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$\mathcal{H}_\infty$ optimization of an enhanced force feedback controller for mitigation of nonlinear vibrations

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Abstract

In this paper, a nonlinear active damping strategy based on force feedback is proposed, where only the modal stiffness ratio of the system when the actuator is installed and removed is required to design an effective controller. Closed-form expressions are derived using the $\mathscr{H}_\infty$ optimization criterion wherein the optimal feedback gains are achieved to minimize the maximal response of the structure. The resonant vibration of a Duffing oscillator is considered to illustrate the proposed tuning of the optimal control gain and the harmonic balance method is employed to approximate the analytical solutions. It is shown that the amplitude of the resonance peaks does not change substantially when the weight of the nonlinearity increases, which means that the response of the coupled system is almost proportional to the forcing amplitude, as it would be the case for a linear system.

Keywords: Force feedback, $\mathscr{H}_\infty$ optimization, Nonlinear vibrations, Harmonic balance method, Closed-form.

1 Motivation

Controlling nonlinear structural vibrations is becoming increasingly important for a various range of engineering applications such as aerospace, medicine and robotics, wherein lightweight materials are considered in the construction of systems in order to meet the increasing demand for fuel efficiency or smaller actuators [1], [2]. However, this will naturally lead to the fact that the resonances are lightly damped and to the presence of some geometric nonlinearities from large deformations. The resulting unwanted nonlinear vibrations thus become the main concern, limiting the success of these applications. One key characteristic of nonlinear vibrations is their frequency-energy dependence which in other words means that the frequency of the nonlinear oscillations depends intrinsically on the motion amplitudes [3]. As a consequence, the mature linear damping-enhanced approaches based on the superposition principle such as tuned mass dampers and piezoelectric shunting [4]–[6] (passive solutions) or direct velocity feedback, integral acceleration and force feedback controllers (active solutions) [7] are no longer remaining effective in the presence of strong nonlinearities.

This paper investigates the potential of a nonlinear enhanced force feedback (EFF) controller for the mitigation of nonlinear vibrations. One important issue with such a control system is how the physical properties of the actuator, for example its stiffness and the gain of the feedback loop, should be set to optimally control the vibrations of the host structure. An $\mathscr{H}_\infty$ optimization criterion is used to derive the optimal feedback gains of the EFF controller, and accordingly the optimal gain is set to minimize the maximum steady state response of the structure. The objective of the study is to understand its working principle and validate the control efficiency through a harmonic balance analysis.

2 Mathematical model and $\mathscr{H}_\infty$ optimization

The system under investigation is shown in Figure 1. The structure is modeled as an undamped single degree of freedom (dof) system, defined through a lumped mass m_1, a linear spring k_1 and a cubic spring k_3. It is excited by a harmonic force $F = F_d \cos(\omega t)$. A force actuator with its stiffness k_2 is placed in parallel to the passive mount and a force sensor is mounted between the actuator and the structure to capture the force delivered by the actuator to the mass. The control loop is implemented by feeding the force sensor output through a novel nonlinear controller $C(F_s)$ to drive the actuator.

The governing equations of the system read:

$$m_1\ddot{x} + k_1 x + k_3 x^3 = F_d \cos(\omega t) + F_s \tag{1}$$

$$F_s = -C(F_s) - k_2 x \tag{2}$$

In a manner analogous to the nonlinear tuned mass damper proposed in [3], the nonlinear controller $C(\cdot)$ is formed as $C(\cdot) = g_1 \int(\cdot)dt + g_2 \iint(\cdot)dt + g_3\left(\iint(\cdot)dt\right)^3$ in order to realize a virtual nonlinear tuned mass damper using force sensors. The $\mathscr{H}_\infty$ optimization criterion is employed to optimize the controller $C(\cdot)$ aiming to minimize the maximum magnitude of the frequency response of the system under consideration. In this context, the magnitude of the driving point receptance $|x/F|$ is taken as the performance index.

