Entanglement robustness against particle loss in multiqubit systems

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When some of the parties of a multipartite entangled pure state are lost, the question arises whether the residual mixed state is also entangled, in which case the initial entangled pure state is said to be robust against particle loss. Here, we investigate this entanglement robustness for N-qubit pure states. We identify *exhaustively* all entangled states that are fragile, i.e., not robust, with respect to the loss of any single qubit of the system. We also study the entanglement robustness properties of symmetric states and put these properties in the perspective of the classification of states with respect to stochastic local operations assisted with classic communication (SLOCC classification).

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Fragility (Robustness) with respect to the loss of a subset of an N-partite system

Definition [1] :

An N-partite entangled state $|\psi\rangle$ is *fragile* (*robust*) with respect to the loss of a given subset $\mathcal{S}$ of the particles if

$$\rho_{\neg\mathcal{S}}(\psi) \equiv \mathrm{Tr}_{\mathcal{S}}(|\psi\rangle\langle\psi|)$$

is separable (entangled).

Fragility with respect to the loss of a single qubit

Theorem :

An entangled N-qubit state $|\psi\rangle$ is fragile with respect to the loss of *any* qubit that is part of a given subset $\mathcal{A}$ of the qubits iff

$$|\psi\rangle = \sqrt{p}\,|e_1,\dots,e_N\rangle + \sqrt{1-p}\,|e'_1,\dots,e'_N\rangle,$$

where

- $0 < p < 1$,
- $|e_i\rangle$, $|e'_i\rangle$ $(i = 1,\dots,N)$ are normalized single-qubit states with $|e'_i\rangle \perp |e_i\rangle, \forall i \in \mathcal{A}$ and, if $\#\mathcal{A} = 1$, $|e'_j\rangle \neq |e_j\rangle$ for at least one qubit $j \notin \mathcal{A}$.

Corollary 1 :

An entangled N-qubit state $|\psi\rangle$ is fragile with respect to the loss of *any* qubit iff

$$|\psi\rangle = a|e_1,\dots,e_N\rangle + b\,|e_1^\perp,\dots,e_N^\perp\rangle,$$

where, $\forall i$, $|e_i\rangle \perp |e_i^\perp\rangle$ and $a,b \in \mathbb{C} : |a|^2 + |b|^2 = 1$.

Theorem :

The entangled N-qubit states that are fragile with respect to the loss of any qubit all belong to the *same* SLOCC class, namely that of the $|\mathrm{GHZ}_N\rangle$ state

$$|\mathrm{GHZ}_N\rangle \equiv (|0,\dots,0\rangle + |1,\dots,1\rangle)/\sqrt{2}$$

$\longrightarrow$ Not anymore true when considering fragility with respect to the loss of *more* than one qubit: Fragile states are NOT restricted in a single SLOCC class in this case.

Fragility in the symmetric subspace

Corollary 2 :

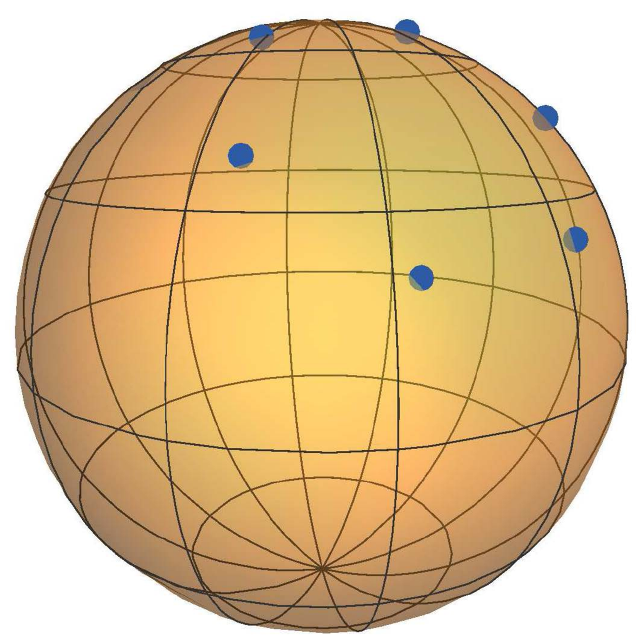
A symmetric entangled N-qubit state $|\psi_S\rangle$ is fragile with respect to the loss of *any* qubit iff

$$|\psi_S\rangle = a|e,\dots,e\rangle + b\,|e^\perp,\dots,e^\perp\rangle,$$

where $|e\rangle \perp |e^\perp\rangle$ and $a,b \in \mathbb{C} : |a|^2 + |b|^2 = 1$.

Theorem :

A symmetric entangled N-qubit state $|\psi_S\rangle$ is fragile with respect to the loss of *any* qubit iff its Majorana points are at the vertices of a regular N-sided polygon in whichever plane intersecting the Bloch sphere.



Robustness in the symmetric Dicke state SLOCC classes

k-excitation N-qubit Dicke states (k = 0, …, N)

$$|D_N^{(k)}\rangle = \binom{N}{k}^{-\frac{1}{2}} \sum_\pi |0,\dots,0,1,\dots,1\rangle,$$

where the multiqubit states in the sum contain k qubits in state $|1\rangle$, and π denotes all permutations of the qubits leading to different terms in the sum. For $k = 0,\dots,\lfloor N/2\rfloor$, the Dicke states $|D_N^{(k)}\rangle$ are SLOCC inequivalent and define in the symmetric subspace the SLOCC classes $\mathcal{D}_{N-k,k}$ [2].

Theorem [3] :

All entangled Dicke states $|D_N^{(k)}\rangle$ are robust with respect to the loss of any number t of qubits $(t \leq N-2)$.

Theorem and conjecture :

This robustness property holds for all symmetric states in the Dicke state SLOCC class $\mathcal{D}_{N-1,1}$. We conjecture based on strong numerical evidence that the same holds for all symmetric states in *all* other Dicke state SLOCC classes $\mathcal{D}_{N-k,k}$ $(k = 2,\dots,\lfloor N/2\rfloor)$.

References

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