Kinematics-Based Modelling of Deep Transfer Girders in Reinforced Concrete Frame Structures

Jian LIU

LIÈGE université
Sciences Appliquées

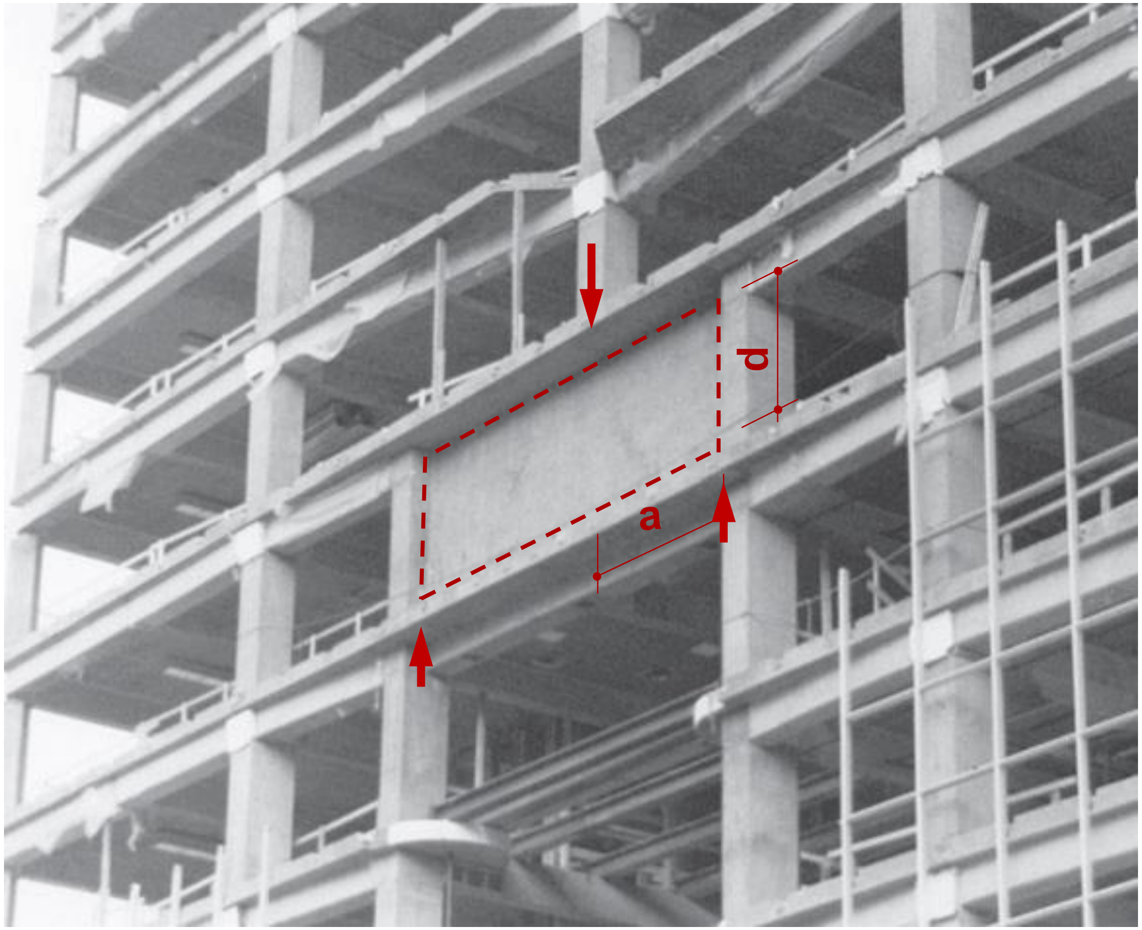
Liège, Belgium
14-06-2019

Outline

1. Background and objectives
2. Comparative study on models for deep beams
3. Macroelement for complete shear behaviour of deep beams
4. Mixed-type modelling with slender and deep beam elements
5. Shear strength of deep beams with openings
6. Conclusions and future work

Background and Objectives

Characteristics of deep transfer girders



(Photo by J. G. MacGregor.)

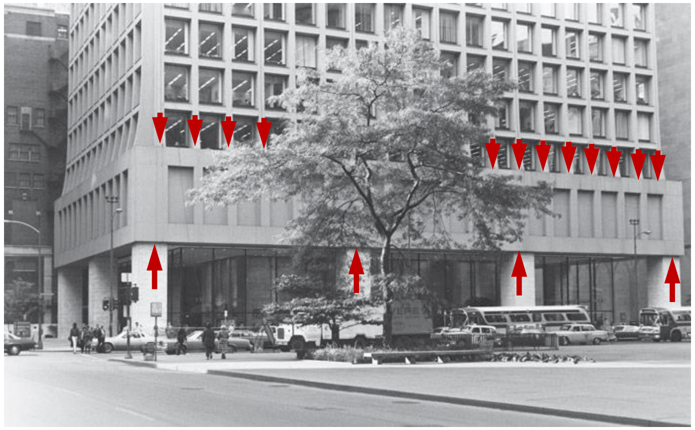
- Transfer heavy loads from discontinuous columns/ walls
- Small aspect ratio: $a/d \leq 2.5$
- Crucial to structural safety

Deep transfer girders in structures


(By Evan Bentz, Toronto, 2008)


(Train station of Leuven)


(Grand Chancellor Hotel, New Zealand. By Kam et al., 2011)

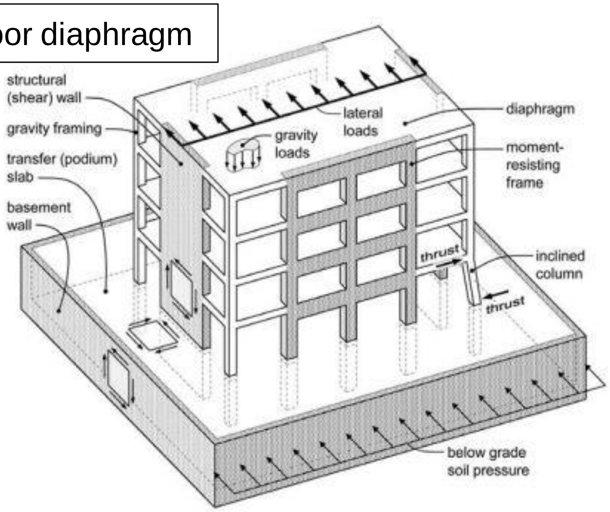

(Brunswick building, Chicago: J. G. MacGregor)

Other application of deep beams

Cap beams

(https://civildigital.com/the-five-major-parts-of-bridges-concrete-span-bridge/)

Floor diaphragm



(https://iarjset.com/upload/2017/march-17/IARJSET%2024.pdf)

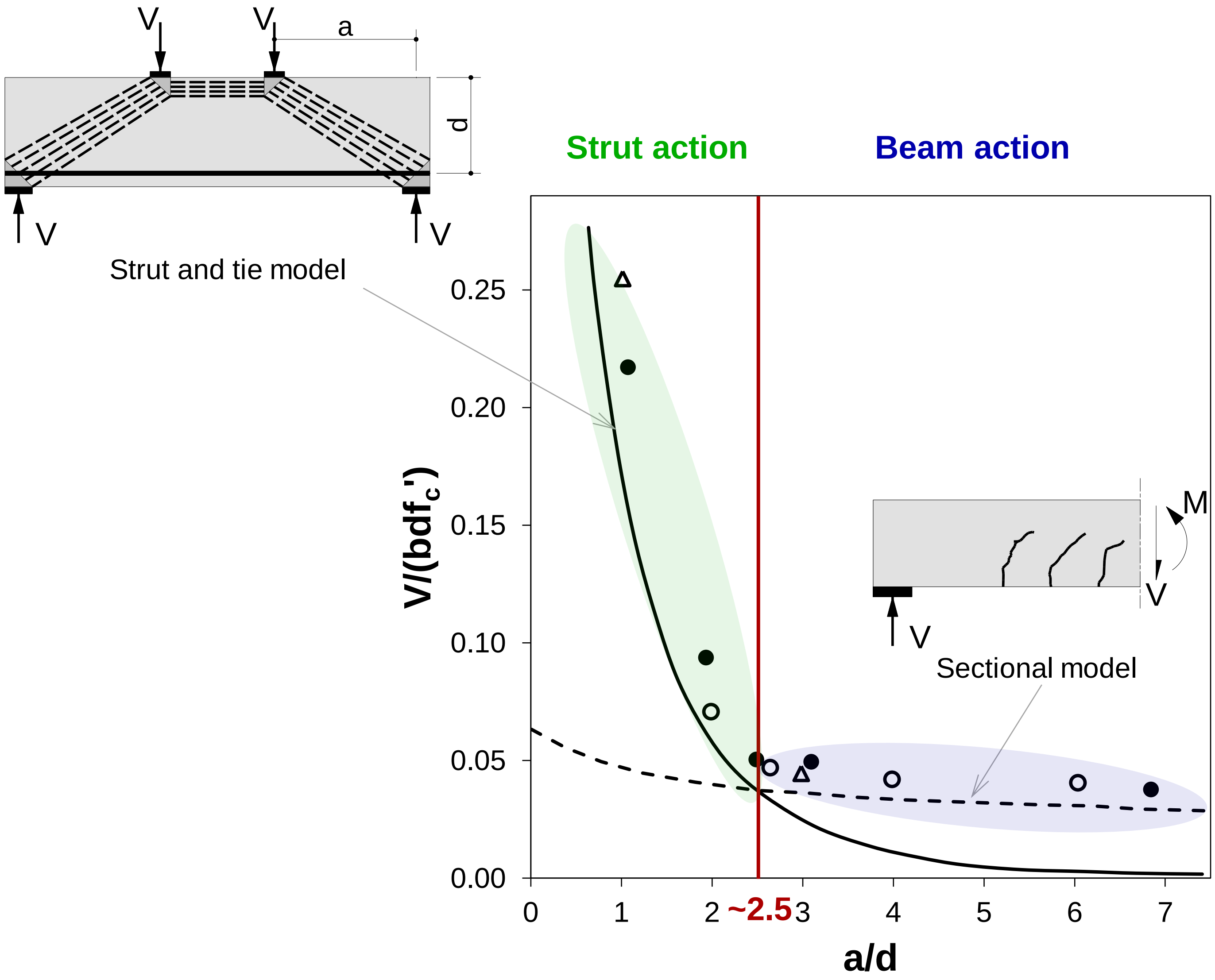
Strip footings

(https://www.kore-system.com/blog_list/insulation-series-what-type-of-foundation-is-right-for-me/)

Raft footings

(https://photo.xuite.net/hspsj60440/4103822/1.jpg)

Difference between slender and deep beams



f_c' = 27.2 MPa

$a_{g,max}$ = 19 mm

d = 538 mm

b = 155 mm

A_s = 2277 mm^2

f_y = 372 MPa

Plate size:

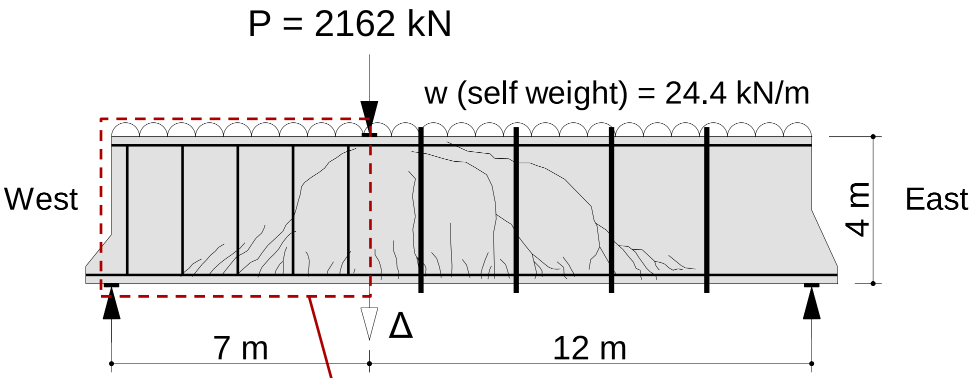
● 152×152×25mm^3

△ 152×229×51mm^3

○ 152×76×9.5mm^3

(tests by Kani in 1979, adapted from Collins and Mitchell, 1997)

