Reallocation of Resources in Multidivisional Firms: A Nonparametric Approach*

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Abstract

Recently, increasing research attention has been paid to studying and modelling the efficiency behaviour of multidivisional firms. In this paper, we propose a simple and intuitive nonparametric approach for cost minimising in multidivisional firms when resources can be reallocated across firms and/or divisions. In particular, we consider that resource reallocation occurs through the price system. One distinguishing feature of our approach is that no price information is required to evaluate cost efficiency. This feature clearly increases the practical usefulness of our methodology given that prices are generally unknown when resource reallocation is possible. We consider various extensions of our framework, present a simulation, and apply our methodology to the Chinese manufacturing sector using a firm-level dataset.

Keywords: resource reallocation; multidivisional firms; cost efficiency; shadow price; Chinese manufacturing industry.

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1 Introduction

Nonparametric efficiency analysis has been widely used to benchmark firms by comparing their performance to that of other firms operating in a similar technological environment. This type of analysis has grown in popularity given its ease of use. That is, only the input–output data are required, no assumptions about the technology are needed, and efficiency measurements are obtained by solving simple linear programs. In practice, nonparametric efficiency analysis is used to identify inefficient behaviour with the aim of enabling profit improvement. Although research in this field has made significant progress in recent years, it is typically assumed that firms are monolithic and mutually independent units. However, both these assumptions are often violated in reality. Contrary to the first assumption, firms are often divided into lower-level units (e.g., divisions and departments); contrary to the second assumption, firms form various kinds of partnerships. In these contexts, tailored measurements of efficiency are required. In particular, these measurements should consider the possibility of reallocation of resources across decision-making units. In this paper, we enrich conventional analysis by allowing for this possibility.

We consider a group of firms, each with a multidivisional structure (M-form). An M-form firm has multiple autonomous divisions, each of which is responsible for a particular product, region or market segment. The pioneers of this widely used organisational form are DuPont, Sears Roebuck, Standard Oil, and General Motors (Williamson, 1985; Milgrom and Roberts, 1992; Roberts, 2007). For example, in the late 1990s, DuPont consisted of divisions based on the product lines (e.g., specialty fibres, pharmaceuticals and nutritional products) (Whittington & Mayer, 2000). More prominent examples include Hilton Hotels, PepsiCo, Hewlett-Packard, and The Walt Disney Company (Griffin, 2010). In the efficiency literature, M-form has been explicitly used in different contexts (Athanassopoulos, 1998; Cherchye et al., 2013; Cherchye et al., 2015, 2016; Walheer, 2018c, 2018f, 2018g). Further, several efficiency models have been developed to account for the organisational structure of firms in a more general way. Castelli et al. (2008) review this literature and identify three main types of models: 'shared-flow models', which allow subunits of a firm to share some inputs and/or outputs (Cook et al., 2000; Tsai & Molinero, 2002); 'multilevel models', in which the inputs (outputs) of a firm are not necessarily the inputs (outputs) of its subunits (Staat, 2002; Cook & Green, 2005); and 'network models', in which the output of one subunit may be the input of another subunit (Prieto & Zofio, 2007; Kao, 2009; Färe et al., 2007; Chen et al., 2013). Our approach shares common features of shared-flow models, but our focus is different: in particular, we are interested in efficiency gains due to reallocation of inputs across divisions (and firms) rather than in accounting for the firm's internal structure only. In addition, these models do not usually consider economies of scope, which is a crucial element in the organisation of large firms with diversified outputs.

To account for economies of scope, we distinguish two types of inputs: division-specific and firm-specific inputs. Division-specific inputs are inputs that are used by each division separately. That is, these inputs are rival and exclusive to the individual divisions. This type of input has been considered in various efficiency models (e.g., Färe et al., 2007; Tone & Tsutsui, 2009; Walheer, 2016a, 2016b, 2018f, 2018h; Ding et al., 2017; Silva, 2018). Examples of division-specific inputs are employees, machines or other resources used exclusively by each division. In contrast, firm-specific inputs are inputs that are shared by all the divisions in a firm. That is, these inputs are nonrival and nonexclusive to the divisions of the firm. This type of input has been considered in various efficiency models (e.g., Salerian & Chan, 2005; Despic et al., 2007; Cherchye et al., 2013; Cherchye et al., 2016; Walheer, 2018b, 2018c, 2018g).

From the perspective of the firm divisions, firm-specific inputs have a public nature. Examples of firm-specific inputs include infrastructure, corporate staff and general managers.

We consider the possibility of reallocation of inputs across divisions and entire firms. Previous research on reallocation of inputs in nonparametric efficiency analysis includes Nesterenko and Zelenyuk (2007), Mayer and Zelenyuk (2014), and Cherchye et al. (2017). In line with our approach, these research endeavours develop group-efficiency measurements while considering the possibility of reallocation of resources. However, these works examine only two limiting cases of reallocation: free reallocation and no reallocation. In contrast, our approach allows for intermediate cases. Another difference is that the previous research considers either reallocation within a firm or between firms. However, we combine and extend both possibilities. Clearly, the distinction between firm-specific and division-specific inputs presents different features in relation to potential reallocation at both the firm level and the division level. In particular, we consider the reallocation of division-specific inputs across divisions of a single firm, as well as across divisions of different firms. We also allow for the reallocation of firm-specific inputs across firms.

Allocation of inputs is also considered in the models that use a centralised decision maker. One branch of this literature considers the problem of the allocation of firm-level inputs among individual units (Cook & Zhu, 2005; Li et al., 2009; Lin, 2011; Du et al., 2014). In line with our approach, some of this research consider various constraints on input reallocation (Beasley, 2003; Korhonen & Syrjanen, 2004; Pachkova, 2009). Another branch of literature considers a centralised decision maker whose aim is to minimise overall input consumption (or alternatively, to maximise overall output production) of all individual units (Du et al., 2010; Hosseinzadeh Lotfi et al., 2010; Yu et al., 2013; Molinero et al., 2014). While the assumption of a centralised allocation of resources is usually justified for firms and organisations, particularly those with a rigid hierarchical structure, this assumption is less applicable in industries where no central planner can be identified. Moreover, multidivisional firms often make use of internal capital and labour markets to allocate resources. Even in cases when firms do not use explicit prices and rely on administrative decisions, there are always implicit prices associated with these decisions. Therefore, following Kuosmanen et al. (2006), Kuosmanen et al. (2010), Fang (2013), and Fang and Li (2015), our approach assumes that allocation of resources both across and within firms occurs through the price system.

Our approach emphasises applicability to practical problems, and therefore, we incorporate several realistic assumptions that are missing in many previous works. Above all, we consider the economic objective of firms. Earlier research has adopted an engineering approach, focusing on technical efficiency, which measures the distance of a firm's actual performance from the frontier of the reconstructed production-possibility sets (Debreu, 1951; Shephard, 1953, 1970). While technical efficiency is undoubtedly important, it usually represents only one component of a firm's inefficient behaviour (Farrell, 1957). Moreover, ignoring the economic objective of the firm may induce bias in the efficiency analysis. For example, firms are typically assumed to maximise profit through employing cost minimisation. To account for this, cost-based efficiency measures have been introduced (for recent research on cost or profit efficiency, see Sahoo & Tone, 2013; Sahoo et al., 2014; Boussemart et al., 2015; Cherchye et al., 2016; Walheer & Zhang, 2018). Accordingly, in our analysis, we assume that firms are cost minimisers.

