LIÈGE université
Sciences Appliquées

Structural & Stochastic Dynamics
Urban & Environmental Engineering

Estimation of modal correlation coefficients in wind buffeting spectral analysis

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2nd National Conference of the Romanian Association for Wind Engineering

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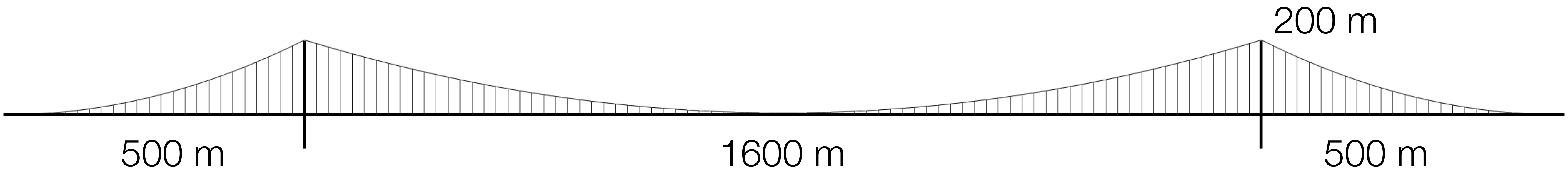
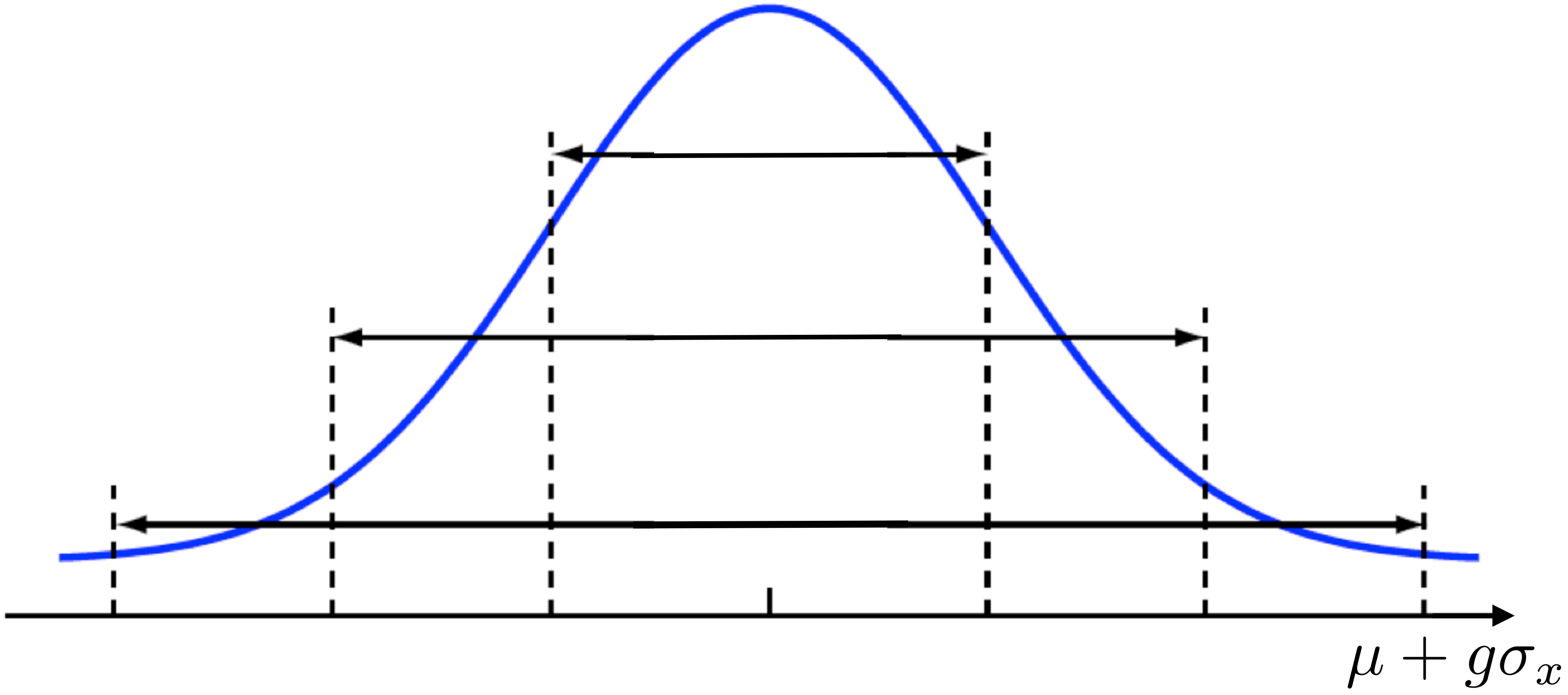
Spectral analysis in the modal basis

- Wind speed PSD instead of synchronous wind series
- Modal truncation, M modes << N dofs



1) Matrices in nodal basis $\longrightarrow$ natural frequencies $\overline{\omega}$ and mode shapes $\boldsymbol{\phi}$
2) Generalized matrices $\mathbf{S}_f, \mathbf{M}, \mathbf{K}$ and $\mathbf{C}$ $\rightarrow$ $\mathbf{H}(\omega) = \left(-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K}\right)^{-1}$
3) PSD matrix of modal amplitudes $\mathbf{S}_q = \mathbf{H}\mathbf{S}_f\mathbf{H}^*$
4) Modal amplitudes statistics $\sigma_{q,mn} = \int_{-\infty}^{\infty} \mathrm{S}_{q,mn}(\omega)\mathrm{d}\omega = \int_{-\infty}^{\infty} \mathrm{H}_m(\omega)\mathrm{S}_{f,mn}(\omega)\mathrm{H}_n^*(\omega)\mathrm{d}\omega$
5) Combine modal statistics to obtain nodal statistics

Modal combination techniques



Variance of design quantities → extreme value distribution

$$\sigma^2_{x,i} = \sum_{m=1}^{M}\sum_{n=1}^{M}\phi_{im}\phi_{in}\sigma_{q,mn} = \underbrace{\boxed{\underbrace{\boxed{\sum_{\substack{m=1\\n=m}}^{M}\phi^2_{im}\sigma^2_{q,m}}}_{\text{SRSS}} + \sum_{m=1}^{M}\sum_{\substack{n=1\\n\neq m}}^{M}\phi_{im}\phi_{in}\sigma_{q,mn}}}_{\text{CQC}}$$

