Output, input, and undesirable output interconnections in Data Envelopment Analysis: convexity and returns-to-scale*

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Abstract

Increasing attention has been given to the development of specific techniques to deal with interconnections between outputs, inputs, and undesirable outputs for Data Envelopment Analysis (DEA) models. These techniques offer the advantages of improving the realism and the flexibility of DEA models; two aspects of crucial importance to convince practitioners about the attractiveness and the reliability of DEA models. In this paper, we propose a unifying methodology coherent with previous works to model these interconnections. We suggest treating the outputs as the fundamental component of the production process by modelling every output individually. This gives us the option of considering the interconnections with the inputs and the undesirable outputs. In particular, we make a distinction between undesirable outputs/inputs that are due to/used by all the outputs, and those that are due/allocated to specific outputs. Attractively, our methodology also offers the option of setting a different returns-to-scale assumption for each output-specific production process, and to choose between different types of convexity. We demonstrate the usefulness of our methodology with the case of the US electricity plants producing fossil and non-fossil electricity generation.

Keywords: Data Envelopment Analysis, interconnections; undesirable outputs; returns-to-scale; convexity; electricity.

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1 Introduction

Data Envelopment Analysis (DEA; after Charnes, Cooper and Rhodes (1978)) is a nonparametric technique that evaluates the efficiency of a Decision Making Unit (DMU) by comparing its input-output performance to that of other DMUs operating in a similar technological environment.[1] DEA is nonparametric in nature since it reconstructs the production possibility set using the observed input-output combinations of the evaluated DMUs. To avoid a trivial reconstruction and to match with the common practice in production theory, regularity conditions, captured by technology axioms, are imposed to the reconstructed production possibility set. In particular, the initial DEA model of Charnes, Cooper and Rhodes (1978), also known as the CCR model, is based on the assumptions of free disposability of the inputs and the outputs, convexity of the production possibility set, and constant returns-to-scale.[2]

In many situations, not only are inputs and outputs present in the production process, but also undesirable outputs. Undesirable outputs are directly (inter)connected with the outputs since the former are only present in the production process because the latter are produced. While different treatments have been suggested in the literature to include undesirable outputs in DEA models, they all acknowledge these interconnections.[3] A first treatment is to use tailored technology axioms for modelling these interconnections (such as weak disposability introduced by Färe et al (1989) and null-jointness introduced by Färe and Grosskopf (2004)); not without criticisms and debates (see Kuosmanen (2005), Färe and Grosskopf (2009), Kuosmanen and Podinovski (2009), Cherchye, De Rock and Walheer (2015), and Forsund (2018)). A second option is to apply a mathematical transformation to the undesirable outputs (see Scheel (2001), Sieford and Zhu (2002), and Cherchye, De Rock and Walheer (2015)). Next, tailored efficiency measurements can be used when undesirable outputs are present (see Chung, Färe and Grosskopf (1997), Färe and Grosskopf (2004), and Tone and Tsutsui (2011)). Finally, modelling undesirable outputs as inputs has also been suggested (see Reinhard, Lovell, and Thijssen (2000), and Hailu and Veeman

[1] See, for example, Färe, Grosskopf and Lovell (1994), Cooper, Seiford and Zhu (2004), Cooper, Seiford and Tone (2007), Fried, Lovell and Schmidt (2008), and Cook and Seiford (2009) for reviews.

[2] See Färe and Primont (1995) for more discussion about regularity conditions of DEA models.

[3] Recent contributions on DEA models with undesirable outputs could be found in Liu et al (2010), Chen (2014), Maghbouli, Amirteimoori and Kordrostami (2014), Bi et al (2015), Cherchye, De Rock and Walheer (2015), Liu et al (2015), and Izadikhak and Saen (2018); for reviews, refer, for example, to Zhou, Ang and Poh (2008) and Dakpo, Jeanneaux and Latruffe (2016).

(2001); and Färe and Grosskopf (2003) and Hailu (2003) for more discussion about this modelling).

Besides, the output-undesirable output interconnections, the input-output interconnections have also been extensively studied. Different types of inputs reflecting these interconnections have been considered in the literature. A first category of inputs are those used to produce all (or a subset of) the outputs. These types of inputs have been considered in different DEA models by, for example, Salerian and Chan (2005), Despic, Despic and Paradi (2007), Cherchye et al (2013), Cherchye, De Rock and Walheer (2015, 2016), Ding et al (2017), and Walheer (2018 b, c, d, e). These inputs could also be interpreted as public goods (they are non-rival and non-exclusive to the output production processes), and, therefore, they give rise to economies of scale (see Panzar and Willig (1977)) and of scope in the production process (see Panzar and Willig (1981) and Nehring and Puppe (2004)). As such, these inputs form a prime economic motivation to produce more than one output. Next, a second category of inputs are those that are allocated to every output. These types of input have been considered in different DEA models by, for example, Färe and Grosskopf (2000), Färe, Grosskopf and Whittaker (2007), Tone and Tsutsui (2009), Cherchye et al (2013), Walheer (2016a, b, 2018a, f), Silva (2018), and Walheer and Zhang (2018). Finally, a third category of inputs are those that are proportional to the outputs. These types of input have been considered in different DEA models by, for example, Podinovski (2004a, c, 2009), Podinovski et al (2014), Podinovski and Husai (2017), and Podinovski, Olesen and Sarrico (2018).

All of the above approaches try to enhance the realism of the DEA analysis by integrating information on the internal production structure. As a consequence, these approaches offer the advantage of increasing the flexibility of DEA models. These two aspects of DEA models are of great importance for practitioners as it offers the option to consider more empirical studies. Also, it improves the attractiveness and the reliability of DEA models. Improving the realism and the flexibility of DEA models also occurs though proposing less restrictive technology axioms. Clearly, fewer assumptions represent an improvement; as for any nonparametric techniques. While free disposability of the inputs and the outputs seems to be a commonly accepted assumption (expect when undesirable outputs are involved in the production process), several extensions have been proposed for the convexity and returns-to-scale aspects.

Banker, Charnes and Cooper (1984) have extended the CCR model to the variable returns-to-scale case. Their model is also known as the BCC model.[4] In many cases, specially in multiple output scenarios, choosing between these two extreme options may be complex. Podinovski (2004a) has provided a DEA model combining the constant and variable returns-to-scale assumptions. In particular, he suggested partitioning the input-output into those that could be reduced proportionally (i.e. constant returns-to-scale) and those that cannot (i.e. variable returns-to-scale).[5] Tulkens (1993) proposed a DEA model without the assumption of convexity.[6] He named his model the Free Disposable Hull (FDH). Again, in many cases, choosing between these two extreme cases may be a difficult task. Intermediate convexity assumptions have been developed by Petersen (1990) and Bogetoft (1996). They suggested DEA models relying on partial or relaxed convexity. Intuitively, convexity is maintained but at its minimal level.[7] In practice, this allows us to model economies of scale and of scope (see also Section 2.1).

In this paper, we propose a unifying methodology coherent with previous works to model the interconnections between the outputs, the inputs, and the undesirable outputs. In particular, we suggest treating the outputs as the central component of the production process by modelling every output individually. Therefore, it gives us the option of considering the interconnections with the inputs and the undesirable outputs. We consider that some undesirable outputs/inputs are due to/used by all the outputs, while others are due to/allocated to specific outputs. It turns out that our model naturally improves the realism and the flexibility required for empirical studies. Attractively, it does not come with the disadvantage of requiring additional assumptions for the production process. In fact, our model is less demanding in terms of assumptions about the production process, and it offers the option of setting a different returns-to-scale assumption for each output-specific production process,

[4]See, for example, Podinovski (2004b, c, 2018), Tone and Sahoo (2006), Lozano and Villa (2010), Tone (2011), Alirezaee, Hajinezhad and Paradi (2018), and Perez-Lopez, Prior and Zafra-Gomez (2018) for DEA models with returns-to-scale; and Banker et al (2004), Banker et al (2011), and Sahoo and Tone (2015) for reviews.