In addition, our approach is realistic in the sense that its various features are mutually compatible. For example, we consider firms with multiple outputs. This feature is closely connected to the possibility of reallocation of resources; indeed, one motive for establishing a multioutput firm is the possibility of being able to transfer resources (Teece, 1980, 1982; Roberts & Saloner, 2013). Likewise, the existence of firm-specific inputs forms a prime economic motivation for firms to consider having more than one division because having multiple divisions means firms can benefit from economies of scale and scope by using these inputs (Panzar & Willig, 1977, 1981; Tone & Sahoo, 2003; Nehring & Puppe, 2004). Finally, a multidivisional structure goes hand in hand with product variety (Hoskisson, 1987; Hoskisson & Hitt, 1988): indeed, each division is responsible for the production of different outputs, where the term 'different' may refer to various criteria, including geography, market segmentation, or simply physical difference of the products. The link between the structure and product variety is particularly strong when the geography criterion is used (Mansfield et al., 1978).

Despite these realistic features, our approach retains the advantages of conventional efficiency analysis (Farrell, 1957; Varian, 1984); in particular, it is nonparametric, and therefore no functional specification of the production function is needed. Another attractive feature of our approach is that it does not require data about input prices. This is important not only because data on prices are often unavailable, but also because inclusion of prices may result in various biases (e.g., due to temporary supply shocks) (Farrell, 1957; Cooper et al., 1996; Camanho & Dyson, 2005). In some contexts, explicit prices may not even exist (Leleu & Briec, 2009). For example, solar-power plants and aviation activities use sunlight and weather (respectively) as inputs. Although these and similar inputs enter production function (Dormady et al., 2019; Richmond et al., 2007), they are not traded in markets and thus have no explicit monetary price. These various complications are even more important when reallocation is considered: complete and detailed information, particularly about within-firm reallocation is not available, and explicit prices do not exist if the allocation occurs through administrative decisions. There are also no explicit prices for the firm-specific inputs employed by individual divisions in production. Rather, just as with public goods, there is only unobservable willingness to pay for these inputs.

Finally, our methodology can also be considered a consistent extension of the aggregation procedure for group introduced by Färe and Zelenyuk (2003) and extended in various directions in research such as Färe et al. (2004), Zelenyuk (2006, 2016), Peyrache (2013, 2015), Färe and Karagiannis (2017), Rogge (2018), and Walheer (2018a, 2018f). In line with this literature, our aggregate efficiency measurements are defined exclusively by the entity-specific counterparts. More specifically, the industry-level efficiency measurements can be obtained in terms of their firm-level counterparts, and the firm-level measures can be obtained in terms of their division-level counterparts. In a sense, our approach extends the standard methodology to situations where reallocation across divisions and firms is considered. These efficiency measurements have practical use for firms and organisations characterised by a multidivisional structure. Low values of these efficiency measurements indicate potential efficiency gains from the reallocation of resources at various organisational levels.

The remainder of this paper is organised as follows. Section 2 defines our notion of cost-efficiency measurement for multidivisional firms. The section demonstrates how this measurement can be computed without observing the technology or the prices. It also describes how we allow for reallocation between divisions and firms, and how the cost-efficiency measurement can be computed in that case. The section then discusses various extensions of our methodology. Section 3 presents a simulation and an empirical application of our approach to the Chinese manufacturing sector. Section 4 concludes the paper.

2 Methodology

We consider a group of n multidivisional firms. Every division $k \in \{1, \ldots, K\}$ of each firm $t \in \{1, \ldots, n\}$ is in charge of producing the outputs $\mathbf{y}_k^t$ using two types of inputs: the firm-specific inputs, captured by the vector $\mathbf{z}^t$, and the division-specific inputs, captured by the vector $\mathbf{x}_k^t$. We use $\mathbf{p}^t$ and $\mathbf{w}_k^t$ to denote the prices of the firm-specific and division-specific inputs for every division k in firm t, respectively. We consider that divisions are cost minimisers. This represents a very general setting that fits many empirical contexts. The following framework can quite easily be extended to a revenue-maximising or profit-maximising setting.

Given the feature that the firm is multidivisional, three levels of reallocation are possible: (1) the reallocation of firm-specific inputs across firms; (2) the reallocation of division-specific inputs across firms; (3) the reallocation of division-specific inputs across divisions. An example of the first level of reallocation is the migration of managers among firms or a purchase of a chain of stores by one company from another. An example of the first level of reallocation is the migration of specialised workers, or purchases of equipment among divisions of different firms. An example of the first level of reallocation is transfers of division managers within the firm or allocation of resources through internal capital markets.

We begin our analysis by defining our notion of cost-efficiency measurement for multidivisional cost-minimiser firms when no reallocation of inputs is possible. This will establish our benchmark case. We then demonstrate how to extend this framework when resource reallocation is considered between divisions, and then between firms. Finally, we discuss several extensions of our approach.

2.1 No reallocation

To capture the multidivisional nature of firms, we define the technology at the firm level. It is given for a particular division k in firm t as follows:

$$I_k^t(\mathbf{y}_k^t) = \left\{(\mathbf{z}, \mathbf{x}_k) \mid \text{ division } k \text{ can produce } \mathbf{y}_k^t \text{ using } (\mathbf{z}, \mathbf{x}_k)\right\}. \tag{1}$$

$I_k^t(\mathbf{y}_k^t)$ contains all the input combinations for division k in firm t that allow for producing $\mathbf{y}_k^t$. We assume that these sets fulfil general requirements to conduct a cost-efficiency analysis (see Färe & Primont (1995), and particularly, Varian (1984) and Tulkens (1993) for a discussion of cost setting). In particular, we assume that these sets are nested and monotone. That is, we assume that the inputs and outputs are freely disposable. We note that convexity is not required. This is attractive given that generally, convexity seems to be a stringent assumption that does not always match with real data.

We then introduce an additional concept to define our minimal cost function for multidivisional firms: the (division-specific) implicit prices for the firm-specific inputs. These prices, denoted by $\mathbf{p}_k^t$ for each division k in firm t, capture the willingness of the divisions to pay for the firm-specific inputs. That is, they can be interpreted as representing the value of marginal product (i.e., an additional increase of output $\mathbf{y}_k^t$ [in monetary terms] associated with an additional unit of the firm-specific inputs). These implicit input prices can be connected to the initial prices as follows:

$$\mathbf{p}^t = \sum_{k=1}^{K} \mathbf{p}_k^t. \tag{2}$$

This condition is intuitively similar to the condition for Lindahl prices in the case of efficient public-goods provision. That is, Pareto-efficient provision of public goods requires that the Lindahl prices add up to the aggregate prices. It turns out that the condition stated in (2) implies that the amount of firm-specific inputs hired by the firm is Pareto efficient (i.e., that $\mathbf{p}_k^t$ captures the divisions' true willingness to pay). Note that a similar condition has been considered in recent previous works about multidivisional firms with firm-specific inputs (Cherchye et al., 2013; Cherchye et al., 2016; Walheer, 2018b, 2018c, 2018g). The condition (2) corresponds to the situation when divisions cooperate to minimise the cost of the firm. However, in reality, given the public character of the firm-specific inputs, each division may free ride on the provision of these inputs by other divisions. Consequently, the firm-specific inputs may turn out to be underprovided. We believe that Pareto efficiency embodied in the condition (2) is a coherent assumption when considering the possibility of reallocation given that reallocation presupposes a level of cooperation among divisions. Indeed, if divisions can reach an agreement on reallocation of resources, they are also likely to agree on the Pareto-efficient amount of the firm-specific inputs. Another justification for using the condition (2) is that we want to focus on the efficiency caused by barriers due to misallocation of resources while ignoring other possible causes.[1] Another simplification is that the firm-specific inputs are considered completely nonrival while in reality, they may be rival to some extent. For example, when services of the finance department are used by one division, there are fewer services of the finance department at the disposal of another division. To account for such cases, we assume