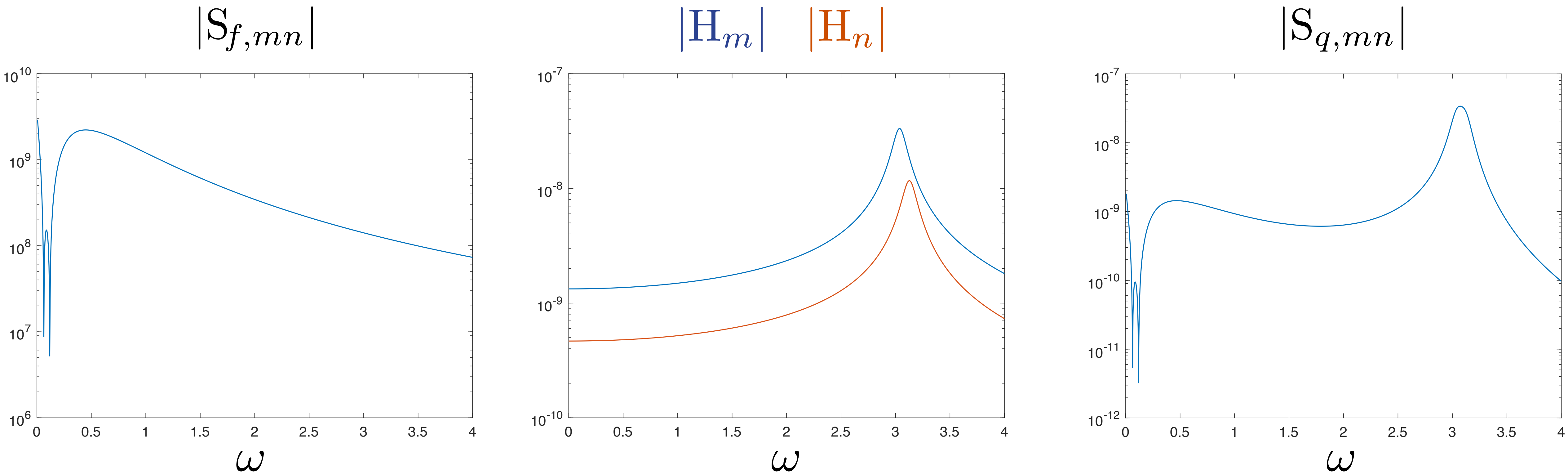
Covariances are not always negligible !

Exact computation of modal statistics

$$\sigma_{q,mn} = \int_{-\infty}^{\infty} \mathrm{H}_m(\omega)\mathrm{S}_{f,mn}(\omega)\mathrm{H}_n^*(\omega)\mathrm{d}\omega$$

$$\mathbf{H}(\omega) = \left(-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K}\right)^{-1}$$

$$\mathbf{S}_f(\omega) = \boldsymbol{\phi}^T\mathbf{S}_p(\omega)\boldsymbol{\phi}$$ (M x N)(N x N)(N x M)



A lot of integration points at which a big matrix is projected in the modal basis

TIME CONSUMING

Need fast and accurate estimations of variances and covariances

B/R decomposition

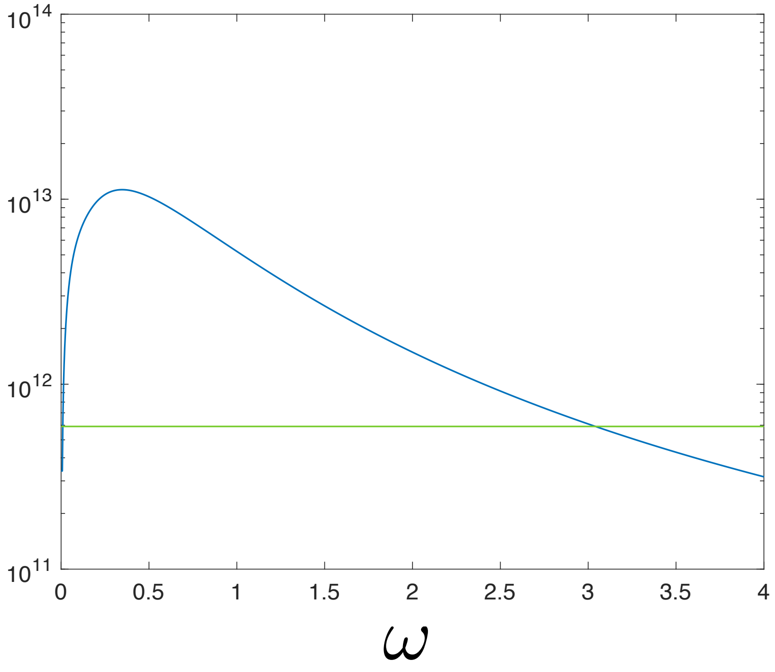
Intuitive variance estimation by Davenport in 1961

$$\sigma^2_{q,m} = \int_{-\infty}^{\infty} \left|\mathrm{H}_m(\omega)\right|^2 \mathrm{S}_{f,m}(\omega)\mathrm{d}\omega \approx \mathrm{B}_m + \mathrm{R}_m$$

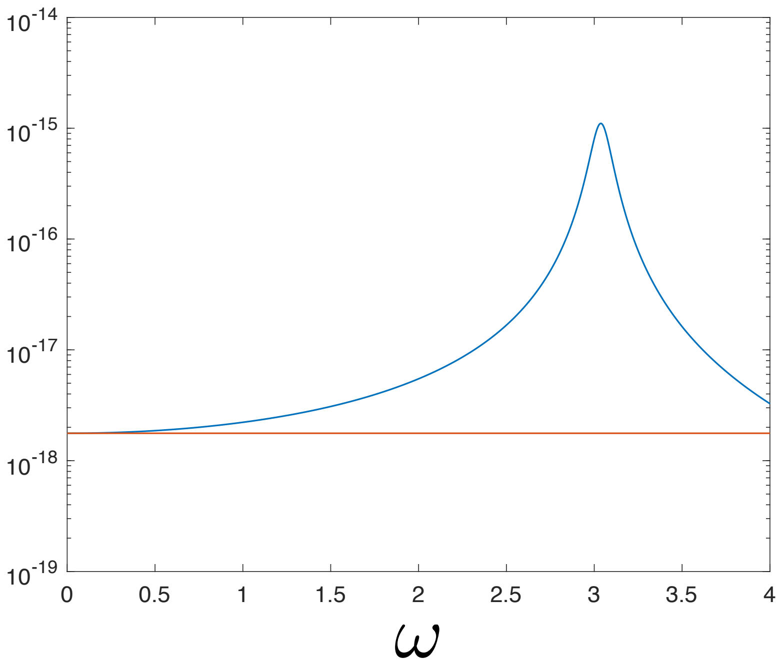
Background $\omega \ll \overline{\omega}_m$ $$\left|\mathrm{H}_m(\omega)\right|^2 \approx \frac{1}{k_m^2} \qquad \mathrm{B}_m = \frac{\sigma^2_{f,m}}{k_m^2}$$