[5]See Podinovski (2009), Podinovski et al (2014), Afsharian, Ahn and Alirezaee (2015), Podinovski and Husai (2017), and Podinovski, Olesen and Sarrico (2018) for extensions.

[6]At this point, we remark that Afriat (1972) was the first to introduce efficiency analysis without the assumption of convexity for single output case (and using different terminology).

[7]See, for example, Bogetoft, Tama and Tind (2000), Dekker and Post (2001), Kuosmanen (2001, 2003), Briec, Kerstens and Vanden Eeckaut (2004), Agrell et al (2005), Podinovski (2005), Ehrgott and Tind (2009), and Podinovski and Kuosmanen (2011) for DEA models without and with partial convexity.

and to choose between different types of convexity.

We apply our model to the case of the US electricity plans that use two inputs: total assets and fuel quantity to produce two (desirable) outputs: fossil and non-fossil electricity. The fossil electricity generation implies the presence of undesirable outputs in the production process. Our model offers several advantages when evaluating the efficient behaviour of the plants. One, we can take the interconnections between the outputs, the inputs, and the greenhouse gases into account. In particular, fuel input is not used to produce non-fossil electricity, and the greenhouse gas emissions are due to the fossil electricity production only. Two, different returns-to-scale assumption may be assumed for each type of electricity. In particular, while constant returns-to-scale seems acceptable for fossil energy, we may argue that variable returns-to-scale is more adequate for non-fossil electricity. Three, overall convexity may be seen as a too strict assumption for the plant production process. As such, the option of relying on a relaxed convexity assumption is clearly an advantage in this context. All in all, our methodology gives the advantage of improving the realism and the flexibility of the evaluation exercise, and thus proposes more reliable results.

The rest of the paper is structured as follows. Section 2 presents our methodology, Section 3 presents our empirical study to the US electricity plants, and Section 4 presents our conclusions.

2 Methodology

We consider a production technology that uses N inputs, captured by the vector $\mathbf{X} = (x^1, \ldots, x^N)' \in \mathbb{R}^N_+$, to produce M (desirable) outputs, captured by the vector $\mathbf{Y} = (y^1, \ldots, y^M)' \in \mathbb{R}^M_+$. We also assume that K undesirable outputs, captured by the vector $\mathbf{U} = (u^1, \ldots, u^K)' \in \mathbb{R}^K_+$, are present in the production process. We start by presenting our method to model the interconnections between the outputs, the inputs, and the undesirable outputs. To do so, we consider the outputs as the central constituent of the production process by modelling each output production process separately. This gives us the option to consider several types of inputs and undesirable outputs. Next, we show how to define and reconstruct the technology when these interconnections are taken into account. Finally, we present the efficiency measurements.

2.1 Modelling interconnections

Interconnections. Following our previous discussion, we partition the inputs into different categories. The main distinction is between inputs that are allocated to specific output production processes, and those that are not. For the former case, we denote $a_i^m \in [0,1]$, with $\sum_{m=1}^{M} a_i^m = 1$, to represent the fraction of the i-th input quantity that is used to produce output m. Examples of this type of input include employees and resources allocated to specific output production processes, and tools used to produce certain outputs. We consider that the inputs that are not allocated to a specific output production process are used jointly to produce all (or a subset of) outputs. Examples of this type of input include factories, infrastructure, machines and human capital. These inputs constitute a prime economic motivation to produce multiple outputs; as the DMUs, in general, benefit from economics of scale (see Panzar and Willig (1977)) and scope (see Panzar and Willig (1981) and Nehring and Puppe (2004)).

We suggest using an information vector, denoted $\mathbf{A}^m \in \mathbb{R}_+^N$, to summarize the interconnections between the inputs and output m. $\mathbf{A}^m$ is defined for output m as follows:

$$(\mathbf{A}^m)_i = \begin{cases} 1 & \text{if input } i \text{ is jointly used to produce output } m, \\ a_i^m & \text{if input } i \text{ is allocated to output } m, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

A similar distinction can be done for the undesirable outputs. Firstly, those that are present in the production process because of the production of all (or a subset) of outputs. Examples of this type of undesirable output include greenhouse gas emissions and waste due to the joint production of all (or a subset of) outputs. Next, those that can be distributed between the different output production process. For that case, we use b_k^m, with $\sum_{m=1}^{M} b_k^m = 1$, as the fraction of undesirable output k that is due to output m. Examples of this type of undesirable output include waste or emissions due to specific outputs. We remark that undesirable outputs are not limited to environmental outputs; it also includes, for example, non-performing loans for a bank.

Again, we can rely on an information vector to capture the interconnections between the outputs and the undesirable outputs. Let $\mathbf{B}^m \in \mathbb{R}_+^K$ be the vector capturing the interconnections between the undesirable outputs and output m; it is given as follows:

$$(\mathbf{B}^m)_k = \begin{cases} 1 & \text{if undesirable output } k \text{ is jointly produced by output } m, \\ b_k^m & \text{if undesirable output } k \text{ is due to output } m, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

As a last remark, we point out that in some contexts, the interconnections between the outputs, inputs, and undesirable outputs may not (or partially) be observed. In that case, different methods have been suggested to recover such information. We refer, for example, to Li et al (2009), Yu, Chern and Hsiao (2013), Du et al (2014), and Walheer (2016b). Knowing or not how the outputs, the inputs, and the undersirable outputs are interconnected only impacts the practical aspect of our method, not the definitions of the concepts (See Section 2.3).

Output-specific production processes. Using, the information vectors, we can define the inputs used to produce output m, denoted $\mathbf{X}^m \in \mathbb{R}_+^N$; and the undesirable outputs due to the production of output m, denoted $\mathbf{U}^m \in \mathbb{R}_+^K$. In fact, it suffices to proceed to an element-to-element product between the initial input or undesirable output vector and the information vector to obtain the input or undesirable output vector for output m. Formally, we obtain the following (where $\odot$ stands for the Hadamard (or element-by-element) product):

$$\mathbf{X}^m = \mathbf{A}^m \odot \mathbf{X}. \tag{3}$$

$$\mathbf{U}^m = \mathbf{B}^m \odot \mathbf{U}. \tag{4}$$

We can simplify our notation by regrouping the output y^m and its undesirable counterparts $\mathbf{U}^m$ into a common vector:

$$\mathbf{Z}^m = \begin{bmatrix} y^m \\ g(\mathbf{U}^m) \end{bmatrix} \in \mathbb{R}_+^{1+K}. \tag{5}$$

We remark that the undesirable outputs are modified through the function $g(\cdot)$. This function, introduced by Cherchye, De Rock and Walheer (2015), captures the undesirable nature of these outputs. Clearly, different transformations are possible, but two conditions must be fulfilled: $g(\mathbf{0}) = 0$ and $g(\mathbf{U}^m)$ decreases when $\mathbf{U}^m$ increases.

Popular transformations are the opposite and reciprocal value: $g(\mathbf{U}^m) = -\mathbf{U}^m$ and $g(\mathbf{U}^m) = 1/\mathbf{U}^m$. Refer, for example, to Scheel (2001), Zhou, Ang and Poh (2008), and Cherchye, De Rock and Walheer (2015) for more discussion. We do not specify the transformation here, but rather leave that choice to the practitioners. See also our empirical application in Section 3.