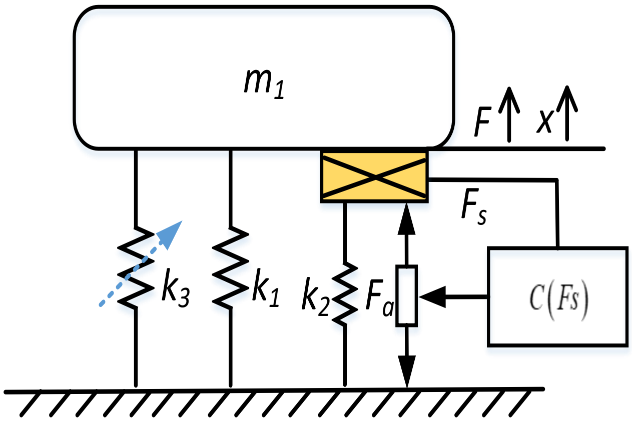

Figure 1 The scheme of the system under consideration

Substituting the proposed form of $C(\cdot)$ into Eqs. (1) and (2), and normalizing the resulting equations, yields:

$$y_1'' + y_1 + \alpha y_1^3 - y_2'' = \cos(\Omega\tau) \tag{3}$$

$$y_2'' + g_{11}y_2' + g_{22}y_2 + \alpha\beta y_2^3 + \mu y_1 = 0 \tag{4}$$

where $g_{11} = g_1/\omega_1$, $g_{22} = g_2/\omega_1^2$, $\alpha = k_3 F_d^2/k_1^3$, $\beta = g_3/k_3$, $y_1 = x_1/F_d$, $y_2 = \iint F_s dt / m_1 F_d$, $\mu = k_2/k_1$, $\omega_1^2 = k_1/m_1$, $\tau = \omega_1 t$ and $\Omega = \omega/\omega_1$. It is shown that the forcing amplitude appears only in the expression of the nonlinear coefficients.

The next objective is to determine the optimal control parameters namely g_{11}, g_{22} and β in terms of the stiffness ratio μ and the primary nonlinear coefficient α . Due to the cubic terms, no explicit solutions can be found for Eqs. (3) and (4). As reported in [8]–[10], harmonic solutions can be used to approximate the exact solutions in a good agreement. In the following, a one-term harmonic balance approximation $y_1 = A_1 \cos(\Omega\tau) + B_1 \sin(\Omega\tau)$ and $y_2 = A_2 \cos(\Omega\tau) + B_2 \sin(\Omega\tau)$ is assumed as the solutions. Substituting the above ansatz into Eqs. (3) and (4), and applying the approximations $\cos^3(\Omega\tau) \approx 3/4\cos(\Omega\tau)$ and $\sin^3(\Omega\tau) \approx 3/4\sin(\Omega\tau)$, then balancing cosine and sine terms, yields the system of polynomial equations:

$$-A_1\Omega^2 + A_1 + A_2\Omega^2 + 3/4\alpha A_1\left(A_1^2 + B_1^2\right) = 1 \tag{5}$$

$$-B_1\Omega^2 + B_1 + B_2\Omega^2 + 3/4\alpha B_1\left(A_1^2 + B_1^2\right) = 0 \tag{6}$$

$$-A_2\Omega^2 + g_{11}\Omega B_2 + \mu A_1 + g_{22} A_2 + 3/4\alpha\beta A_2\left(A_2^2 + B_2^2\right) = 0 \tag{7}$$

$$-B_2\Omega^2 - g_{11}\Omega A_2 + \mu B_1 + g_{22} B_2 + 3/4\alpha\beta B_2\left(A_2^2 + B_2^2\right) = 0 \tag{8}$$

Up to now, it is still not possible to find explicit solutions of Eqs. (5)-(8), we thus expand the harmonic coefficients A_i and B_i into series with respect to the primary nonlinear coefficient α , i.e. $A_1 = A_{11} + \alpha A_{12}$, $B_1 = B_{11} + \alpha B_{12}$, $A_2 = A_{21} + \alpha A_{22}$ and $B_2 = B_{21} + \alpha B_{22}$.

Substituting the above ansatz into Eqs. (5)-(8), and collecting the resulting expressions with respect to the order of the parameter α , then after omitting the expressions whose orders are higher than α^1 , yields:

$$\left(-A_{11} + A_{21}\right)\Omega^2 + A_{11} - 1 = 0 \tag{9}$$

$$1/4\left(-4A_{12} + 4A_{22}\right)\Omega^2 + 3/4A_{11}^3 + 3/4A_{11}B_{11}^2 + A_{12} = 0 \tag{10}$$

$$\left(-B_{11} + B_{21}\right)\Omega^2 + B_{11} = 0 \tag{11}$$

$$1/4\left(-4B_{12} + 4B_{22}\right)\Omega^2 + 3/4A_{11}^2 B_{11} + 3/4B_{11}^3 + B_{12} = 0 \tag{12}$$