[1] When the firm-specific inputs are not provided in a Pareto-efficient way, we may state another condition in which the subscript j refers to the $j-$th entry of the vector:

$$(\mathbf{p}^t)_j = \max_k (\mathbf{p}_k^t)_j. \tag{3}$$

That is, the highest division-specific implicit prices must equal the observed prices of the firm-specific inputs. Intuitively, only the divisions with the highest willingness to pay will provide the firm-specific input, while the divisions with lower willingness to pay will free ride. For example, see Cherchye et al. (2014) for further discussion. To define our concepts when a Pareto-efficient provision is not assumed, it suffices to replace the condition stated in (2) by the condition stated in (3) when defining the concept of minimal cost for firms.

that firm-specific inputs are available in sufficient amounts so that their use by one division does not involve opportunity costs across the firm.

We now have all the necessary notations to define our minimal cost function for multidivisional firms. It is given for a particular firm t:

$$C^t = \sum_{k=1}^{K} C_k^t = \min_{(\mathbf{z},\mathbf{x}_1,\ldots,\mathbf{x}_K)} \left\{ \sum_{k=1}^{K} \mathbf{p}_k^{t\prime}\mathbf{z} + \mathbf{w}_k^{t\prime}\mathbf{x}_k \mid \forall k \in \{1,\ldots,K\} : (\mathbf{z},\mathbf{x}_k) \in I_k^t(\mathbf{y}_k^t) \right\}. \tag{4}$$

That is, minimal cost for firm t is defined as the sum of the division-specific minimal costs, such that each division k in firm t can produce the output quantity $\mathbf{y}_k^t$. This again highlights the multidivisional nature of the firms. When $C^t = \sum_{k=1}^{K} \mathbf{p}_k^{t\prime}\mathbf{z}^t + \mathbf{w}_k^{t\prime}\mathbf{x}_k^t$, it means that the outputs are produced with minimal cost in firm t, while $C^t < \sum_{k=1}^{K} \mathbf{p}_k^{t\prime}\mathbf{z}^t + \mathbf{w}_k^{t\prime}\mathbf{x}_k^t$ reflects potential cost savings. At this point, it is worth indicating that C^t is defined so that all prices (this includes the implicit input prices for the firm-specific inputs) are observed. That is, it also implies that C^t is mathematically dependent on the input prices (i.e., it is a function of the input prices). We consider below a more useful cost-efficiency measurement that does not assume that the implicit input prices are observed (see (7)).

It is usual to rely on a goodness-of-fit measurement to quantify the potential cost savings of cost-inefficient firms. In particular, we make use of the ratio defined by Farrell (1957). It is given for firm t by:

$$CE^t = \frac{\sum_{k=1}^{K} C_k^t}{\sum_{k=1}^{K} \mathbf{p}_k^{t\prime}\mathbf{z}^t + \mathbf{w}_k^{t\prime}\mathbf{x}_k^t}. \tag{5}$$

By construction, CE^t is below unity. A value of one implies cost-efficient behaviour for firm t, while cost-inefficient behaviour is captured by a smaller value. In practice,

CE^t can be computed using the following linear programming:

$$CE^t = \max_{\substack{C_t^k \in \mathbb{R}_+ \\ k\in\{1,\ldots,K\}}} \frac{\sum_{k=1}^K C_k^t}{\sum_{k=1}^K \mathbf{p}_k^{t'}\mathbf{z}^t + \mathbf{w}_k^{t'}\mathbf{x}_k^t}$$

$$\text{s.t.} \quad \forall k \in \{1,\ldots,K\} : C_k^t \leq \mathbf{p}_k^{t'}\mathbf{z}^s + \mathbf{w}_k^{t'}\mathbf{x}_k^s \text{ for all } s \in \{1,\ldots,n\} : \mathbf{y}_k^s \geq \mathbf{y}_k^t. \tag{6}$$

That is, the constraints verify that every division producing more output quantities than the evaluated division k in firm t (i.e., all divisions k in firms s, such that $\mathbf{y}_k^s \geq \mathbf{y}_k^t$) use higher costs than division k in firm t (i.e., $C_k^t \leq \mathbf{p}_k^{t'}\mathbf{z}^s + \mathbf{w}^{t'}\mathbf{x}_k^s$). Note that this type of program dates back to Varian (1984), and is very popular for nonparametric efficiency analysis. Finally, note that in this programming variable, returns-to-scale is assumed, which represents the less restrictive case (e.g., Walheer, 2018e).

While the definition of CE^t is perfectly fine from a theoretical perspective, it clearly lacks practical usefulness. Indeed, the (division-specific) implicit input prices are generally not observed for empirical studies. In that case, we suggest relying on shadow prices. In practice, we evaluate the firms in the best possible way, in that we allow 'benefit of the doubt' in the absence of true price information. That is, we select the prices that maximise the cost-efficiency measurement of the firms. We define our shadow cost-efficiency measurement for firm t as follows:

$$SCE^t = \max_{\substack{\mathbf{p}_k^t \in \mathbb{R}_+ \\ k\in\{1,\ldots,K\}}} \left\{ CE^t \mid \mathbf{p}^t = \sum_{k=1}^K \mathbf{p}_k^t \right\}. \tag{7}$$

That is, the shadow cost-efficiency measurement for firm t is defined when selecting the most favourable implicit input prices, such that Pareto-efficient provision of the firm-specific inputs between divisions is fulfilled. As explained below, we impose

Pareto-efficient provision because intuitively, we may see that assumption as a necessary condition for considering the possibility of reallocation of inputs. (In addition, as explained, it is straightforward to define a shadow cost-efficiency measurement when Pareto-efficient provision of firm-specific inputs is not assumed; see Footnote 1). It turns out that SCE^t defines a natural upper bound for the (theoretical) cost-efficiency measurement CE^t (i.e., $SCE^t \geq CE^t$). In practice, SCE^t can be computed using the following linear programming:

$$SCE^t = \max_{\substack{C_t^k \in \mathbb{R}_+, \mathbf{p}_k^t \in \mathbb{R}_+ \\ k \in \{1,\ldots,K\}}} \frac{\sum_{k=1}^K C_k^t}{\sum_{k=1}^K \mathbf{p}_k^{t\prime} \mathbf{z}^t + \mathbf{w}_k^{t\prime} \mathbf{x}_k^t}$$

$$\text{s.t.} \quad \forall k \in \{1,\ldots,K\} : C_k^t \leq \mathbf{p}_k^{t\prime} \mathbf{z}^s + \mathbf{w}_k^{t\prime} \mathbf{x}_k^s \text{ for all } s \in \{1,\ldots,n\} : \mathbf{y}_k^s \geq \mathbf{y}_k^t,$$

$$\mathbf{p}^t = \sum_{k=1}^K \mathbf{p}_k^t. \tag{8}$$

The linear programming in (8) is very similar to the linear programming in (6). The only differences are that in (8), the (division-specific) implicit input prices are not observed, and we have an additional constraint. This is why these prices are variables in (8) and not in (6). $SCE^t = 1$ implies that (shadow) cost efficiency is reached, while smaller values reflect greater (shadow) cost-inefficient behaviour. As a final comment, we highlight that infeasibility is avoided in (8) because the denominator of the objective function is always strictly greater than zero. This is ensured by noticing that the input quantities are by construction strictly greater than zero, and we impose the condition that the shadow prices are strictly greater than zero (see the condition under the max operator in (8)).