All in all, it means that when the interconnections are taken into consideration we move from an overall or aggregation representation of the production process $\langle \mathbf{Z}, \mathbf{X} \rangle$, where $\mathbf{Z} = \begin{bmatrix} \mathbf{Y} & g(\mathbf{U}) \end{bmatrix}' \in \mathbb{R}_+^{M+K}$, to a multi-output representation $\langle \mathbf{Z}^m, \mathbf{X}^m \rangle$, for $m = 1, \ldots, M$.

Illustrative example. We propose a simple example to illustrate how the information vectors are used in practice. We consider DMUs that produce three outputs y^1, y^2, and y^3 using employees x^1, machines x^2 and a factory x^3. Also, we assume that employees are allocated to the output production process (30%, 30%, and 40%), and the machines are only used to produce y^2 and y^3. Finally, we assume that greenhouse gas emissions u^1, due to the joint production of the three outputs, and specific waste u^2, due to the joint production of outputs 1 and 2, are also present in the production process. Figure 1 summarizes the production process of the DMUs.

Figure 1: Illustrative example − production process

At the overall or aggregate production level, we obtain:

$$\mathbf{Y} = \begin{bmatrix} y^1 \\ y^2 \\ y^3 \end{bmatrix}, \mathbf{U} = \begin{bmatrix} u^1 \\ u^2 \end{bmatrix}, \mathbf{Z} = \begin{bmatrix} y^1 \\ y^2 \\ y^3 \\ g(u^1) \\ g(u^2) \end{bmatrix}, \text{ and } \mathbf{X} = \begin{bmatrix} x^1 \\ x^2 \\ x^3 \end{bmatrix}. \tag{6}$$

Next, we obtain the following information vectors:

$$\mathbf{A}^1 = \begin{bmatrix} 30\% \\ 0 \\ 1 \end{bmatrix}, \mathbf{A}^2 = \begin{bmatrix} 30\% \\ 1 \\ 1 \end{bmatrix}, \text{ and } \mathbf{A}^3 = \begin{bmatrix} 40\% \\ 1 \\ 1 \end{bmatrix}. \tag{7}$$

$$\mathbf{B}^1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \mathbf{B}^2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \text{ and } \mathbf{B}^3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{8}$$

Finally, we obtain the inputs and undesirable outputs for each output ($m = 1, 2, 3$) as follows:

$$\mathbf{X}^1 = \mathbf{A}^1 \odot \mathbf{X} = \begin{bmatrix} 30\% * x^1 \\ 0 \\ x^3 \end{bmatrix}, \mathbf{X}^2 = \begin{bmatrix} 30\% * x^1 \\ x^2 \\ x^3 \end{bmatrix}, \text{ and } \mathbf{X}^3 = \begin{bmatrix} 40\% * x^1 \\ x^2 \\ x^3 \end{bmatrix}. \tag{9}$$

$$\mathbf{U}^1 = \mathbf{B}^1 \odot \mathbf{U} = \begin{bmatrix} u^1 \\ u^2 \end{bmatrix}, \mathbf{U}^2 = \begin{bmatrix} u^1 \\ u^2 \end{bmatrix}, \text{ and } \mathbf{U}^3 = \begin{bmatrix} u^1 \\ 0 \end{bmatrix}. \tag{10}$$

$$\mathbf{Z}^1 = \begin{bmatrix} y^1 \\ g(\mathbf{U}^1) \end{bmatrix} = \begin{bmatrix} y^1 \\ g(u^1) \\ g(u^2) \end{bmatrix}, \mathbf{Z}^2 = \begin{bmatrix} y^2 \\ g(u^1) \\ g(u^2) \end{bmatrix}, \text{ and } \mathbf{Z}^3 = \begin{bmatrix} y^2 \\ g(u^1) \\ 0 \end{bmatrix}. \tag{11}$$

2.2 Define and estimate the technology

Technology sets. Following our previous discussion and our multi-output representation of the production process, it is natural to characterize the technology for each output m individually. We define the production possibility set for output m as follows:

$$T^m = \left\{(\mathbf{X}^m, \mathbf{Z}^m) \in \mathbb{R}_+^{N+1+K} \mid \mathbf{X}^m \text{ can produce } \mathbf{Z}^m\right\}. \tag{12}$$

T^m contains all the combinations of inputs $\mathbf{X}^m$ and outputs and undesirable outputs $\mathbf{Z}^m$ that are technically feasible. These sets are, in general, interconnected since inputs and undesirable outputs appear in several production possibility sets. As we want to consider relaxed convexity assumptions, we define the input and output sets for output m as:

$$I^m(\mathbf{Z}^m) = \left\{\mathbf{X}^m \in \mathbb{R}_+^N \mid (\mathbf{X}^m, \mathbf{Z}^m) \in T^m\right\}. \tag{13}$$

$$P^m(\mathbf{X}^m) = \left\{\mathbf{Z}^m \in \mathbb{R}_+^{1+K} \mid (\mathbf{X}^m, \mathbf{Z}^m) \in T^m\right\}. \tag{14}$$

Again, these sets are interconnected as inputs and undesirable outputs are present in several input and output sets. It turns out that the overall or aggregate production possibility set is given by:

$$T = \left\{ (\mathbf{X}, \mathbf{Z}) \in \mathbb{R}_+^{N+M+K} \mid \forall m \in \{1, \ldots, M\} : (\mathbf{X}^m, \mathbf{Z}^m) \in T^m \right\}. \tag{15}$$

In words, the overall production possibility set contains all inputs and outputs that are feasible in each output-specific production possibility set. This definition clearly reveals the interconnections between the output-specific production processes. The corresponding overall or aggregate input and output sets are defined as follows:

$$I(\mathbf{Z}) = \left\{ \mathbf{X} \in \mathbb{R}_+^{N} \mid \forall m \in \{1, \ldots, M\} : \mathbf{X}^m \in I^m(\mathbf{Z}^m) \right\}. \tag{16}$$

$$P(\mathbf{X}) = \left\{ \mathbf{Z} \in \mathbb{R}_+^{M+K} \mid \forall m \in \{1, \ldots, M\} : \mathbf{Z}^m \in P^m(\mathbf{X}^m) \right\}. \tag{17}$$

Technology axioms. In practice, the technology, captured here by the production possibility set, is unobserved. As explained in the Introduction, an advantage of DEA is that it does not assume any functional form for the production possibility set, but rather reconstructs the production possibility set using the data. Nevertheless, to avoid a trivial reconstruction, regularity conditions, captured by technology axioms, are imposed. The initial DEA model of Charnes, Coooper and Rhodes (1978) is based on the technology axioms of free disposability of the inputs and the outputs, a convex production possibility set, and constant returns-to-scale.

The distinguishing feature of our method is that we impose these axioms for the output-specific production possibility sets (i.e. T^m, for $m = 1, \ldots, M$); and not for the overall production possibility set (i.e. T). In fact, imposing these axioms for the T^m's is less demanding, and thus less restrictive, than imposing the similar axioms for T. Intuitively, this comes from the observation that, in general, there are no specific connections between the output-specific production possibility sets and the overall production possibility set. This fact is an immediate consequence of the interconnections between the outputs, the inputs, and the undesirable outputs, and is formally captured by (15).

We take three examples to illustrate this important fact. One, in a macroeconomic context, Walheer (2016a, b) proposed to study economic growth of countries using sector-based indicators. This author assumes that the sector-specific production possibility sets satisfied the four axioms; this does not mean that the country-specific production possibility sets fulfilled the same conditions. This also comes from the interconnections between sectors considered by this author (e.g. the education and fiscal systems). It turns out that sector-specific modellings are, in general, less demanding in terms of assumptions than country-specific modellings. Next, Cherchye et al (2013) and Cherchye, De Rock and Walheer (2016) studied cost and profit efficient behaviour of a large multi-division service firm. They assume that the division-specific production possibility sets meet the four axioms; but it does not imply that this is the case for the firm-level production possibility set. Again, this also comes for the interconnections between the divisions (e.g. divisions use common inputs). Finally, Cherchye De Rock and Walheer (2015) and Walheer (2018d, e), in the context of electricity plants producing different types of electricity, assume that the production possibility set of each electricity type respects the axioms. It does not imply that the plant-level production possibility set also respects these axioms.