$$-A_{22}\Omega^2 + g_{11}\Omega B_{22} + 1/4\left(3A_{21}^3 + 3A_{21}B_{21}^2\right)\beta + \mu A_{12} + g_{22}A_{22} = 0 \tag{13}$$

$$\left(-\Omega^2 + g_{22}\right)A_{21} + g_{11}\Omega B_{21} + \mu A_{11} = 0 \tag{14}$$

$$\left(-\Omega^2 + g_{22}\right)B_{21} - g_{11}\Omega A_{21} + \mu B_{11} = 0 \tag{15}$$

$$-B_{22}\Omega^2 - g_{11}\Omega A_{22} + 1/4\left(3A_{21}^2 B_{21} + 3B_{21}^3\right)\beta + \mu B_{12} + g_{22}B_{22} = 0 \tag{16}$$

Solving for A_{ij} and B_{ij} ($i = 1,2$, $j = 1,2$) from Eqs. (9)-(16), the resulting solutions are found to be in terms of the control gains g_{11}, g_{22}, β and the normalized frequency Ω. Due to the complexity, these expressions are not given here. Nevertheless, the absolute value of the normalized frequency response $|Q(\Omega)|$ between the response $x(\Omega)$ and the applied force $F_d\cos(\Omega\tau)$ can be expressed as:

$$|Q(\Omega)| = \sqrt{{A_{11}}^2 + {B_{11}}^2 + 2\alpha\left(A_{11}A_{12} + B_{11}B_{12}\right) + \alpha^2\left({A_{12}}^2 + {B_{12}}^2\right)} \tag{17}$$

It can be derived that there exists two fixed points for the linear system with the linear controller ($\alpha = 0$ and $\beta = 0$), meaning that the frequency response of the driving point receptance with different feedback gains intersects at two invariable frequency locations. The linear version of the proposed controller $C(\cdot)$ ($\beta = 0$) can be found in more details in [11]. With the $\mathscr{H}_\infty$ optimization criterion, the responses at the two fixed points are sought to be equal and maximized. The following optimal control gains have thus been obtained:

$$g_{11\_opt} = \sqrt{\frac{\left(\left(2\sqrt{\mu}\sqrt{2}+\mu+2\right)P1 + \left(-2\sqrt{\mu}\sqrt{2}+\mu+2\right)P2 + \left(\mu-2\right)^2\right)\mu}{2\left(\mu-2\right)^2}} \tag{18}$$

$$g_{22\_opt} = 1 - \frac{\mu}{2} \tag{19}$$

$$\Omega_{fixed1} = \frac{\sqrt{2}\sqrt{\sqrt{2\mu}+2}}{2} \tag{20}$$

$$\Omega_{fixed2} = \frac{\sqrt{2}\sqrt{-\sqrt{2\mu}+2}}{2} \tag{21}$$

where $P1 = \sqrt{-4\mu^{3/2}\sqrt{2} - 8\sqrt{\mu}\sqrt{2} + \mu^2 + 12\mu + 4}$ and $P2 = \sqrt{4\mu^{3/2}\sqrt{2} + 8\sqrt{\mu}\sqrt{2} + \mu^2 + 12\mu + 4}$.

Substituting Eqs. (18)-(21) into Eq. (17) and solving for β to remain the equal peaks at the fixed points, yields

$$\beta = \frac{2}{\mu} - \frac{3}{2} + \frac{99}{256}\mu + \mathrm{O}\left(\mu^{3/2}\right) \tag{22}$$

In fact, Eq. (22) represents a simpler and more easily interpretable relation which is the Taylor series expansion of the exact solution with respect to the stiffness ratio μ given $\mu \ll 1$.

Up to now, the derivation of the explicit expressions for forming the novel controller $C(\cdot)$ is complete wherein the optimal feedback gains are given in Eqs. (18), (19) and (22) respectively. The control effectiveness of the proposed control strategy is checked for an example system where the stiffness ratio μ is set to 0.1 and the primary nonlinear coefficient α is varied between 0.0001 and 0.008. Substituting Eqs. (18), (19) and (22) into Eqs. (3) and (4) for the different combinations of α and μ, the resulting nonlinear equations are computed using a path-following algorithm combining shooting and pseudo-arclength continuation [12]. This algorithm provides a very accurate numerical solution to the equations of motion. Figure 2 (b) illustrates that the frequency response of the Duffing oscillator with the linear controller is strongly detuned in the presence of strong nonlinearities i.e. $\alpha = 0.008$. Conversely, Figure 2 (a) shows that the nonlinear controller can compensate, to a large extend, the detuned equal peaks for values of α ranging from 0.0001 to 0.008. Although the control effectiveness degrades in terms of the differences between the two peaks when α is large, Eq. (22) can be updated using approximated solutions with more harmonics. Another interesting observation is that the amplitude of the resonance peaks does not change substantially when α increases, which means that the response of the coupled system is almost proportional to the forcing amplitude, as it would be the case for a linear system.