Resonant $\omega \approx \overline{\omega}_m$, $\xi \ll 1$ $$\mathrm{S}_{f,m}(\omega) \approx \mathrm{S}_{f,m}(\overline{\omega}_m) \qquad \mathrm{R}_m = \frac{S_{f,m}(\overline{\omega}_m)}{k_m^2}\frac{\pi\overline{\omega}_m}{\xi}$$

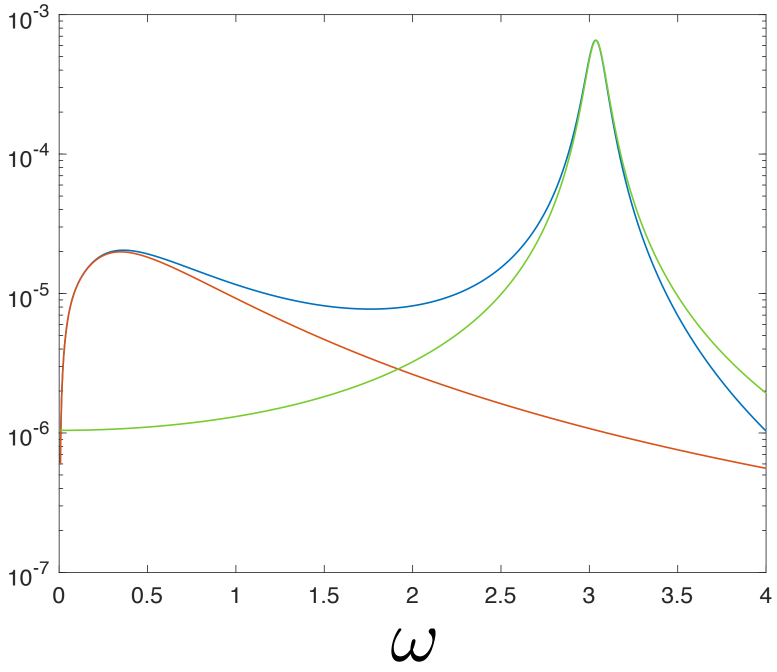
$$\mathrm{S}_{f,m}$$



$$\left|\mathrm{H}_m\right|^2$$



$$\mathrm{S}_{q,m}$$



Multiple Timescale Spectral Analysis

- Generalization of B/R decomposition $\sigma_{q,mn} \approx \mathrm{B}_{mn} + \mathrm{R}_{mn}$
- Mathematical explanation based on perturbation methods
- Validation of covariance estimations by (Gu, 2009) or (Denoël, 2009)

Background

$$\omega \ll \overline{\omega}_m$$

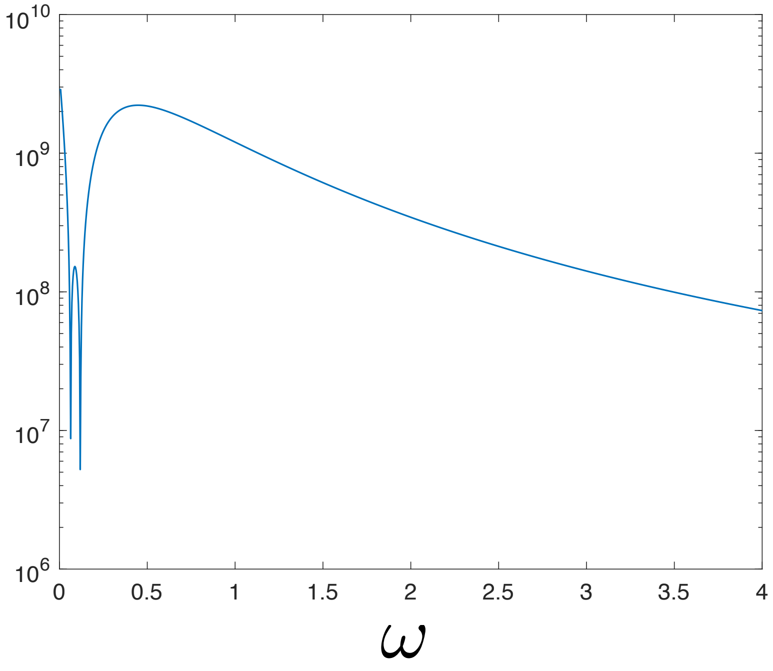
$$\omega \ll \overline{\omega}_n$$

$$\mathrm{B}_{mn} = \frac{\sigma_{f,mn}}{k_m k_n}$$

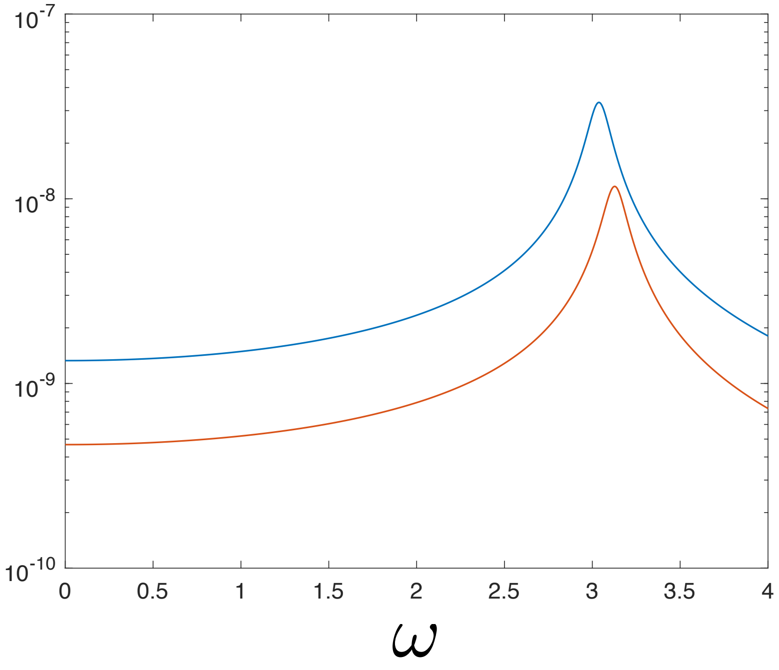
Resonant

$$\omega \approx \frac{\overline{\omega}_m + \overline{\omega}_n}{2} \qquad \begin{matrix} \xi \ll 1 \\ \varepsilon \ll 1 \end{matrix} \qquad \varepsilon = \frac{\overline{\omega}_n - \overline{\omega}_m}{\overline{\omega}_m + \overline{\omega}_n}$$