We obtain the following four technology axioms:

T1 (output free disposability): $(\mathbf{X}^m, \mathbf{Z}^m) \in T^m$ and $\mathbf{Z}^m \geq \mathbf{Z}^{m'} \implies (\mathbf{X}^m, \mathbf{Z}^{m'}) \in T^m$.

T2 (input free disposability): $(\mathbf{X}^m, \mathbf{Z}^m) \in T^m$ and $\mathbf{X}^{m'} \geq \mathbf{X}^m \implies (\mathbf{X}^{m'}, \mathbf{Z}^m) \in T^m$.

T3 (convexity): $(\mathbf{X}^m, \mathbf{Z}^m) \in T^m$ and $(\mathbf{X}^{m'}, \mathbf{Z}^{m'}) \in T^m \implies \forall \lambda \in [0,1] : \lambda(\mathbf{X}^m, \mathbf{Z}^m) + (1-\lambda)(\mathbf{X}^{m'}, \mathbf{Z}^{m'}) \in T^m$.

T4 (returns-to-scale): $(\mathbf{X}^m, \mathbf{Z}^m) \in T^m \implies k \times (\mathbf{X}^m, \mathbf{Z}^m) \in T^m, \forall k \in K^m(rts)$.

In words, *T1* says that if $\mathbf{X}^m$ can produce $\mathbf{Z}^m$, then it can also produce less output, $\mathbf{Z}^{m'}$. Next, *T2* means that more inputs never reduces the outputs. *T3* states that, if two input-output associations $(\mathbf{X}^m, \mathbf{Z}^m)$ and $(\mathbf{X}^{m'}, \mathbf{Z}^{m'})$ are feasible, then any convex combinations of the two associations are also feasible. Finally, *T4* says that if $\mathbf{X}^m$ can produce $\mathbf{Z}^m$, $k \times \mathbf{X}^m$ can produce $k \times \mathbf{Z}^m$, with k restricted in the set $K^m(rts) \subseteq \mathbb{R}_0^+$. In particular, we use $K^m(crs) = \mathbb{R}_0^+, K^m(drs) = (0,1], K^m(irs) = [1,\infty)$ and $K^m(vrs) = \{1\}$ for the constant, decreasing, increasing and variable returns-to-scale assumptions. We remark that axiom *T4* naturally gives the option to consider returns-to-scale assumptions at the output level.

While these axioms are less strict, convexity of the output-specific production possibility sets remains a strong assumption for many empirical studies. Moreover, this assumption is not always required (for example, when cost or revenue is of interest, see also our discussion in Section 3). As such, we also consider two additional axioms when only the input or the output sets are assumed to be convex:

T5 (convex input set): $\mathbf{X}^m \in I^m(\mathbf{Z}^m)$ and $\mathbf{X}^{m'} \in I^m(\mathbf{Z}^m) \implies \forall \lambda \in [0,1] : \lambda \mathbf{X}^m + (1-\lambda)\mathbf{X}^{m'} \in I^m(\mathbf{Z}^m)$.

T6 (convex output set): $\mathbf{Z}^m \in P^m(\mathbf{X}^m)$ and $\mathbf{Z}^{m'} \in P^m(\mathbf{X}^m) \implies \forall \lambda \in [0,1] : \lambda \mathbf{Z}^m + (1-\lambda)\mathbf{Z}^{m'} \in P^m(\mathbf{X}^m)$.

Axiom *T5* states that, if two inputs $\mathbf{X}^m$ and $\mathbf{X}^{m'}$ can produce $\mathbf{Z}^m$, then any convex combination $\lambda \mathbf{X}^m + (1-\lambda)\mathbf{X}^{m'}$ can also produce the same output. Axiom *T6* says the same, but when considering the output side of the production process: if two outputs $\mathbf{Z}^m$ and $\mathbf{Z}^{m'}$ can be produced by $\mathbf{X}^m$, then any convex combination $\lambda \mathbf{Z}^m + (1-\lambda)\mathbf{Z}^{m'}$ can also produce the same input. Again, we point out that these two axioms are weaker than assuming a convex input set $I(\mathbf{Z})$ or a convex output set $P(\mathbf{X})$.

Finally, we remark that additional axioms can be assumed, as for example weak disposability or local returns-to-scale assumptions. See also our discussion at the end of this Section when we compare our empirical technology sets with existing sets in the literature.

Empirical technology sets. The empirical counterparts of the production possibility, the input, and the output sets start with the observation of a data set. Suppose we observe data for J DMUs. For each DMU $j \in \{1, \ldots, J\}$, we observe the outputs $\mathbf{Y}_j$ (with y_j^m the quantity produced of output m), the undesirable outputs $\mathbf{U}_j$ and the inputs $\mathbf{X}_j$. We assume that we also observe the information vectors $\mathbf{A}_j^m$ and $\mathbf{B}_j^m$, for $j = 1, \ldots, J$ and $m = 1, \ldots, M$. It implies that we observe, for every DMU j and output m, $\mathbf{X}_j^m = \mathbf{A}_j^m \odot \mathbf{X}_j$ and $\mathbf{U}_j^m = \mathbf{B}_j^m \odot \mathbf{U}_j$; and thus $\mathbf{Z}_j^m = \begin{bmatrix} y_j^m & g(\mathbf{U}_j^m) \end{bmatrix}'$. All in all, we observe the following data set D:

$$D = \{(\mathbf{Z}_j^1, \ldots, \mathbf{Z}_j^M, \mathbf{X}_j^1, \ldots, \mathbf{X}_t^M) \mid j = 1, \ldots, J\}. \tag{18}$$

DEA is based on two important principles to reconstruct the technology. One, DEA assumes that what we observe is certainly feasible. That is, if we observe the data set D, then these observed inputs can certainly produce these observed outputs. Two, DEA is based on the "minimum extrapolation" principle. This principle states that the empirical reconstructed set must be the smallest empirical construction that is consistent with the chosen technology axioms.

We start by presenting the empirical counterpart of the output-specific production possibility set in (12) when axioms *T1-T4* are imposed:

$$\widehat{T}^m = \left\{ (\mathbf{X}^m, \mathbf{Z}^m) \in \mathbb{R}_+^{N+1+K} \;\middle|\; \begin{array}{l} \sum_{s=1}^J \lambda_s^m \mathbf{X}_s^m \leq \mathbf{X}^m; \sum_{s=1}^J \lambda_s^m \mathbf{Z}_s^m \geq \mathbf{Z}^m; \\ (\lambda_1^m, \ldots, \lambda_s^m, \ldots, \lambda_J^m)' \in \Lambda^m(rts). \end{array} \right\}. \tag{19}$$

The set $\Lambda^m(rts)$ captures the chosen returns-to-scale assumption for the production process of output m. In particular, $\Lambda^m(rts)$ is defined, when constant, decreasing, increasing, or variable returns-to-scale is picked, as follows :

$$\Lambda^m(crs) = (\mathbb{R}_0^+)^J. \tag{20}$$

$$\Lambda^m(drs) = \left\{ (\lambda_1^m, \ldots, \lambda_J^m)' \in (\mathbb{R}_0^+)^J \mid \sum_{s=1}^J \lambda_s^m \leq 1 \right\}. \tag{21}$$

$$\Lambda^m(irs) = \left\{ (\lambda_1^m, \ldots, \lambda_J^m)' \in (\mathbb{R}_0^+)^J \mid \sum_{s=1}^J \lambda_s^m \geq 1 \right\}. \tag{22}$$

$$\Lambda^m(vrs) = \left\{ (\lambda_1^m, \ldots, \lambda_J^m)' \in (\mathbb{R}_0^+)^J \mid \sum_{s=1}^J \lambda_s^m = 1 \right\}. \tag{23}$$