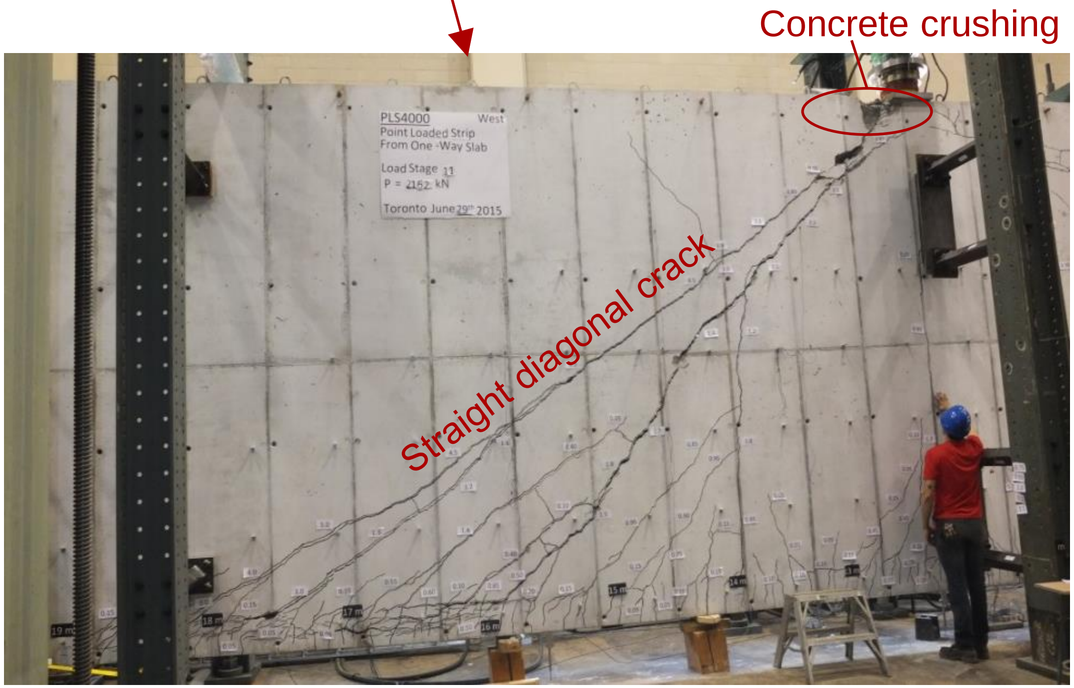
Shear behaviour of deep transfer girders

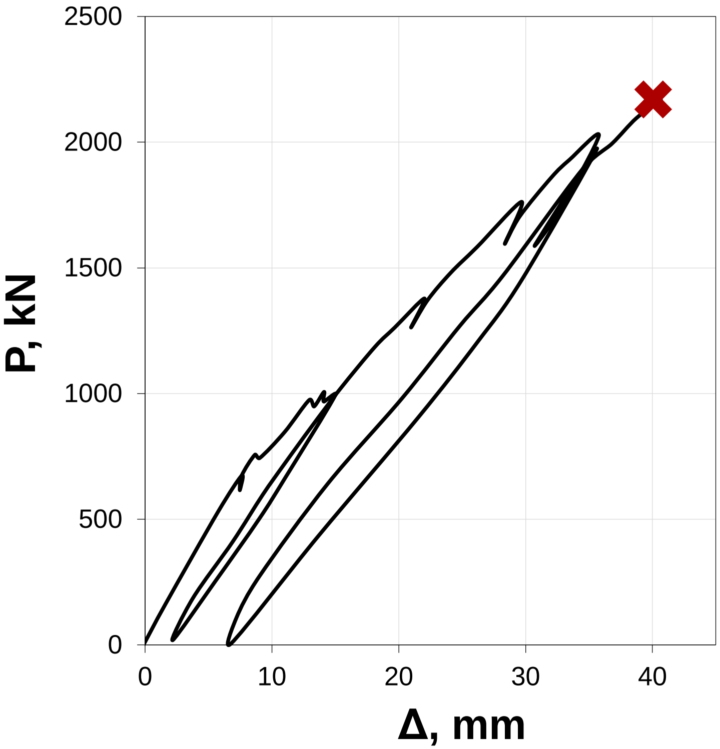


f_c = 40 MPa a_g = 14 mm

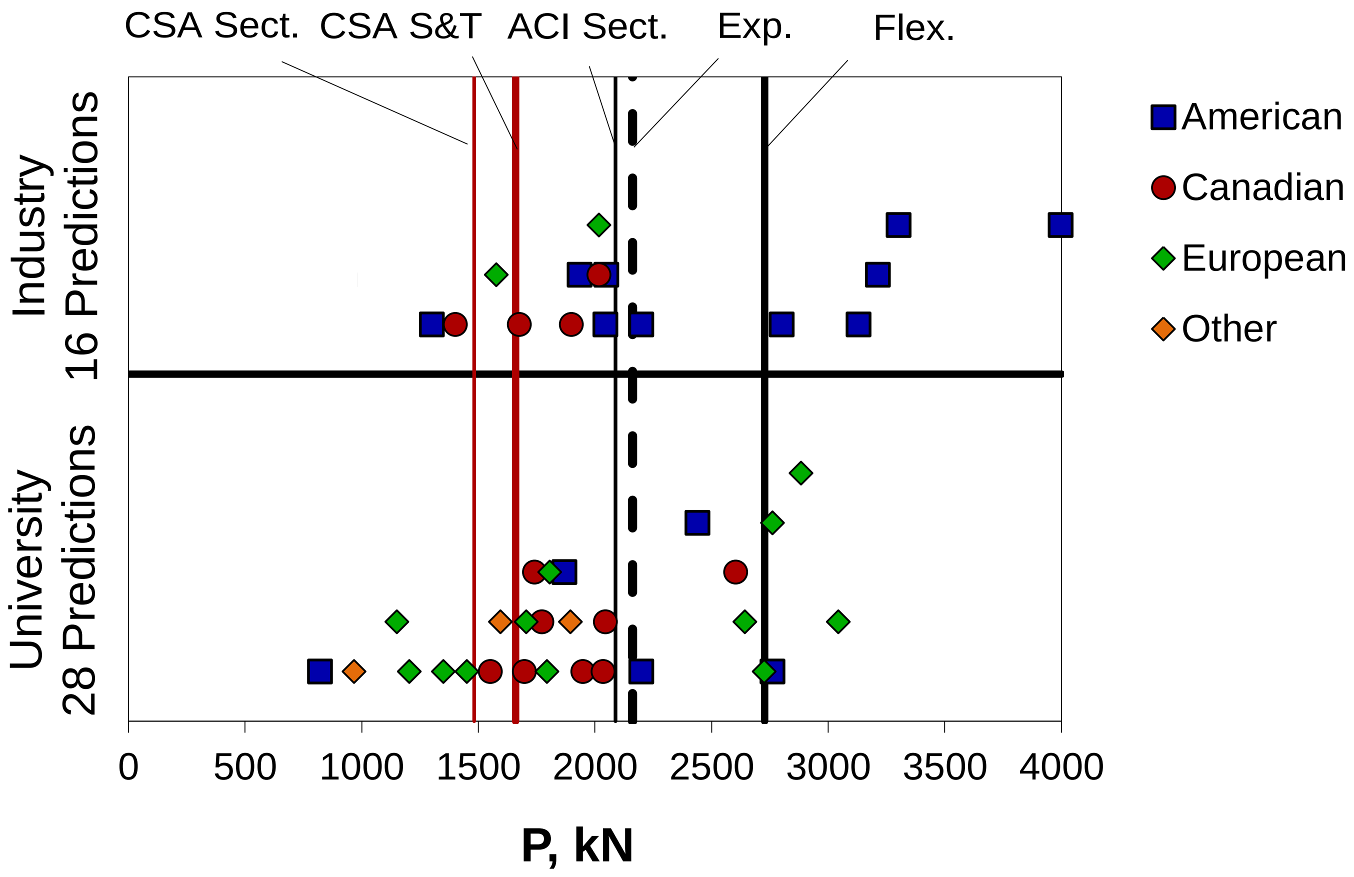
d = 3840 mm b = 250 m

ρ_l = 0.656% f_y = 573 MPa





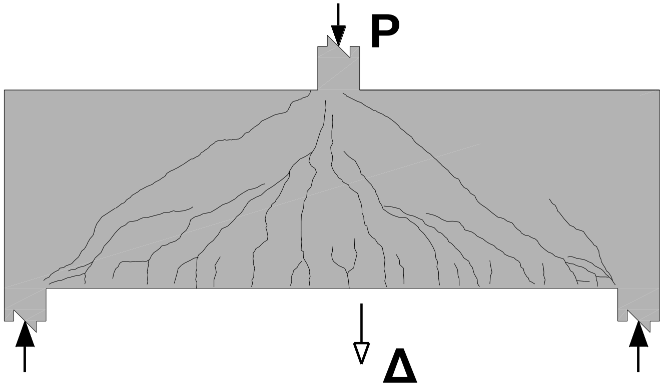
Difficulty in predicting shear strength of deep transfer girders



Objective 1): To evaluate the accuracy of existing models for shear resistance of deep beams by using a large database of laboratory tests.

Complete shear response of deep transfer girders

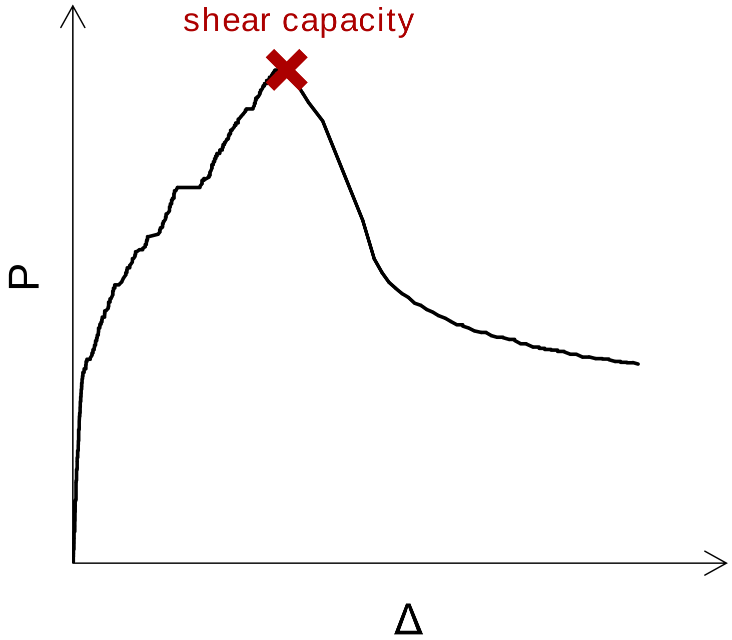
- **Deep transfer girder**



Objective 2):

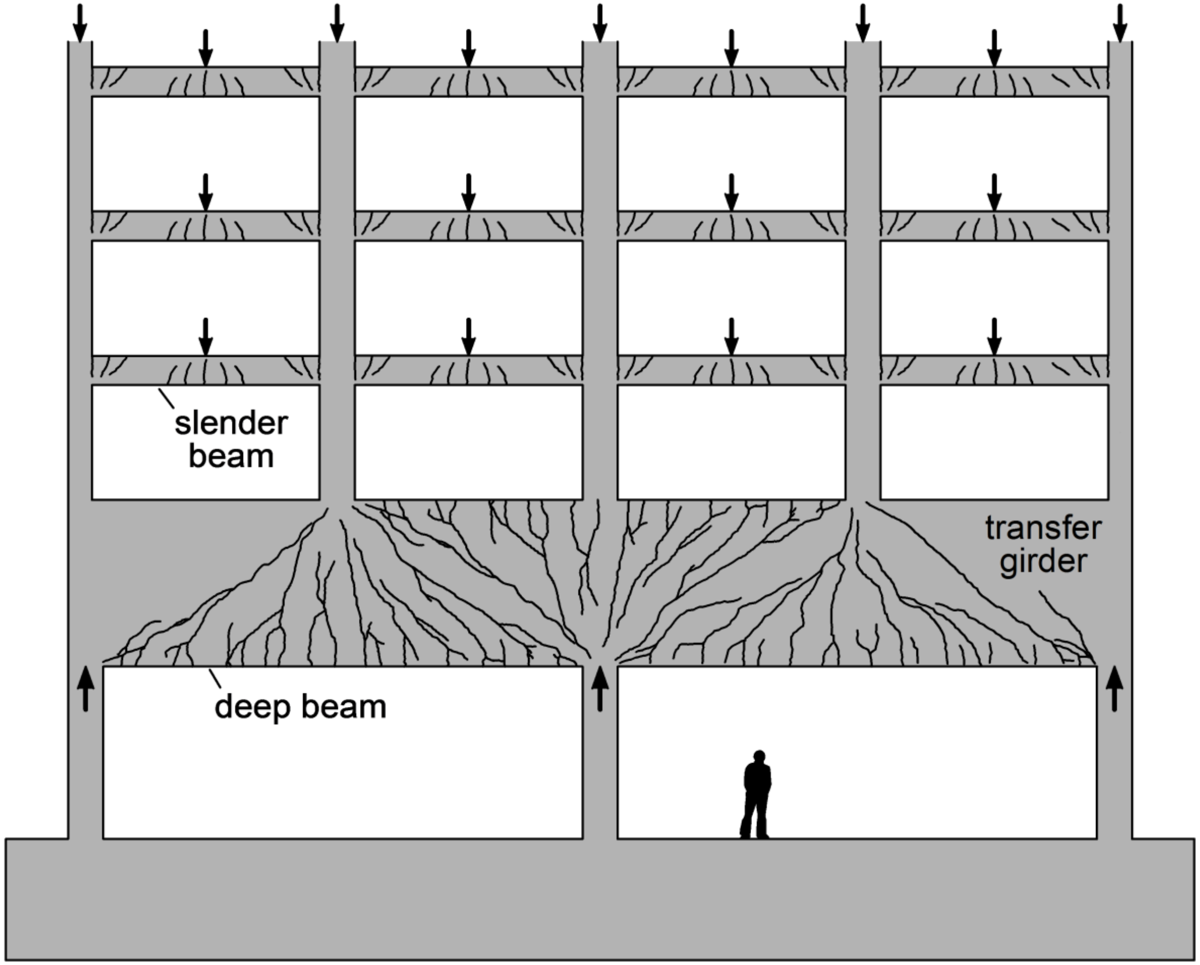
To develop 1D element for deep beams combining accuracy and efficiency.