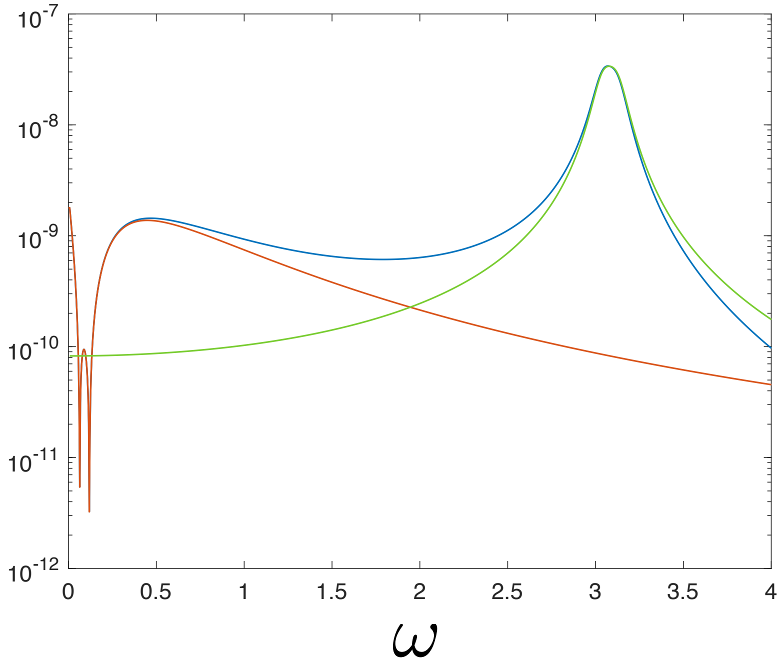
$$\mathrm{R}_{mn} = \frac{1}{k_m k_n} \mathrm{Re}\left[S_{f,mn}\left(\frac{\omega_m + \omega_n}{2}\right) \frac{\omega_m + \omega_n}{2} \frac{\xi - i\varepsilon}{\varepsilon^2 + \xi^2} \frac{\pi}{2} \right]$$