It turns out that the overall empirical production possibility set (see (15)), when *T1-T4* are imposed on every output-specific production process, is given by:

$$\widehat{T} = \left\{ (\mathbf{X}, \mathbf{Z}) \in \mathbb{R}_+^{N+M+K} \mid \forall m \in \{1, \ldots, M\} : (\mathbf{X}^m, \mathbf{Z}^m) \in \widehat{T}^m \right\}. \tag{24}$$

Equivalently, we can rewrite the empirical overall production possibility set as follows:

$$\widehat{T} = \left\{ (\mathbf{X}, \mathbf{Z}) \in \mathbb{R}_+^{N+M+K} \;\middle|\; \begin{array}{l} \forall m = 1, \ldots, M: \\ \sum_{s=1}^J \lambda_s^m \mathbf{X}_s^m \leq \mathbf{X}^m; \sum_{s=1}^J \lambda_s^m \mathbf{Z}_s^m \geq \mathbf{Z}^m; \\ (\lambda_1^m, \ldots, \lambda_s^m, \ldots, \lambda_J^m)' \in \Lambda^m(rts). \end{array} \right\}. \tag{25}$$

We remark that, by construction, we have that $\widehat{T}^m \subseteq T^m$ following the "minimum extrapolation" principle. In words, $\widehat{T}^m$ provides a useful inner bound approximation of T^m (see Cherchye, De Rock and Walheer (2016) for a formal proof when variable returns-to-scale is assumed). It also implies that $\widehat{T} \subseteq T$.

We continue by presenting the empirical counterparts of the input and output sets when partial or relaxed convexity is assumed (i.e. axiom *T5* or *T6* instead of axiom *T4*). Following Petersen (1990) and Bogetoft (1996), we define a function that scales up or down the outputs/inputs, depending on the returns-to-scale assumption chosen, to make two DMUs comparable. It is defined for output m and DMU j as follows:

$$\alpha_s^m(\mathbf{Z}_j^m; rts) = \inf\left\{\alpha \in K^m(rts) \mid \alpha \mathbf{Z}_s^m \geq \mathbf{Z}_j^m\right\}. \tag{26}$$

$$\beta_s^m(\mathbf{X}_j^m; rts) = \sup\left\{\beta \in K^m(rts) \mid \beta \mathbf{X}_s^m \leq \mathbf{X}_j^m\right\}. \tag{27}$$

The possible values for $\alpha_s^m(\mathbf{Z}_t^m; rts)$ depend directly on the returns-to-scale assumption chosen trough the set $K^m(rts)$. We recall that $K^m(crs) = \mathbb{R}_0^+, K^m(drs) = (0,1], K^m(irs) = [1,\infty)$ and $K^m(vrs) = \{1\}$ for the constant, decreasing, increasing and variable returns-to-scale assumptions. If such a value exists, $\alpha_s^m(\mathbf{Z}_j^m; rts)$ gives the factor by which the value of $\mathbf{Z}_s^m$ should be scaled to make it comparable with $\mathbf{Z}_j^m$. In a similar vein, $\beta_s^m(\mathbf{X}_j^m; rts)$ gives the factor by which $\mathbf{X}_s^m$ should be scaled to make it comparable with $\mathbf{X}_j^m$. If it is not possible to find such factors, i.e. no comparison is possible, we set $\alpha_s^m(\mathbf{Z}_j^m; rts) = +\infty$ and $\beta_s^m(\mathbf{X}_j^m; rts) = -\infty$.

The smallest empirical construction of the input set in (13) under axioms *T1-T3* and *T5* is given by:

$$\widehat{I}^m(\mathbf{Z}_j^m) = \left\{ \mathbf{X}^m \in \mathbb{R}_+^N \;\middle|\; \begin{array}{l} \sum_{s=1}^J \lambda_s^m \alpha_s^m(\mathbf{Z}_j^m; rts)\mathbf{X}_s^m \leq \mathbf{X}^m; \\ \forall s : \lambda_s^m(\alpha_s^m(\mathbf{Z}_j^m; rts)\mathbf{Z}_s^m - \mathbf{Z}_j^m) \geq 0; \\ \sum_{s=1}^J \lambda_s^m = 1;\text{ and } \forall s : \lambda_s^m \geq 0. \end{array} \right\}. \tag{28}$$

The corresponding empirical overall input set (see (16)) is thus given by:

$$\begin{aligned} \widehat{I}(\mathbf{Z}_j) &= \left\{\mathbf{X} \in \mathbb{R}_+^N \mid \forall m \in \{1,\ldots,M\} : \mathbf{X}^m \in \widehat{I}^m(\mathbf{Z}_j^m)\right\}, \\ &= \left\{ \mathbf{X} \in \mathbb{R}_+^N \;\middle|\; \begin{array}{l} \forall m = 1,\ldots,M : \\ \sum_{s=1}^J \lambda_s^m \alpha_s^m(\mathbf{Z}_j^m; rts)\mathbf{X}_s^m \leq \mathbf{X}^m; \\ \forall s : \lambda_s^m(\alpha_s^m(\mathbf{Z}_j^m; rts)\mathbf{Z}_s^m - \mathbf{Z}_j^m) \geq 0; \\ \sum_{s=1}^J \lambda_s^m = 1;\text{ and } \forall s : \lambda_s^m \geq 0. \end{array} \right\}. \end{aligned} \tag{29}$$

As for the production possibility set, the following relationship holds true for the empirical and theoretical input sets: $\widehat{I}^m(\mathbf{Z}_j^m) \subseteq I^m(\mathbf{Z}_j^m)$ and $\widehat{I}(\mathbf{Z}_j) \subseteq I(\mathbf{Z}_j)$. We refer to Cherchye et al (2013) for a formal proof when variable returns-to-scale is assumed.

Finally, when axioms *T1-T3* and *T6* are assumed, we obtain the following empirical output requirement sets (see (14) and (17)):

$$\widehat{P}(\mathbf{X}_j^m) = \left\{ \mathbf{Z}^m \in \mathbb{R}_+^{1+M} \;\middle|\; \begin{array}{l} \sum_{s=1}^J \lambda_s^m \beta_s^m(\mathbf{X}_j^m; rts)\mathbf{Z}_s^m \geq \mathbf{Z}^m; \\ \forall s : \lambda_s^m(\beta_s^m(\mathbf{X}_j^m; rts)\mathbf{X}_j^m - \mathbf{X}_s^m) \geq 0; \\ \sum_{s=1}^J \lambda_s^m = 1; \text{ and } \forall s : \lambda_s^m \geq 0. \end{array} \right\}. \tag{30}$$

and

$$\begin{aligned} \widehat{P}(\mathbf{X}_j) &= \left\{ \mathbf{Z} \in \mathbb{R}_+^{M+K} \mid \forall m \in \{1, \ldots, M\} : \mathbf{Z}^m \in P^m(\mathbf{X}_j^m) \right\}, \\ &= \left\{ \mathbf{Z} \in \mathbb{R}_+^{M+K} \;\middle|\; \begin{array}{l} \forall m = 1, \ldots, M : \\ \sum_{s=1}^J \lambda_s^m \beta_s^m(\mathbf{X}_j^m; rts)\mathbf{Z}_s^m \geq \mathbf{Z}^m; \\ \forall s : \lambda_s^m(\beta_s^m(\mathbf{X}_j^m; rts)\mathbf{X}_j^m - \mathbf{X}_s^m) \geq 0; \\ \sum_{s=1}^J \lambda_s^m = 1; \text{ and } \forall s : \lambda_s^m \geq 0. \end{array} \right\}. \end{aligned} \tag{31}$$

Clearly, we also have the two following relationships: $\widehat{P}^m(\mathbf{X}_j^m) \subseteq P^m(\mathbf{X}_j^m)$ and $\widehat{P}(\mathbf{X}_j) \subseteq P(\mathbf{X}_j)$. The proof can be immediately adapted from the proof of Cherchye et al (2013) for the input sets.

