CONFINEMENT AND CUT-OFF: A MODEL FOR THE PION QUARK DISTRIBUTION FUNCTION

P. STASSART, F. BISSEY, J.-R. CUDELL, J. CUGNON, M. JAMINON AND J.-P. LANSBERG

Université de Liège, Département de Physique, Institut de Physique B.5, Sart Tilman, B-4000 LIEGE 1, Belgium

The pion structure function is investigated in a simple pseudo-scalar coupling model of pion and constituent quark fields. The imaginary part of the forward Compton scattering amplitude is evaluated. We show that the introduction of non-perturbative effects, linked through a cut-off to the size of the pion, allows the reproduction of important features of the pion quark distribution function.

While perturbative QCD is consistent with the Q^2 evolution of the structure functions provided by deep inelastic scattering (DIS) experiments[1], it is not able to predict the structure function at an initial value from which the Q^2 dependence can be evaluated as these functions depend on non-perturbative effects, like confinement or chiral symmetry breaking.

Phenomenological quark models based on chiral symmetry breaking properties, successful in describing low-energy properties of hadrons, are expected to help us understand the connection between DIS data and non-perturbative aspects. However, regularization procedures needed to connect the hadron to the quark loop vary from model to model and their choice has an impact on the structure functions that can be extracted [2].

In this talk, we try to avoid this problem by considering a simple model of the pion, in which the $q\overline{q}\pi$ vertex is described by a simple pseudoscalar coupling, and where possible singularities are buried in the mass and coupling parameters, so that the diagrams needed to calculate the pion structure function yield a finite value for the imaginary part of the amplitude. More details are to be found in Ref[3]. The interaction Lagrangian reads:

$$L_{int} = ig\left(\overline{\psi}\vec{\tau}\gamma_5\psi\right).\vec{\pi} \tag{1}$$

where g is the quark hadron coupling. The relevant diagrams, up to first order in the fine structure constant α_s and to second order in g, can be classified as "box" diagrams,where one quark connects the photon lines and "cross" diagrams,where all quark lines stand between photon and hadron vertices.The imaginary part of the amplitudes from each diagram i reads

$$\Im T_{i\mu\nu} = Cg^2 \int d^4k t_{i\mu\nu} D_1 D_2 D_3 D_4 \tag{2}$$

where $t_{i\mu\nu}$ is the fermionic trace other the loop i and D_{1-4} are the fermion propagators or the cuts on each quark line in the loop. Constant C accounts for flavour, charge and momentum integration factors. Summing all diagrams we can then identify the structure functions W_1 and W_2

$$W_{\mu\nu} = \frac{1}{2\pi}\Im T_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1 + \left(p_\mu - q_\mu \frac{p.q}{q^2}\right)\left(p_\nu - q_\nu \frac{p.q}{q^2}\right) W_2 \tag{3}$$

At this point, to interpret the pion as a collection of partons with a probability distribution, we should need that the cross diagrams be suppressed by a power of Q^2 compared to the box diagrams. However this is not what we get. In the small m_π large Q^2 limit, these contributions are

$$W_1^{box} = \frac{5g^2}{24\pi^2} \ell n \left[\frac{2(1-x)\nu}{mq^2} - 1\right]; W_1^{cross} = \frac{5g^2}{24\pi^2} \tag{4}$$

A reason for this is that the pion we use has not a finite size. Imposing it by requiring that the relative four-momentum squared of the quarks inside the pion be limited to a maximum value Λ^2, one limits the momentum transfer in the case of the cross diagrams, as no line joins the photon quark vertices without going through a cut-off limited quark-pion vertex, whereas the momentum may be transferred without such a limitation in the box diagrams where one quark line joins the quark-photon vertices directly. The cross diagrams contribute now as higher twists, and the remaining box contribution can be interpreted in terms of parton distributions.

We fix g by imposing that there be only two constituent quarks in the pion, i.e. $\int_0^1 v(x)dx = \frac{1}{2}$ where $v(x)$ is the valence quark distribution.

We can then calculate the momentum fraction carried by the quarks

$$2\langle x \rangle = 4\int_0^1 xv(x)dx = \frac{\int_0^1 F_2(x)dx}{\int_0^1 F_1(x)dx} \tag{5}$$

as the model yields naturally the Callan Gross relation $F_2 = 2xF_1$.

In the parton model, as $Q^2 \to \infty$, $\langle 2x \rangle$ should be equal to one. This is what we get if we do not impose that the pions have a finite size, i.e. for $\Lambda \to \infty$. However, when we use a physical pion, the results, shown in Fig. 1, display a plateau at $Q^2 > 2$ GeV^2, where the momentum fraction

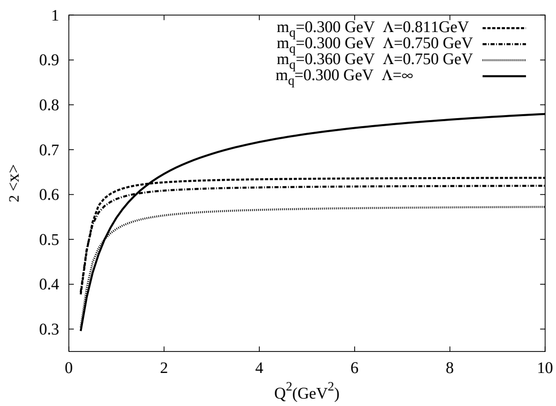

Figure 1. Momentum fraction carried by the quarks inside the neutral pion (Eq.5), as a function of Q^2 , for the parameter values displayed on top.

saturates at a value which stays below 0.55 and 0.65 for conservative values of m = 300 MeV, Λ = 800 MeV ; g = 3.8 is in very close agreement with the value obtained[4] in a NJL model for the same parameter values, which in that case correspond to setting the correct value for f_π.

We have displayed how a cut-off is needed to represent the pion as a physical particle, from which structure functions can be deduced. When quarks behave as free particles, the sum rule holds, whereas in the physical pion case, at least one of the quarks remains off-shell. This implies that the quark momentum transfer is reduced by the imposition of the cut-off at the vertex, i.e., it is suppressed by the non-perturbative effects which the cut-off stands for. Higher twist terms disappear then for $Q^2 > 2$ GeV^2. The quark momentum fraction is reduced somewhat more than in the case of other hadrons. This is presumably due to the Goldstone nature of the pion, as in the $m \to 0$ limit the momentum sum rule is recovered.

References

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