In practice, cost-inefficient firms can reduce their costs by either reducing the amounts of some of the inputs and/or by changing the input mix. In the following section, we decompose the shadow cost-efficiency measurement SCE^t to identify potential gains from reallocation of inputs across divisions.

2.2 Reallocation between divisions

We now consider various possibilities of reallocation. The intuition behind our approach is as follows: assume that inputs are homogeneous and reallocation is costless. Then price differences represent arbitrage opportunities that can be profitably exploited. Arbitrage activities will then lead to equalisation of prices across divisions and/or firms. Accordingly, when reallocation is possible, we impose an additional condition that prices of the inputs that can be reallocated are equal. Moreover, when prices are equal, it means that inputs are substitutable, which we may see as a necessary condition to consider reallocation of resources. First, we consider reallocation of division-specific inputs across divisions of a firm. Reallocation of firm-specific inputs is considered in Section 2.3.

Unfortunately, prices are often not observable, particularly for resources reallocated within firms. If the inputs are allocated via administrative decisions rather than via markets, prices do not exist at all. Therefore, in the absence of any price information, we again suggest using the benefit-of-the-doubt approach. That is, we find the most favourable prices, such that reallocation is possible. We then define our division-reallocation shadow cost-efficiency measurement for firm t as follows:

$$DSCE^t = \max_{\substack{\mathbf{w}_k^t \in \mathbb{R}_+ \\ k \in \{1,\dots,K\}}} \left\{ SCE^t \mid \forall k \in \{1,\dots,K\} : \mathbf{w}_k^t = \mathbf{w}^t \right\}. \tag{9}$$

That is, $DSCE^t$ gives a cost-efficiency measurement for firm t, such that common (most favourable) division-specific input prices are selected (captured by the additional constraint: $\forall k \in \{1,\dots,K\} : \mathbf{w}_k^t = \mathbf{w}^t$). The benefit-of-the-doubt spirit is clearly present in that definition given that selected prices maximise the cost efficiency of the firm. $DSCE^t$ can be computed by adding the common price constraint to (8):

$$
\begin{aligned}
DSCE^t = & \max_{\substack{C_t^k \in \mathbb{R}_+, \mathbf{p}_k^t \in \mathbb{R}_+, \mathbf{w}_k^t \in \mathbb{R}_+ \\ k \in \{1,\dots,K\}}} \frac{\sum_{k=1}^K C_k^t}{\sum_{k=1}^K \mathbf{p}_k^{t\prime} \mathbf{z}^t + \mathbf{w}_k^{t\prime} \mathbf{x}_k^t} \\
\text{s.t.} \quad & \forall k \in \{1,\dots,K\} : C_k^t \leq \mathbf{p}_k^{t\prime} \mathbf{z}^s + \mathbf{w}_k^{t\prime} \mathbf{x}_k^s \text{ for all } s \in \{1,\dots,n\} : \mathbf{y}_k^s \geq \mathbf{y}_k^t, \\
& \mathbf{p}^t = \sum_{k=1}^K \mathbf{p}_k^t, \\
& \forall k \in \{1,\dots,K\} : \mathbf{w}_k^t = \mathbf{w}^t. \qquad (10)
\end{aligned}
$$

As a result, (10) is a nonlinear program because variables appear at both the numerator and the denominator of the objective function. Fortunately, we can make (10) linear by using a simple transformation. In practice, we set the denominator equal unity: $\sum_{k=1}^K \mathbf{p}_k^{t\prime} \mathbf{z}^t + \mathbf{w}_k^{t\prime} \mathbf{x}_k^t = 1$. This transformation was made popular by Charnes et al. (1978) for nonparametric efficiency analysis. The resulting linear program to compute $DSCE^t$ is defined as follows:

$$
\begin{aligned}
DSCE^t = & \max_{\substack{C_t^k \in \mathbb{R}_+, \mathbf{p}_k^t \in \mathbb{R}_+, \mathbf{w}_k^t \in \mathbb{R}_+ \\ k \in \{1,\dots,K\}}} C^t \\
\text{s.t.} \quad & \forall k \in \{1,\dots,K\} : C_k^t \leq \mathbf{p}_k^{t\prime} \mathbf{z}^s + \mathbf{w}_k^{t\prime} \mathbf{x}_k^s \text{ for all } s \in \{1,\dots,n\} : \mathbf{y}_k^s \geq \mathbf{y}_k^t, \\
& \mathbf{p}^t = \sum_{k=1}^K \mathbf{p}_k^t, \\
& \forall k \in \{1,\dots,K\} : \mathbf{w}_k^t = \mathbf{w}^t, \\
& \sum_{k=1}^K \mathbf{p}_k^{t\prime} \mathbf{z}^t + \mathbf{w}_k^{t\prime} \mathbf{x}_k^t = 1. \qquad (11)
\end{aligned}
$$

Intuitively, when reallocation is possible, only more cost-inefficient behaviour can be found. It turns out that $DSCE^t \leq SCE^t$. When $DSCE^t = SCE^t$, it means that no cost-inefficient behaviour has resulted from a reallocation issue between divisions. In contrast, $DSCE^t < SCE^t$ reflects that potential costs can be saved if reallocation between divisions is considered. To quantify this potential gain, we can define the concept of division-reallocation cost-efficiency-gain measurement for firm t:

$$DEG^t = \frac{SCE^t}{DSCE^t}. \tag{12}$$

Since $DSCE^t \leq SCE^t$, we obtain that DEG^t is greater than unity. A value of one indicates that no cost gain is possible when considering reallocation between divisions. Larger values for DEG^t indicate the opposite. Using the concept of division-reallocation cost-efficiency-gain measurement, we can obtain a useful decomposition of our shadow cost-efficiency measurement for firm t:

$$SCE^t = DEG^t \times DSCE^t. \tag{13}$$

That is, this equation decomposes the shadow cost-efficiency measurement into two parts: the cost-efficiency measurement when reallocation between divisions is possible, and the division-reallocation cost-efficiency gain measurement. Finally, note that $DSCE^t$ is also a shadow cost-efficiency measurement because it is based on shadow prices.

To illustrate the usefulness of the decomposition (13), consider a firm i, for which $SCE^i = DSCE^i = 0.6$. Given that $SCE^i < 1$, this firm is cost inefficient. However, this inefficiency cannot be reduced by reallocation of inputs across the divisions because $SCE^i = DSCE^i$ and consequently, $DEG^i = 1$. Therefore, managers of firm i should look for other ways to improve their cost efficiency. In the following section, we consider potential efficiency gains from reallocation of inputs across firms.