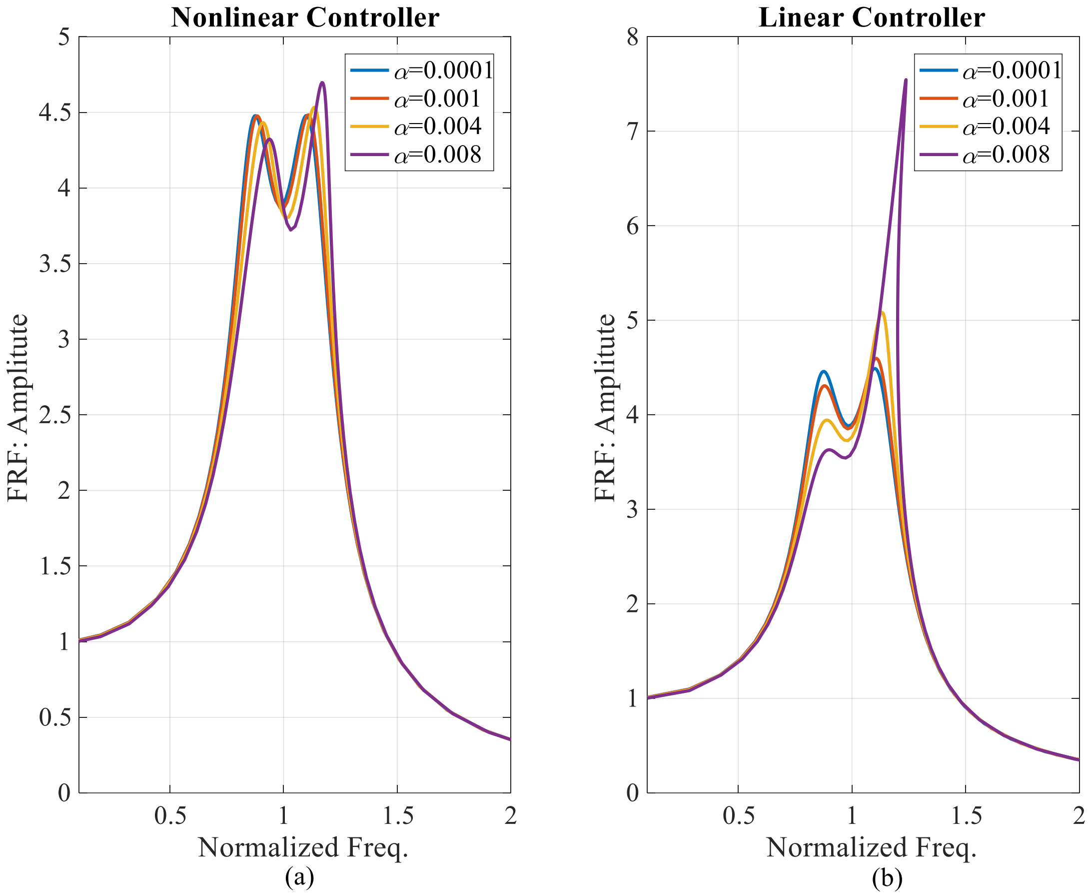

Figure 2 Frequency response of an example system where $\mu = 0.1$ and the primary nonlinear coefficient α varies between 0.0001 and 0.008

3 Conclusions

A nonlinear active damping strategy based on force feedback is proposed, where only the modal stiffness ratio of the system when the actuator is installed and removed is required to design an effective controller. Closed-form expressions are derived using the $\mathscr{H}_\infty$ optimization criterion wherein the optimal feedback gains are achieved to minimize the maximal response of the structure. The resonant vibration of a Duffing oscillator is considered to illustrate the proposed tuning of the optimal control gains and the harmonic balance method is employed to approximate the analytical solutions. It is shown that the amplitude of the resonance peaks does not change substantially when the weight of the nonlinearity increases, which means that the response of the coupled system is almost proportional to the forcing amplitude, as it would be the case for a linear system.

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