$|\mathrm{S}_{f,mn}|$



$|\mathrm{H}_m|$ $|\mathrm{H}_n|$



$|\mathrm{S}_{q,mn}|$

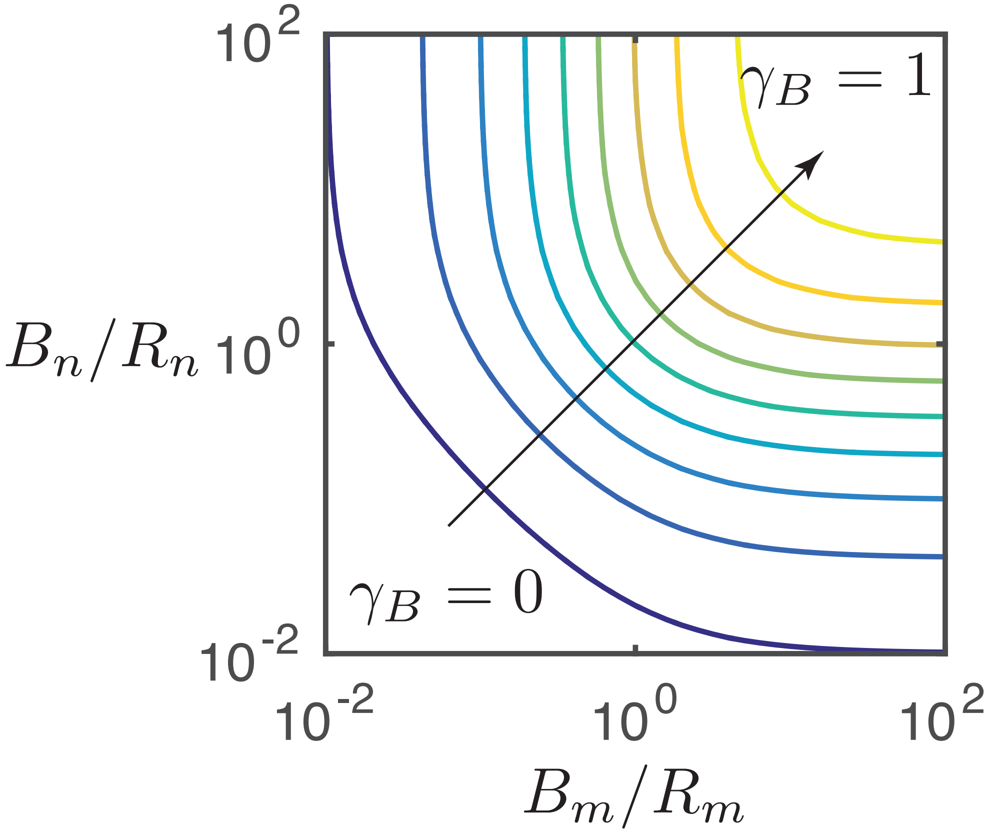
Modal correlation coefficient

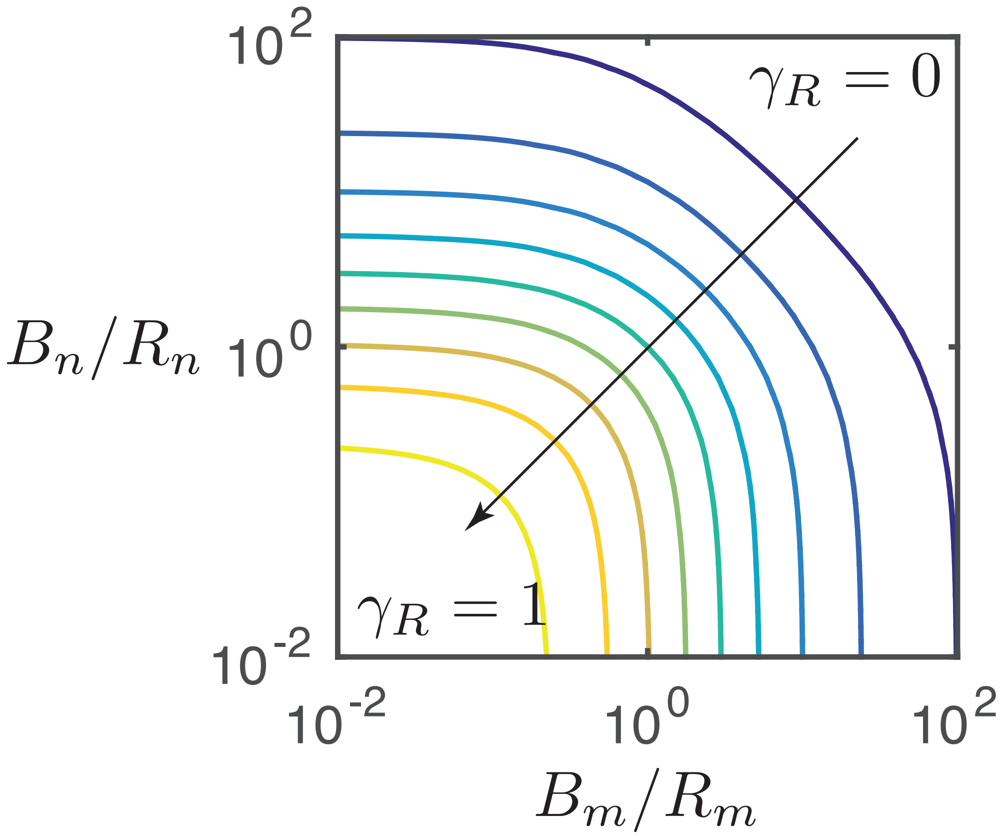
$$\sigma_{q,mn} \approx \mathrm{B}_{mn} + \mathrm{R}_{mn}$$

$$\rho_{q,mn} \approx \gamma_B \rho_{f,mn} + \gamma_R \,\mathrm{Re}[\overline{\Gamma}\theta]$$

$$\gamma_B = \frac{1}{\sqrt{1+b_m^{-1}}\sqrt{1+b_n^{-1}}}$$

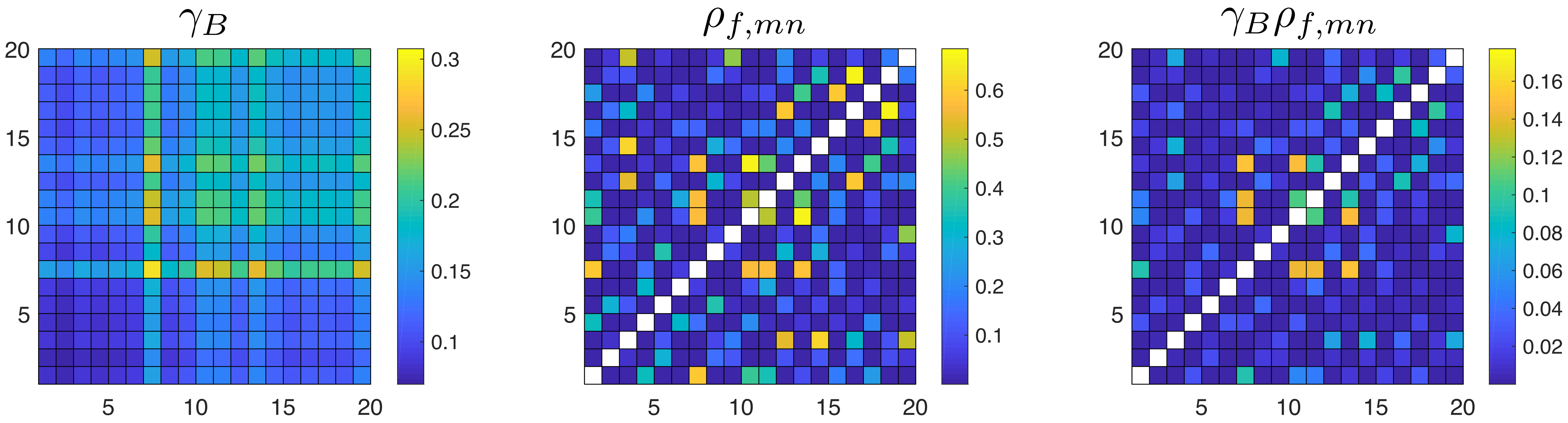
$$\gamma_R = \frac{1}{\sqrt{1+b_m}\sqrt{1+b_n}}$$

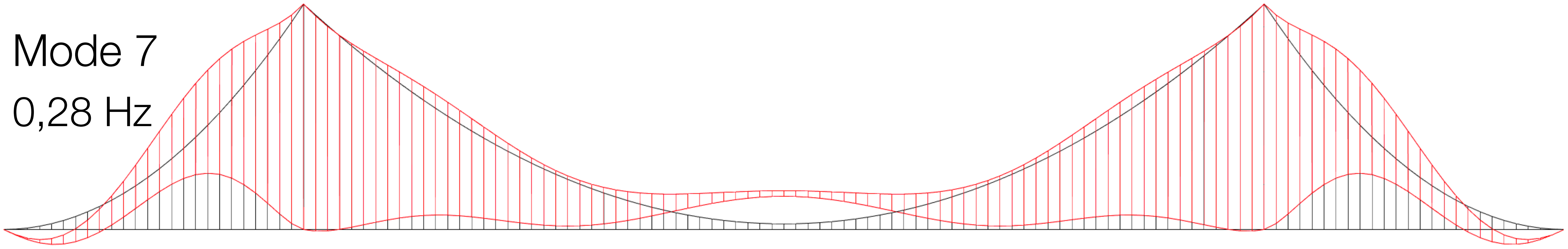




Background coefficient

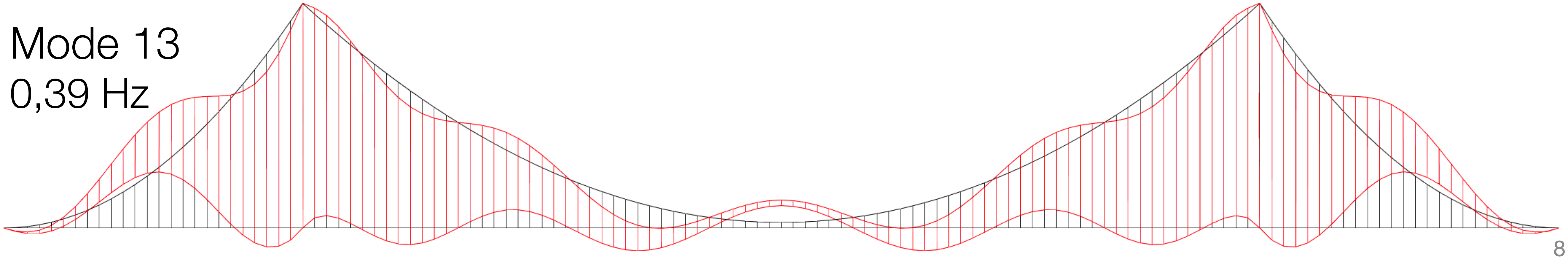
$$\rho_{q,mn} = \gamma_B \rho_{f,mn} + \gamma_R \, \mathrm{Re}[\overline{\Gamma}\theta]$$