- **Complete shear response**



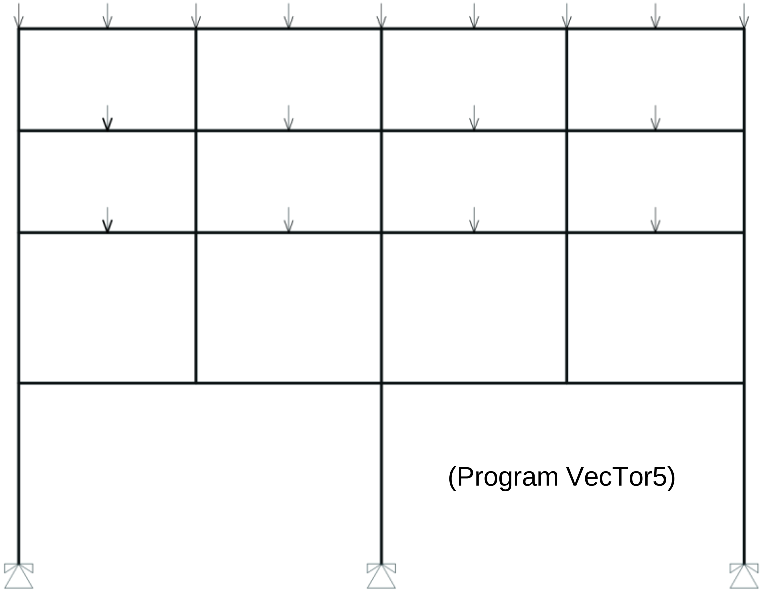
- Serviceability
- Ductility
- Resilience
- Structure-soil interaction
- …

Large frame structure with deep transfer girders



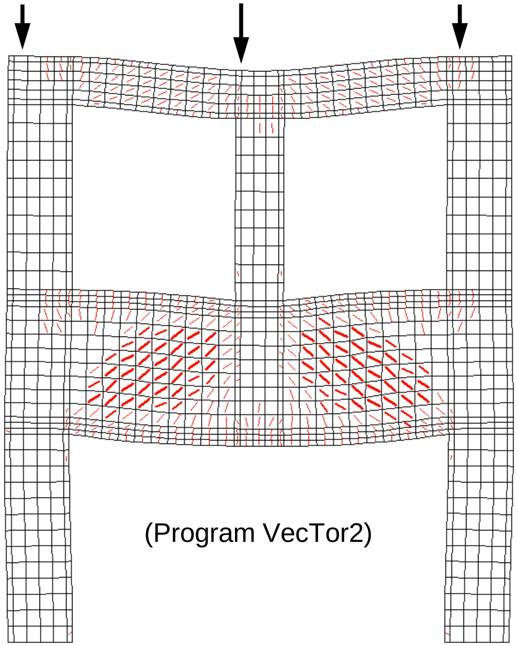
Modelling of frame structures with deep transfer girders

- **Model with 1D frame elements**


(Program VecTor5)

- computationally efficient
- inaccurate for deep beams

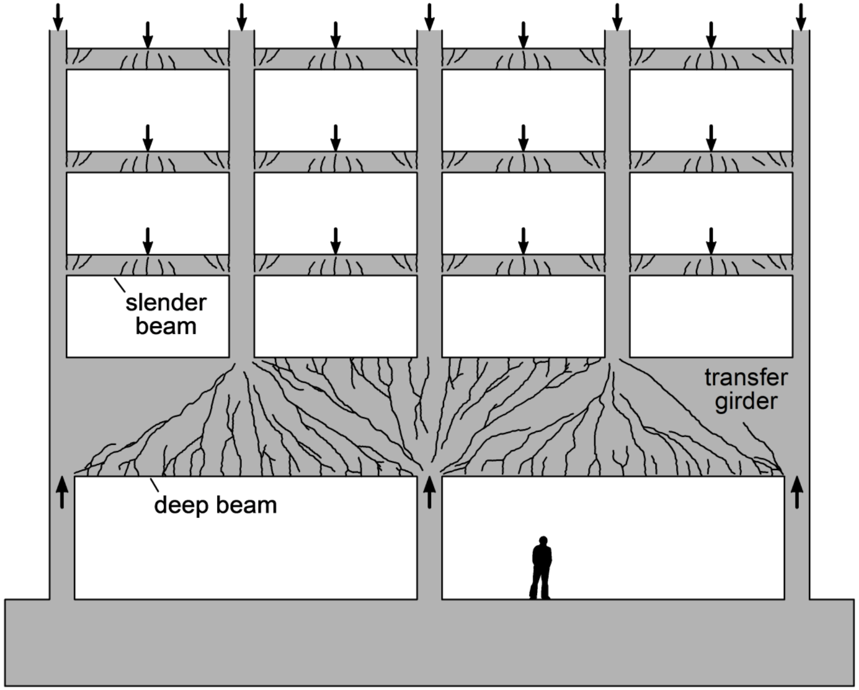
- **Model with 2D elements**


(Program VecTor2)

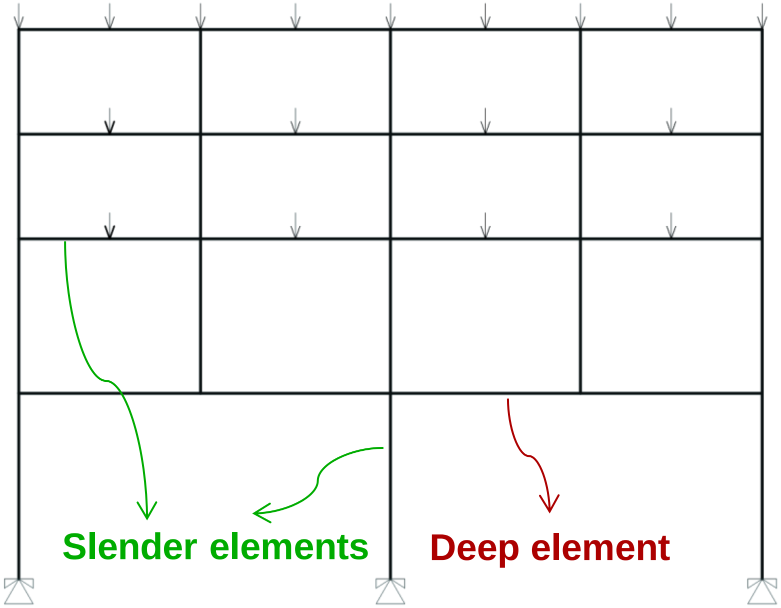
- complex for large structures
- suitable for deep beams

Modelling of large structures with deep beams

- **Large frame structure with deep transfer girder**

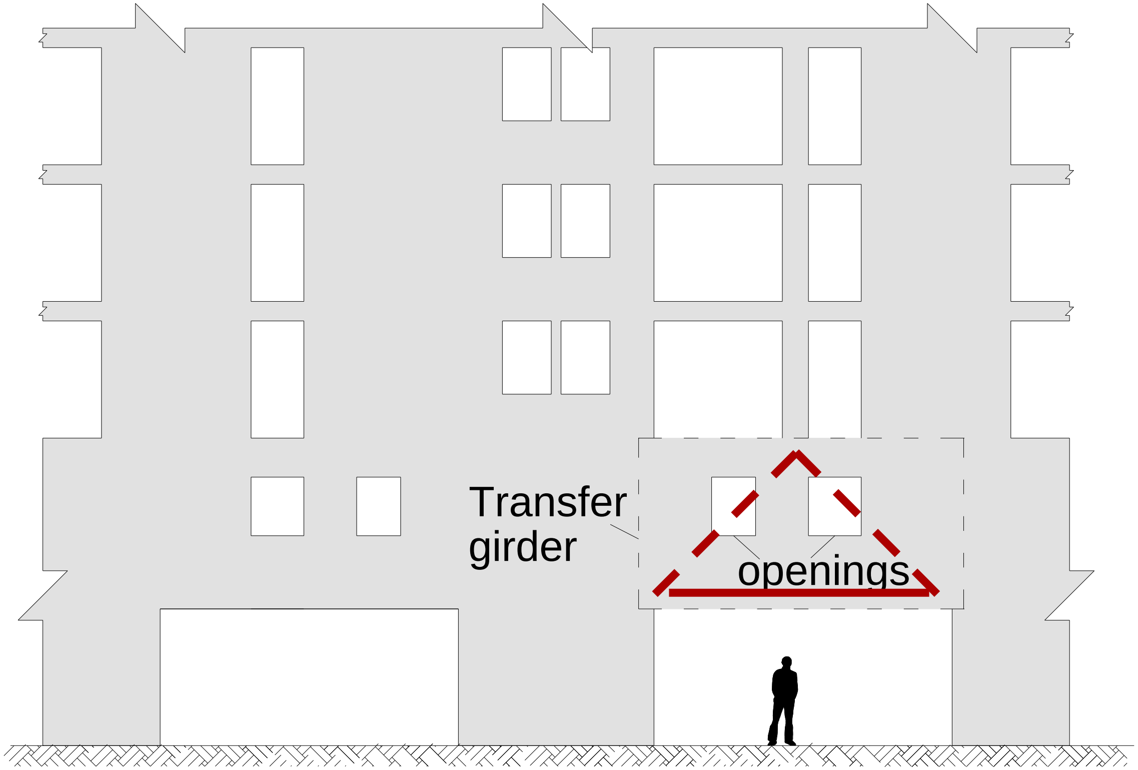


- **Model with 1D slender and deep elements**



Objective 3): To integrate the new model into a framework of frame structures with both slender and deep elements.

Deep transfer girder with web openings

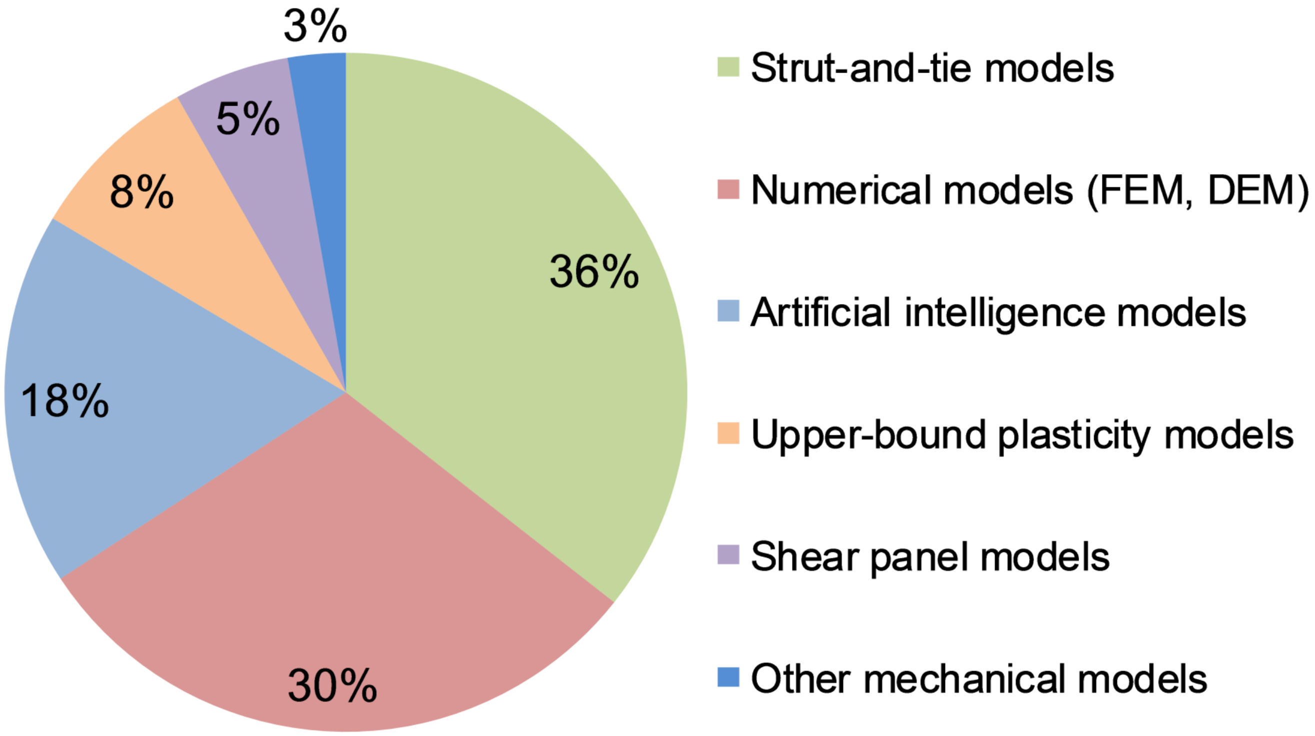


Objective 4): To propose a model to predict the shear capacity of RC deep beams with web openings.

