ESTIMATION OF MODAL CORRELATION COEFFICIENTS IN WIND BUFFETING ANALYSIS

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Abstract: The dynamic response of large and slightly damped civil engineering structures subjected to buffeting winds is usually studied in the modal basis and in the frequency domain. These choices respectively reduce the analysis to a few modes instead of many degrees-of-freedom and provide a simple estimation of the statistics of the modal responses as a sum of background and resonant components, avoiding generation of long time series. In many cases, the proper combination of modal responses, which is necessary to estimate design quantities, requires taking account of the correlation between the modal responses. This paper provides a review of available techniques to estimate the covariances of modal responses with the same objectives of rapidity and accuracy as the background-resonant decomposition, developed by Davenport for the computation of variances.

Keywords: Correlation, Complete Quadratic Combination, Multiple Timescale Spectral Analysis

1. INTRODUCTION

Large and slightly damped civil engineering structures, such as bridges or roof structures, are designed to withstand extreme displacements, internal forces and ground reactions induced by buffeting winds. In order to determine these extreme design quantities, the buffeting analysis of the structure is often conducted in the frequency domain. Generations of long wind speed time series are therefore avoided and replaced by the treatment of fundamental statistical information. The analysis of the structure is also usually achieved in a modal basis, to take advantage of the modal truncation and thus reduce the complexity of the model studied from many degrees-of-freedom to a few modes, which are considered uncoupled in this paper since it focuses on the effects of modal correlations only.

The variance $\sigma_{x,i}^2$ of design quantities x at each node i is essential to determine extreme value distributions and is obtained by combining variances $\sigma_{q,m}^2$ and covariances $\sigma_{q,mn}$ of modal amplitudes following one of the two rules derived from the random vibration theory, where ϕ_{im} is the shape of mode m at node i:

$$\sigma_{x,i}^2 = \sum_{m=1}^{M}\sum_{n=1}^{M} \phi_{im}\phi_{in}\sigma_{q,mn} = \boxed{\underbrace{\sum_{\substack{m=1\\n=m}}^{M} \phi_{im}^2\sigma_{q,m}^2}_{\text{SRSS}} + \sum_{m=1}^{M}\sum_{\substack{n=1\\n\neq m}}^{M} \phi_{im}\phi_{in}\sigma_{q,mn}}_{\text{CQC}} \tag{1}$$

The CQC (complete quadratic combination) is more precise but much more demanding than the SRSS (square root of the sum of the squares) as it accounts for modal covariances. Indeed, as shown in Eq. 2 where H_m is the transfer function associated to mode m, the computation of each additional term is expensive because the numerical integration of such a function with sharp peaks requires a lot of integration points. Moreover, the most consuming operation has to be repeated at each of these numerous points and consists in projecting the power spectral density (PSD) matrix of the loading $\boldsymbol{S_p}$ into the modal basis to form the PSD matrix of generalized forces $\boldsymbol{S_f}$.

$$\sigma_{q,mn} = \int_{-\infty}^{\infty} H_m(\omega)\, S_{f,mn}(\omega)\, H_n^*(\omega)\, \mathrm{d}\omega \quad \text{with} \quad \boldsymbol{S_f}(\omega) = \boldsymbol{\phi}^T \boldsymbol{S_p}(\omega)\boldsymbol{\phi} \tag{2}$$

2. FAST AND ACCURATE ESTIMATIONS OF VARIANCES AND COVARIANCES

In the 1960's, Davenport [1] first developed the background-resonant (B/R) decomposition which approximates very accurately the variance $\sigma_{q,m}^2$ using only one projection of the PSD at the natural frequency ω_m of mode m. He intuitively replaced the transfer function by a constant in the quasi-static regime and did the same for the PSD in the resonant regime.

$$\sigma_{q,m}^2 = \frac{\sigma_{f,m}^2}{k_m^2} + \frac{S_{f,mm}(\omega_m)}{k_m^2}\frac{\pi\omega_m}{2\xi} \tag{3}$$

Then, in 2009, Denoël [2] and Gu [3] followed independently the intuition of Davenport to formulate a precise and fast approximation of covariances. Finally, in 2015, Denoël provided a mathematical explanation for these intuitive developments with the Multiple Timescale Spectral Analysis [4]. Covariances are therefore estimated by Eq. 4 where k_m and ξ_m are respectively the generalized stiffness and the damping ratio in mode m but, for the sake of conciseness here, the same damping ratio ξ is used for all modes.

$$\sigma_{q,mn} = \underbrace{\frac{\sigma_{f,mn}}{k_m k_n}}_{B_{mn}} + \underbrace{\frac{1}{k_m k_n}\mathrm{Re}\left[S_{f,mn}\left(\frac{\omega_m+\omega_n}{2}\right)\frac{\omega_m+\omega_n}{2}\frac{\xi - i\varepsilon}{\varepsilon^2+\xi^2}\frac{\pi}{2}\right]}_{R_{mn}} \quad \text{with} \quad \varepsilon = \frac{\omega_n - \omega_m}{\omega_m + \omega_n} \tag{4}$$

Considering the continuity and the smoothness of the PSD in the resonant regime, the number of projections needed to evaluate the covariances further decreases when $S_{f,mn}((\omega_m + \omega_n)/2)$ is replaced by $(S_{f,mn}(\omega_m) + S_{f,mn}(\omega_n))/2$ and the expression of the correlation coefficient of modal amplitudes $\rho_{q,mn}$ is reduced to Eq. 5, where b_m, $\Gamma_{mn}(\omega)$, $\rho_{f,mn}$ are respectively the background-to-resonant ratio B_{mm}/R_{mm} in mode m, the coherence function and the correlation coefficient of generalized forces.

$$\rho_{q,mn} = \gamma_B\,\rho_{f,mn} + \gamma_R\,\mathrm{Re}[\bar{\Gamma}\,\theta] \text{ with } \begin{array}{ll} \gamma_B = \dfrac{1}{\sqrt{1+b_m^{-1}}\sqrt{1+b_n^{-1}}} & \bar{\Gamma} = \dfrac{\Gamma_{mn}(\omega_m) + \Gamma_{mn}(\omega_n)}{2} \\ \gamma_R = \dfrac{1}{\sqrt{1+b_m}\sqrt{1+b_n}} & \theta = \dfrac{\omega_m+\omega_n}{\sqrt{\omega_m\omega_n}}\dfrac{\xi - i\varepsilon}{\varepsilon^2+\xi^2}\dfrac{\xi}{2} \end{array} \tag{5}$$

3. CONCLUSIONS

Equation 5 defines a combination technique which takes covariances into account and offers the advantages of both the accurate CQC and the fast SRSS techniques. The correlation coefficient of modal responses $\rho_{q,mn}$ corresponds to the weighted sum of two expected limit values: if the response is background in both modes, $\rho_{q,mn}$ is equal to the correlation coefficient of generalized forces; if the response is resonant in both modes, $\rho_{q,mn}$ is significant when, all at once, natural frequencies are close and generalized forces are coherent at natural frequencies.

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