Mode 7

0,28 Hz

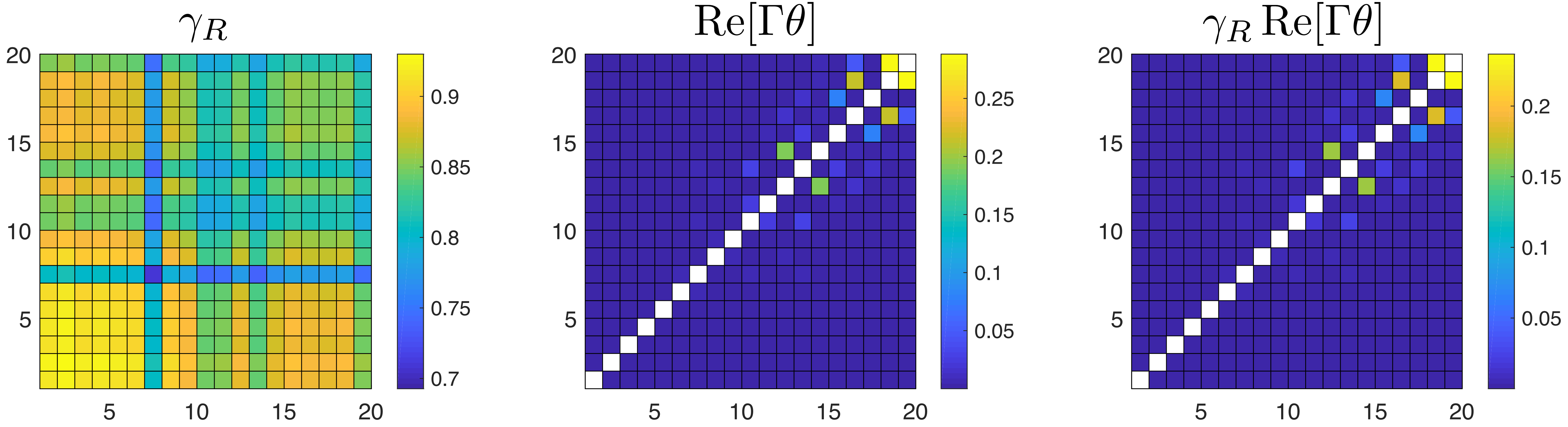


Mode 13

0,39 Hz

Resonant coefficient

$$\rho_{q,mn} = \gamma_B \rho_{f,mn} + \gamma_R \operatorname{Re}[\overline{\Gamma}\theta]$$



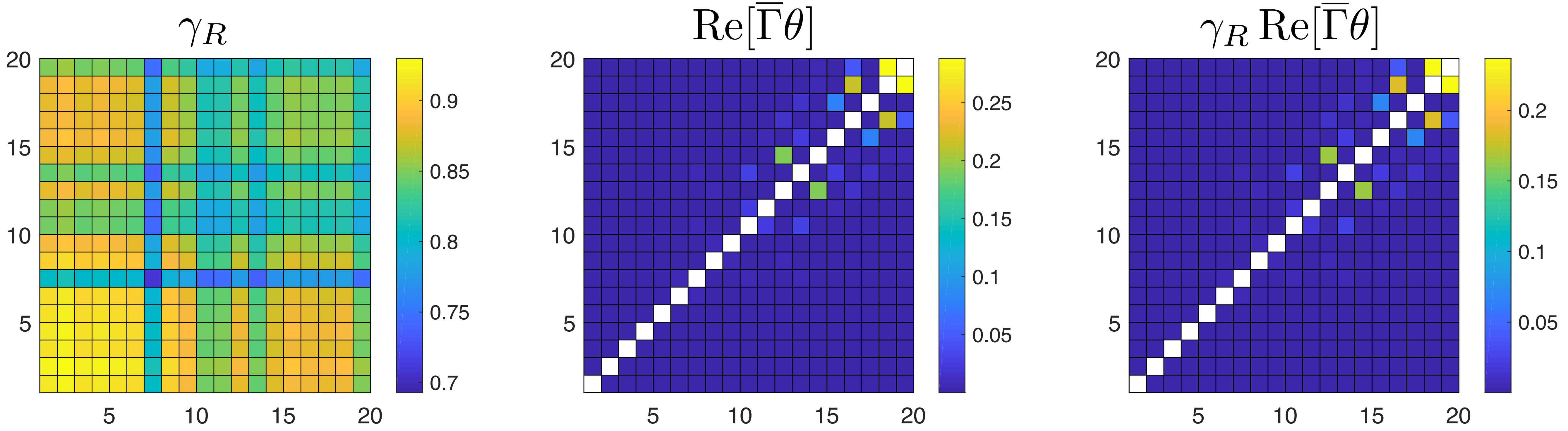
$$\overline{\Gamma} = \frac{X}{\sqrt{S_{f,mm}(\omega_m)\, S_{f,nn}(\omega_n)}}$$

$$X = \mathrm{S}_{f,mn}\left(\frac{\overline{\omega}_m + \overline{\omega}_n}{2}\right)$$

$$\theta \; = \frac{\omega_m + \omega_n}{2\sqrt{\omega_m \omega_n}} \frac{\xi^2 - i\varepsilon\xi}{\varepsilon^2 + \xi^2}$$

