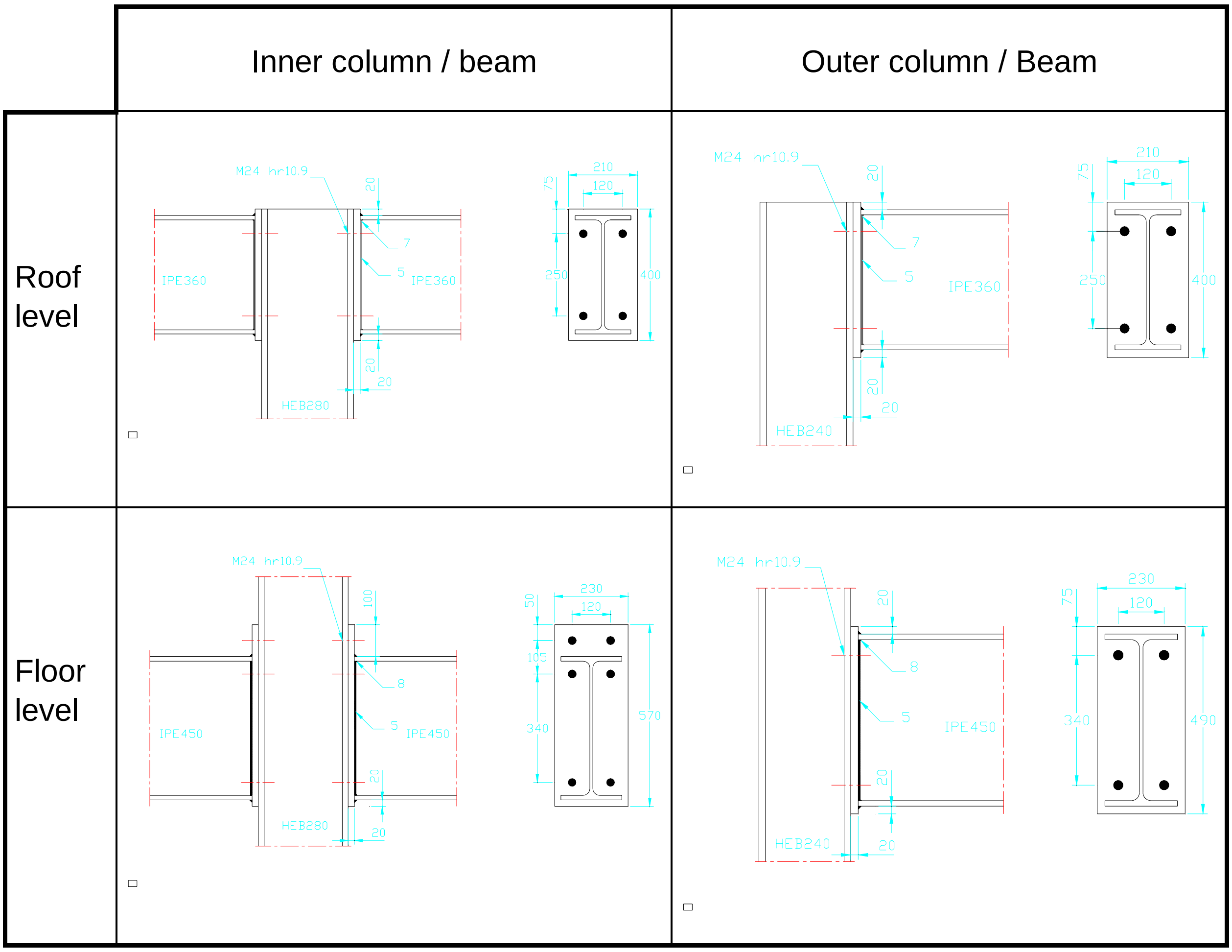


Table 11.6 Joint details (semi-rigid joints)

	Inner columns		Outer columns	
Joint	Floor level	Roof level	Floor level	Roof level
$S_{j,ini}$ (kNm/rad)	66755	25593	40992	23720
$S_{j,ini}/2$ (kNm/rad)	33377	12797	20496	11860
M_{Rd} (kNm)	220.5	115.8	151.3	94.8
$2m_{rd}/3$ (kNm)	147.0	77.8	100.9	63.2

Table 11.7 Joint characteristics (semi-rigid joints)

At the floor level :

The rigidity of the beam floor IPE 400 (span of 6.5 m) is :

$$\frac{EI_b}{L_b} = \frac{210000 \times 33740 \times 10^4}{6500} \times 10^{-6} = 10901\,\text{kNm/rad}$$

Criterion at the outer columns :

$$\frac{S_{j,ini}}{\frac{EI_b}{L_b}} = \frac{40992}{10901} = 3.8 \quad \langle\, 25$$

$\Uparrow$ Semi-rigid joint.

Criterion at the inner columns :

$$\frac{S_{j,ini}}{\frac{EI_b}{L_b}} = \frac{66755}{10901} = 6.1 \quad \langle\, 25$$

$\Uparrow$ Semi-rigid joint.

At the roof level :

The rigidity of the beam roof IPE 360 (span 6.5 m) is :

$$\frac{EI_b}{L_b} = \frac{210000 \times 16270 \times 10^4}{6500} \times 10^{-6} = 5256\,\text{kNm/rad}$$

Criterion at the outer columns :

$$\frac{S_{j,ini}}{\frac{EI_b}{L_b}} = \frac{23720}{5256} = 4.5 \quad \langle\, 25$$

$\Uparrow$ Semi-rigid joint.

Criterion at the inner columns :

$$\frac{S_{j,ini}}{\frac{EI_b}{L_b}} = \frac{25593}{5256} = 4.9 \quad \langle\, 25$$

$\Uparrow$ Semi-rigid joint.

As it might be expected, all the unstiffened joints must be classified as semi-rigid joints.

11.4.4 Structural analysis

A linear elastic analysis is conducted for both the ultimate and serviceability limit states. For this purpose, the joints are characterised by their nominal stiffness $S_{j,ini}/2$.

For the sake of simplicity, the mechanical properties of all the joints are computed based on a single value β=1 of the transformation parameter. It is recalled that the transformation parameter accounts for the influence of the flexibility of the column web panel on both the design moment resistance and rotational stiffness of the joints; therefore, the actual value of β at each joint depends on the actual moments and shear forces relative to each load combination case. Assuming β=1 for all the joints of the frame and for all the load combination cases results in a significant simplification in the global analysis and in a slight underestimation of both the strength and stiffness of these joints.

11.4.5 Design checks of the frame with semi-rigid joints

11.4.5.1 Serviceability limit state

At the serviceability limit state :

The maximum horizontal deflection of all the storeys is 11.9 mm.

$\cong$ 11.7 mm (*))

The maximum horizontal deflection of the structure as a whole is 19.9 mm.

(< 21 mm)

The maximum vertical deflection of the floor beams is 10.6 mm.

(< 26 mm)

The maximum vertical deflections of the roof beam is 12.1 mm.

(< 32.5 mm)

(*) For the calculation of the deflections, the nominal stiffness of the joints was used. The actual values will be larger because some joints will experience moments less than $2/3\ M_{Rd}$ and would then have a stiffness of $S_{j,ini}$. Therefore the slight excedence of the horizontal deflection at the 1st floor level, based on the nominal stiffness of the joints is acceptable.

11.4.5.2 Ultimate limit state

11.4.5.2.1 Sway classification of the frame

The classification of the frame subject to a given load combination case is investigated by computing the ratio of the total design vertical load V_{sd} acting on the frame and the elastic critical load V_{cr} of the frame for the sway buckling mode *(Eurocode 3 - 5.2.6.2 (6))* :

$$\frac{V_{Sd}}{V_{cr}} = Max\left[\frac{\delta}{h}\frac{V}{H}\right]_i$$

with :

i designation of the storey i;

V_{Sd} design value of the total vertical load;

V_{cr} elastic critical load of the frame for the sway buckling mode;

δ horizontal displacement at the top of the storey relative to the bottom of this storey obtained from a first order elastic analysis;

h storey height;

H total horizontal reaction at the bottom of the storey;

V total vertical reaction at the bottom of the storey.

The classification of the frame subject to each of the load combination cases is summarised in Table 11.7.

Because all the values of the ratio V_{Sd}/V_{cr} are smaller than *0,25*, the frame behaves as a sway frame. As a result, the effects of sway must be considered in the design checks of the ultimate limit state, whatever the load combination case.

According to *Eurocode 3*, three methods are available for this purpose *(Eurocode 3 - 5.2.6.2)* :

- Second order analysis;
- First order analysis with amplification of the sway moments, provided that $\left(V_{Sd}/V_{cr} \langle\ 0.25\right)$;
- First order analysis with sway mode buckling lengths.

For the sake of simplicity, the second method, i.e. the first order analysis with amplification of the sway moments, is selected. The sway moments are those associated with the horizontal translation of the top of the storey relative to the bottom of the same storey. In this worked example (symmetrical structure which is symmetrically loaded), the sway moments are due to the horizontal loads only, being understood that the latter include the equivalent forces for imperfections.