2.3 Reallocation between firms

To consider the option of reallocating inputs between firms, we must first define cost efficiency at the firm level. A natural counterpart of firm-level efficiency defined in (5) is given by the following equation (see Färe et al., 2004; Färe & Karagiannis, 2017; Walheer, 2018a):

$$CE = \frac{\sum_{t=1}^{n} \sum_{k=1}^{K} C_k^t}{\sum_{t=1}^{n} \sum_{k=1}^{K} \mathbf{p}_k^{t\prime} \mathbf{z}^t + \mathbf{w}_k^{t\prime} \mathbf{x}_k^t}. \tag{14}$$

When minimal cost coincides with actual cost for every division and firm (i.e., $\forall k, \forall t$: $C_k^t = \mathbf{p}_k^{t\prime} \mathbf{z}^t + \mathbf{w}_k^{t\prime} \mathbf{x}_k^t$), the group of firms is found to be cost efficient with a cost-efficiency measurement of unity ($CE = 1$). When $CE < 1$, it means that greater cost-inefficient behaviour is detected for the group.

Our previous discussion about the empirical usefulness of the theoretical cost efficiency and observability of implicit input prices holds true for the group-level cost-efficiency measurement. A shadow cost-efficiency measurement is defined as follows:

$$SCE = \max_{\substack{\mathbf{p}_k^t \in \mathbb{R}_+ \\ k \in \{1,\ldots,K\}, t \in \{1,\ldots,n\}}} \left\{ CE \mid \forall t \in \{1, \ldots, n\} : \mathbf{p}^t = \sum_{k=1}^{K} \mathbf{p}_k^t \right\}. \tag{15}$$

This measure is analogous to the shadow cost-efficiency measure for the firm level in (7). Again, we use the benefit-of-the-doubt approach and select prices that maximise the cost-efficiency measurement. Therefore, we have $SCE \geq CE$. In practice,

SCE can be computed using the following linear programming:

$$SCE = \max_{\substack{C_t^k \in \mathbb{R}_+, \mathbf{p}_k^t \in \mathbb{R}_+ \\ k \in \{1,\ldots,K\}, t \in \{1,\ldots,n\}}} \frac{\sum_{t=1}^n \sum_{k=1}^K C_k^t}{\sum_{t=1}^n \sum_{k=1}^K \mathbf{p}_k^{t\prime} \mathbf{z}^t + \mathbf{w}_k^{t\prime} \mathbf{x}_k^t}.$$

$$\text{s.t.} \quad \forall t \in \{1,\ldots,n\}, \forall k \in \{1,\ldots,K\} : C_k^t \leq \mathbf{p}_k^{t\prime} \mathbf{z}^s + \mathbf{w}_k^{t\prime} \mathbf{x}_k^s \text{ for all } s \in \{1,\ldots,n\} : \mathbf{y}_k^s \geq \mathbf{y}_k^t,$$

$$\forall t \in \{1,\ldots,n\} : \mathbf{p}^t = \sum_{k=1}^K \mathbf{p}_k^t. \tag{16}$$

This linear programming is analogous to the to the programming (8) used for the firm level. We now consider the possibility of reallocation of inputs. For the group of firms, we consider two types of reallocation: reallocation of the firm-specific inputs across firms and reallocation of division-specific inputs across divisions of all firms. Therefore, we impose conditions on the prices of both inputs, $\mathbf{w}_k^t$ and $\mathbf{p}^t$. Just like before, we argue that reallocation implies equality of these prices across firms. We also continue to assume that these prices are not observable. We then define the total-reallocation shadow cost-efficiency measurement for a group of firms as follows:

$$TSCE = \max_{\substack{\mathbf{w}_k^t \in \mathbb{R}_+, \mathbf{p}^t \in \mathbb{R}_+ \\ k \in \{1,\ldots,K\}, t \in \{1,\ldots,n\}}} \left\{ SCE \mid \forall t \in \{1,\ldots,n\}, \forall k \in \{1,\ldots,K\} : \mathbf{w}_k^t = \mathbf{w} \wedge \mathbf{p}^t = \mathbf{p} \right\}. \tag{17}$$

TSCE can be computed using the following (normalised) linear programming:

$$
\begin{aligned}
TSCE = & \max_{\substack{C_t^k \in \mathbb{R}_+, \mathbf{p}_k^t \in \mathbb{R}_+, \mathbf{w}_k^t \in \mathbb{R}_+, \mathbf{p}^t \in \mathbb{R}_+ \\ k \in \{1,\dots,K\}, t \in \{1,\dots,n\}}} \sum_{t=1}^{n} \sum_{k=1}^{K} C_k^t \\
\text{s.t.} \quad & \forall k \in \{1,\dots,K\}, t \in \{1,\dots,n\} : C_k^t \leq \mathbf{p}_k^{t'} \mathbf{z}^s + \mathbf{w}_k^{t'} \mathbf{x}_k^s \text{ for all } s \in \{1,\dots,n\} : \mathbf{y}_k^s \geq \mathbf{y}_k^t, \\
& \forall t \in \{1,\dots,n\} : \mathbf{p}^t = \sum_{k=1}^{K} \mathbf{p}_k^t, \\
& \forall t \in \{1,\dots,n\} : \mathbf{p}^t = \mathbf{p}, \\
& \forall t \in \{1,\dots,n\}, \forall k \in \{1,\dots,K\} : \mathbf{w}_k^t = \mathbf{w}, \\
& \sum_{t=1}^{n} \sum_{k=1}^{K} \mathbf{p}_k^{t'} \mathbf{z}^t + \mathbf{w}_k^{t'} \mathbf{x}_k^t = 1. \qquad (18)
\end{aligned}
$$

Again, we have that $TSCE \leq SCE$. To quantify the potential gain from reallocation across firms, we define the total-reallocation cost-efficiency-gain measurement:

$$TEG = \frac{SCE}{TSCE} \qquad (19)$$

A value of *TEG* greater than one indicates the possibility of efficiency gains from reallocation. The shadow cost-efficiency measurement can then be conveniently decomposed as:

$$SCE = TEG \times TSCE \qquad (20)$$

So far, our assumptions about reallocation were somewhat extreme because we assumed that all kinds of reallocation are possible. In particular, we assumed that not only firms but also all divisions in the group agree to reallocate. Nevertheless, we can adapt our previous cost-efficiency measurements to several other cases. For example, we consider below the case when firms agree to reallocate, but divisions do not. More specifically, we consider the possibility of reallocating firm-specific inputs across firms, division-specific inputs within firms, but not division-specific inputs across firms. Other examples and further discussion are provided in Section 2.4. The firm-reallocation shadow cost-efficiency measurement is given for the group of firms by:

$$FSCE = \max_{\substack{\mathbf{w}_k^t \in \mathbb{R}_+, \mathbf{p}^t \in \mathbb{R}_+ \\ k \in \{1,\dots,K\}, t \in \{1,\dots,n\}}} \left\{ SCE \mid \forall t \in \{1,\dots,n\} : \mathbf{w}_k^t = \mathbf{w}_k \wedge \mathbf{p}^t = \mathbf{p} \right\}. \tag{21}$$