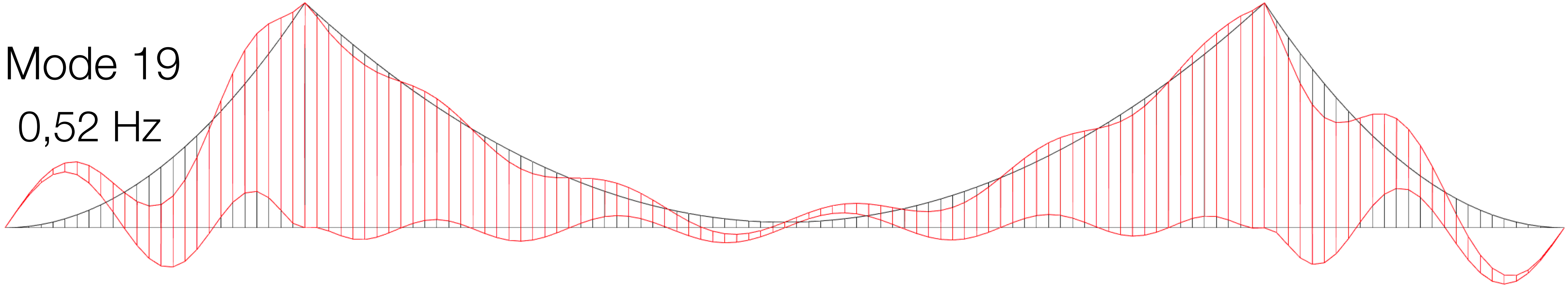
Resonant coefficient

$$\rho_{q,mn} = \gamma_B \rho_{f,mn} + \gamma_R \operatorname{Re}[\overline{\Gamma}\theta]$$



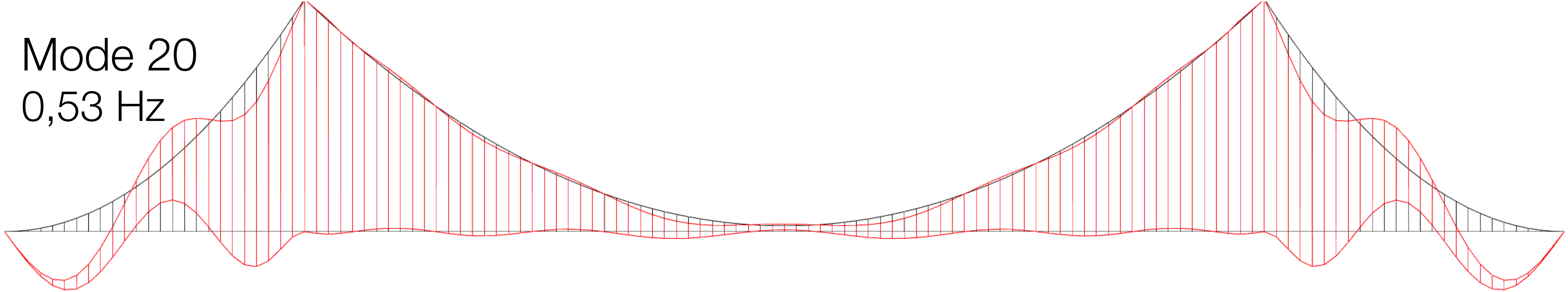
Mode 19

0,52 Hz



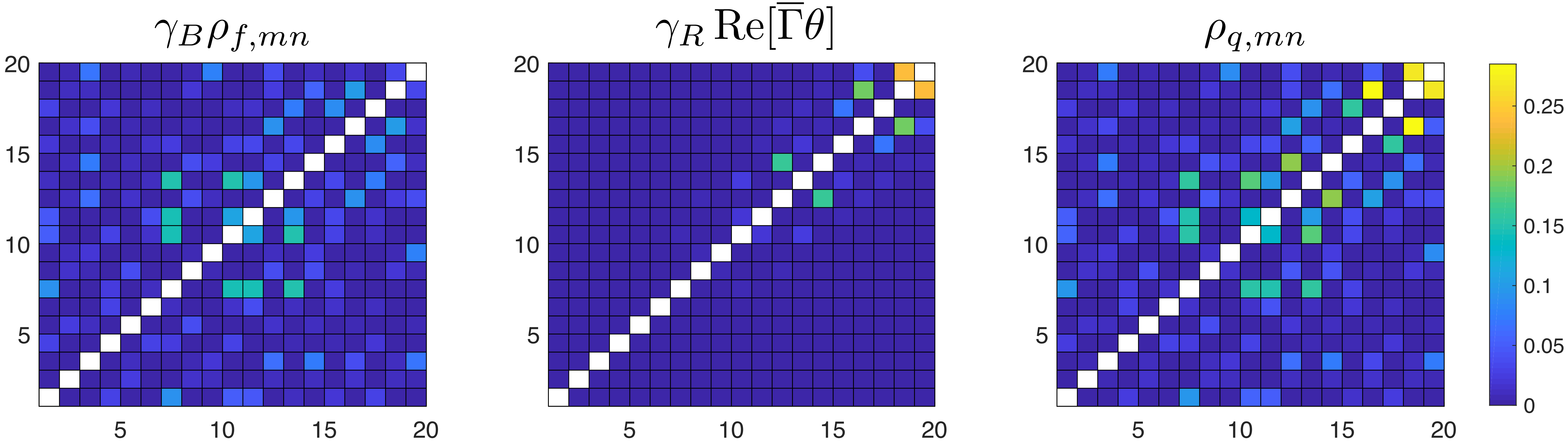
Mode 20

0,53 Hz



B + R coefficient

$$\rho_{q,mn} = \gamma_B \rho_{f,mn} + \gamma_R \, \mathrm{Re}[\overline{\Gamma}\theta]$$



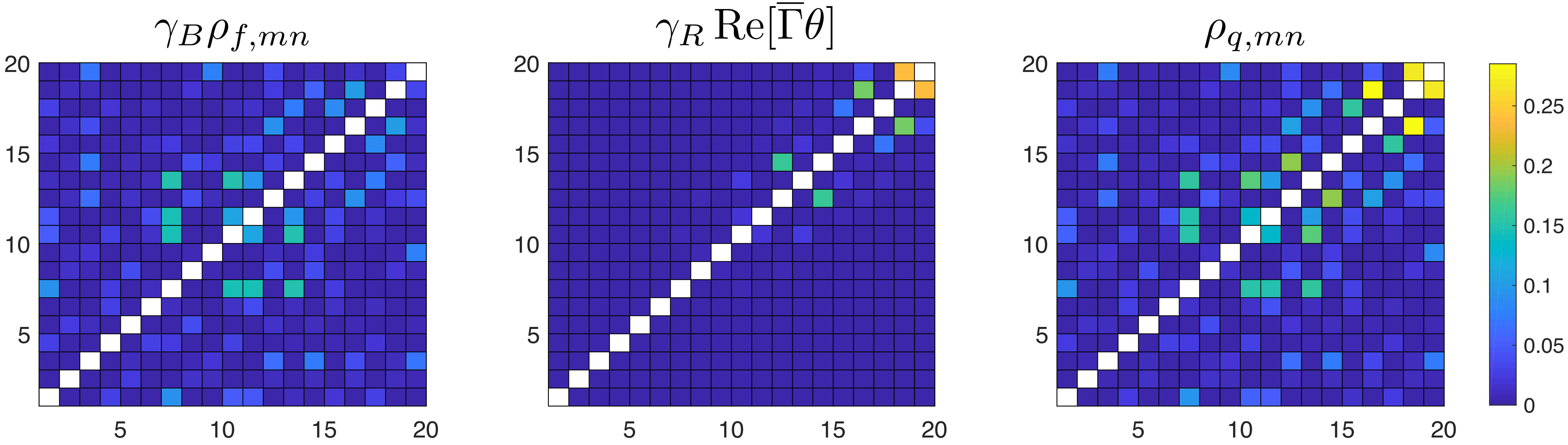
$$\overline{\Gamma} = \frac{X}{\sqrt{S_{f,mm}(\omega_m)\, S_{f,nn}(\omega_n)}}$$