Load combination	Storey	δ (mm)	V (kN)	H (kN)	$\frac{\delta}{h}\frac{V}{H}$
Case 1					
	Roof	4.2	526.5	17.4	0.04
	2nd floor	8.7	1316.3	51.4	0.06
	1st floor	19.5	2106	85.5	**0.14**
Case 2					
	Roof	2.2	702	2.2	0.03
	2nd floor	1.0	2018.3	6.4	0.09
	1st floor	2.5	3334.5	10.6	**0.22**
Case 3					
	Roof	0.3	585	1.9	0.03
	2nd floor	0.9	1725.8	5.7	0.08
	1st floor	2.1	2691	8.7	**0.18**
Case 4					
	Roof	0.3	643.5	2.1	0.03
	2nd floor	0.9	1569	5.1	0.08
	1st floor	2.1	2709.8	8.7	**0.19**
Case 5					
	Roof	3.9	684.5	16.3	0.05
	2nd floor	8.2	1948	48.7	0.09
	1st floor	18.5	3211.7	81.1	**0.21**
Case 6					
	Roof	3.9	579.2	16.0	0.04
	2nd floor	8.1	1684.9	47.9	0.08
	1st floor	18.1	2632.5	79.2	**0.17**
Case 7					
	Roof	3.9	631.8	16.2	0.04
	2nd floor	8.1	1579.5	47.6	0.08
	1st floor	18.1	2685.2	79.4	**0.17**

Table 11.8 Frame classification (semi-rigid joints)

The value of the amplification factor to be applied to the sway moments is computed for each load combination case, in accordance with *Eurocode 3 - 5.2.6.2.(3)*. The results are given in Table 11.8.

	$\frac{V_{Sd}}{V_{cr}}$	$\alpha = \frac{1}{1 - \frac{V_{Sd}}{Vcr}}$
Load combination case 1	0.14	1.16
Load combination case 2	0.22	1.28
Load combination case 3	0.18	1.22
Load combination case 4	0.19	1.23
Load combination case 5	0.21	1.27
Load combination case 6	0.17	1.21
Load combination case 7	0.17	1.21

Table 11.9 Amplification factor (semi-rigid joints)

The amplification factor can be considered as affecting the value of the partial safety factors on the horizontal actions. Then Table 11.9 shall be substituted for Table 11.2. Proceeding this way is especially recommended because first order analysis permits the use of the principle of superposition. Thus the internal forces and moments for the load combination cases of Table 11.9 can be easily obtained and be used afterwards for the design checks of members and joints.

Load combination	G	W	I1	I2	I3	Equivalent horizontal forces
Case 1	1.35	1.74				1.16
Case 2	1.35		1.5			1.28
Case 3	1.35			1.5		1.22
Case 4	1.35				1.5	1.23
Case 5	1.35	1.72	1.35			1.27
Case 6	1.35	1.63		1.35		1.21
Case 7	1.35	1.63			1.35	1.21

Table 11.10 Equivalent horizontal forces (semi-rigid joints)

11.4.5.2.2 Beam design

IPE 450 beam at the floor levels

Actions

The load combination case 4 is the most critical for this beam. The bending moment at mid-span is 241 kNm.

The maximum shear at any cross-section of this beam is 240 kN.

Properties of the IPE 450 section

h = 450 mm b_f = 190 mm t_w = 9.4 mm t_f = 14.6 mm

r = 21 mm

I_z = 1676 cm^4 I_t = 6718 cm^4 I_w = 0.79 dm^6

Steel grade : *S235*, the flange thickness is less than 40 mm => f_y =235 N/mm^2

Section classification in bending *(Eurocode 3 - 5.3)*

$$\varepsilon = \left(\frac{235}{f_y}\right)^{0.5} = 1$$

Flange check : $\frac{c}{t_f} = 6.51 \leq 10\varepsilon$ ↑ Flange is class 1.

Web check : $\frac{d}{t_w} = 40.3 \leq 72\varepsilon$ ↑ Web is class 1.

All elements are class 1 : ↑ Cross-section is class 1.

Shear resistance *(Eurocode 3 - 5.4.6)*

Shear area : A_v = 50.85 cm^2

Shear resistance : $V_{pl.Rd} = \dfrac{A_v \dfrac{f_y}{\sqrt{3}}}{\gamma_{M0}} = 690\ kN$ and $\dfrac{V_{Sd}}{V_{pl.Rd}} = 0.35$

As this ratio is less than 0.5, the effect of shear on the moment resistance can be neglected.

Member resistance : lateral torsional buckling

Normally the concrete slab prevents lateral torsional buckling of the beams. However, at the erection stage, a check of the lateral stability of the beams is often required; the loads for this case are less than the ultimate limit state load.

(Eurocode 3 - Table F.1.2)

k=1 (assumption of simple supports for weak-axis bending)

⇑ $C_1 = 1.285$ $C_2 = 1.562$ $C_3 = 0.753$

(Eurocode 3 - F.1.3.(1))

$z_j = 0$ (symmetrical section)

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{(kL)^2} \left\{ \left[\left(\frac{k}{k_w} \right)^2 \frac{I_w}{I_z} + \frac{(kL)^2 GI_t}{\pi^2 EI_z} + \left(C_2 z_g \right)^2 \right]^{0.5} - C_2 z_g \right\}$$

$k_w = 0.5$ (fixed warping end condition) and $z_g = 0.225$ m => $M_{cr} = 279.2$ kNm

(*Eurocode 3 - F2.1.1 (1) and (2))*

$$\lambda_{LT} = \left(\frac{\pi^2 EW_{pl.y}}{M_{cr}} \right)^{0.5} = 112.4$$

$$\overline{\lambda}_{LT} = \left(\frac{\beta_w W_{pl.y} f_y}{M_{cr}} \right)^{0.5} \quad \beta_w = 1 \qquad ⇑ \qquad \overline{\lambda}_{LT} = 0.3734 \leq 0.4$$

There is thus no reduction for lateral torsional buckling. That means the member resistance is given by the moment resistance of the cross-section.

(Eurocode 3 - 5.4.5)

$$W_{pl.y} = 1702 \text{ cm}^3$$

$$M_{c.Rd} = M_{pl.Rd} = \frac{W_{pl.Rd} f_y}{\gamma_{M0}} = \frac{1702x235x10^{-3}}{10} = 400 \text{ kNm}$$

⇑ Satisfactory.

11.4.5.2.3 Column design

Inner HEB 280 column

Actions

The maximum axial forces and moments in the inner columns are those relative to load combination case 5. The distributions of the internal forces in these columns are given in Figure 11.5.

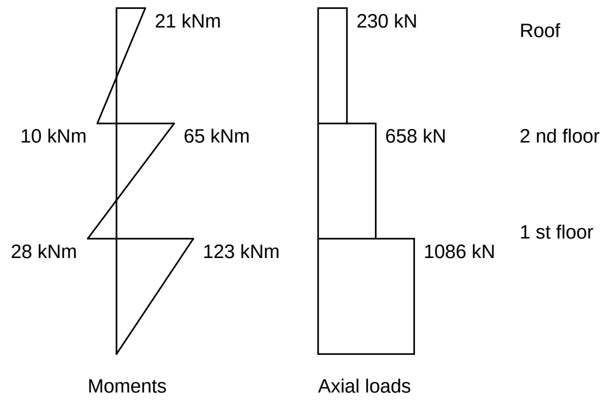


Figure 11.5

The maximum shear force in the column is 40 kN.

Properties of HEB280 section

h = 280 mm b_f = 280 mm t_w = 10.5 mm t_f = 18 mm r = 24 mm

Steel grade *S235* and the flange thickness is less than 40 mm

=> f_y = 235 N/mm^2

Section classification in bending *(Eurocode 3 - 5.3)*

$$\varepsilon = \left(\frac{235}{f_y}\right)^{0.5} = 1$$

Flange check : $\frac{c}{t_f} = 7.78 \leq 10\varepsilon$ $\Rightarrow$ Flange is class 1.

Web check : $\frac{d}{t_w} = 18.67 \leq 33\varepsilon$ $\Rightarrow$ Web is class 1.

All elements are class 1 $\Rightarrow$ Cross-section class 1.

Shear resistance *(Eurocode 3 - 5.4.6)*

Shear area : $A_v = 4109\ mm^2$

Shear resistance : $V_{pl.Rd} = \dfrac{A_v \dfrac{f_y}{\sqrt{3}}}{\gamma_{M0}} = 557.5 \text{ kN}$ and $\dfrac{V_{Sd}}{V_{pl.Rd}} = 0.07$

As this is less than 0.5, the effect of shear on the moment resistance is neglected.

Resistance of the cross-section *(Eurocode 3 - 5.4.8.1)*

The criteria to be fulfilled is : $M_{Sd} \le M_{N.y.Rd}$

For standard rolled section : $M_{N.y.Rd} = 1.11(1-n)M_{pl.y.Rd}$

$$n = \frac{N_{Sd}}{N_{pl.Rd}} = 0.352 \quad \text{wherefrom} \quad M_{N.y.Rd} = 264.5 \text{ kNm}$$

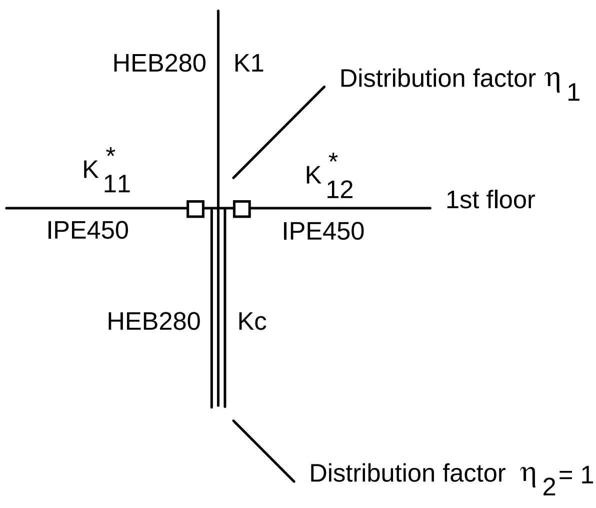
$\Uparrow$ Satisfactory.