Comparative Study on Models for Shear Strength of RC Deep Beams

Classification of models

73 existing models published between 1987 and 2014:



Comparative study procedure

Collect a large database of tested deep beams

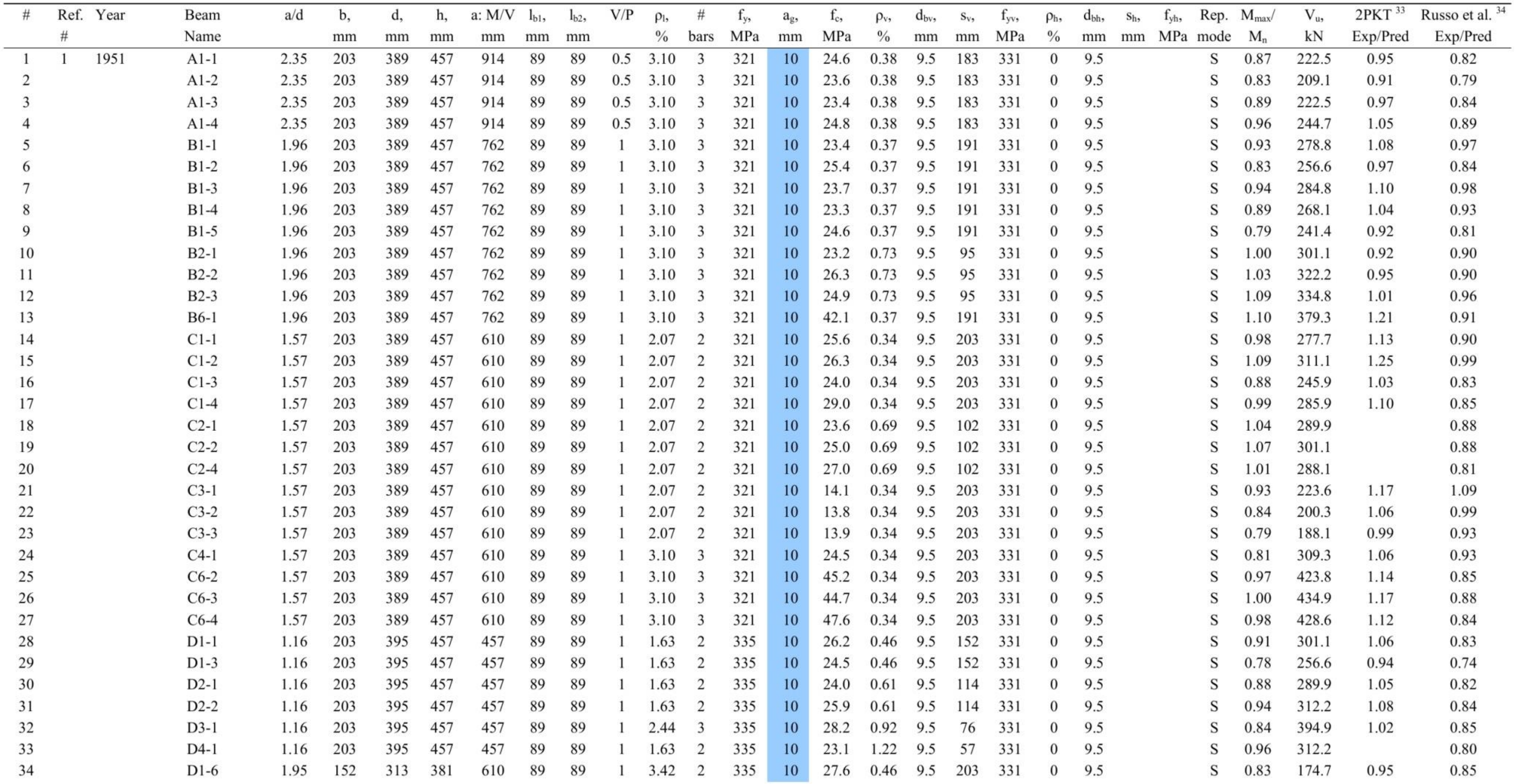
↓

Implement ten representative models

↓

Predicted vs. experimental shear strength

Database of 574 RC deep beams

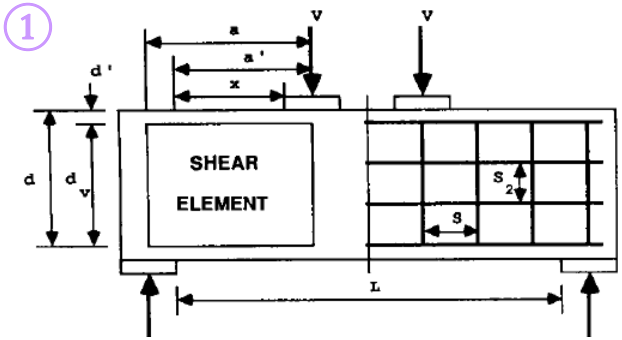


#	Ref. #	Year	Beam Name	a/d	b, mm	d, mm	h, mm	a: M/V mm	l_{b1}, mm	l_{b2}, mm	V/P	ρ_l, %	# bars	f_y, MPa	a_g, mm	f_c, MPa	ρ_v, %	d_{bv}, mm	s_v, mm	f_{yv}, MPa	ρ_h, %	d_{bh}, mm	s_h, mm	f_{yh}, MPa	Rep. mode	M_{max}/ M_n	V_u, kN	2PKT [33] Exp/Pred	Russo et al. [34] Exp/Pred
1	1	1951	A1-1	2.35	203	389	457	914	89	89	0.5	3.10	3	321	10	24.6	0.38	9.5	183	331	0	9.5			S	0.87	222.5	0.95	0.82
2			A1-2	2.35	203	389	457	914	89	89	0.5	3.10	3	321	10	23.6	0.38	9.5	183	331	0	9.5			S	0.83	209.1	0.91	0.79
3			A1-3	2.35	203	389	457	914	89	89	0.5	3.10	3	321	10	23.4	0.38	9.5	183	331	0	9.5			S	0.89	222.5	0.97	0.84
4			A1-4	2.35	203	389	457	914	89	89	0.5	3.10	3	321	10	24.8	0.38	9.5	183	331	0	9.5			S	0.96	244.7	1.05	0.89
5			B1-1	1.96	203	389	457	762	89	89	1	3.10	3	321	10	23.4	0.37	9.5	191	331	0	9.5			S	0.93	278.8	1.08	0.97
6			B1-2	1.96	203	389	457	762	89	89	1	3.10	3	321	10	25.4	0.37	9.5	191	331	0	9.5			S	0.83	256.6	0.97	0.84
7			B1-3	1.96	203	389	457	762	89	89	1	3.10	3	321	10	23.7	0.37	9.5	191	331	0	9.5			S	0.94	284.8	1.10	0.98
8			B1-4	1.96	203	389	457	762	89	89	1	3.10	3	321	10	23.3	0.37	9.5	191	331	0	9.5			S	0.89	268.1	1.04	0.93
9			B1-5	1.96	203	389	457	762	89	89	1	3.10	3	321	10	24.6	0.37	9.5	191	331	0	9.5			S	0.79	241.4	0.92	0.81
10			B2-1	1.96	203	389	457	762	89	89	1	3.10	3	321	10	23.2	0.73	9.5	95	331	0	9.5			S	1.00	301.1	0.92	0.90
11			B2-2	1.96	203	389	457	762	89	89	1	3.10	3	321	10	26.3	0.73	9.5	95	331	0	9.5			S	1.03	322.2	0.95	0.90
12			B2-3	1.96	203	389	457	762	89	89	1	3.10	3	321	10	24.9	0.73	9.5	95	331	0	9.5			S	1.09	334.8	1.01	0.96
13			B6-1	1.96	203	389	457	762	89	89	1	3.10	3	321	10	42.1	0.37	9.5	191	331	0	9.5			S	1.10	379.3	1.21	0.91
14			C1-1	1.57	203	389	457	610	89	89	1	2.07	2	321	10	25.6	0.34	9.5	203	331	0	9.5			S	0.98	277.7	1.13	0.90
15			C1-2	1.57	203	389	457	610	89	89	1	2.07	2	321	10	26.3	0.34	9.5	203	331	0	9.5			S	1.09	311.1	1.25	0.99
16			C1-3	1.57	203	389	457	610	89	89	1	2.07	2	321	10	24.0	0.34	9.5	203	331	0	9.5			S	0.88	245.9	1.03	0.83
17			C1-4	1.57	203	389	457	610	89	89	1	2.07	2	321	10	29.0	0.34	9.5	203	331	0	9.5			S	0.99	285.9	1.10	0.85
18			C2-1	1.57	203	389	457	610	89	89	1	2.07	2	321	10	23.6	0.69	9.5	102	331	0	9.5			S	1.04	289.9		0.88
19			C2-2	1.57	203	389	457	610	89	89	1	2.07	2	321	10	25.0	0.69	9.5	102	331	0	9.5			S	1.07	301.1		0.88
20			C2-4	1.57	203	389	457	610	89	89	1	2.07	2	321	10	27.0	0.69	9.5	102	331	0	9.5			S	1.01	288.1		0.81
21			C3-1	1.57	203	389	457	610	89	89	1	2.07	2	321	10	14.1	0.34	9.5	203	331	0	9.5			S	0.93	223.6	1.17	1.09
22			C3-2	1.57	203	389	457	610	89	89	1	2.07	2	321	10	13.8	0.34	9.5	203	331	0	9.5			S	0.84	200.3	1.06	0.99
23			C3-3	1.57	203	389	457	610	89	89	1	2.07	2	321	10	13.9	0.34	9.5	203	331	0	9.5			S	0.79	188.1	0.99	0.93
24			C4-1	1.57	203	389	457	610	89	89	1	3.10	3	321	10	24.5	0.34	9.5	203	331	0	9.5			S	0.81	309.3	1.06	0.93
25			C6-2	1.57	203	389	457	610	89	89	1	3.10	3	321	10	45.2	0.34	9.5	203	331	0	9.5			S	0.97	423.8	1.14	0.85
26			C6-3	1.57	203	389	457	610	89	89	1	3.10	3	321	10	44.7	0.34	9.5	203	331	0	9.5			S	1.00	434.9	1.17	0.88
27			C6-4	1.57	203	389	457	610	89	89	1	3.10	3	321	10	47.6	0.34	9.5	203	331	0	9.5			S	0.98	428.6	1.12	0.84
28			D1-1	1.16	203	395	457	457	89	89	1	1.63	2	335	10	26.2	0.46	9.5	152	331	0	9.5			S	0.91	301.1	1.06	0.83
29			D1-3	1.16	203	395	457	457	89	89	1	1.63	2	335	10	24.5	0.46	9.5	152	331	0	9.5			S	0.78	256.6	0.94	0.74
30			D2-1	1.16	203	395	457	457	89	89	1	1.63	2	335	10	24.0	0.61	9.5	114	331	0	9.5			S	0.88	289.9	1.05	0.82
31			D2-2	1.16	203	395	457	457	89	89	1	1.63	2	335	10	25.9	0.61	9.5	114	331	0	9.5			S	0.94	312.2	1.08	0.84
32			D3-1	1.16	203	395	457	457	89	89	1	2.44	3	335	10	28.2	0.92	9.5	76	331	0	9.5			S	0.84	394.9	1.02	0.85
33			D4-1	1.16	203	395	457	457	89	89	1	1.63	2	335	10	23.1	1.22	9.5	57	331	0	9.5			S	0.96	312.2		0.80
34			D1-6	1.95	152	313	381	610	89	89	1	3.42	2	335	10	27.6	0.46	9.5	203	331	0	9.5			S	0.83	174.7	0.95	0.85

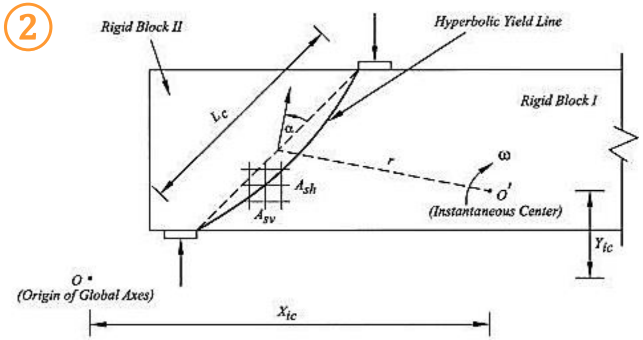
...