	Error max	CPU
$X = \mathrm{S}_{f,mn}\left(\frac{\overline{\omega}_m + \overline{\omega}_n}{2}\right)$	0,0102	25
$X = \sqrt{\mathrm{S}_{f,mn}(\overline{\omega}_m)\, \mathrm{S}_{f,mn}(\overline{\omega}_n)}$	0,0103	1
$X = \frac{\mathrm{S}_{f,mn}(\overline{\omega}_m) + \mathrm{S}_{f,mn}(\overline{\omega}_n)}{2}$	0,0108	1

Conclusions

$$\sigma_{x,i}^2 = \underbrace{\underbrace{\sum_{\substack{m=1\\n=m}}^{M} \phi_{im}^2\sigma_{q,m}^2}_{\text{SRSS}} + \sum_{m=1}^{M}\sum_{\substack{n=1\\n\neq m}}^{M} \phi_{im}\phi_{in}\rho_{q,mn}\sigma_{q,m}\sigma_{q,n}}_{\text{CQC}}$$

$$\rho_{q,mn} = \gamma_B \rho_{f,mn} + \gamma_R \,\mathrm{Re}[\overline{\Gamma}\theta]$$



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Thank you !
Questions ? Comments ?

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Modal correlation coefficient

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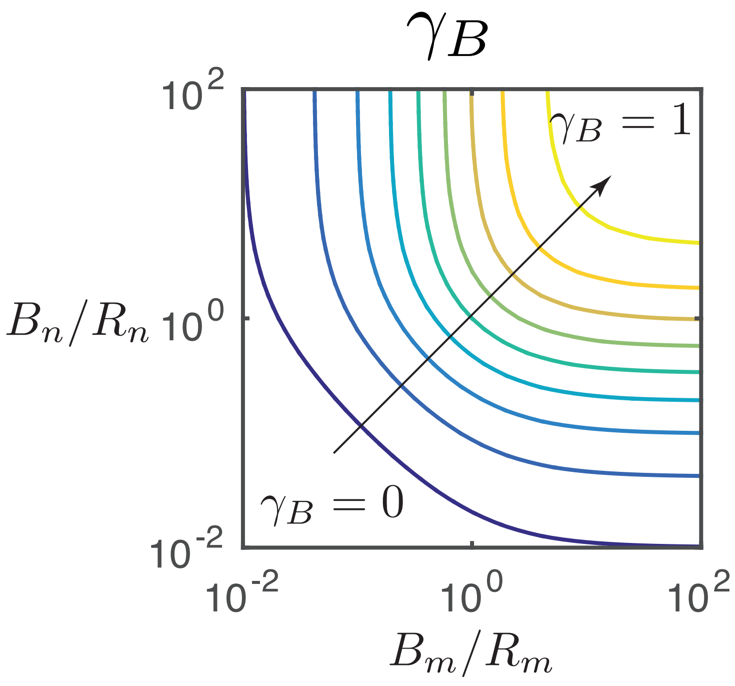
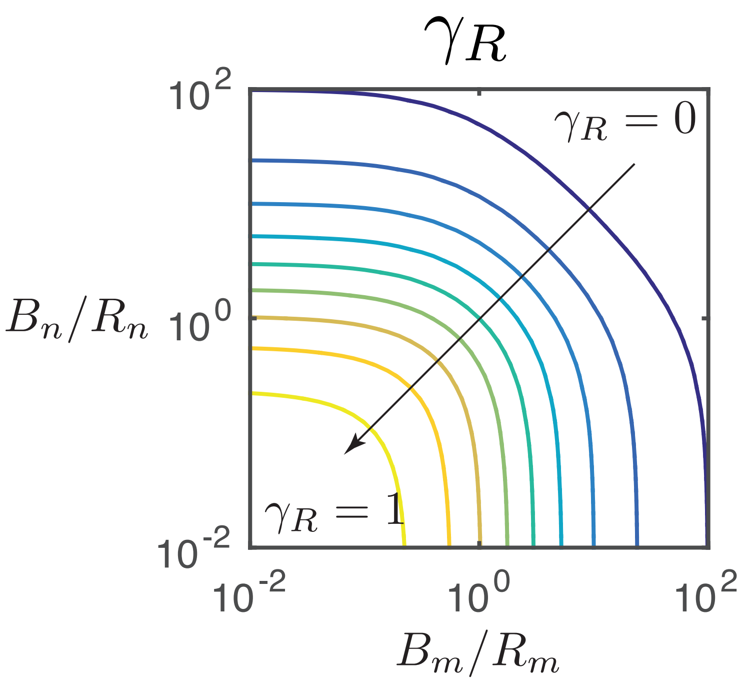
$$\sigma^2_{q,m} = \frac{\sigma^2_{f,m}}{k^2_m} + \frac{S_{f,mm}\left(\overline{\omega}_m\right)}{k^2_m} \frac{\pi}{2} \frac{\overline{\omega}_m}{\xi} \qquad \sigma^2_{q,n} = \frac{\sigma^2_{f,n}}{k^2_n} + \frac{S_{f,nn}\left(\overline{\omega}_n\right)}{k^2_n} \frac{\pi}{2} \frac{\overline{\omega}_n}{\xi}$$





$$\gamma_B = \frac{1}{\sqrt{1 + b_m^{-1}}\sqrt{1 + b_n^{-1}}}$$

$$\gamma_R = \frac{1}{\sqrt{1 + b_m}\sqrt{1 + b_n}}$$

$$\theta = \frac{\omega_m + \omega_n}{2\sqrt{\omega_m \omega_n}} \frac{\xi^2 - i\varepsilon\xi}{\varepsilon^2 + \xi^2}$$

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