Member resistance : flexural buckling *(Eurocode 3 - 5.5.4 (1))*

The section should satisfy : $\dfrac{N_{Sd}}{\chi_{min} A \dfrac{f_y}{\gamma_{M1}}} + \dfrac{k_y M_{y.Sd}}{W_{pl.y} \dfrac{f_y}{\gamma_{M1}}} \le 1$

χ_{min} is the lesser of χ_y and χ_z (*Eurocode 3 - 5.5.1*) , where χ_y and χ_z are the reduction factors determined as follows *(Eurocode 3 - Annex E)* :

Computation of l_y , the in-plane buckling length :



K_c : column stiffness coefficient :

$$K_c = \frac{I_c}{L} = \frac{19270}{350} 55.06 \text{ cm}^3$$

K_1 : stiffness coefficient for the adjacent length of column :

$$K_1 = \frac{I_c}{L} = \frac{19270}{350} 55.06 \text{ cm}^3$$

K_{11}^* , K_{12}^* : effective stiffness coefficient of the beam with semi-rigid joints :

$$K_{11}^* = K_{12}^* = \frac{I_{b,eff}}{L_b}$$

$I_{b,eff}$: second moment of area of the beam with semi-rigid joints :

$$I_{b,eff} = \frac{I_b}{1 + 3\left(\dfrac{2EI_b}{L_b \dfrac{S_{j,ini}}{2}}\right)}$$

I_b : second moment of area of the IPE450 beam section;

$S_{j,ini}$: initial stiffness of the joint;

$S_{j,ini}/2$ nominal stiffness of the joint.

Thus, one has :

$$I_{b,eff} = \frac{33740}{1 + 3\left(\dfrac{2 \times 210000 \times 33740 \times 10^4}{6500 \times \dfrac{66755 \times 10^6}{2}}\right)} = 17043 \text{ cm}^4$$

$$K_{11}^* = K_{12}^* = \frac{17043}{650} = 26.22 \text{ cm}^3$$

$$\eta_1 = \frac{K_c + K_1}{K_c + K_1 + K_{11}^* + K_{12}^*} = \frac{55.06 + 55.06}{55.06 + 55.06 + 26.22 + 26.22} = 0.68$$

$$\frac{l_y}{L_c} = 0.5 + 0.14(\eta_1 + \eta_2) + 0.055(\eta_1 + \eta_2)^2 = 0.5 + 0.14 \times 168 + 0.055 \times 2.82 = 0.89$$

Therefore, $l_y = 0.89 \times 3500 = 3116$ mm

$$\lambda_y = \frac{l_y}{i_y} = 25.7 \qquad \lambda_1 = 93\varepsilon = 93 \qquad \bar{\lambda}_y = \frac{\lambda_y}{\lambda_1}(\beta_A)^{0.5} = 0.277$$

$\chi_y = 0.976$ (curve *b*)

The effective out-of-plane buckling length is taken as the system length : $l_z = 3500$ mm

$\lambda_z = \dfrac{l_z}{i_z} = 49.4$ $\qquad \lambda_1 = 93\varepsilon = 93$ $\qquad \bar{\lambda}_z = \dfrac{\lambda_z}{\lambda_1}(\beta_A)^{0.5} = 0.531$

$\chi_z = 0.828$ (curve *c*)

$\chi_{min} = \chi_z = 0.828$

$k_y = 1 - \mu_y \dfrac{N_{Sd}}{\chi_y A f_y} \leq 15$ $\qquad$ where $\mu_y = \bar{\lambda}_y\left(2\beta_{My} - 4\right) + \dfrac{W_{pl.y} - W_{el.y}}{W_{el.y}} \leq 0.9$

$\beta_{My} = 18$ and $\mu_y = -0.0096$ $\qquad \Uparrow \qquad k_y = 1003$

$$\frac{N_{Sd}}{\chi_{min} A \dfrac{f_y}{\gamma_{M1}}} + \frac{k_y M_{y.Sd}}{W_{pl.y} \dfrac{f_y}{\gamma_{M1}}} = 0.47 + 0.38 = 0.85$$

$\Uparrow$ Satisfactory.

Member resistance : lateral torsional buckling *(Eurocode 3 - 5.5.4 (2))*

The section should satisfy : $\qquad \dfrac{N_{Sd}}{\chi_z A \dfrac{f_y}{\gamma_{M1}}} + \dfrac{k_{Lt} M_{y.Sd}}{W_{pl.y} \dfrac{f_y}{\gamma_{M1}}} \leq 1$

(Eurocode 3 - table F.1.1)

k=1 and $\psi = 0$ $\qquad$ => $\qquad$ $C_1 = 1.879$ $\quad C_2 = 0$ $C_3 = 0.939$

(Eurocode 3 - F.2.2.(1))

$z_j = 0$ (symmetrical section) and $z_g = 0$ (end moment loading)

$$\lambda_{LT} = \frac{\dfrac{L}{i_{LT}}}{C_1^{0.5}\left[1 + \dfrac{\left(\dfrac{L}{a_{LT}}\right)^2}{25.66}\right]^{0.25}} = 30.21$$

(*Eurocode 3 - F2.1(1))*

$$\bar{\lambda}_{LT} = \frac{\lambda_{LT}}{\lambda_1}\left(\beta_w\right)^{0.5} = 0.325 \leq 0.4$$

No allowance for lateral torsional buckling is necessary.

⇑ Satisfactory.

11.4.5.2.4 Joint Design

Inner beam-to-column joints

Floor level joint (HEB280 / IPE450)

The load combination case 5 is found the most critical for this joint. The maximum amplified bending moment applied to a joint is *217 kNm* and the shear force is 240 kN.

For this joint :

Design moment resistance :	220.5 kNm	>	217 kNm
Shear resistance :	443.3 kN	>	240 kN

⇑ Satisfactory

Roof level (HEB280 / IPE360**)**

The load combination case 5 is found the most critical for this joint. The maximum amplified bending moment applied to a joint is *82* kNm and the shear force is 119 kN.

For this joint :

Design moment resistance :	115.8 kNm	>	82 kNm
Shear resistance :	161.4 kN	>	119 kN

⇑ Satisfactory

Outer beam-to-column joints

Floor level (HEB240 / IPE450**)**

The load combination case 5 is found the most critical for this joint. The maximum amplified bending moment applied to a joint is *145* kNm and the shear force is *221* kN.

For this joint :

Design moment resistance :	151.3 kNm	>	145 kNm

Shear resistance : 282.4 kN > 221 kN

↑ Satisfactory.

Roof level (HEB240 / IPE360)

The load combination case 5 is found the most critical for this joint. The maximum amplified bending moment applied to a joint is 66 kNm and the shear force is 115 kN.

For this joint :

Design moment resistance : 94.8 kNm > 66 kNm

Shear resistance : 161.4 kN > 115 kN

↑ Satisfactory.

11.5 Conclusion

Clearly the unbraced frame can get a substantial advantage from the semi-rigid concept. Though the semi-rigidity of the joints requires an increase in the member sizes, because drifts and deflections would be larger than in the solution with rigid joints, significant cost savings may be expected from the use of much less expensive (unstiffened) joints.

Eurocode 3 permits the use of the semi-rigid design procedure. This implementation, compared to most previous standards, has been hampered up to now by the lack of appropriate methods of global analysis and design tools. The latter are now becoming readily available and their use in the daily practice is therefore a matter of technology transfer and further code recognition.

CHAPTER 12

WORKED EXAMPLE 3

DESIGN OF AN INDUSTRIAL TYPE BUILDING

12.1 Frame geometry

The skeletal structure of a two bay pinned-base pitched portal frame with haunches for an industrial building is shown in Figure 12.1.

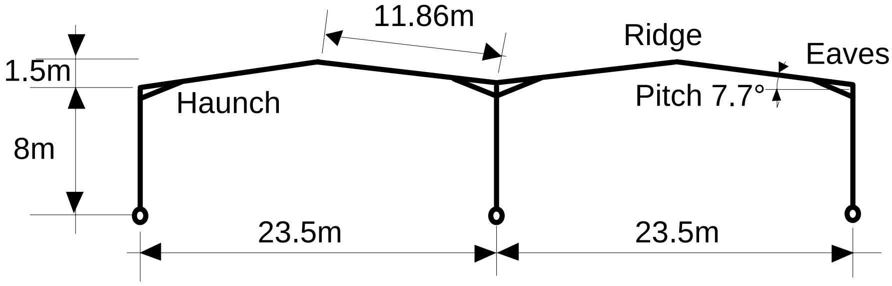


Figure 12.1 Frame geometry

The outside dimensions of the building, including the cladding, are :

Width :	48 m
Height :	10 m
Length :	60,5 m

The portal frames, which are at 6,0 m intervals, have pinned-base 8 m high columns at centrelines of 23,5 m and have rafters sloped at 7,7° with a centreline ridge height of 9,5 m above ground level.

Haunches are used for the joints of the rafters to the columns.

12.2 Objectives and design strategy

The principal objective is to aim at *global economy*, without increasing the design effort in any significant manner. A *traditional approach* to the design of the structure including the joints is adopted initially.

The traditional approach (see Chapter 2) is taken here to describe when the design of the joints is carried out once the global analysis and the design of the members have been accomplished. With such a separation of the task of designing the joints from those of analysing the structure and designing the structural members, it is possible that they are carried out by different people who may either be within the same company or, in some cases, may be part of another company.

It has been usual for designers to put web stiffeners in the columns so as to justify the usual assumption that the rafter-to-column joints are rigid. It is recognised that eliminating these stiffeners simplifies the joint detailing and reduces fabrication costs. Although the removal of the stiffeners may have an impact on the member sizes required, in particular those of the columns, this is not always the case.