Comparison. In this last part, we position our empirical reconstructions to existing empirical reconstructions in the DEA literature. In fact, our empirical reconstructions share close relationships to several well-established reconstructions.

One connection is with the initial DEA models of Charnes, Cooper and Rhodes (1978). In fact, our empirical reconstruction of the production possibility sets in (25) corresponds to their reconstruction when only one (desirable) output is involved in the production process and constant returns-to-scale is assumed (when there is only one output, interconnections do not exist). When there are more than one output in the production process, our two empirical reconstructions differ. The main differences are that our construction gives the option to model interconnections and choose different returns-to-scale assumptions for every output-specific production process, while their construction assumes constant returns-to-scale for the overall production process and ignores the interconnections. A similar discussion holds true when comparing our empirical reconstruction to the reconstruction of Banker, Charnes and Cooper (1984).

The main difference is that they assume variable returns-to-scale at the aggregate level.

Next, there is also a direct connection with the empirical reconstructions of the input and output sets with those of Petersen (1990) and Bogetoft (1996). In fact, when there is only one (desirable) output in the production process (29) and (31) correspond to their reconstruction. We point out a slight difference even in that case; we give the option to consider increasing returns-to-scale, while this option is not possible in their model. Clearly, when more outputs are involved, our two reconstructions do not correspond anymore; and the advantages of ours are to provide more flexibility and improve the realism (i.e. modelling interconnections and output-specific returns-to-scale assumption).

Afterwards, our empirical constructions share the spirit of those considered by Podinovski (2004a) and followers. As in their model, different returns-to-scale assumptions may be considered. One main difference is that they consider that some inputs and outputs are proportional, implying constant returns-to-scale, while others are not, implying variable returns-to-scale. Our modelling is more general in the sense that it does not asked to partition input-output into two categories; but rather it allows us to consider various types of interconnections. Moreover, the empirical reconstructions of Podinovski (2004a) and followers are based on very general technology axioms; while our modelling is based on very standard and well-established technology axioms.

Finally, there are also connections with recent works that have opted for the output-specific modelling of the production process. In particular, the models developed in Cherchye et al (2013), Cherchye, De Rock and Walheer (2015, 2016), and Walheer (2018b, c, d, e). One, our model generalized their empirical reconstructions by considering more possible interconnections between the outputs, the inputs, and the undesirable outputs (only Cherchye, De Rock and Walheer (2015) have considered the presence of undesirable outputs). Two, our model is more general since it considers different returns-to-scale assumptions (only variable returns-to-scale is considered in their model). Three, different types of convexity is possible in our model (they rely on partial convexity only for the input set).

As a final remark, we point out that our method is not based on the assumption of weak disposability (or on any other particular technology axioms when undesirable outputs are involved, such as null-jointness) of the undesirable outputs. We believe that this is an appealing feature of the proposed approach since several issues have been pointed out when assuming weak disposability (see, for example, Kuosmanen (2005), Färe and Grosskopf (2009), Kuosmanen and Podinovski (2009), Cherchye, De Rock and Walheer (2015), and Forsund (2018)).[8] Moreover, assuming weak disposability implies imposing more (unverifiable) structure about the production process. It turns out that our model remains very close to the initial DEA models, while offering several additional advantages. Nevertheless, if other technology axioms are required for empirical studies, they can fairly easily be incorporated into our methodology.

2.3 Efficiency measurements

Different types of efficiency measurements can be considered for evaluating the good practice of the DMUs.[9] In this Section, we present the case when potential input reduction is of interest, and convexity is only imposed for the input sets. This choice is also motivated by our empirical study (see Section 3).

In general, input-oriented efficiency is evaluated as the distance of the DMU's input vector to the isoquant. A natural indicator in our context is the radial input distance function introduced by Shephard (1970). It is given for a particular DMU j as follows:

$$D_j = D_j(\mathbf{Z}_j^1, \ldots, \mathbf{Z}_j^M, \mathbf{X}_j^1, \ldots, \mathbf{X}_j^M) = \max \left\{ \phi \mid \forall m \in \{1, \ldots, M\} : \left(\frac{\mathbf{X}_j^m}{\phi} \right) \in I^m(\mathbf{Z}_j^m) \right\}. \tag{32}$$

D_t the largest equiproportionate factor by which the inputs can be reduce and still produced the output quantity. D_j is greater than one, with a value of one reflecting an efficient behaviour, i.e. inputs are at their minimal level given the outputs. We remark that D_j corresponds to Shephard's (1970) distance function only for one-output cases; when there are more than one output in the production process, they differ. This follows from the specificities of our methodology (see our discussion at the end of Section 2.2). In practice, it is more convenient to work with a technical efficiency measurement. In fact, the (input) distance function is reciprocal to the

[8]Note that even when weak disposability is assumed, opting for a relaxed convexity approach is also of interest. See Podinovski and Kuosmanen (2011).

[9]For example, radial efficiency measurements, non-radial efficiency measurements, hybrid efficiency measurements, directional distance functions, etc. All these efficiency measurements can be considered here.

(input-oriented) Debreu (1951) – Farell (1957) technical efficiency measurement. It is given for DMU j by:

$$TE_j = TE_j(\mathbf{Z}_j^1, \ldots, \mathbf{Z}_j^M, \mathbf{X}_j^1, \ldots, \mathbf{X}_j^M) = \min\{\theta \mid \forall m \in \{1, \ldots, M\} : \theta \mathbf{X}_j^m \in I^m(\mathbf{Z}_j^m)\}. \tag{33}$$

As TE_j is the inverse of D_j, it is, by construction, smaller than one. When it is one, it means that DMU j produces outputs with the minimal level of inputs. When it is smaller than one, it implies some potential input savings (for constant outputs). Again, we highlight that TE_j only corresponds to the definition of Debreu (1951) and Farell (1957) when one output is considered (we refer to our discussion at the end of Section 2.2).

As defined, TE_j (and thus D_j) is not directly useful as it is based on the theoretical input sets. We can obtain the empirical counterpart by using the empirical input sets in (33):

$$\widehat{TE}_j = \widehat{TE}_j(\mathbf{Z}_j^1, \ldots, \mathbf{Z}_j^M, \mathbf{X}_j^1, \ldots, \mathbf{X}_j^M) = \min\{\eta \mid \forall m \in \{1, \ldots, M\} : \eta \mathbf{X}_j^m \in \widehat{I}^m(\mathbf{Z}_j^m)\}. \tag{34}$$

$\widehat{TE}_j$ has to be interpreted as TE_j; the only difference is that they are based on empirical and theoretical inputs sets, respectively. It turns out that $TE_j \leq \widehat{TE}_j$. In words, TE_j is a lower bound for $\widehat{TE}_j$. This immediately follows from the relationship between the empirical and theoretical input sets discussed previously: $\widehat{I}^m(\mathbf{Z}_j^m) \subseteq I^m(\mathbf{Z}_j^m)$, for $m = 1, \ldots, M$.

