Kapitza stabilization of a repulsive Bose-Einstein condensate in an oscillating optical lattice

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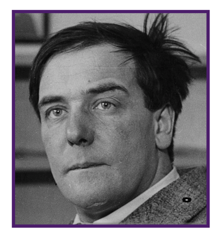
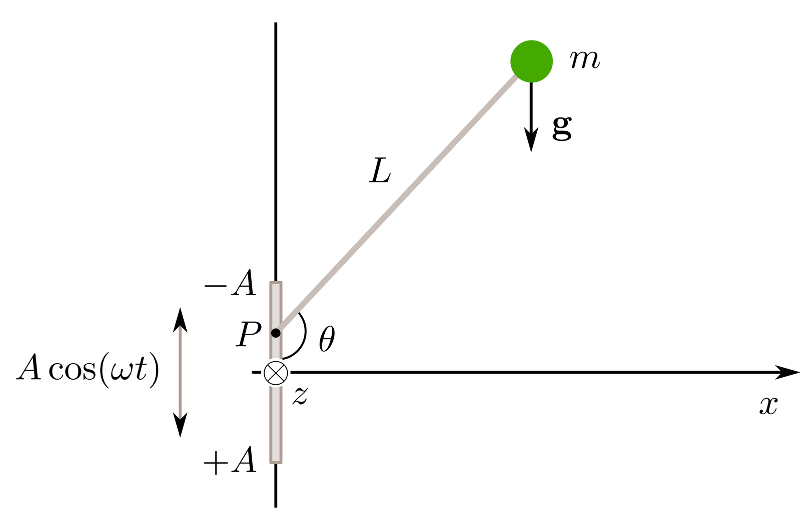
Inverted driven pendulum

Pendulum submitted to an oscillating force of frequency ω with zero average value:



Piotr Kapitza (1894-1984)



Natural oscillation frequency: $\omega_0 = \sqrt{\frac{g}{L}}$

Reduced acceleration: $\varepsilon = \frac{A\omega^2}{g} = \frac{A}{L}\left(\frac{\omega}{\omega_0}\right)^2$

Moment of inertia: $I = mL^2$

Gravitational potential energy: $U_L = mgL = I\omega_0^2$

Hamiltonian: $H = \frac{p_\theta^2}{2I} - U_L \cos\theta\,(1 + \varepsilon\cos(\omega t))$

Effective potential and stabilization

→ timescale separation when the pivot point vibrates with a small amplitude ($A \ll L$) and with a large frequency ($\omega \gg \omega_0$):

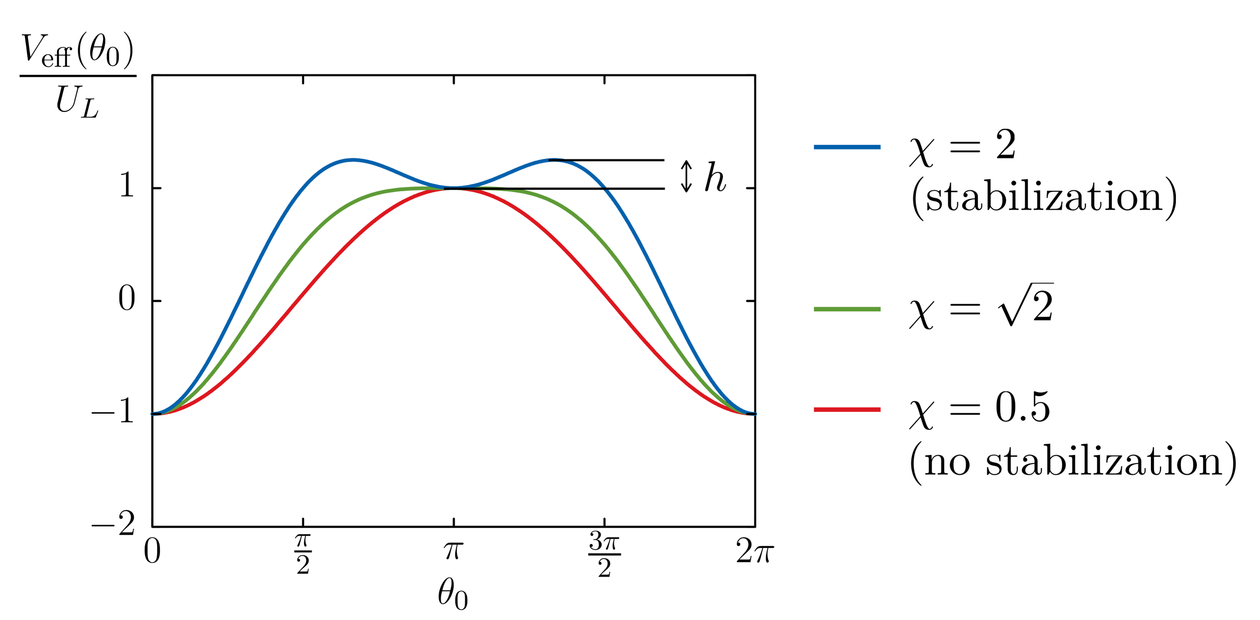
$$\theta(t) = \theta_0(t) + \xi(t) \quad \text{with} \quad |\xi(t)| \ll 2\pi$$