Therefore, the strategy chosen here is to assume that *economy can be achieved by the elimination of the web stiffeners in the columns*. For the chosen structure, it is shown that the joint detailing can be simplified without any modification in the member sizes being required and without violating the initial assumption about the rigid nature of the joints. To achieve this end, the methods provided in the *Eurocode 3* for the design of the moment resistant joints are used.

12.3 Design assumptions and requirements

12.3.1 Structural bracing

The structure is unbraced in its plane.

In the longitudinal direction of the building, i.e. normal to the plane of the portals, bracing is provided so that the purlins act as out-of-plane support points to the frame. It is therefore assumed that the top of each column is held in place against out-of-plane displacement and that the lateral support provided for the rafter is adequate to prevent lateral torsional buckling in it.

12.3.2 Structural analysis and design of the members and joints

A widely used elastic linear elastic analysis was adopted for the ultimate and serviceability limit states. Elastic analysis is particularly suited since plastic hinge behaviour in the members or the joints is not considered.

At the final stage of the design, an allowance was made in the analysis for the increased section properties of the rafter over the length of the haunches.

In accordance with the principle that elastic analysis is valid up to the formation of the first plastic hinge in the structure, the plastic design resistances of the member cross-sections and of the joints can be used for the verification of the ultimate limit states.

The traditional assumption that joints are *rigid* is adopted. This assumption is verified.

12.3.3 Materials

Hot-rolled standard sections are used for the members.

For the members, the haunches, the end-plates, the base-plates and any stiffeners, an *S275* steel to *EN 10025*, with a yield strength of 275N/mm² and an ultimate strength of 430N/mm², is adopted.

The bolts are Class 10.9 with mechanical characteristics according to *EN 20898-1* and *EN 20898-2*.

12.3.4 Partial safety factors on resistance

The values of the partial safety factors on resistance are as follows :

γ_{M0} = 1,1 for the resistance of cross-sections;

γ_{M1} = 1,1 for the buckling resistance of members;

γ_{M2} = 1,25 for the resistance of net sections;

γ_{Mb} = 1,25 for the resistance of bolts;

γ_{Mw} = 1,25 for the resistance of welds.

12.3.5 Loading

12.3.5.1 Basic loading

While the loads given are typical for a building of this type, they should be taken as indicative since the values currently required at the present time in different countries vary. These differences concern wind and snow loading mainly.

Rather than apply the relevant parts of *ENV 1991-Basis of Design*, which either are recently available or are still under discussion, the French loading standards were used to determine the design load intensities and their distribution on the structure. The building is situated in a rather exposed location for wind.

For simplicity, the self-weight of the cladding plus that of its supporting purlins is considered to act as a uniformly distributed load on the frame perimeter.

Permanent actions	
Roofing self-weight	Purlins: 0,15 kN/m Cladding with insulation: 0,2 kN/m^2
Wall self-weight	Cladding with insulation: 0,2 kN/m^2

Variable action Permanent and variable actions	
Wind load	Wind pressure of 0,965 kN/m^2 at 10 metres from the ground level.
Snow load	Roof under 0,44 kN/m^2

Table 12.1 Permanent and variable actions

12.3.5.2 Basic load cases

The basic load cases are schematised in Table 12.2.

12.3.5.3 Load combination cases

12.3.5.3.1 Ultimate load limit state combinations

The simplified load combination cases of *Eurocode 3 -Chapter 2* are adopted.

Thus, the following ultimate load limit state combinations cases have been examined :

1.35 G + 1.5 W (2 combinations)

1.35 G + 1.5 S (4 combinations)

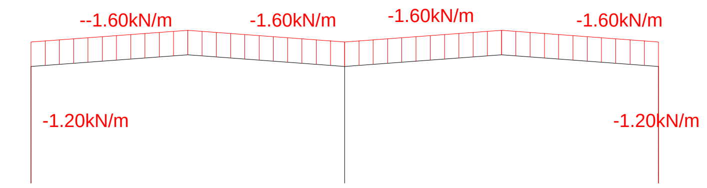
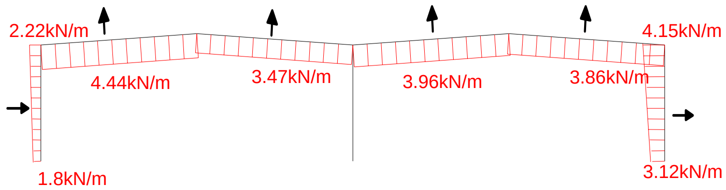
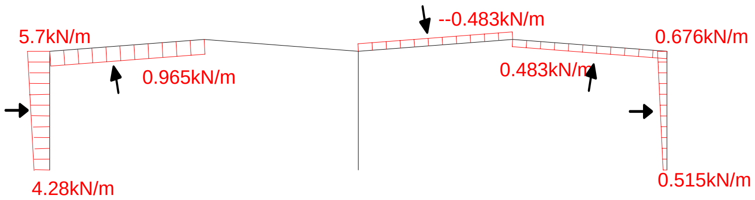
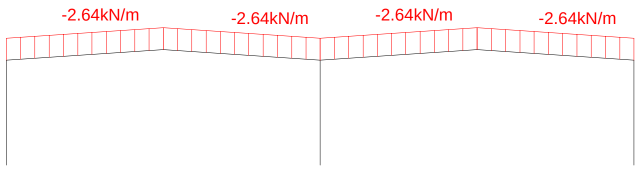
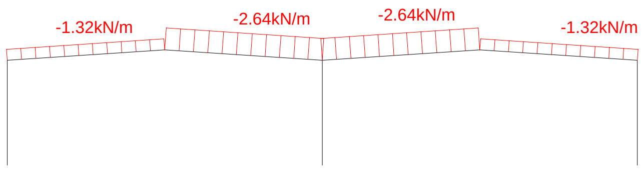
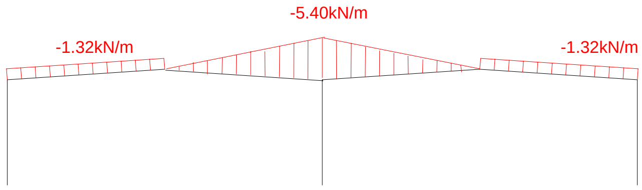
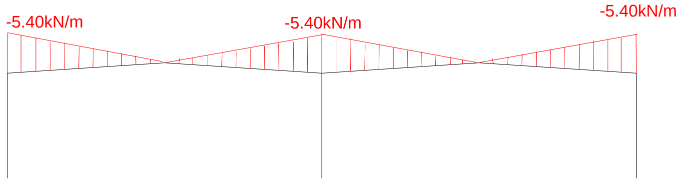
1.35 G + 1.35 W + 1.35 S (8 possible combinations).

where *G* is the permanent loading, *W* is the wind loading and *S* is the snow loading.

12.3.5.3.2 Serviceability limit state requirements and load combinations

According to *Eurocode 3-4.2.2(1) and Table 4.1*, the limit for the maximum vertical deflection of the roof is:

$$\delta_{max} \le \frac{L}{200}$$ where L is the span of a rafter.

Dead load other than portal self-weight (G1)⁎	
Internal Wind Pressure (W1)	
Internal Wind Suction (W2)	
Snow (S1)	
Snow (S2)	
Snow (S3)	
Snow (S4)	

⁎ *The self-weight of the frame structural members(G2) is added to the self-weight from the cladding and purlins (G1) to give the total dead load (G).*

Table 12.2 Basic load cases

According to *Eurocode 3 - 4.2.2(4),* the limit for the horizontal displacement of a portal frame without a gantry crane is :

$\delta_{horiz} \leq \frac{h}{150}$ where *h* is the height of the column at the eaves.

The following serviceability limit state combination cases have been examined:

- Maximum vertical deflection at the ridge (mid-span of each bay):

  1.0 G + 1.0 S1

  1.0 G + 1.0 S4

- Maximum horizontal deflection at the eaves:

  1.0 G + 1.0 W1

  1.0 G + 1.0 W2

12.3.5.4 Frame imperfections

The sway imperfections are derived from the following formula (*see Eurocode 3- 5.2.4.3(1)*):

$$\phi = k_c k_s \phi_0$$

where :

$$k_c = \sqrt{0.5 + \frac{1}{n_c}} \leq 1$$

$$k_s = \sqrt{0.2 + \frac{1}{n_s}} \leq 1$$

For the present structure we have:

$n_c = 3$ (number of full height columns per plane);

$n_s = 1$ (number of the story in the frame);

wherefrom :

$$\phi_0 = \frac{1}{200}$$

$$k_c = \sqrt{0.5 + \frac{1}{3}} = 0{,}913$$

$k_s = \sqrt{0.2 + 1}$ = 1,095 > 1.0 therefore take 1,0

$$\phi = (0{,}913) x (1{,}0) . \frac{1}{200} = \frac{1}{219}$$

All the columns are assumed to have an inclination of ϕ so that the eaves and the ridges are initially displaced laterally, as shown in Figure 12.2, by an horizontal distance of :

$\phi . h = \frac{8000}{219} = 36{,}5\text{mm}$ at the eaves and $\phi . h = \frac{9500}{219} = 43{,}4\text{mm}$ at the ridge.

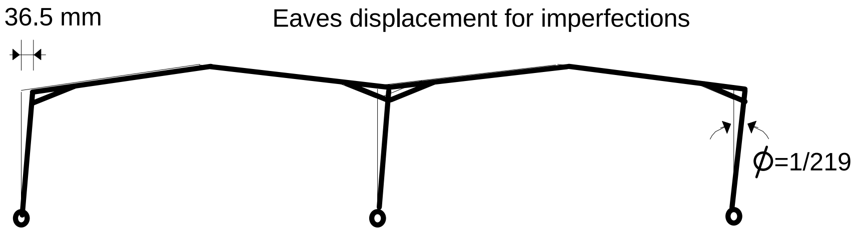


Figure 12.2 Global frame imperfections

12.4 Preliminary design

12.4.1 Member selection

It was decided to use standard hot-rolled sections.