This measurement can be computed for the group of firms as follows:

$$\begin{aligned} FSCE = & \max_{\substack{C_t^k \in \mathbb{R}_+, \mathbf{p}_k^t \in \mathbb{R}_+, \mathbf{w}_k^t \in \mathbb{R}_+, \mathbf{p}^t \in \mathbb{R}_+ \\ k \in \{1,\dots,K\}, t \in \{1,\dots,n\}}} \sum_{t=1}^{n} \sum_{k=1}^{K} C_k^t \\ \text{s.t.} \quad & \forall t \in \{1,\dots,n\}, k \in \{1,\dots,K\} : C_k^t \leq \mathbf{p}_k^{t\prime} \mathbf{z}^s + \mathbf{w}_k^{t\prime} \mathbf{x}_k^s \text{ for all } s \in \{1,\dots,n\} : \mathbf{y}_k^s \geq \mathbf{y}_k^t, \\ & \forall t \in \{1,\dots,n\} : \mathbf{p}^t = \sum_{k=1}^{K} \mathbf{p}_k^t, \\ & \forall t \in \{1,\dots,n\} : \mathbf{p}^t = \mathbf{p}, \\ & \forall t \in \{1,\dots,n\} : \mathbf{w}_k^t = \mathbf{w}_k, \\ & \sum_{t=1}^{n} \sum_{k=1}^{K} \mathbf{p}_k^{t\prime} \mathbf{z}^t + \mathbf{w}_k^{t\prime} \mathbf{x}_k^t = 1. \end{aligned} \tag{22}$$

The programming (22) is very similar to the programming (18). The only difference is that in (18), prices of all division-specific inputs are the same because all reallocation is possible (i.e., $\mathbf{w}_k^t = \mathbf{w}$), while in (22), the prices of these inputs are the same only within a given firm, but not necessarily across the firms because only reallocation within a firm is possible (i.e., we have $\mathbf{w}_k^t = \mathbf{w}_k$). As before, we have that $FSCE \leq SCE$. We can again define the firm-reallocation cost-efficiency-gain measurement:

$$FEG = \frac{SCE}{FSCE} \tag{23}$$

The values of FEG greater than one indicate potential gains from reallocation of firm-specific inputs across firms and/or division-specific inputs across divisions within firms. The shadow cost-efficiency measurement can be now decomposed as:

$$SCE = FEG \times FSCE \tag{24}$$

That is, FEG is the firm-reallocation cost-efficiency-gain measurement, and $FSCE$ is the firm-only cost-efficiency measurement (i.e., when divisions do not agree to reallocate inputs across firms). We can now also relate the all-reallocation and firm-only-reallocation measurements as follows:

$$FDEG = \frac{FSCE}{TSCE} \tag{25}$$

$FDEG$ is the firm-division-reallocation cost-efficiency-gain measurement (i.e., the potential gains if the divisions also agree to reallocate division-specific inputs across firms when the firms are reallocating only firm-specific inputs). Naturally, we have $FSCE \geq TSCE$ making $FDEG$ larger than unity. Finally, combining the previous equations (24) and (25), we obtain another useful decomposition of our shadow cost-efficiency measurement for the group of firms:

$$SCE = FEG \times FDEG \times TSCE \tag{26}$$

This equation relates the (shadow) cost efficiency when no reallocation is possible

SCE to the cost efficiency when reallocation is possible at all levels $TSCE$. The difference between both measurements is decomposed into two parts: first, cost-efficiency gain due to firms only, FEG; second, additional cost-efficiency gain due to divisions, $FDEG$. Finally, we highlight that all the cost-efficiency measurements in (26) are based on shadow prices.

The practical usefulness of the decomposition (26) is illustrated by the following example. Assume that $SCE > TSCE$, which indicates the possibility of a cost savings from reallocation. Assume further that $FEG = 1$. Therefore, there are no efficiency gains from reallocating firm-specific inputs across firms or from reallocating division-specific inputs within firms. At the same time, $FDEG > 1$, which means that there are efficiency gains from reallocating division-specific inputs across firms. One possible way to improve cost efficiency is through the acquisition of one or more divisions of one firm by another firm, followed by the reallocation of division-specific inputs between the old and the newly acquired divisions.

2.4 Extensions

Our previous considerations can be readily tailored to the needs of various empirical problems. We discuss here several extensions of our methodology. We also refer to our simulations and our empirical analysis in Section 3 to illustrate how to apply and extend our general definitions.

Price restriction. While our previous measurements are attractive because they do not rely on the price observation, they may be considered extreme for some practical cases. This can easily be solved by considering additional constraints for the prices in the linear programmings. These additional constraints are used to increase the realism of the computed shadow prices. In practice, partial price information can be known (Cherchye et al., 2016; Walheer, 2008c), or restrictions on the prices can be imposed, for example, absolute multiplier restrictions (Dyson and Thanassoulis, 1988; Pachkova, 2009); cone ratio restrictions (Charnes et al., 1990; Thompson et al., 1995; Talluri & Yoon, 2000); and assurance regions (Thompson et al., 1990; Allen et al., 1997). Cook and Seiford (2009) provide an extensive review of this literature.

Non-reallocatable inputs. Our previous measurements assume that all inputs are reallocatable. However, in reality, this assumption may not hold true for some inputs (e.g., buildings or other forms of physical capital). When some inputs are not reallocatable, it suffices to maintain the price difference for that particular input in the linear programmings. That is, the following conditions will not hold for these inputs: $\mathbf{p}^t = \mathbf{p}$ (firm-specific inputs), $\mathbf{w}_k^t = \mathbf{w}_k$, or $\mathbf{w}_k^t = \mathbf{w}$ (division-specific inputs). Similar modification can be adopted when only some divisions and/or firms agree to reallocate resources.

Transaction costs. In practice, reallocating inputs may be costly. Transaction costs and other barriers can be incorporated as restrictions on the ratio of prices. In our framework, if reallocation is possible and costless, the prices of two substitutable inputs are the same (i.e., their ratio is equal to one). With costly reallocation, this ratio is equal to some constants that are different from one. The lower the cost of reallocation, the closer this ratio is to unity. Kuosmanen et al. (2006) provide further discussion of this case.

Technical and allocative efficiency. An important aspect of the efficiency behaviour of firms is their ability to transform inputs into outputs, which is also referred to as 'technical efficiency'. As shown by Farrell (1957), technical and structural (or economic) efficiency are related by a simple equation:

$$CE = TE \times AE, \tag{27}$$

where AE represents the inefficiency due to nonoptimal allocation of inputs given the prices. Our previous cost-efficiency measurements can all be decomposed into these two parts. This decomposition, not considered here for the sake of compactness, may reveal interesting patterns; in particular, the decomposition of the total-reallocation cost-efficiency measurements. See, for example, Nesterenko and Zelenyuk (2007) and Mayer and Zelenyuk (2014) for further discussion about decomposing revenue efficiency when reallocation of firm-specific inputs is allowed.

