Estimation of confidence regions for random excursion sets with application to large-scale ice-sheet simulations

Kevin Bulthuis[1, 2*], Maarten Arnst[1], Frank Pattyn[2]

[1]Aerospace and Mechanical Engineering
Université de Liège
Allée de la Découverte 9, Quartier Polytech 1, 4000 Liège, Belgium
kevin.bulthuis@uliege.be, maarten.arnst@uliege.be

[2]Laboratory of Glaciology
Université Libre de Bruxelles
Av. F.D. Roosevelt 50, 1050 Brussels, Belgium
fpattyn@ulb.ac.be

ABSTRACT

In many applications, including in evaluations of failure regions and in geophysical hazard assessment, we are interested in evaluating excursion sets, that is, regions in the spatial domain where the response function exceeds some critical value. Determining such excursion sets in the presence of uncertainties in a model is an interesting problem connected with the theory of random sets.

A first issue connected with random excursion sets is in defining confidence regions that can properly represent the uncertainty in the excursion sets . Here, we adopt a definition based on a generalization of the concept of confidence regions based on previous works by [Bolin, 2015] and [French, 2015]. An outer or inner confidence region is defined as a region that contains or is contained in the excursion set with a given level of probability, respectively.

Such confidence regions are approximated numerically as optimal subsets within a parametric family of subsets with the appropriate coverage probability, which provides nested approximations for the confidence regions. A second issue, related to this numerical approximation of confidence regions, stems from the numerical approximation of the coverage probability, which may prove challenging for computationally intensive models and small probability levels. Here, we explore methods based on a hybrid surrogate-based approach [Li, 2010] and subset simulation [Au, 2001] to evaluate the coverage probability.

We apply this methodology to the evaluation of confidence regions for the retreat of grounded ice in large-scale simulation of the Antarctic ice sheet subject to parametric uncertainties.

References

[1] [Au, 2001] Au, S.-K. and Beck, J. L. (2001). Estimation of small failure probabilities in high dimensions by subset simulation. Probabilistic Engineering Mechanics, 16(4):263–277.
[2] [Bolin, 2015] Bolin, D. and Lindgren, F. (2015). Excursion and contour uncertainty regions for latent gaussian models. Journal of the Royal Statistical Society: Series B (Statistical Methodology), 77(1):85–106.
[3] [French, 2015] French, J. P. and Hoeting, J. A. (2015). Credible regions for exceedance sets of geostatistical data. Environmetrics, 27(1):4–14.
[4] [Li, 2010] Li, J. and Xiu, D. (2010). Evaluation of failure probability via surrogate models. Journal of Computational Physics, 229(23):8966–8980.