When resistance is the only determining factor, it is usually possible in a two bay portal frame of this kind to have a smaller column section size for the central column than for the eaves columns. However, in this case the use of similar columns throughout was justified since the wind loads are quite high and serviceability requirements on horizontal deflections are an important consideration in the choice of the member sections. Doing this provided a column-rafter combination with adequate overall structural stiffness and strength and furthermore insured that, despite the fact that the columns are unstiffened, the assumption of *rigid* joints is not violated.

Taking these considerations into account, the following member section sizes were chosen:

Columns : IPE 550

Beams (rafters) : IPE 400

12.4.2 Joint selection

A flush end-plate bolted haunch joint is used for the rafter-to-column joints. The haunch is obtained by welding a part of an IPE 400 section to the bottom flange at the ends of each IPE 400 rafter. The height of the section is increased from 403,6mm (flange-to-flange allowing for the beam slope of 7,7°) to 782,6mm. The haunch extends 1,5 m along the length of the rafter.

An extended end-plate bolted joint is used at the mid-span of the rafters, i.e. at the ridge joints.

12.5 Classification of the frame as non-sway

The global analysis was conducted using a first-order elastic analysis and assuming rigid joints. Only the results for the two more critical load combination cases are given (see Table 12.3).

U.L.S. Load combin. case	**Load effect**	**Eaves column (base)**	**Eaves Column (top)**	**Central column (base)**	**Central Column (top)**	**Haunch at eaves column**	**Beam at haunch**	**Beam at mid-span**	**Haunch at central column**
1.35 G +1.5 S1	M (kNm)	0.0	291.8	0.0	6.44	291.8	220.80	+121.7	326.45
	N (kN)	105.1	80.87	179.7	168.53	45.96	45.14	35.5	46.3
	V (kN)	36.53	36.42	0.83	0.78	75.87	68.85	6.63	78.79
1.35 G +1.35 S1 +1.35 W2	M (kN/m)	0.0	328.57	0.0	91.99	328.57	237.99	+111.9	344.58
	N (kN)	101.26	77.16	171.42	160.24	47.00	55.12	35.41	56.28
	V (kN)	44.49	37.95	11.53	11.47	71.99	65.50	11.75	75.42

Table 12.3 Internal efforts for the most critical load combination cases at ULS

According to *Eurocode 3 - 5.2.5.2(4),* an unbraced frame can be classified as non-sway for a given load if the following criterion is satisfied :

$$\frac{\delta}{h}.\frac{V}{H} \leq 0.1$$

where :

δ horizontal displacement at the top of the storey, relative to the bottom of the storey;

h storey height;

H total horizontal reaction at the bottom of the storey;

V total vertical reaction at the bottom of the storey.

In the following, only the most critical load combination case was considered : dead load + snow.

Note :
The method of Eurocode 3 is not strictly valid for single storey pitched portal frames. The reason is that the compression in the beams (rafters) is not properly accounted for when the beams are at a pitch. Furthermore, since the eaves columns are subject to quite large, but opposing, lateral displacements, there is a difficulty of correct interpretation.
Either some adaptation of the method is needed or a more sophisticated method is required.
A number of more suited approaches are therefore presented to examine the sway stability of the structure.

a) Method using the lateral stiffness of the frame

It can be observed that the criterion can be reorganised as follows :

$$\frac{\delta}{h}.\frac{V}{H} = \left[\frac{\delta}{H}\right]_{mean}.\left[\frac{V}{h}\right] = \left[\frac{1}{Stiffness}\right].\left[\frac{V}{h}\right] \leq 0.1$$

The method given here involves the mean lateral stiffness of the structure corresponding to a horizontal load at the eaves level. The technique introduces the effect of the axial load in the rafters. The horizontal load has been shared between the columns as shown in Figure 12.3.

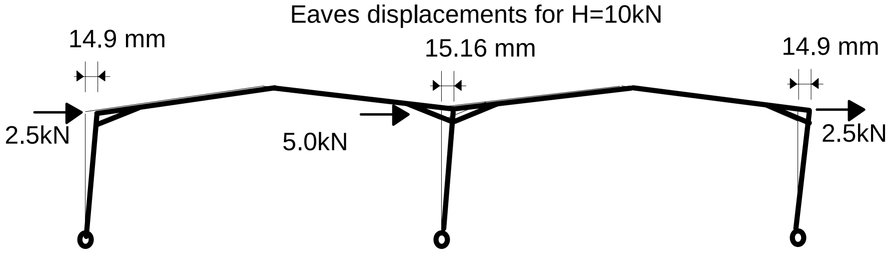


Figure 12.3 Frame lateral stiffness

The value of V corresponds to the ultimate limit state load combination case involving the maximum vertical load in the columns, which is easy to estimate prior to any analysis.

In the first-order elastic analysis for the vertical loads and the lateral displacement, the initial sway imperfections have been included.

The average lateral displacement at the eaves (see Figure 12.3) for a total horizontal load H of 10 kN is 15,0 mm (δ_{mean}).

The examination of the ultimate load cases indicates that the maximum value of the sum V of the axial loads in the three columns is 389,5 kN, which is for the gravity loading plus snow loading combination case.

The storey height being 8 m, we obtain :

$$\frac{\delta_{mean}}{H}.\frac{V}{h} = \frac{15,0}{10}.\frac{389,5}{8000} = 0,073 < 0,1$$

According to this approach, the structure can be classified as *non-sway.*

b) Method of weighted average column chord rotation

In this approach, which is the subject of a forthcoming publication by Y. Galea of CTICM, the individual loading cases can be examined by using an average value of the column chord rotation, which is weighted to account for the axial load in each column. Since an average weighted column chord rotation must be considered, the algebraic sum of the weighted chord rotations is calculated.

$$\varphi_{mean} = \left[\frac{\delta}{h}\right]_{mean} = \frac{\Sigma\varphi_i.N_i}{\Sigma N_i}$$, where the sum is over all columns in a storey, axial load in each being N_i.

For the load combination case 1, the horizontal load is that for the imperfections only. This load is taken as :

$H = V/\Phi = V/219$ so that $V/H = 219.$

We obtain for load combination case 1 :

$$\frac{\delta}{h}.\frac{V}{H} = \left[\frac{\delta}{h}\right]_{mean}.\frac{V}{H} = \frac{\left[\frac{-19,21}{8000}\right] x\, 104,77 + \left[\frac{+2,57}{8000}\right] x\, 179,72 + \left[\frac{+24,27}{8000}\right] x\, 105,01}{104,77 + 179,72 + 105,01} x\; 219$$

$$= 0,07$$

The structure can be classified as *non-sway* according to this method.

c) Method using a specialised analysis to determine the critical load

Another approach to evaluate the sensitivity of the structure to second-order effects is to obtain the value of V_{cr} for each ultimate limit state. The value of V_{cr} can be obtained by an analysis using specially developed computer programmes, a number of which are commercially available.

From such an analysis for the load combination case 1, we obtain :

$$V_{sd} / V_{cr} = 1/13{,}202 = 0{,}076.$$

d) Method using a special formula to determine the critical load (Horne and Davies)

For hand calculations, use can be made of formulae relevant to this type of structure proposed by Horne and Davies (see *Plastic design of single-storey pitched roof portal frames to Eurocode 3*, by King C.M., Technical report n°147, The Steel Construction Institute).

Two separate cases need to be examined:

- Eaves column plus rafter;
- Central column plus one rafter on each side.

The formula for truly pinned bases is as follows for the eaves column-rafter case :

$$\alpha_{cr} = \frac{V_{cr}}{V_{Sd}} = \frac{3EI_r}{s\left[0{,}3P_r . s + \left(1 + \frac{1{,}2}{R}\right) P_c . h\right]}$$

$$= \frac{3 x\, 210 x\, 231{,}3.}{11{,}86 x \left[0{,}3 x\, 45 x\, 11{,}86 + \left(1 + \frac{1{,}2}{4{,}3}\right) x\, 92{,}9 x\, 8\right]} = 11{,}06$$

where :

P_c and P_r axial compression loads in the column and in the rafter respectively;

R ratio of the column flexural stiffness to the rafter flexural stiffness;

s length of the rafter along the slope (eaves to ridge-apex = 11.86 m);

h height of the column (base to eaves = 8 m);

E Young modulus (210000 N/mm²);

I_r second moment of area of the rafter in the frame plane (I_y = $231{,}3.10^6$ mm^4).

For the eaves column-rafter case we obtain :

$$R = \frac{\frac{I_c}{h}}{\frac{I_r}{s}} = \frac{I_c . s}{I_r . h} = \frac{67116{,}5 \, x \, 11{,}86}{23128{,}4 \, x \, 8} = 4{,}3$$

The values of the average axial loads for the load combination case concerned are 92,9kN and 45kN for the external column and rafter respectively.

For the internal column-rafter case, the values are 174kN(column) and 45kN(rafter). A similar but slightly modified formula gives the following result :

$$\alpha_{cr} = 9{,}8$$

The inverse of the result is to be compared to the values given by the other methods:

$1/\alpha_{cr} = 1/11{,}06$ = 0,09 for the eaves column-rafter case;

$1/\alpha_{cr} = 1/9{,}8$ = 0,10 for the central column-rafter case.