Ten implemented models

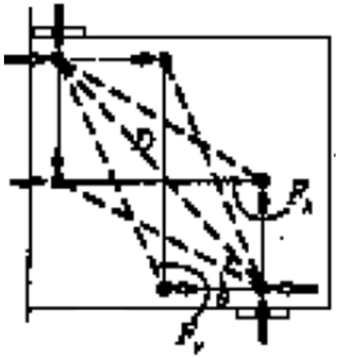
①



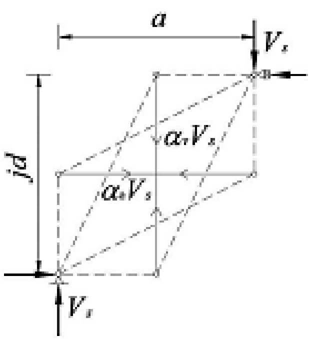
②



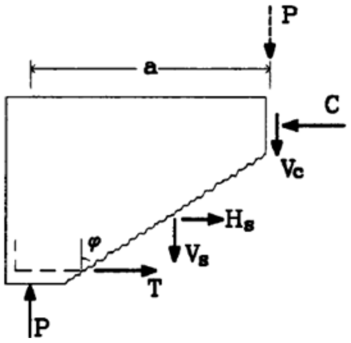
③



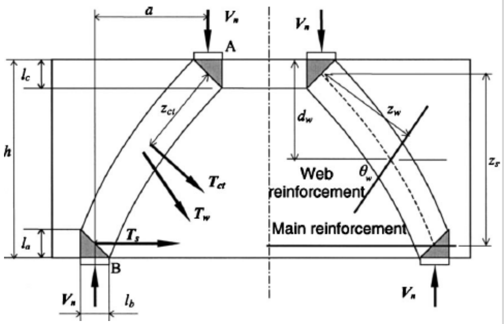
④



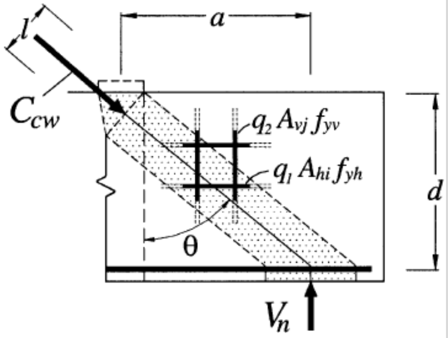
⑤



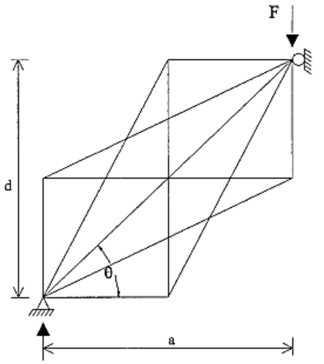
⑥



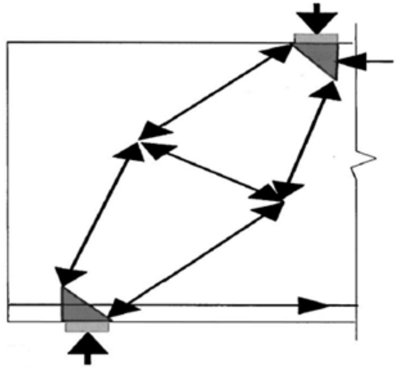
⑦



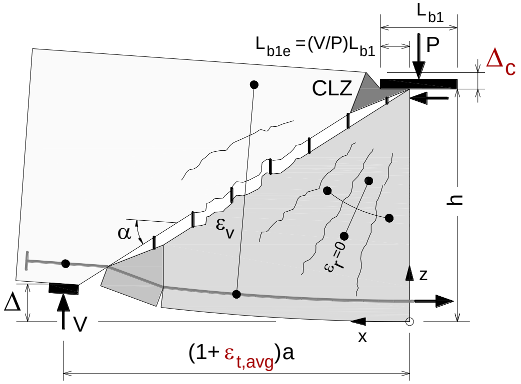
⑧



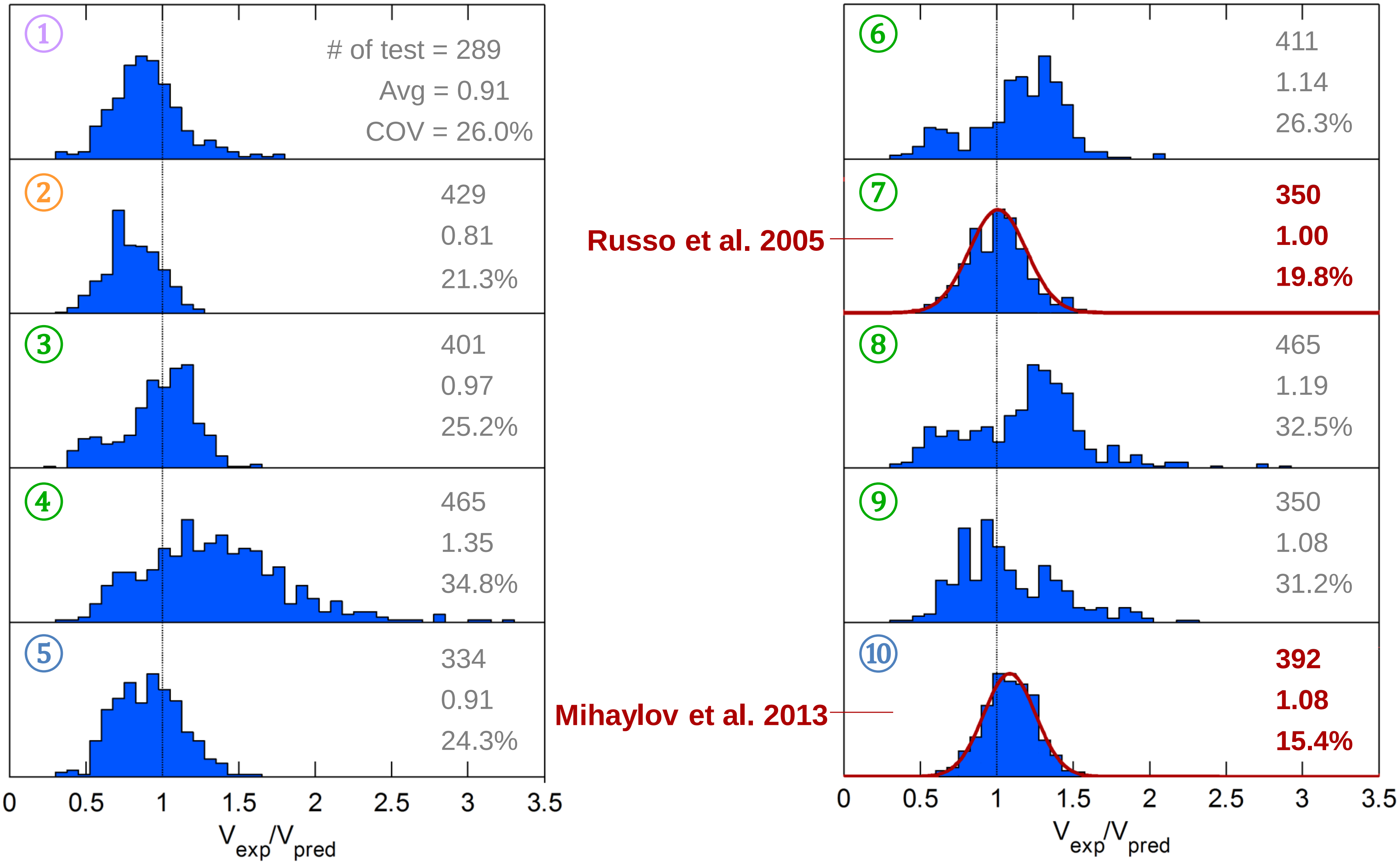
⑨



⑩



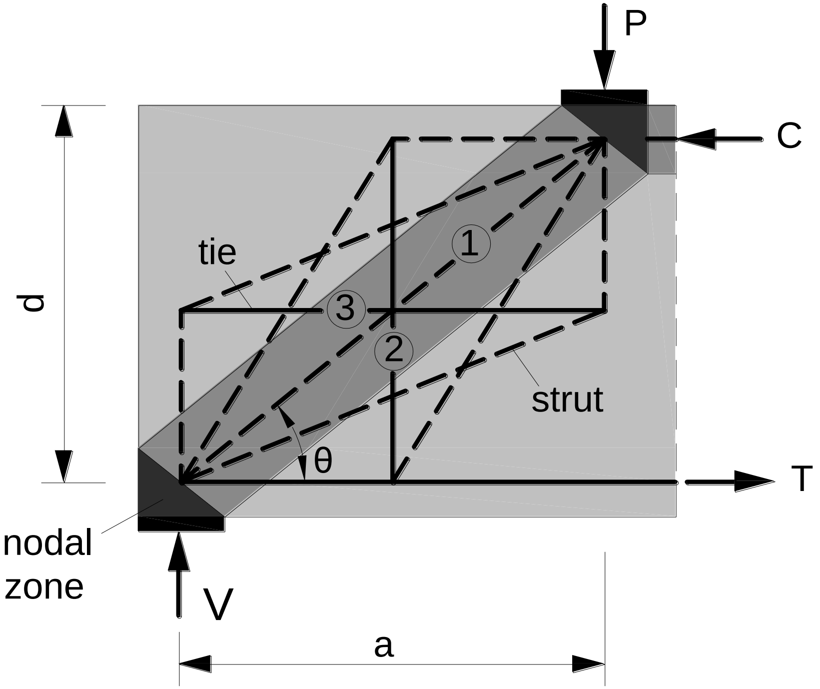
Shear strength predictions



Russo et al. 2005

Mihaylov et al. 2013

Strut-and-tie model by Russo et al., 2005



① — Diagonal strut

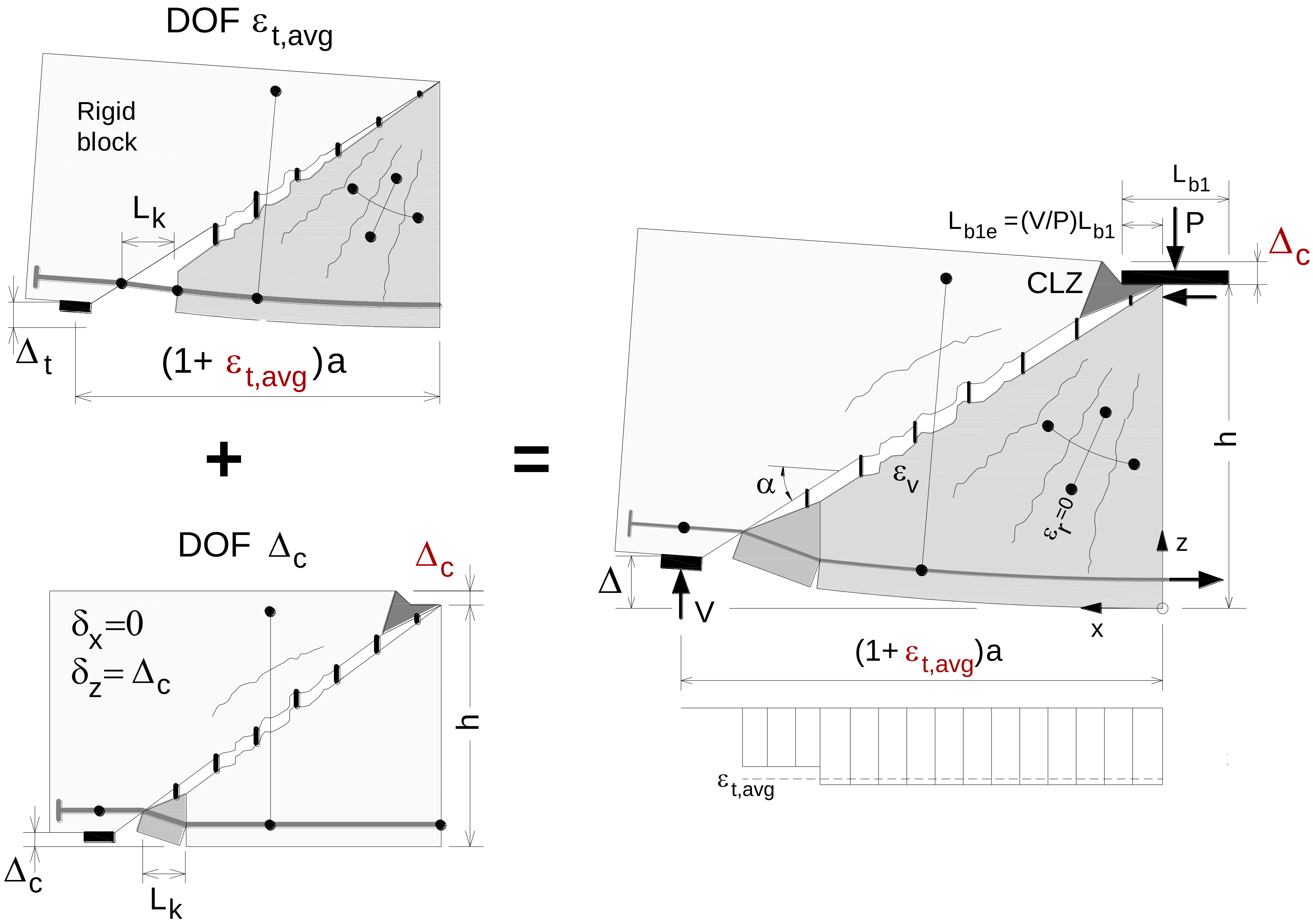
② — Vertical web reinforcement