$\theta_0(t)$: slowly varying component

$\xi(t) = \frac{A}{L}\sin\theta_0(t)\cos(\omega t)$: rapidly varying component

→ Euler-Lagrange equation for the slow component with the effective potential:

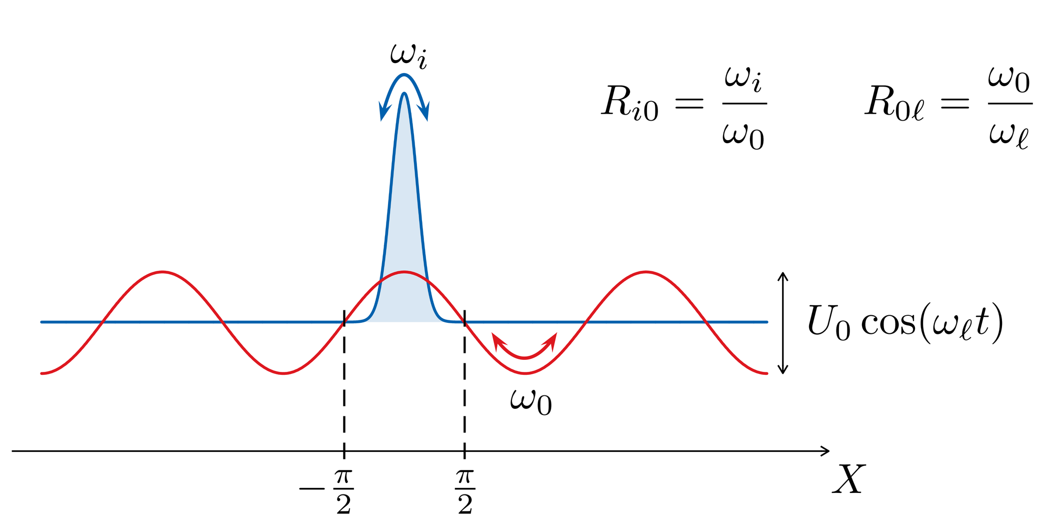
$$V_{\text{eff}}(\theta_0) = \frac{\overline{F^2}}{2m\omega^2} = -U_L\left(\cos\theta_0 - \frac{\chi^2}{4}\sin^2\theta_0\right) \text{ with } \chi = \frac{A}{L}\frac{\omega}{\omega_0} = \varepsilon\frac{\omega_0}{\omega}$$