This method appears to be conservative, probably because it does not account for the stabilising effect of the haunches. It indicates that the structure *cannot be strictly considered as non-sway*; however since the result is close to the required criteria and because the method is conservative, it can be accepted to allow a non-sway classification.

e) Second-order elastic analysis to integrate the second-order effects

The last approach possible is to carry out a second-order elastic analysis. The structure has been thus analysed and the results show that second-order effects are negligible, thus confirming the validity of the methods of assessment used above.

12.6 Design checks of members

According to *Eurocode 3 - 4.2.2(1) and Table 4.1*, the limit for the maximum vertical deflection of the roof under the service loads is:

$$\delta_{max} \leq \frac{L}{200} = \frac{23500}{200} = 117{,}5\text{mm}$$

Since the vertical deflection of 61,25 mm < 117,5 mm, the condition is satisfied.

According to *Eurocode 3 - 4.2.2(4),* the limit for the horizontal displacement, under the service loads, of a portal frame without a gantry crane is :

$$\delta_{horiz} \leq \frac{h}{150} = \frac{8000}{150} = 53{,}3\,\text{mm}$$

Since the maximum lateral displacement is 42,35 mm < 53,3 mm, the condition is satisfied.

Detailed verifications at the ultimate limit state (sections and lateral stability of rafters, sections and stability of columns) have been carried out using the *Eurocode 3-TOOLS* suite of programmes. These calculations show that the design is fully satisfactory. In order to reduce the volume of this worked example and not to duplicate checks that have yet been illustrated in the two previous examples, the detailed results are not reproduced here.

12.7 Joint design and joint classification

The joints are designed according to *Eurocode 3-(revised) Annex J.*

12.7.1 Joint at the mid-span of the beam

The joint at the ridge is subjected to the following extreme design loading situation:

Load combination case	Load effects	
1	Positive moment M_{Sd} (kNm) :	121,77
	Axial force (compression) N_{Sd} (kN)	-46,3
	Shear force V_{Sd} (kN)	6,63

Table 12.4 Load effects at mid-span of the beam

The axial load plastic resistance of the IPE 400 beam is :

$$N_{pl} = \frac{A\,x\,f_y}{\gamma_{M0}} = \frac{46.5\,x\,10^2\,x\,275}{11x\,10^3} = 1162{,}5\,\text{kN}$$

Since the axial loads are always smaller than 10% of axial load plastic resistance N_{pl} of the IPE 400 beam section, it can be assumed that the design resistances of the joints are unaffected by them. Shear forces at this location are also small.

The extended end-plate joint of Figure 12.4 has been designed according to *Eurocode 3-(revised) Annex J*, with the aid of the *DESIMAN* program.

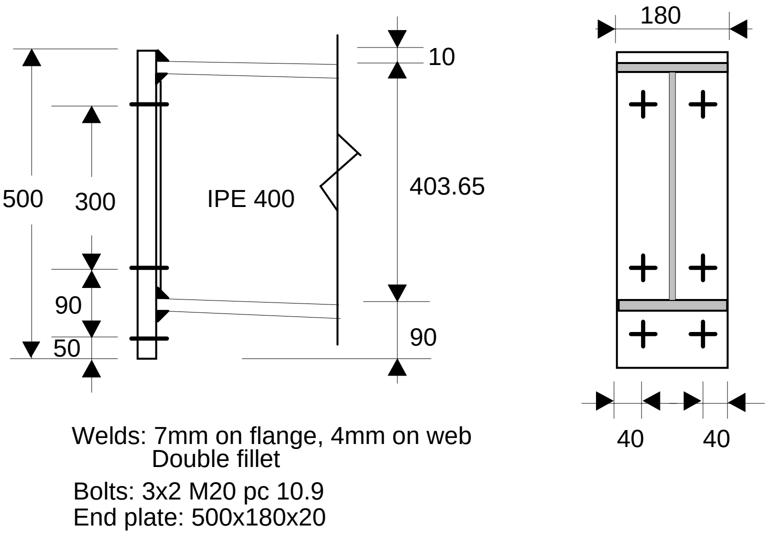


Figure 12.4 Beam ridge end-plate joint

a) Resistance to positive moments and associated shear forces

The characteristics of the mid-span end-plate joint and joint to positive moments are as follows:

Moment resistance :	$M_{j.Rd}$	= 235,6 kNm.
Shear resistance :	$V_{j.Rd}$	= 107.7kN.
Initial joint stiffness :	$S_{j.ini}$	= 273219 kNm/radian.
Nominal joint stiffness :	S_j	= 91073 kNm/radian.

Since $M_{Sd} < M_{j.Rd}$, the joint has adequate resistance in bending.

Since $V_{Sd} < V_{j.Rd}$, the joint has adequate shear resistance.

b) Joint classification

This joint can be classified as rigid if the following criterion of *Eurocode-(revised) Annex J* for an unbraced frame is met :

$$\frac{S_{j.ini} L_b}{EI_b} \geq 25$$

In the present case, the length L_b has to be taken as the developed length of the rafter, i.e. 23,71 m. The rigidity of the IPE 400 beam over a span of 23.71 m is :

$$\frac{EI_b}{L_b} = \frac{210000\,x\,23128.4\,x\,10^4}{23710} = 2049\;10^6\,Nmm = 2049\;\text{kNm}$$

Thus, for the mid-span ridge joint we obtain for the positive moment:

$$\frac{S_{j.ini}L_b}{EI_b} = \frac{273219}{2049} = 133{,}4$$

which meets the criterion for a *rigid* joint classification.

The bending resistance of the IPE 400 beam is :

$$M_{pl} = \frac{W_{pl}f_y}{\gamma_{M0}} = \frac{1307{,}1x\,10^3\,x\,275}{1{,}1x\,10^6} = 326{,}78\;10^6\,Nmm = 326{,}78\,k\text{Nm}$$

Since $\frac{M_{j.Rd}}{M_{pl}} = \frac{121{,}77}{326{,}78} = 0{,}37 < 1{,}0$, the joint has a *partial-strength* classification.

12.7.2 Haunch joint at the beam-to-column joint

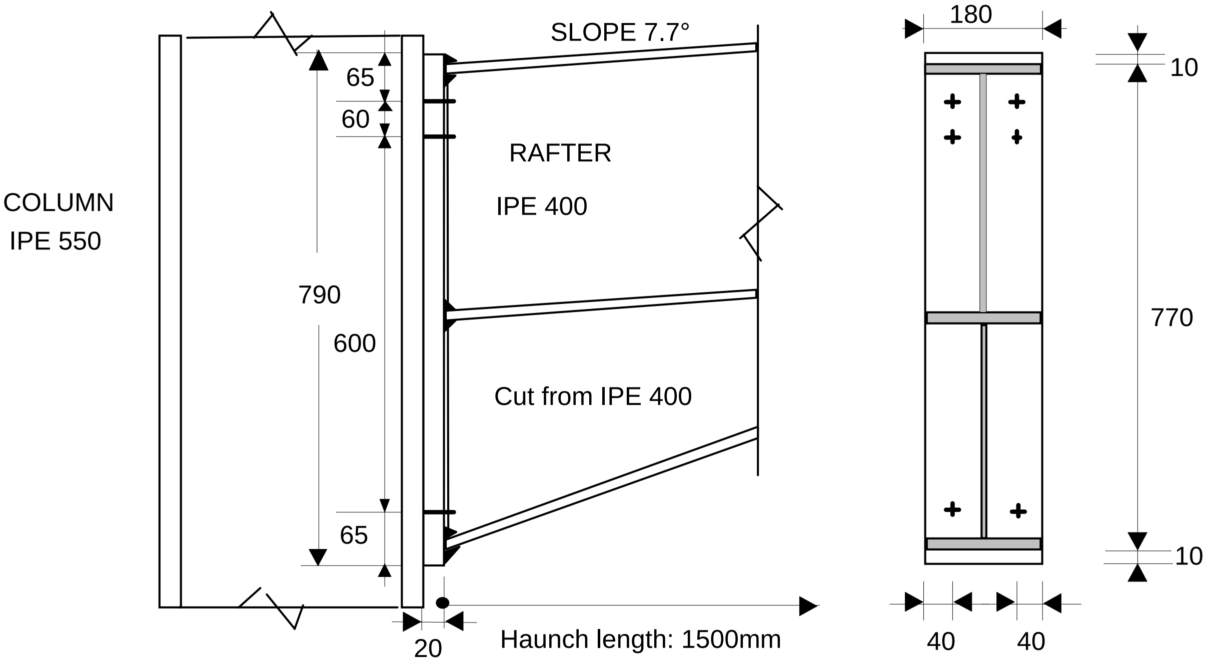
The beam-to-column joint is subjected to the two following extreme design loading situations :

Load combination case	Loads effects	
2 (at eaves column)	Negative moment M_{Sd} (kNm)	328,57
	Axial force (tension) N_{Sd} (kN)	77,16
	Shear force V_{Sd} (kN)	37,95
2 (at central column)	Negative moment M_{Sd} (kNm)	344,58
	Axial force (compression) N_{Sd} (kN)	56,28
	Shear force V_{Sd} (kN)	75,42

Table 12.5 Load effects at the beam-to-column joint

Since the axial loads are always smaller than 10% of the axial load plastic resistance N_{pl} of the IPE 400 beam section, it can be assumed that the design resistance of the joints is unaffected by them.

The joint of Figure 12.5 was designed with the aid of the DESIMAN program.