③ — Horizontal web reinforcement

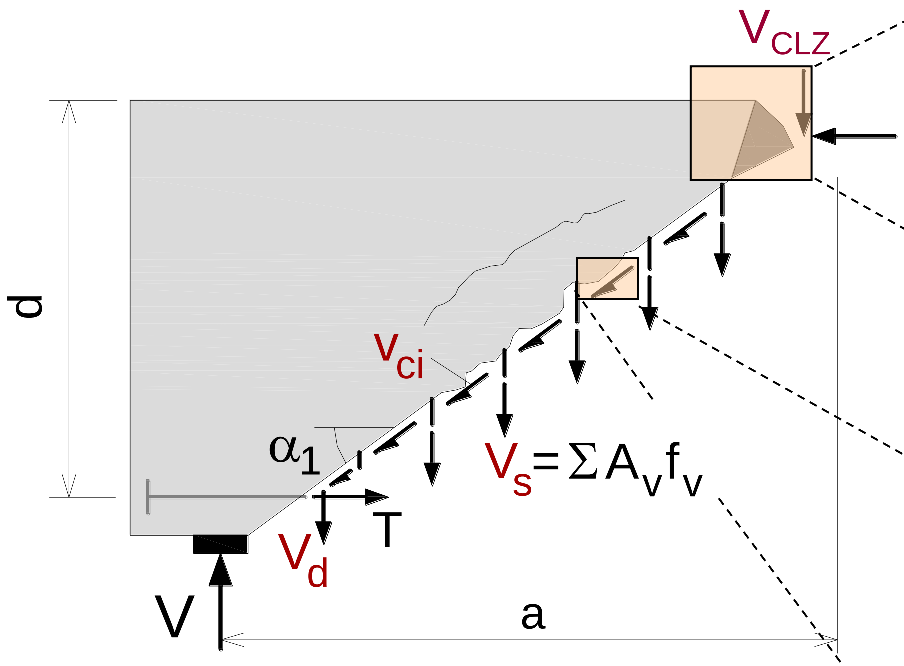
$$v = \frac{V}{bd} = v_c + v_w$$

$$v = \underset{0.76}{c_1}(\underline{k\chi f_c\cos\theta} + \underline{\underset{0.25}{c_2}\rho_h f_{yh}\cot\theta} + \underline{\underset{0.35}{c_3}\frac{a}{d}\rho_v f_{yv}})$$

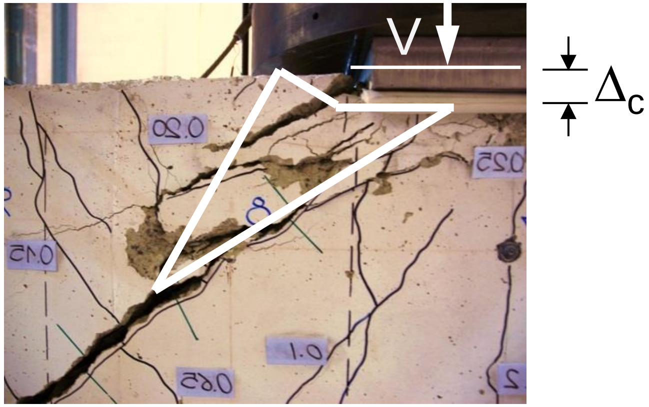
Two-parameter kinematic theory (2PKT) by Mihaylov et al., 2013



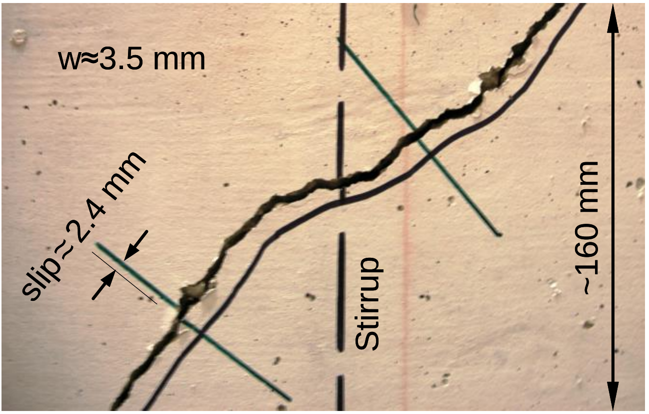
Shear components and solution procedure in 2PKT



Critical Loading Zone

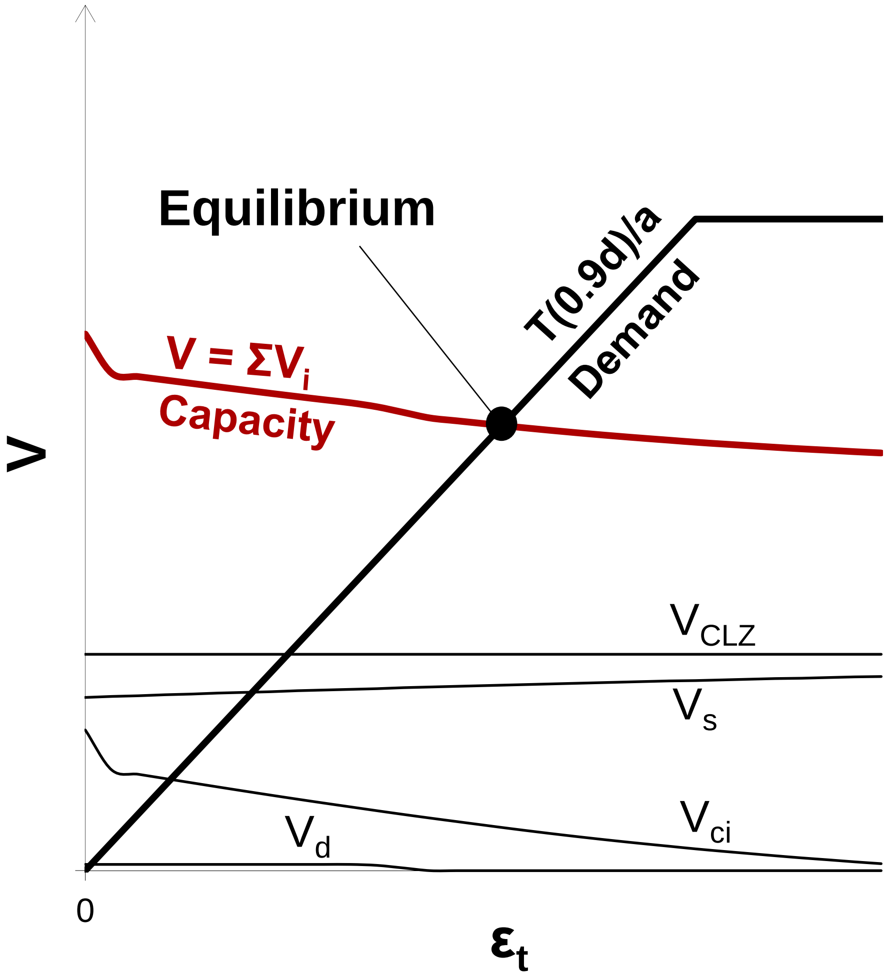


Aggregate interlock



$$V = \underbrace{V_{CLZ} + V_s + V_{ci} + V_d}_{\text{Vertical equilibrium}} = \underbrace{T(0.9d)/a}_{\text{Moment equilibrium}}$$