BEC in oscillating optical lattice

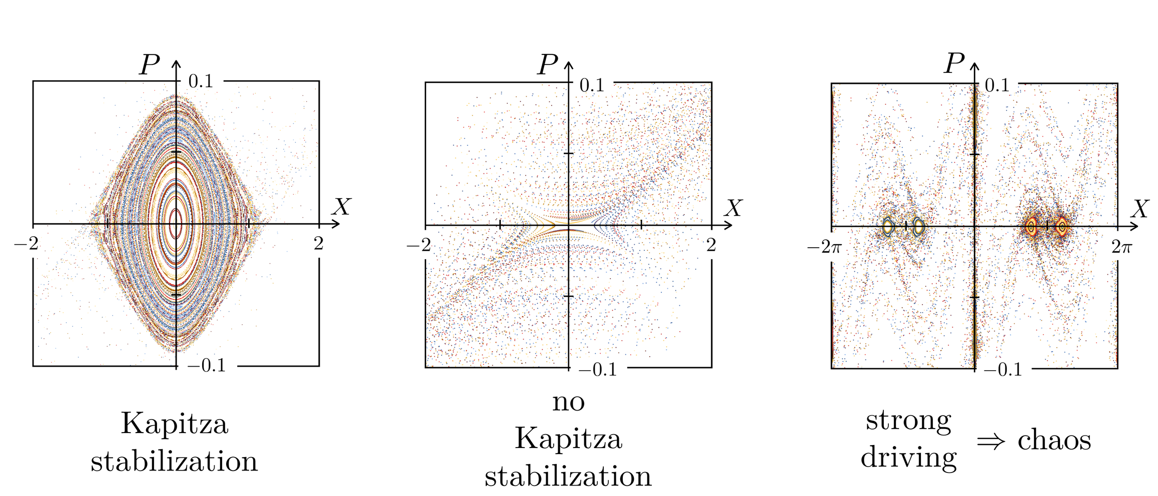
Idea: repulsive atom-atom interactions give rise to a mean-field potential with an unstable fixed point that can be transformed into a stable fixed point by subjecting the BEC to an optical lattice with oscillating amplitude (oscillating force)

$R_{i0} = \frac{\omega_i}{\omega_0} \qquad R_{0\ell} = \frac{\omega_0}{\omega_\ell}$



Classical dynamics

Hamiltonian: $\frac{H}{U_0} = 8P^2 \overbrace{- R_{i0}^2 X^2}^{\text{inverted harmonic potential}} + \underbrace{\cos X \cos(2\pi T)}_{\text{driving}}$



Poincaré sections for the frequency ratios $R_{i0} = 0.075$ and $R_{0\ell} = 0.45$ (left), $R_{i0} = 0.15$ and $R_{0\ell} = 0.2$ (middle), and $R_{i0} = 0.02$ and $R_{0\ell} = 0.7$ (right).

Kapitza stabilization of a repulsive BEC

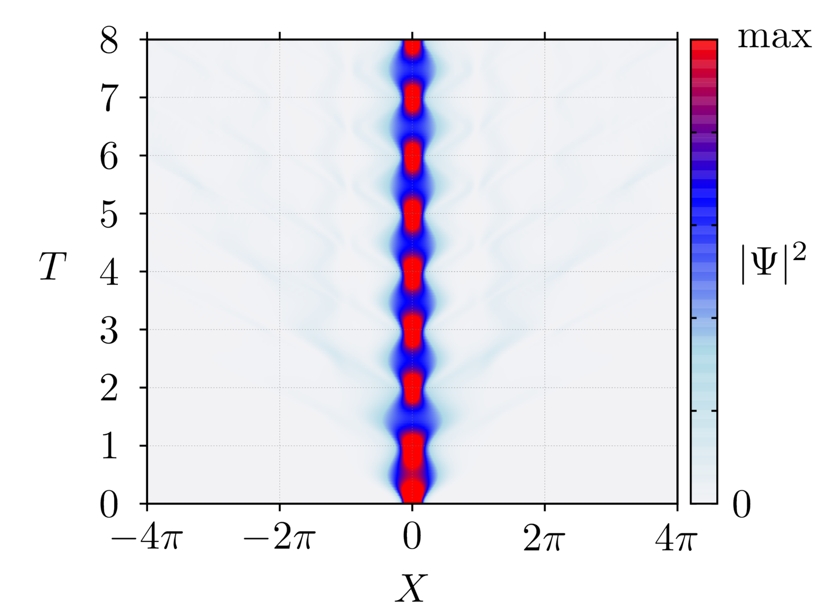
Quantum evolution with GPE

In reduced variables, Gross-Pitaevskii equation reads

$$i\partial_T\Psi = 4\pi R_{0\ell}\left(-\hbar_{\text{eff}}\partial_X^2 + \frac{\cos(2\pi T)\cos X}{8\hbar_{\text{eff}}} + \frac{\bar{g}|\Psi|^2}{2}\right)\Psi$$