Welds: 7mm on flange, 4mm on web
Double fillet
Bolts: 3x2 M20 pc 10.9
End-plate: 790x180x20

Figure 12.5 Beam-to-column end-plate haunch joint

a) Resistance to negative moments and associated shear forces at the eaves column

The characteristics of the haunch joint at the beam to eaves column joint under negative moments are :

Moment resistance :	$M_{j.Rd}$	= 335,8 kNm.
Shear resistance :	$V_{j.Rd}$	= 308 kN.
Initial joint stiffness :	$S_{j.ini}$	= 108640 kNm/radian.
Nominal joint stiffness :	S_j	= 54320 kNm/radian.

The resistance of the joint at the central column joint is similar, the failure mode being column web compression failure. The joint stiffness at this location could be considered as greater for symmetric loading about the central column; it is simpler to consider the joint to have the same stiffness as that of the eaves joint without any significant loss of accuracy.

Since $M_{Sd} < M_{j.Rd}$, the joint has adequate resistance in bending.

Since $V_{Sd} < V_{j.Rd}$, the joint has adequate shear resistance.

b) Resistance to negative moment and associated shear forces at the central column

The characteristics of the joint at this location are as follows :

Moment resistance:	$M_{j.Rd}$	= 360 kNm.
Shear resistance:	$V_{j.Rd}$	= 308 kN.
Initial joint stiffness:	$S_{j.ini}$	= 150537 kN.m/radian.
Nominal Rigidity:	S_j	= 75268 kN.m/radian.

Since $M_{Sd} < M_{j.Rd}$, the joint has adequate resistance in bending.

Since $V_{Sd} < V_{j.Rd}$, the joint has adequate shear resistance.

c) Joint classification

This joint can be classified as *rigid* since :

$$\frac{S_{j,ini} L_b}{EI_b} = \frac{108640}{2049} = 53 \quad > 25$$

and
$$\frac{S_{j,ini} L_b}{EI_b} = \frac{150537}{2049} = 73{,}5 > 25$$

Since for negative moments $\frac{M_{j.Rd}}{M_{pl}} = \frac{335{,}8}{326{,}8} = 1{,}03 > 1{,}0$, the joint is a *full-strength* joint.

12.8 Conclusions

An analysis of the structure accounting for the semi-rigid characteristics of the beam-to-column joints was also been carried out. It shows only a slight reduction in the moments at the beam-to-column joints with a corresponding slight increase in the mid-span moments. The small change in the moments obtained reflects the fact that the joints are quite rigid despite the absence of lateral stiffeners in the columns.

If horizontal web stiffeners were used, a smaller central column could be adopted and the eaves columns could be reduced to an IPE 500. However IPE 450 rafters are needed to avoid excessive loading in the column. As a result, this solution is not necessarily more economical in steel weight than the IPE

550 column/IPE 400 rafter solution; in addition it involves extra fabrication costs due to the use of column web stiffeners.

The other commonly used strategy for obtaining global economy is to use plastic design, but designs so obtained will usually require the more costly stiffened joints and, probably, a greater design effort. Which approach leads to the most economical solution can be determined only by the fabricator and/or designer.

It was decided to omit shear and horizontal web stiffeners in the column so as to provide the potential of economy in fabrication and in erection by simplification of the joint detailing. The possibility that the joints can be semi-rigid is therefore permitted, a priori. However it is demonstrated that by a judicious choice of members of sufficient strength and rigidity, the joints can be considered as rigid. The central column member size has been dictated in part by the absence of column web stiffeners, but the rafter and the eaves columns have not been affected by this option in joint detailing.

ADDITIONAL READING

European standards

Centre Européen de Normalisation
Eurocode 1: Basis of design and actions on structures
ENV 1991-1
CEN, Bruxelles.

Centre Européen de Normalisation
Eurocode 3 - Design of steel structures - Part 1.1 : General rules and rules for buildings
ENV 1993-1-1
CEN, Bruxelles.

Centre Européen de Normalisation
Eurocode 3 - Design of steel structures - Part 1.1 : General rules and rules for buildings - *Annex J (revised)*: Joints in building frames
CEN, Bruxelles.

European Standard EN 10025: Hot-rolled products of non-alloy structural steels-Technical delivery conditions.

European Standard EN 10113: Hot-rolled products in weldable fine grain structural steels.

European references

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European Research Activity in the Field of Semi-Rigid Joints and Frames
Edited by R. Zandonini, Technical Committee 10 - Structural Connections
ECCS, Brussels, April 1995.

Reference list of scientific research reports and publications.

European Convention for Constructional Steelwork
Analysis and Design of Steel Frames with Semi-Rigid Joints
Technical Committee 8 - Structural Stability, WG 8.1/8.2
ECCS, Brussels, Publication n° 67, 1992.

Background information and worked examples. Contains further references.

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Books covering all the aspects of steel and composite construction; some chapters devoted more especially to the design of joints.

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Joints in Steel construction: Moment Connections.
SCI, SilwoodPark, Ascot, Berks, 1995.

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Joints in simple construction - Volume 1: Design methods (2d edition)
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SCI, SilwoodPark, Ascot, Berks, 1992.

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Collection of research papers.

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Semi-rigid connections in steel frmes.
Mc Graw-Hill Inc., 1992.

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American Society of Civil Engineers
Connection flexibility and steel frames
ASCE, New-York, 1985.

Collection of papers dealing with design considerations, behaviour of connections, elastic buckling behaviour of flexibly-connected frames, strength and flexibility of flexibly-jointed frames and joint behaviour under cyclic loading

In French

Lescouarc'h, Y. (*)

Les pieds de poteaux articulés en acier. Dispositions constructives, méthodes de calcul, standardisation.
CTICM, Saint-Remy-les-Chevreuse, June, 1982.

Book devoted to pinned base-plate joints.

Lescouarc'h, Y. (*)
Les pieds de poteaux encastrés en acier. Dispositions constructives, méthodes de calcul, standardisation.
CTICM, Saint-Remy-les-Chevreuse, J une, 1988.

Book devoted to fixed base-plate joints.

CIMsteel EUREKA 130
Connection design and detailing - Engineering basis documents.
SCI, Silwood Park, Ascot, Berks, Publication n°151.

Methods for computer integrated manufacturing for constructional steelwork; amongst the set of CIMsteel publications , one is dealing with connection design and detailing (engineering basis doicuments).

A.P.K.
Construction métallique et mixte acier-béton. Volumes 1 et 2.
Eyrolles, Paris, 1996.

Books covering all the aspects of steel and composite construction; some chapters devoted more especially to the design of joints in accordance with *Eurocode 3-(revised) Annex J*.

Collection CTICM
Assemblages flexionnels en acier selon l'Eurocode 3
Outils de calcul pour les assemblages rigides et semi-rigides.
CTICM, Saint-Remy-les-Chevreuse, édition 1996.

Simplified version of the application rules of *Eurocode-(revised) Annex J*, design tables for beam-to-column end-plate and flange-cleated joints and for beam-to-beam end-plate joints (Tables established with the values of the safety factors for strength given in *French DAN* to *Eurocode 3*).

CRIF, Construction métallique et Département M.S.M. , Université de Liège
L'Eurocode 3 et les assemblages en acier: Aides de calcul pour assemblages rigides et semi-rigides.
CRIF-MSM, Liège, 1995.

Simplified version of the application rules of *Eurocode-(revised) Annex J*, design tables for beam-to-column end-plate and flange-cleated joints and for beam-to-beam end-plate joints (Tables established with the values of the safety factors for strength given in *Eurocode 3*).

Centre Suisse de la Construction Métallique (*)
Assemblages par plaques frontales et boulons HR - Assemblages de poutrelles par doubles cornières - Appuis de poutres sur raidisseurs.
SZS, Zurich, Publication C9.1, 1983.

Design tables for beam-to-column joints with bolted end-plates and beam-to-beam joints with web cleated connections including joint detailing and design resistances.

In German

ÖSTV (Österreichischer Stahlbauverband)
SZS (Schweizerische Zentralstelle für Stahlbau)
Rahmentragwerke in Stahl unter besonderer Berücksichtigung der steifenlosen
Bauweise, Wien, Zürich, Oktober 1987.

Theoretical background, examples and design tables for the design of steel building frames with "economical" joints.

Oberegge, O.; Hockelmann, H.-P. (*)
Bemessungshilfen für profilorientiertes Konstruieren
Stahlbau-Verlags GmbH, 2. Auflage, 1992

Design tables for member and joint design in steel building frames including rolled section properties, standard joint detailing and design resistances. (Remark: The tables for joint design are an updated version of the "DASt-Ringbuch: Typisierte Verbindungen")

Schweizerische Zentralstelle für Stahlbau (*)
Stahlbaupraxis - Strinplattenverbindungen und Trägeranschlüsse,
SZS, Zurich, Publication C 9.1, Zürich, 1983.

Design tables for beam-to-column joints with bolted end-plates and beam-to-beam joints with web cleated connections including joint detailing and design resistances.

Petersen, Ch.
Stahlbau
F. Vieweg & Sohn, Braunschweig, Wiesbaden, 1988

Handbook for steel construction. Contains detailed background information on all relevant topics related to the design of steel structures.

DSTV - Deutscher Stahlbau-Verband
Stahlbau Handbuch
Stahlbau-Verlags GmbH, Köln.

Handbook for engineers in practice. Gives guidelines for practical application in steel construction.

Stahl-Informations-Zentrum
ESDEP - Europäisches Stahlbau-Lehrprogramm, CD-ROM, Version 1.0
SIZ,Düsseldorf, 1995

German version of the ESDEP package (European Steel Design Education Program). The CD-ROM contains about 200 lectures and worked examples related to the design of steel structures. In addition the package includes 1000 slides and 20 videos.

(*) This publication refers to traditional design approaches. It is not considered as a reference which conforms with the design approaches presented in this manual.