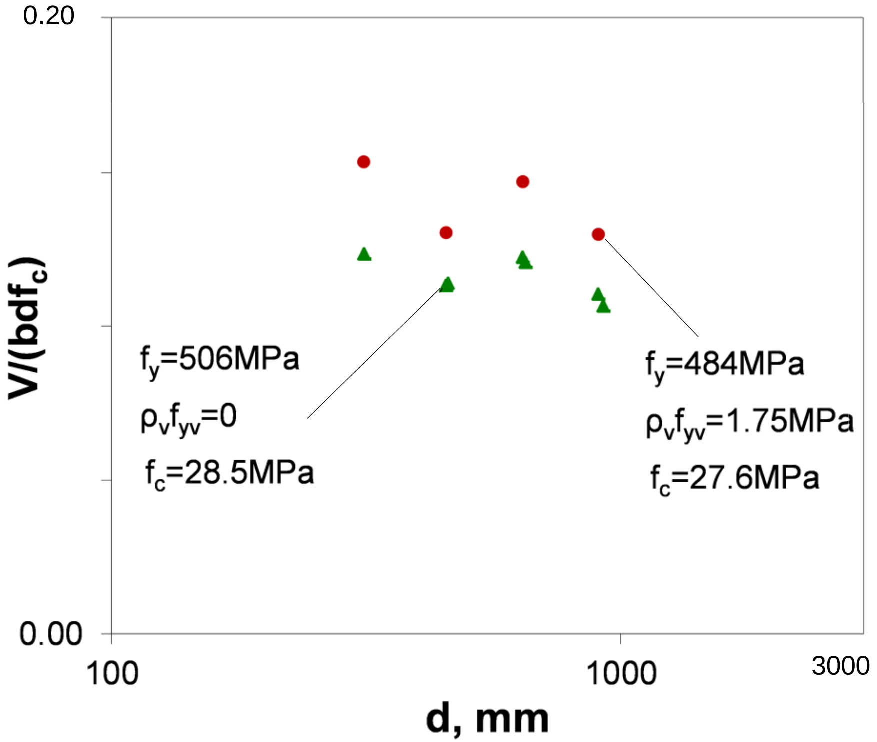
Solution procedure for 2PKT

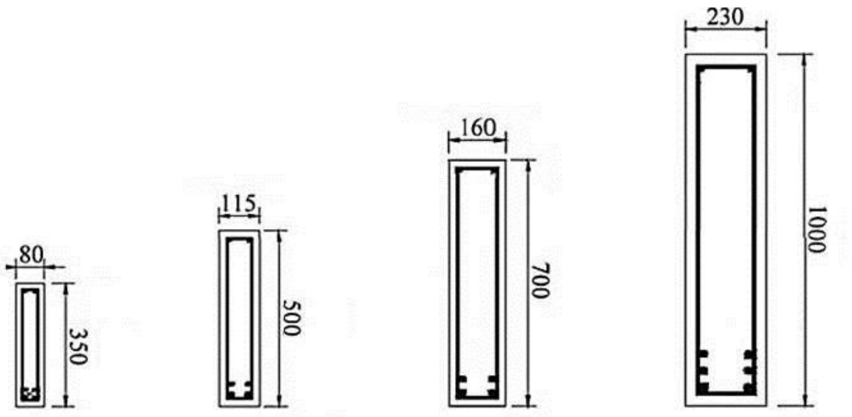


$$V = V_{CLZ} + V_s + V_{ci} + V_d$$

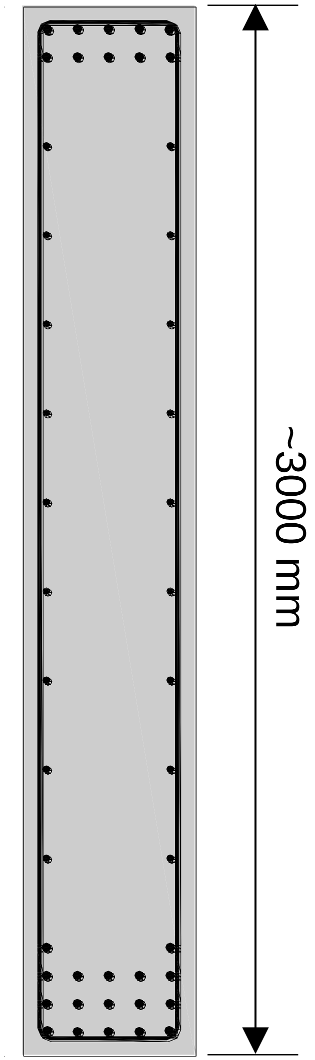
$$= T(0.9d)/a$$

Size effect in shear

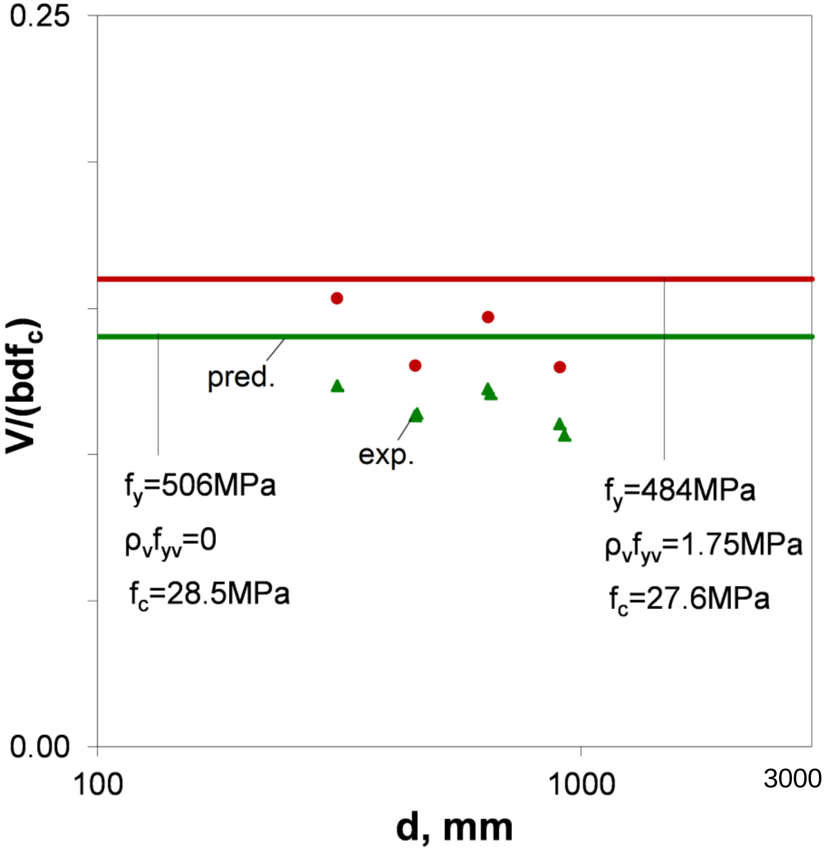




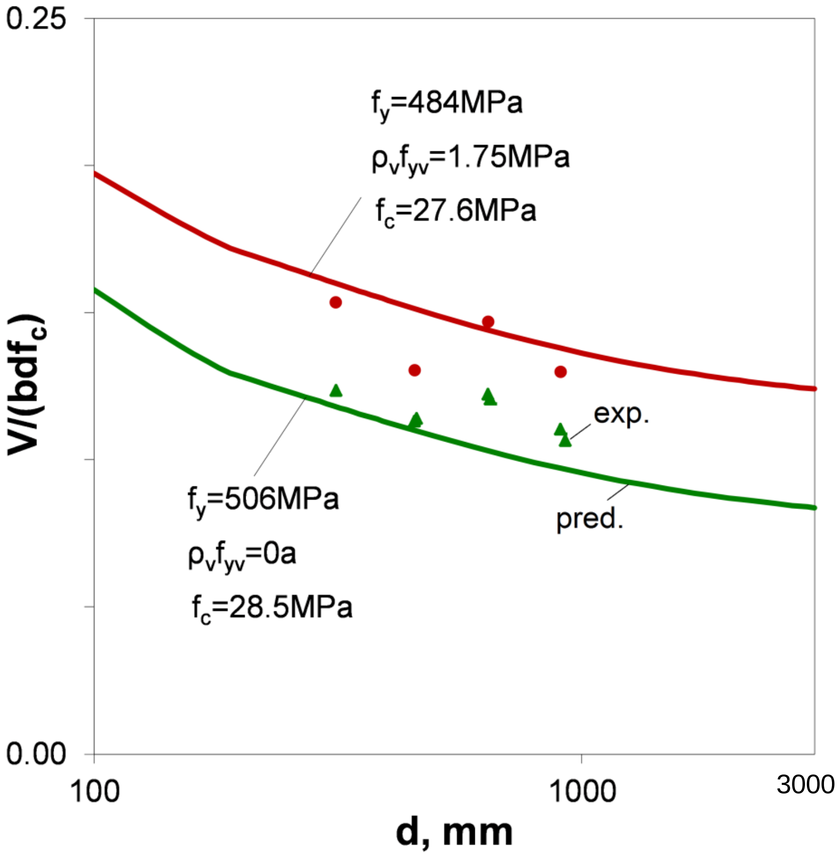
(tested by Zhang & Tan, 2007)



Predicting size effect in shear



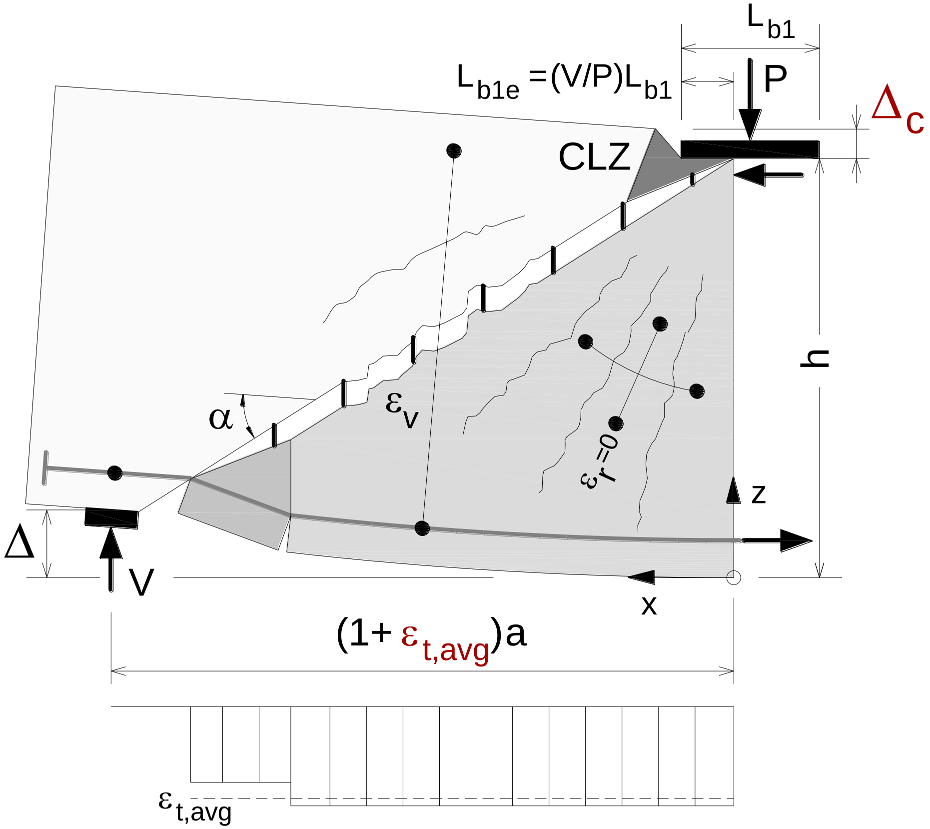
STM by Russo et al., 2005



2PKT by Mihaylov et al., 2013

2PKT by Mihaylov et al. (2013) provides adequate predictions.

Deformation prediction of 2PKT



- **Displacement at (x, z)**

Above the crack:

$$\delta_x(x,z) = \varepsilon_{t,avg}(h-z)\cot\alpha$$

$$\delta_z(x,z) = \varepsilon_{t,avg}x\cot\alpha + \Delta_c$$

Below the crack:

$$\delta_x(x,z) = \varepsilon_{t,avg}x$$

$$\delta_z(x,z) = \frac{\varepsilon_{t,avg}x^2}{h-z}$$

- **Crack width and slip:**

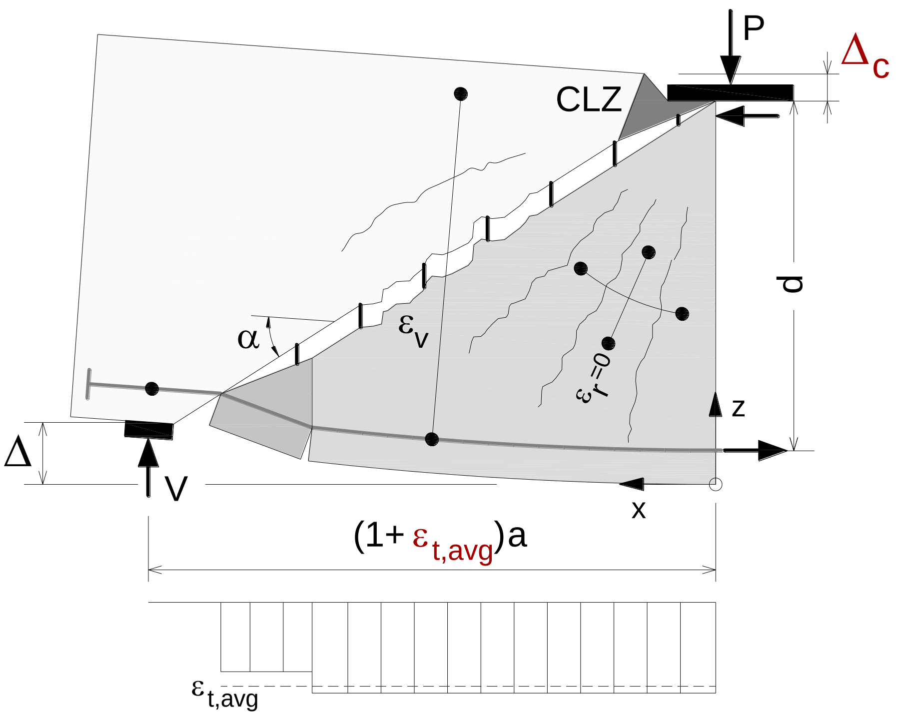
$$w = \varepsilon_{t,avg}\frac{l_k}{2\sin\alpha_1} + \Delta_c\cos\alpha_1$$