Dynamic settings. In some cases, firms are observed during several different periods. In such cases, we may rely on the time-indexed ratio of our measurements to quantify the improvement/regression over time. In particular, when comparing two periods (say b and c), we obtain a ratio of (shadow) cost-efficiency change for the group: $\frac{SCE_c}{SCE_b}$. When $\frac{SCE_c}{SCE_b} > 1$, it means that less cost saving is possible at time c than at time b. That is, the cost-efficient behaviour has improved between the two periods of time. Conveniently, we can obtain a dynamic counterpart of our decomposition in (26):

$$\frac{SCE_c}{SCE_b} = \frac{FEG_c}{FEG_b} \times \frac{FDEG_c}{FDEG_b} \times \frac{TSCE_c}{TSCE_b}. \tag{28}$$

That is, the (shadow) cost-efficiency change is decomposed into the change in cost-efficiency gain due to firms only, the change in additional cost-efficiency gain due to divisions, and the change in shadow cost-efficiency when reallocation is possible at all levels. For example, refer to Walheer (2018c) for further discussion about cost-efficiency change for multidivisional firms over time. A similar procedure can be applied to all other concepts and equations, either individually or in combination with technical and allocative efficiency (see (27)).

3 Simulation and empirical application

To demonstrate the practical usefulness of our methodology, we propose both a simulation and an empirical application to the Chinese manufacturing sector.

3.1 Simulation

We illustrate our results with the following example. Consider three firms, each with two divisions. Following our theoretical framework, each division employs two types of inputs (division-specific and firm-specific inputs) to produce a single output normalised to one. This example could represent three high schools, each with two departments, producing teaching as their only output. Firm-specific input could refer to general management, while division-input could be mathematics and economics teachers. Table 1 presents the quantities of these inputs for each division and each firm.

Table 1: Data for the simulation

Firm 1	Firm 2	Firm 3
Firm-specific inputs		
6	5	4.5
Division-specific inputs		
2	4	3
5	3	7

We then consider various possibilities of reallocation across divisions and/or firms. For example, consider that teachers can freely reallocate. Assuming that they are perfectly substitutable, competition will lead to a uniform price for their services. In particular, the price of teaching increases in some firms and decreases in others.

We start by presenting the case in which reallocation between divisions within firms is considered. The results are presented in Table 2. Clearly, only Firm 1 can gain from reallocating the division-specific inputs across divisions, given that DEG^1 is 1.271 (i.e., greater than one). In Firms 2 and 3, inputs are allocated efficiently (both, DEG^2 and DEG^3 are equal to one). However, Firm 2 is cost inefficient, given that SCE^2 is 0.912 (i.e., less than one).

We then consider reallocation across firms. The results are shown in Table 3. We have SCE equal to unity, revealing cost-efficient behaviour at the group level. $TSCE$ is 0.905, which indicates cost inefficiency when all reallocation is possible. Likewise, $FSCE$ is 0.931, which demonstrates inefficiency when the divisions of different firms do not agree to reallocate division-specific inputs across firms. Moreover, we have $TSCE < FSCE < SCE$. This illustrates that once we allow for more possibilities of reallocation, only more inefficient behaviour can be identified. These inefficiencies are quantified in terms of efficiency gains by $\widehat{FEG}$, $FDEG$ and TEG. All these measurements are greater than one, which indicates gains from reallocation of firm-specific and division-specific inputs across firms. For example, the value of TEG equal to 1.105 demonstrates potential gains from moving from 'no reallocation' to 'all possible reallocations'. Finally, given that $FEG > FDEG$, there is more to be gained from reallocating firm-specific inputs across firms than division-specific inputs across divisions of different firms.

Table 2: Reallocation between divisions

No reallocation	SCE^1	SCE^2	SCE^3
	0.986	0.912	1
Reallocation	$DSCE^1$	$DSCE^2$	$DSCE^3$
	0.718	0.912	1
Efficiency gains	DEG^1	DEG^2	DEG^3
	1.271	1	1

Table 3: Reallocation between firms

SCE	1
$TSCE$	0.905
$FSCE$	0.931
TEG	1.105
FEG	1.074
$FDEG$	1.029

3.2 Empirical application

We now apply our framework to the high-technology manufacturing industry in China. Several earlier studies have analysed this industry from the efficiency perspective. For example, Zhang and Chen (2017) apply an additive network data envelopment analysis (DEA) to five branches of the Chinese high-technology industry: manufacture of medicines, manufacture of aircrafts and spacecraft, manufacture of electronic equipment and communication equipment, manufacture of computers and office equipment, and manufacture of medical equipment and measuring instrument. Kao and Hung (2007) study manufacturing companies in Taiwan focusing on one input only: management performance. Hung, He and Lu (2014) use DEA to evaluate the performance of business groups in Taiwan's semiconductor industry. The Taiwanese high-technology industry is also the focus of Tseng et al. (2009) and Liu and Wang (2008), who analyse the performance of large-size thin-film transistor liquid-crystal display (TFT-LCD) panel companies and semiconductor companies, respectively.

Our data were taken from the firm-level dataset issued by the National Bureau of

Statistics of China: the China Industry Survey (CIS). This dataset included a total of 30 manufacturing sectors from 1999 to 2007. Only three of these manufacturing sectors are considered highly technology intensive: (1) communication-equipment, computers, high and other electronic-equipment manufacturing; (2) meters and business-machinery manufacturing; and (3) pharmaceutical manufacturing. In this study, we restrict our attention to (1) communication-equipment, computers, high and other electronic-equipment manufacturing. In this industry, most firms have a multidivisional structure, which makes our framework directly applicable.

The Chinese manufacturing industry has several specific characteristics. The first is the coexistence of different types of ownership. Usually, registration information is used to define the ownership type as state, collective, private or foreign (e.g., Brandt et al., 2012; Wang & Wang, 2015; Berkowitz et al., 2017). Recently, it has been highlighted that the registration information may not reflect the true ownership status. In particular, firms registered as privately owned are controlled by the government, and would be more appropriately registered as state owned. Thus, Hsieh and Song (2015) suggest defining ownership type using the control rights rather than the registration information. We follow their method for our empirical study.

A second characteristic of the Chinese manufacturing industry is the importance of export-oriented production. Most firms produce goods for both the domestic market and for export, which also justifies their multidivisional form. We recognise that domestic and export outputs are generally produced using different technologies and production factors (or inputs). While previous research has acknowledged that outputs have been modified for foreign consumption before exporting (Bernard and Jensen, 1999), no corresponding distinction has been made on the technology side.

We consider a very simple setting with only two inputs: we assume that every firm uses capital (K) and labour (L) to produce output (Y). This setting, while simple, is the most widely used in empirical macroeconomics, and dates back to Solow (1956); Arrow et al. (1961); Christensen et al. (1973); Berndt & Christensen (1973). Labour is measured by the average annual number of employed people; capital is measured by the value of fixed assets net of depreciation; and output is measured by the value added. While no correction is needed for our measure of labour, which is measured in real terms, correction is needed for output and capital, which are measured in value terms. Although the CIS data do not provide a deflator, industrial output is given in both current and constant prices, which can be used to estimate the inflation rate.

We begin by demonstrating the importance of the different types of ownership in Table 4. As seen in the table, foreign ownership is dominant throughout. The share of private ownership increased, while the share of state and collective ownership decreased during the study period. In addition, the number of state-owned and collectively owned firms also decreased in absolute terms.