GLOSSARY

Wording	**Meaning**
Analysis	See *global analysis.*
Actual stiffness (of a joint)	Best value of the rotational stiffness of a joint, as got from experiments, numerical simulations or accurate computations. See also *rotational stiffness*.
Approximate stiffness (of a joint) $S_{j,app}$	Rough estimation of the initial stiffness of a joint See also *initial stiffness*.
Beam	Member subject to predominant bending due to transverse loads and/or to end moments.
Beam-column	Member subject to combined significant bending and axial forces.
Braced frame	Frame where the sway resistance is provided by a bracing system with a response to in-plane horizontal loads which is sufficiently stiff for it to be acceptably accurate to assume that all horizontal loads are resisted by the bracing system. A steel frame may be classified as braced if the bracing system reduces the horizontal displacements of the frame by at least 80 %.
Characterisation (of a joint)	Determination of the properties of this joint for what regards its stiffness and/or strength and/or possibly rotation capacity.
Classification (of a joint)	Assignment of this joint to a class characterised by its stiffness (pinned, semi-rigid, rigid) and/or by its strength (pinned, partial strength, full strength) and/or ductility.
Column	Member subject to predominant axial compression.
Column web panel	Zone of the column web that is adjacent to the connection and the depth of which is concerned with the spreading of the stress resultants from the beam into the column.

Component	See *joint component.*
Connection	Location at which two members are interconnected, and, by extension, set of the physical joint components which fasten mechanically the connected members.
Continuous joint	Joint which ensures a full rotational continuity between the connected members.
Design approach	Specific frame design and analysis process.
Design bending moment $M_{j,Sd}$	Design bending moment experienced by the joint when the frame is subject to the design loads.
Design methodology	Description and selection of the appropriate design and analysis process.
Design strength $M_{j,Rd}$	Design bending resistance of the joint.
Double-sided joint configuration	Configuration of the joint where the column is fitted with two adjacent beams respectively located at about the same level on either side of the column axis.
Ductility (of a joint)	Measure of the ability of this joint to exhibit rotation capacity.
Elastic critical load parameter λ_{cr}	Amplification factor of the vertical loads producing the elastic buckling of the whole frame.
Elastic (global) analysis	Global analysis performed by assuming an indefinitely linear elastic moment-rotation relationship for members and joints.
Elastic-plastic (global) analysis	Global analysis performed by assuming a moment-rotation relationship composed of a linear elastic range and an infinite yield plateau for members and joints.
Elasto-plastic (global) analysis	Global analysis performed by assuming a non-linear moment-rotation relationship for members and joints.
Factored load	Load obtained by multiplying the relevant service load by an appropriate partial safety factor γ_F and possibly by an appropriate load combination factor ψ.
First-order collapse load parameter λ_p	Amplification factor of the loading components which produces a plastic hinge mechanism, any second-order effect being disregarded.
First-order theory/analysis	Theory where equilibrium is expressed in the non-defored configuration of the structural element or frame when determining the distribution of the internal forces.
Frame	Structural system composed mainly of beams, columns and beam-columns being fitted together by means of connections.
Frame imperfections	Initial out-of-plumb of the vertical members.

Full-strength (joint)		Joint the design resistance moment of which is not less than that of the connected members.
Global analysis (of a frame)		Process aimed at the determination of the distribution of internal forces and displacements in this frame subject to a given loading. See also *elastic analysis, elasto-plastic analysis, elastic-plastic analysis* and *rigid-plastic analysis*.
Idealisation		Simplified representation of the structural response of a section, a structural element, a joint or a frame.
Initial (rotational) stiffness	$S_{j,ini}$	Design rotational stiffness of the joint in the elastic range of its behaviour, i.e. when $M_{Sd} \leq 2/3\ M_{j,Rd}$.
Internal forces		Stress resultants (axial force, shear force(s) and bending moment(s)) in a cross-section.
Joint		Assembly of basic components which enable members to be connected together in such a way that the relevant internal forces can be transferred between them, and, by extension, whole zone concerned with this transfer.
Joint component		Any specific individual part of the joint which makes an identified contribution to one or more of the joint structural properties.
Joint model		Assumption regarding the joint behaviour that enables to account for the effects of the latter on the global analysis. See also *simple joint*, *continuous joint* and *semi-continuous joint.*
Limit state (of a frame)		State beyond which the frame no longer satisfies the design performance requirements, regarding more especially strength, stability or serviceability.
Linear behaviour		Structural response of an element or frame where the internal forces and displacements are proportional to the applied loads, and thus to the load parameter.
Load parameter λ		Multiplier of all the reference loads (factored loads) acting on a frame, so that to produce a proportional increase of the loading. See also *elastic critical load parameter, ultimate load parameter, elastic critical* and *first-order collapse load parameter.*
Load-introduction		Transfer of the axial forces existing in the beam flanges into the column web panel.
Major axis joint		Joint where the beam web is located in the same plane as the column web.
Member		Structural component of a framed structure such as a beam, a column or a beam-column.
Member imperfections		Geometrical (out-of-straightness) and structural (residual stresses) imperfections of a member and, by

	extension, equivalent out-of-straightness of this member.
Minor axis joint	Joint where the beam web is located in the plane perpendicular to the column web.
Modelling (of joint)	See *joint model.*
Modelling (of a frame)	Simplified representation of the structural behaviour of the supports and of the constituent members and joints, made preliminary to the global analysis of this frame.
Nominal (rotational) stiffness S_j	Rotational stiffness of the joint to be used for an elastic global analysis.
Non-linearities	Material and structural behaviours which result in a lack of proportionality between stress and strain and/or between the applied loads and the structural response (internal forces and displacements).
Non-linear behaviour	Structural response of a structural element or frame, which exhibits non-linearities.
Non-sway frame	Frame in which the response to in-plane horizontal loads is sufficiently stiff for it to be acceptable to neglect any additional internal forces or moments arising from horizontal displacements of its storeys.
Non-sway mode (of a frame)	Buckling mode of this frame when the horizontal displacement of any storey is prevented or may be considered as negligible.
Partial safety factor	Coefficient factoring a strength function and/or a loading in order to ensure that the structure remains fit for the use with a given probability.
Partial-strength joint	Moment resistant joint which neither meets the criteria for a full-strength joint nor can be classified as pinned joint.
Pinned joint	Joint which is not at all moment resistant and allows for free rotation between the connected structural members (truly pinned) and, by extension: joint the design rotational stiffness of which is without significative influence on the elastic distribution of internal forces (fairly pinned), or joint the design moment resistance of which is less than 25% of the design moment resistance required for a full-strength joint.
Plastic hinge	Fully yielded member section or joint subject to bending which behaves then as a structural hinge as soon as the plastic resistance has been reached.
Pre-design	See *preliminary design.*
Preliminary design	Determination of member and/or joint mechanical and geometrical properties made preliminary to the global analysis of the structure involving these members and joints.

Reference loads	Loads composing the reference loading to which a load multiplier is applied (usually the reference loads are chosen as the factored loads).
Rigid joint	Joint the rotational stiffness of which is infinite (truly rigid joint), and, by extension, joint of finite rotational stiffness where this relative rotation is without significative influence on the elastic distribution of internal forces (fairly rigid joint).
Rigidity (of a joint)	See *rotational stiffness.*
Rigid-plastic (global) analysis	Plastic global analysis where the elastic strains are fully disregarded compared to the plastic ones.
Rotation capacity	Ability of a member cross-section or joint, which has reached its design strength, to rotate without exhausting the material ultimate strain.
Rotational stiffness (of a joint)	Slope of the characteristic moment-relative rotation characteristic of the joint. See also *initial stiffness*, *nominal stiffness, actual stiffness* and *approximate stiffness.*
Second-order theory/analysis	Theory where the determination of the internal forces is conducted with reference made to the deformed shape of the structural element or frame.
Semi-continuous joint	Joint which ensures a partial rotational continuity between the connected members.
Semi-rigid joint	Joint allowing for a relative rotation between the axes of the connected structural members, which is likely to influence significantly the elastic distribution of the internal forces, and being bending resistant to a certain extent.
Service load	Load for service conditions, possibly affected by an appropriate load combination factor.
Sheared (column web) panel	See *column web panel.*
Simple joint	Joint which prevents from any rotational continuity between the connected members.
Single-sided joint configuration	Configuration where the column is fitted with only one adjacent beam.
Statically determinate structure	Structure where the distribution of the internal forces may be deduced from the sole equilibrium conditions.
Statically indeterminate structure	Structure where the determination of the internal forces needs that use be made not only of equilibrium but also of compatibility conditions.
Strength function	Expression enabling the determination of the presumably ultimate resistance.
Stiffness (of a joint)	See *rotational stiffness*.

Stiffness ratio η	Factor by which the initial rotational stiffness of a joint shall be divided to get a nominal rotational stiffness appropriate for frame analysis as long as the design moment in the joint does not exceed its design resistance $M_{j,Rd}$.
Structural response	Results, in terms of internal forces and displacements, of the global analysis of the structure subject to a given loading.
Sway frame	Frame where the horizontal displacements of the storeys induce significant additional forces or moments and need therefore to be accounted for in the global analysis.
Sway mode	Buckling mode of the frame when allowance is made for the horizontal displacements of the storeys.
Transformation parameter β	Ratio of the shear force in the column web panel to the compressive and tensile forces that are statically equivalent to the adjacent beam-end moment.
Unbraced frame	Frame where the lateral stability is not provided by a bracing system or is provided by a bracing system whose response to in-plane horizontal loads is not sufficiently stiff.
Ultimate load parameter λ_u	Amplification factor of the loading components for which the ultimate limit state of the frame is reached by interaction of plastic hinge formation and sway instability