Attractively, in practice, it suffices to solve a linear program to obtain the distance function and the technical efficiency scores. In particular for DMU $j \in \{1, \ldots, J\}$,

$\widehat{TE}_j$ is obtained as follows:

$$\begin{aligned}
\widehat{TE}_j = \min_{\lambda_s^m \ (m\in\{1,\ldots,M\}, s\in\{1,\ldots,J\})} \eta \\
\forall m \in \{1,\ldots,M\} : \sum_{s=1}^{J} \alpha_s^m(\mathbf{Z}_j^m, rts)\lambda_s^m \mathbf{X}_s^m \leq \eta \mathbf{X}_j^m, \\
\forall s \in \{1,\ldots,J\}, \forall m \in \{1,\ldots,M\} : \lambda_s^m(\alpha_s^m(\mathbf{Z}_j^m; rts)\mathbf{Z}_s^m - \mathbf{Z}_j^m) \geq 0, \\
\forall m \in \{1,\ldots,M\} : \sum_{s=1}^{J} \lambda_s^m = 1, \\
\forall s \in \{1,\ldots,J\}, \forall m \in \{1,\ldots,M\} : \lambda_s^m \geq 0, \\
\eta \geq 0.
\end{aligned} \tag{35}$$

3 Application

We propose to use our new model to tackle the question of the technical efficiency of US electric utilities in the presence of greenhouse gas emissions. This question has already been treated in the efficiency literature. Recent empirical studies include Sarkis and Cordeiro (2012), Sueyoshi and Goto (2012, 2014), Cook, Du and Zhu (2017), and Walheer (2018b, d, e) for the US; Tone and Tsutsui (2007) and Sueyoshi and Goto (2011) for Japan; Korhonen and Syrijanen (2003), Jamasb and Pollitt (2003), and Giannakis, Jamasb and Pollitt (2005) for Europe; Abbott (2006) for Australia; Pombo and Taborda (2006) for Columbia; and Kulshreshtha and Parikh (2002) for India. While these empirical studies make use of different strategies to incorporate the interconnections between electricity generation and greenhouse gases, they all acknowledge the importance of taking these links into account in the efficiency evaluation exercise. Also, another common feature of these empirical studies is their input choice. Indeed, they systematically select input(s) to proxy the total assets and the quantity of fuel used.

A first particularity of our empirical study is to improve the realism of the modelling of the plant production process by partitioning the electricity generated into electricity generated by fossil energies (e.g. coal, oil, gas) and electricity generated by non-fossil energies (e.g. wind, solar, geothermal). This gives us the option to take the interconnections with the inputs and the greenhouse gases into account. A second particularity is that we make use of plant-level data developed by the Environmental

Protection Agency of the US: the eGRID system. We use eGRID 2012 version 1.0 that gives data for 2009. This database provides data for two inputs: nameplate capacity (used as a proxy for total assets), and the quantity of fuel; two outputs: fossil and non-fossil electricity generation; and three undesirable outputs: CO_2, SO_2, and NO_x emissions. Additional variables that we will use to perform a second-stage analysis are also provided by this database.

In the following, we investigate for potential input reductions for the plants. We consider that the demand side (i.e. the number of consumers) of the electricity market is more or less stable (at least in the short run), while plants do not have real incentives to diminish greenhouse gases in the US.[10] In what follows, we first discuss the specificities of our set-up in more detail; specifically with respect to the interconnections, convexity and returns-to-scale assumptions. Subsequently, we present our data and our results.

3.1 Interconnections, convexity, and returns-to-scale

When distinguishing between fossil and non-fossil electricity, several interconnections are naturally present in the production process. One, we cannot assume that the whole electricity is produced by the use of fuel. Clearly, fuel is only used to generate fossil electricity. Two, non-fossil electricity cannot be responsible for the emission of greenhouse gas emissions. We obtain the following setting for every plant: nameplate capacity (x^1) is used to produce non-fossil (y^1) and fossil (y^2) electricity; fuel (x^2) is only used to generate fossil electricity; and three greenhouse gases, CO_2 (u^1), SO_2 (u^2), and NO_x (u^3) are present in the production process as the consequence of the production of fossil electricity. We use $g(u^i) = -u^i$ (for $i = 1, 2, 3$) to transform the greenhouse gases (see Section 2 for more discussion about the possible transformations). Figure 2 summarizes the production process of the plants.

Figure 2: Plant production process

Adopting the notation of Section 2, for each plant we obtain the following output,

[10]See, for example, Cherchye, De Rock and Walheer (2015) for an empirical study of electricity plants when specific targets are specified for the greenhouse gas reductions. These types of targets could fairly easily be integrated into our model.

undesirable output, and input vectors at the overall level:

$$\mathbf{Y} = \begin{bmatrix} y^1 \\ y^2 \end{bmatrix}, \mathbf{U} = \begin{bmatrix} u^1 \\ u^2 \\ u^3 \end{bmatrix}, \mathbf{Z} = \begin{bmatrix} y^1 \\ y^2 \\ g(u^1) \\ g(u^2) \\ g(u^3) \end{bmatrix} = \begin{bmatrix} y^1 \\ y^2 \\ -u^1 \\ -u^2 \\ -u^3 \end{bmatrix}, \text{ and } \mathbf{X} = \begin{bmatrix} x^1 \\ x^2 \end{bmatrix}. \tag{36}$$

Next, given our previous discussion about the interconnections between the two types of electricity, the two inputs, and the three greenhouse gases for the plants, we obtain the following information vectors:

$$\mathbf{A}^1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{A}^2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \mathbf{B}^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \text{ and } \mathbf{B}^2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \tag{37}$$

It turns out that the input and undesirable output vectors associated with non-fossil electricity (output 1) and fossil electricity (output 2) are provided by:

$$\mathbf{X}^1 = \mathbf{A}^1 \odot \mathbf{X} = \begin{bmatrix} x^1 \\ 0 \end{bmatrix}, \text{ and } \mathbf{X}^2 = \begin{bmatrix} x^1 \\ x^2 \end{bmatrix}. \tag{38}$$

$$\mathbf{U}^1 = \mathbf{B}^1 \odot \mathbf{U} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \text{ and } \mathbf{U}^2 = \begin{bmatrix} u^1 \\ u^2 \\ u^3 \end{bmatrix}. \tag{39}$$

$$\mathbf{Z}^1 = \begin{bmatrix} y_1 \\ g(\mathbf{U}^1) \end{bmatrix} = \begin{bmatrix} y^1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \text{ and } \mathbf{Z}^2 = \begin{bmatrix} y^2 \\ g(u^1) \\ g(u^2) \\ g(u^3) \end{bmatrix} = \begin{bmatrix} y^2 \\ -u^1 \\ -u^2 \\ -u^3 \end{bmatrix}. \tag{40}$$

We discuss our choice for the convexity and the returns-to-scale assumption. When undesirable outputs are involved in the production process, the constant and variable returns-to-scale assumptions are the two most popular choices for empirical works (see, for example, Zhou, Ang and Poh (2008)). For our empirical study, we choose to rely on constant returns-to-scale for non-fossil electricity generation, and on variable returns-to-scale for fossil electricity generation; and impose only convexity of the input sets. We denote this case by *VRS-CRS*. Different arguments can be invoked to justify our choices. Firstly, we may argue that fossil energy is proportional to the inputs (similar argument to the one used by Podinovski (2004a) and followers to justify constant returns-to-scale). Indeed, the quantity of fossil electricity produced by each unit of fuel input is known (physic law). As such, doubling the inputs (fuel and nameplate capacity), will also double the fossil electricity production. A similar reasoning does not hold true for non-fossil fossil electricity, as the production of non-fossil electricity also depends on external factors (e.g. the wind or solar exposition). This advocates for choosing a variable returns-to-scale.