Table 4: Descriptive statistics

Year	**State**	**Private**	**Foreign**	**Collective**	**Y**	**L**	**K**
1999	744	608	1,740	484	133,452,510	1,721,318	132,350,103
2000	716	764	1,899	434	170,252,183	1,847,818	147,614,833
2001	625	1,014	1,889	349	170,312,604	1,708,682	158,306,969
2002	613	1,326	2,255	346	252,172,642	2,090,806	230,167,270
2003	548	1,688	2,727	315	334,487,295	2,603,590	24,9641,109
2004	373	1,236	2,300	185	442,325,726	3,147,578	238,093,215
2005	493	3,106	4,502	310	564,670,392	4,238,513	387,543,094
2006	462	3,513	4,848	303	697,479,962	4,939,030	460,356,019
2007	411	4,177	5,518	292	755,978,251	5,600,050	606,365,453

We consider that capital and labour enter differently into the output-production processes. In particular, we model capital as an input used to produce both outputs simultaneously. That is, we consider capital as a firm-specific input. For example, we may think of factories and machines used to produce both domestic and export outputs. The use of capital also generates interdependencies between the two production processes, and thus gives rise to economies of scale and of scope in the production process, forming a prime economic motivation for firms to consider the production of both domestic and export outputs. We then model labour as an input allocated to each output-production process. That is, we consider labour as a division-specific output. In a sense, this modelling of labour recognises that employees cannot be used to produce simultaneously both domestic and export outputs, but, in contrast, it places a constraint on the output-production processes. Thus, labour also generates interdependencies between the output-production processes because they compete for the limited quantity of this input. Unfortunately, data for labour are only provided at the aggregate level in the CIS dataset. We suggest allocating labour by considering both the quantity and the quality of the outputs. That is, we acknowledge that more employees are needed when the quantity and the quality of the outputs increase. Thus, we allocate the number of employees according to the total revenue of each type of output. (These data are available in the CIS dataset.) Indeed, greater quantity implies a higher total revenue, and higher quality is generally more expensive, which also results in a higher total revenue.

We then investigate potential reallocation of the inputs between divisions (labour) and between firms (labour and capital). The results are presented in Tables 5, 6 and 7. Table 5 is based on the setting outlined in Section 2.1. It presents the shadow cost-efficiency measurement, SCE, for each group of firms assuming that no reallocation of inputs is possible. According to this measurement, in all considered years, firms with foreign ownership are the most efficient, followed by privately owned firms. We also see a slight increase in cost efficiency over time for all groups of firms. In Table 6, we examine whether cost efficiency can be improved by reallocation of labour across divisions. That is, we examine the division-reallocation cost-efficiency gain, DEG

(as defined in Section 2.2). For all groups, we identify the potential efficiency gains from reallocating labour across divisions. These gains are largest for the state-owned firms and smallest for the firms with foreign ownership. Nevertheless, for all cases, we observe an improvement in cost-efficiency over time.

Table 5: Cost efficiency without reallocation (*SCE*)

Year	**State**	**Private**	**Foreign**	**Collective**
1999	0.69	0.78	0.81	0.70
2000	0.70	0.77	0.83	0.71
2001	0.69	0.79	0.82	0.72
2002	0.71	0.80	0.84	0.72
2003	0.70	0.81	0.85	0.72
2004	0.71	0.81	0.84	0.70
2005	0.70	0.82	0.84	0.71
2006	0.71	0.82	0.84	0.72
2007	0.72	0.81	0.83	0.73

Table 6: Reallocation between divisions: average cost-efficiency gains (*DEG*)

Year	**State**	**Private**	**Foreign**	**Collective**
1999	1.25	1.19	1.18	1.20
2000	1.24	1.17	1.15	1.20
2001	1.25	1.16	1.15	1.19
2002	1.23	1.14	1.12	1.18
2003	1.24	1.12	1.11	1.18
2004	1.20	1.11	1.12	1.18
2005	1.19	1.11	1.10	1.17
2006	1.18	1.11	1.09	1.16
2007	1.17	1.10	1.08	1.16

We then apply the setting introduced in Section 2.3 to analyse the firm-reallocation cost efficiency behaviour. The results are presented in Table 7. Several interesting facts emerge. First, the gains from moving from 'no reallocation' to 'all possible reallocation', measured by *TEG*, are similar for different groups. Second, collectively owned firms can gain the most (in relative terms) from the reallocation of capital, given that their *FEG* is the highest. Accordingly, for collectively owned firms, the gains from reallocating labour, measured by $FDEG$, are the lowest. Finally, as with the cost-efficiency gains from reallocation at the division level, for the firm-level measurements we observe an improvement in efficiency behaviour over time. This improvement results from improvements in allocation of capital in all groups of firms (as indicated by changes in FEG). In the case of privately and foreign-owned firms, there is also an improvement in allocation of labour (as indicated by changes in $FDEG$).

At a general level, we believe that this empirical analysis convincingly demonstrates the usefulness of our methodology for multidivisional firms when resource reallocation is allowed. In particular, these results can be useful for both managers and policymakers. For example, managers of state-owned and collectively owned firms may consider measures that would improve allocation of resources within their firms. Policymakers may use our methodology to evaluate the effects on firm efficiency of removing internal barriers to migration.

4 Conclusion

We consider the reallocation of various types of inputs across divisions and firms. Our approach has several desirable properties. In particular, it does not rely on information about technology and prices. Our methodology is highly versatile and can be adjusted to account for various situations. For example, it can incorporate transaction costs as well as additional information about prices, and it can be applied in a dynamic setting.

We illustrate the practicality of our approach with a simulation and an application to the Chinese manufacturing industry in the period 1999-2007. Although our application is illustrative, it identifies some interesting patterns. For example, we observe a general improvement in efficiency over time. However, state-owned and collectively

Table 7: Reallocation between firms

Year	**State**	**Private**	**Foreign**	**Collective**
		TEG		
1999	1.31	1.37	1.35	1.30
2000	1.30	1.35	1.34	1.29
2001	1.29	1.33	1.32	1.30
2002	1.28	1.30	1.29	1.28
2003	1.27	1.28	1.28	1.27
2004	1.26	1.26	1.27	1.26
2005	1.24	1.24	1.23	1.25
2006	1.23	1.22	1.22	1.24
2007	1.25	1.18	1.19	1.22
		FEG		
1999	1.19	1.22	1.18	1.25
2000	1.18	1.21	1.17	1.24
2001	1.17	1.20	1.17	1.25
2002	1.16	1.19	1.16	1.23
2003	1.17	1.18	1.15	1.22
2004	1.17	1.18	1.14	1.21
2005	1.16	1.17	1.13	1.20
2006	1.15	1.16	1.12	1.19
2007	1.14	1.14	1.11	1.18
		FDEG		
1999	1.10	1.12	1.14	1.04
2000	1.10	1.12	1.15	1.04
2001	1.10	1.11	1.13	1.04
2002	1.10	1.09	1.11	1.04
2003	1.09	1.08	1.11	1.04
2004	1.08	1.07	1.11	1.04
2005	1.07	1.06	1.09	1.04
2006	1.07	1.05	1.09	1.04
2007	1.10	1.04	1.07	1.03

owned firms remain less cost efficient than privately owned and foreign-owned firms. In addition, we find that all types of firms can gain from the reallocation of physical capital more than they can gain from the reallocation of labour.

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