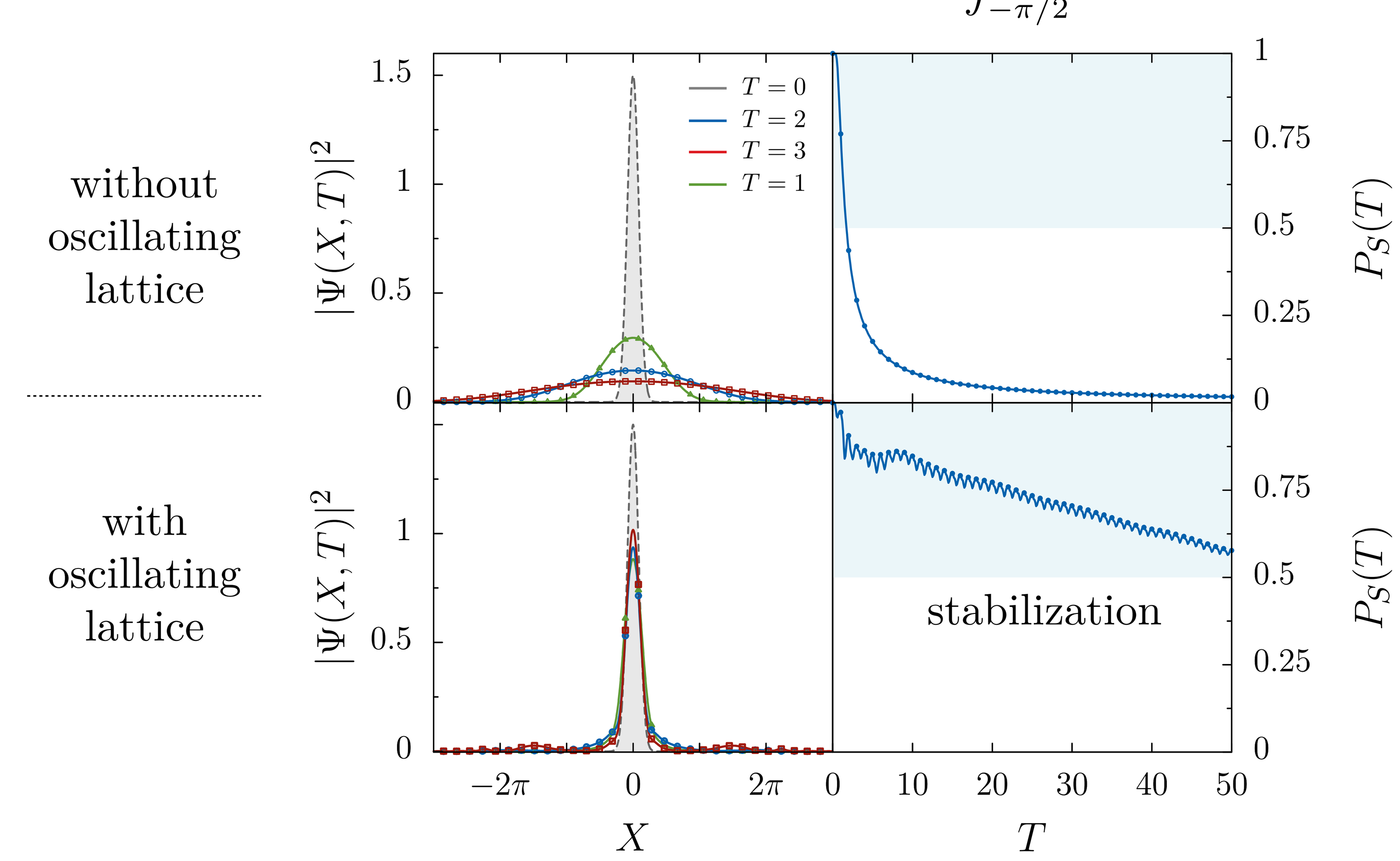
X: position in units of the lattice spacing
T: time in units of the driving period
$R_{0\ell} = \omega_0/\omega_\ell$: frequency ratio
$\hbar_{\text{eff}} \propto U_0^{-1/2}$: effective Planck constant
$\bar{g}$: atom-atom interaction strength