Next, we may also argue that fossil electricity is closer to the perfect competitive market than non-fossil electricity. (Perfect competition is often associated with constant returns-to-scale.) In a sense, fossil electricity production may be assimilated to an 'old technology', while non-fossil electricity production may be assimilated to 'new technology'. As such, more competition occurs for the latter. Finally, we do not see any reason to impose convexity of the overall production possibility set, neither convexity for the output-specific production possibility sets. As we are interested by potential input reductions, convexity of the input sets is a sufficient assumption (see also Cherchye, De Rock and Walheer (2015) and Walheer (2018e) for more arguments).

In an illustrative purpose, we also consider two additional cases. Namely, when constant and variable returns-to-scale are assumed for both types of electricity. We denote these two extra cases by *CRS-CRS* and *VRS-VRS*, respectively.

3.2 Results

We restrict our attention to large plants (i.e. those generating more than 1,000,000 megawatts) that produce both fossil and non-fossil electricity. We end with a sample of 62 plants. While this number may seem small, it only reflects that most plants are built to generate one type of electricity in the US (see also Walheer (2018d) for more discussion). Table 1 reports the corresponding descriptive statistics for the inputs and outputs taken up in our analysis.

We start by computing the input efficiency scores $\widehat{TE}$ when assuming variable returns-to-scale for non-fossil electricity and constant returns-to-scale for fossil electricity (i.e. *VRS-CRS*) using the linear program provided in (38). Next, in an il-

Table 1: Descriptive statistics

	Outputs					Inputs	
	Non-Fossil Electricity (MWh)	Fossil Electricity (MWh)	CO_2 (tons)	SO_2 (tons)	NO_x (tons)	Nameplate Capacity (MW)	Fuel (MMBtu)
Min	660,025	312,500	360,748	332	310	93	490,478
Mean	707,200	507,700	442,100	500	1600	200	4,818,000
Max	19,650,000	11,140,000	11,980,000	20,000	70,000	3,562	117,000,000
Std	2,976,000	1,995,000	1,912,000	200	900	100	1,936,300

lustrative purpose, we recompute $\widehat{TE}$ for the two extra cases (i.e. *VRS-VRS* and *CRS-CRS*). Results are displayed in Table 2.

Table 2: Efficiency scores

	VRS-VRS	*VRS-CRS*	*CRS-CRS*
Min	0.4375	0.0909	0.0601
Mean	0.9472	0.7429	0.6017
Median	1	0.9357	0.6197
Max	1	1	1
St Dev	0.1260	0.3063	0.3629
# Efficient	46	24	16
% Efficient	74.19	38.71	25.81

$\widehat{TE}$ is interpreted as the potential input reduction keeping the electricity generation (and the greenhouse gases) constant. The median/average are 0.9357/0.7429 meaning that 6.43%/25.71% of the input quantities could be saved without impacting the electricity production. Also, 38.71% of the plants are efficient, i.e. they use the optimal level of inputs to produce the outputs. Clearly the returns-to-scale assumption choice for the two types of electricity has a direct impact on the efficiency scores and on the number of efficient plants. We observe from Table 2 that it is more difficult to be efficient when assuming constant returns-to-scale for both types of electricity than when assuming variable returns-to-scale (16 against 46 efficient plants). This observation is also confirmed by the other descriptive statistics. This ranking is intuitive and also observed when comparing the initial DEA models of Charnes, Cooper and Rhodes (1978) and Banker, Charnes and Cooper (1984). Finally, our mixed case (i.e. *VRS-CRS*)) lies between these two extreme cases (24 efficient plants).

We make used of a truncated regression as a second-stage analysis. Our aim is to investigate whether differences in efficiency scores are explained by exogenous factors.[11] The eGRID system provides some relevant variables that may directly affect the efficient behaviour of the plants. The regression equation is given as:

$$\widehat{TE} = \beta_0 + \beta_1 w_1 + \beta_2 w_2 + \beta_3 w_3 + \beta_4 \delta_1 + \beta_5 \delta_2 + \beta_6 \delta_3 + \epsilon, \tag{41}$$

where w_1 is the number of boilers; w_2 is the number of generators; w_3 is the capacity factor; $\delta_1 = 1$ if a plant uses coal to generate fossil electricity, and $\delta_1 = 0$ otherwise; $\delta_2 = 1$ if a plant uses oil to generate fossil electricity, $\delta_2 = 0$ otherwise; and $\delta_3 = 1$ if a plant uses gas to generate fossil electricity, $\delta_3 = 0$ otherwise.

Table 3 provides the values of the regression coefficients and their significant level for the *VRS-CRS* case (we use ***, **, and * to denote the 1%, 5%, and 10% significance levels). Again, in an illustrative purpose, we have also estimated the regression coefficients for the *CRS-CRS* and *VRS-VRS* cases.

Table 3: Truncated regressions

	VRS-VRS	*VRS-CRS*	*CRS-CRS*
$\widehat{\beta}_0$	0.9141***	0.6232***	0.4542***
$\widehat{\beta}_1$	0.0025	0.0682**	0.0941***
$\widehat{\beta}_2$	-0.0117	0.0013	0.0098
$\widehat{\beta}_3$	1.1549***	1.3121***	0.9980***
$\widehat{\beta}_4$	-0.0508	-0.4931**	-0.4441***
$\widehat{\beta}_5$	0.2703*	-0.1426*	-0.3799***
$\widehat{\beta}_6$	-0.0451	-0.1292	-0.0601

Some interesting lessons can be learned from the results given by Table 3. Firstly, whatever the returns-to-scale assumptions chosen, the estimated coefficients $\widehat{\beta}_1$, $\widehat{\beta}_2$ and $\widehat{\beta}_3$ are positive; meaning that the largest plants are, on average, more efficient. Next, plants that use coal, oil or gas are, on average less efficient under the *VRS-CRS* and *CRS-CRS* cases, but not under the *VRS-VRS* case. It means that relying on fossil energy sources reduces the efficient behaviour of the plants. Note that the regression coefficient associated with the coal energy source is the largest.

[11] We remark that there is a debate in the literature about the proper way to evaluate the effect of exogenous factors on efficiency scores. Using a truncated regression seems to be a good compromise. See, for example, Hoff (2007) and McDonald (2009) for more discussion.

4 Conclusion

Data Envelopment Analysis (DEA; after Charnes, Cooper and Rhodes (1978)) is a nonparametric technique that evaluates efficiency of a Decision Making Unit (DMU) by comparing its input-output performance to that of other DMUs operating in a similar technological environment. Recently, increasing attention has be given to the development of specific techniques to deal with interconnections between outputs, inputs, and undesirable outputs for DEA models. These techniques offer the advantages of improving the realism and the flexibility of DEA models; two aspects of crucial importance to convince practitioners about the attractiveness and the reliability of DEA models.

In this paper, we propose a unifying methodology coherent with previous works to model these interconnections. In particular, we suggest treating the outputs as the foundation of the production process by modelling every output individually. This thus gives us the option to consider the interconnections with the inputs and the undesirable outputs, and to make a distinction between different categories of inputs and undesirable outputs. It turns out that our methodology naturally improves the realism and the flexibility of DEA models. These advantages do not come with important drawbacks. In fact, it is the opposite as our methodology also offers the option of setting a different returns-to-scale assumption for each output-specific production process, and to choose between different types of convexity. As such, the realism and flexibility are improved even more.

We demonstrate the usefulness of our methodology with an application to the US electricity plants using a plant-level data set. In particular, we decompose the electricity generation into fossil and non-fossil electricity generation, and model the interconnections with the greenhouse gases and the inputs for each type of electricity. We show that inputs can potentially be saved, and perform a second-stage analysis to explain the (in)efficient behaviours observed.

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