Probability density evolution

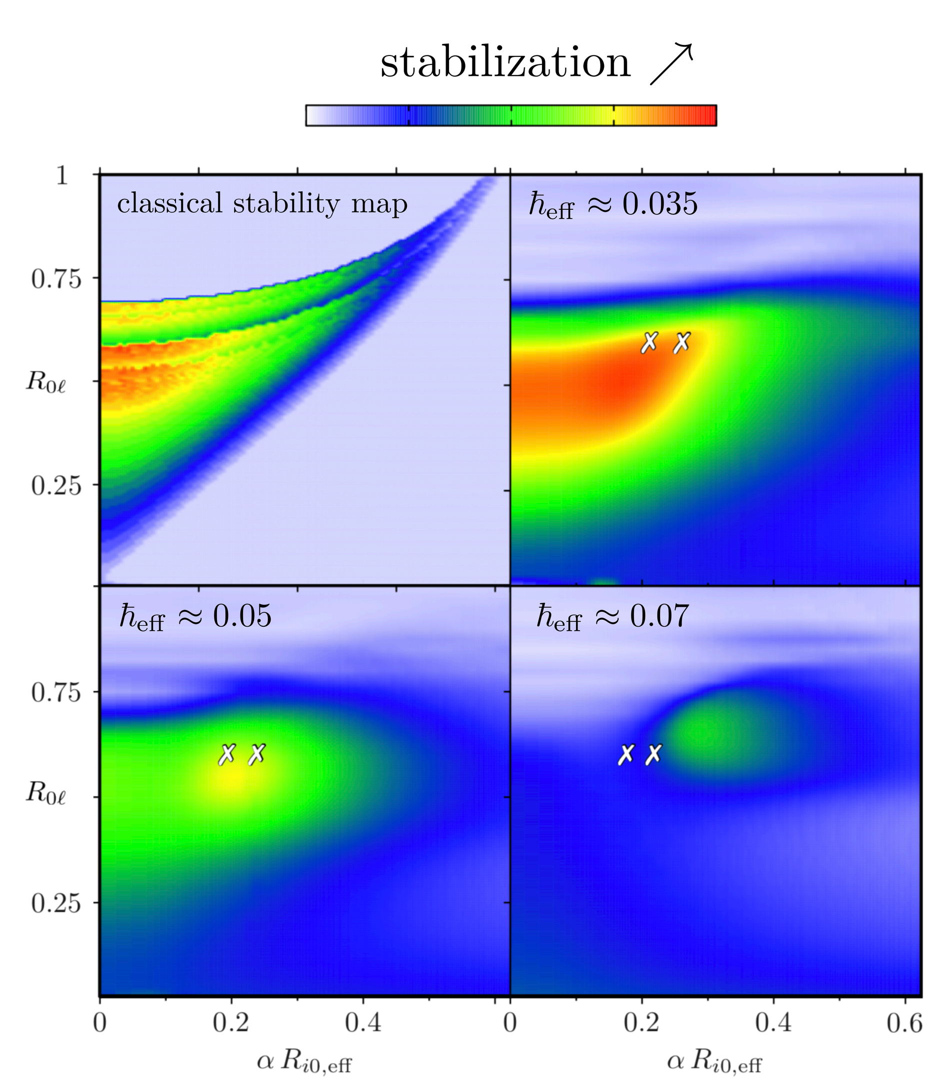


Probability density as a function of position and time for $R_{0\ell} = 0.6$, $\bar{g} = 0.6$, $\hbar_{\text{eff}} \approx 0.035$ and an initial Gaussian wave packet with $R_{i0,\text{eff}} \approx 1.34$.

Probability inside the initial "well": $P_S(T) = \int_{-\pi/2}^{+\pi/2} |\Psi(X,T)|^2 dX$



Stability diagram



Stabilized fraction of the condensate as quantified by $\overline{P_S} = \frac{1}{10}\int_{3T_{\text{sp}}}^{3T_{\text{sp}}+10} P_S(T)dT$ where T_{sp} is a characteristic spreading time of the initial wave packet in the absence of driving ($U_0 = 0$) determined by $P_S^{\varepsilon=0}(T_{\text{sp}}) = 0.75$, as a function of $R_{0\ell}$ and $R_{i0,\text{eff}} = \frac{\omega_{i,\text{eff}}}{\omega_0}$ (multiplied by a scaling coefficient $\alpha = 0.2$) with $\frac{\omega_{i,\text{eff}}}{\omega_0} \propto \sqrt{\frac{\hbar_{\text{eff}}\bar{g}}{\tilde{\sigma}^3}}$ for an initial Gaussian wave packet of width $\tilde{\sigma}$.

References

See J. Martin, B. Georgeot, D. Guéry-Odelin and D. L. Shepelyansky, Phys. Rev. A **97**, 023607 (2018) & arXiv:1709.07792 and references therein