$$s = \Delta_c\sin\alpha_1$$

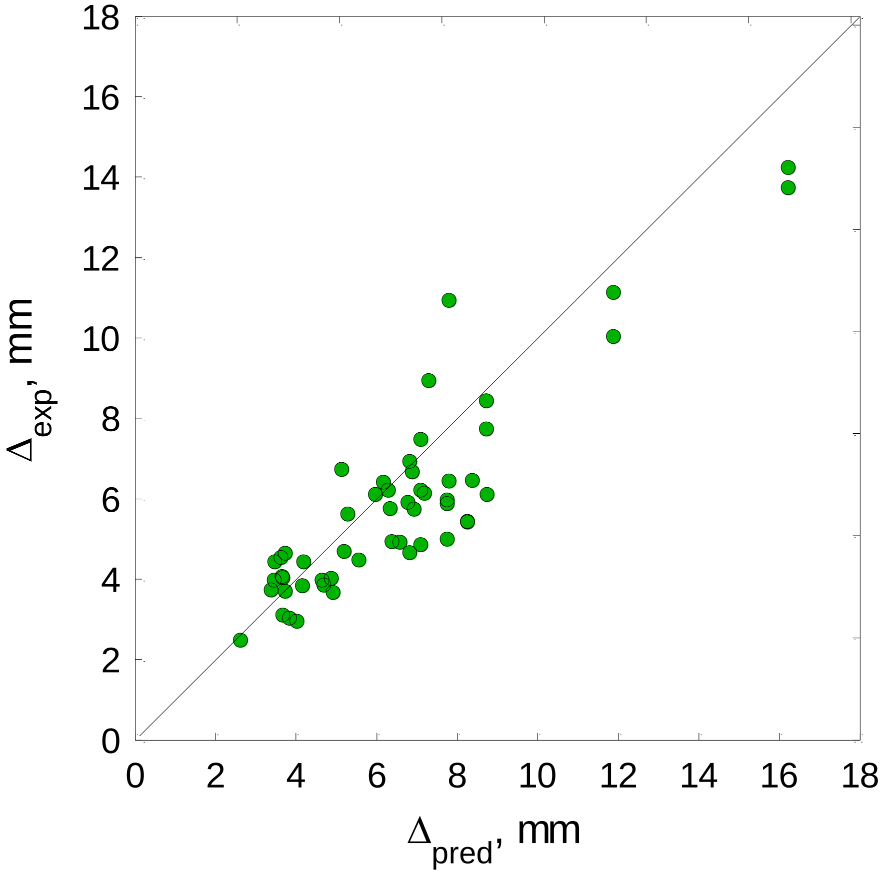
- **Deflection:**

$$\Delta = \Delta_c + \varepsilon_{t,avg}a\cot\alpha$$

Predicted displacement capacity, 53 tests

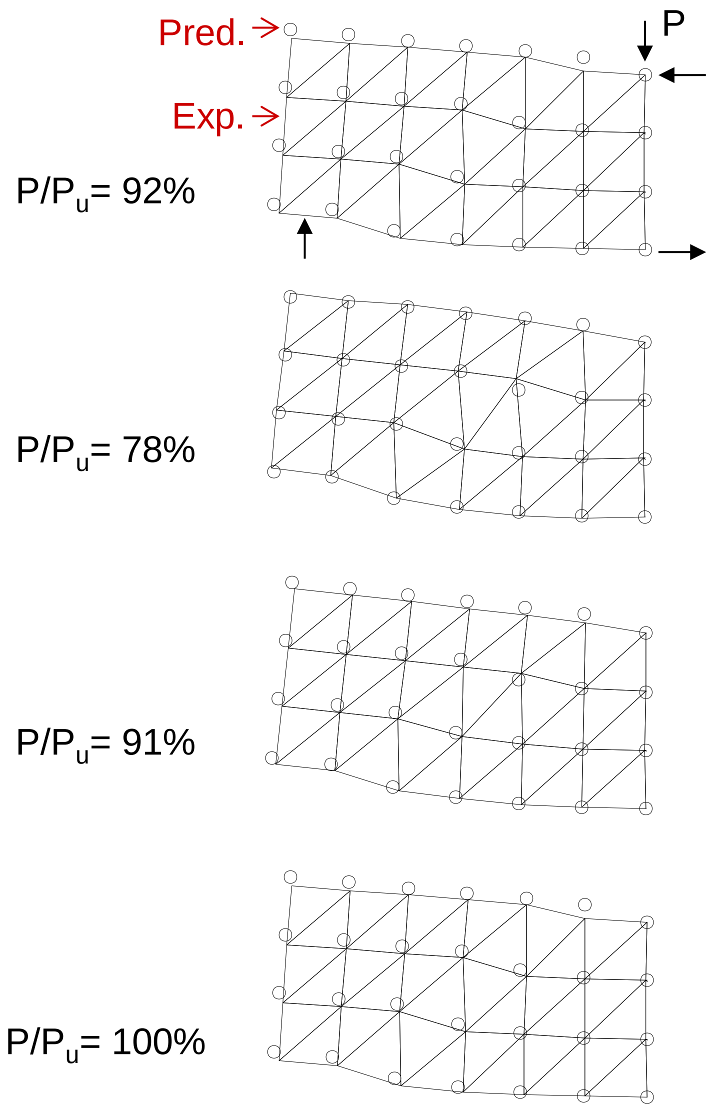


$$\Delta = \Delta_c + \varepsilon_{t,avg} a \cot\alpha$$

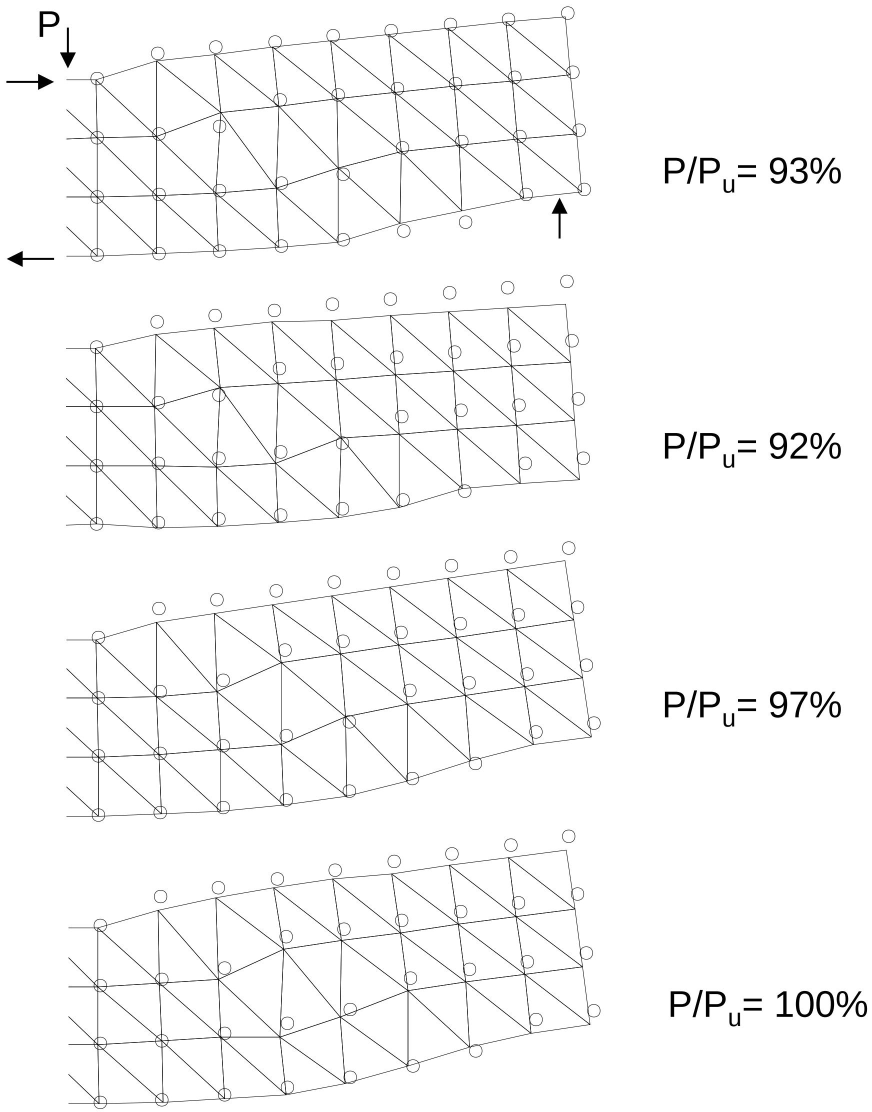


Predicted deformed shapes

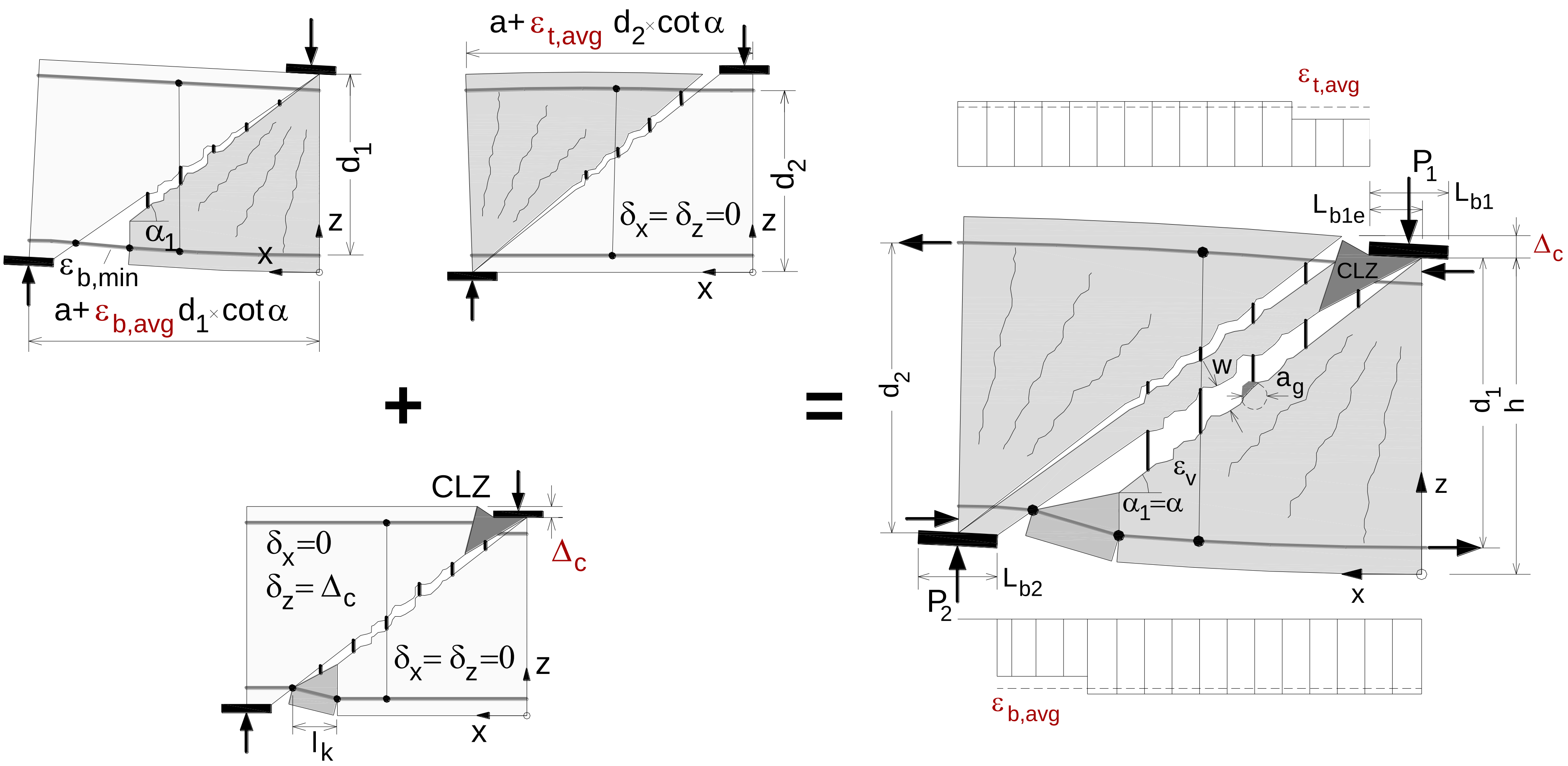
- **a/d = 1.55**



- **a/d = 2.29**

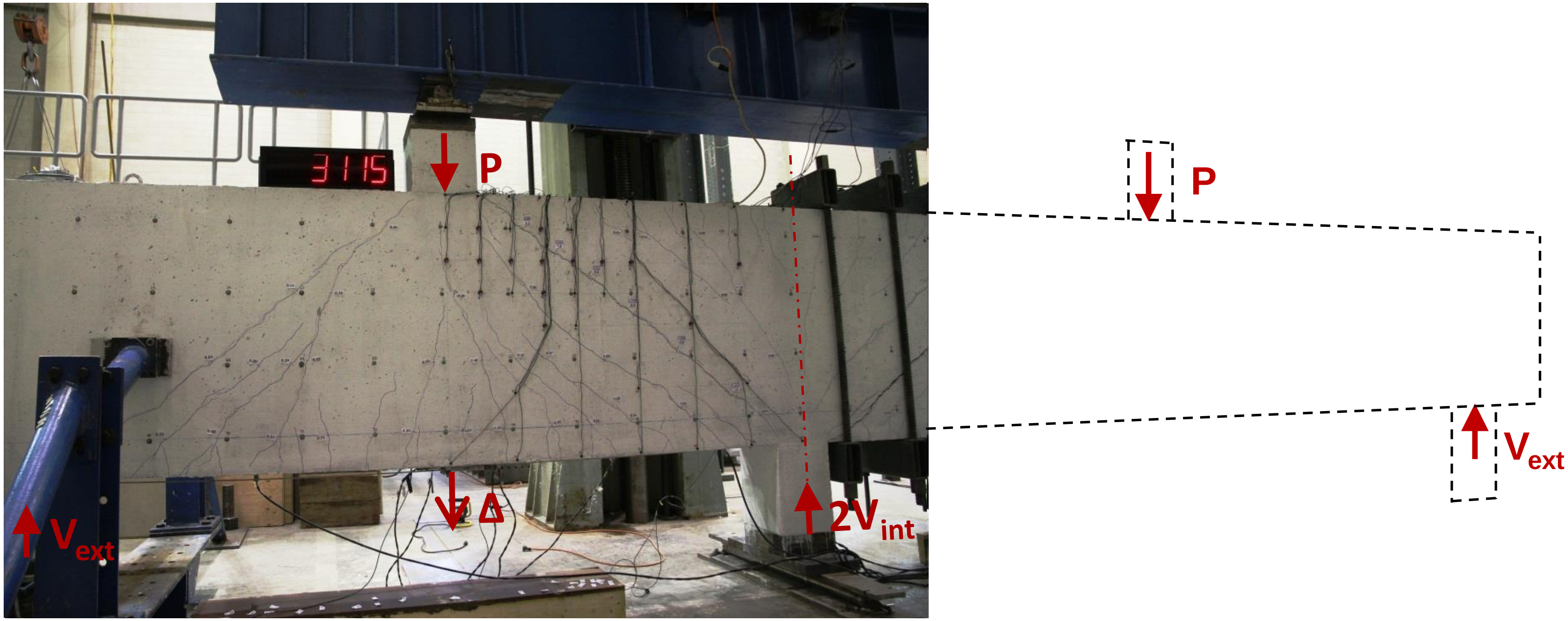


Three-parameter kinematic theory (3PKT) by Mihaylov et al., 2015



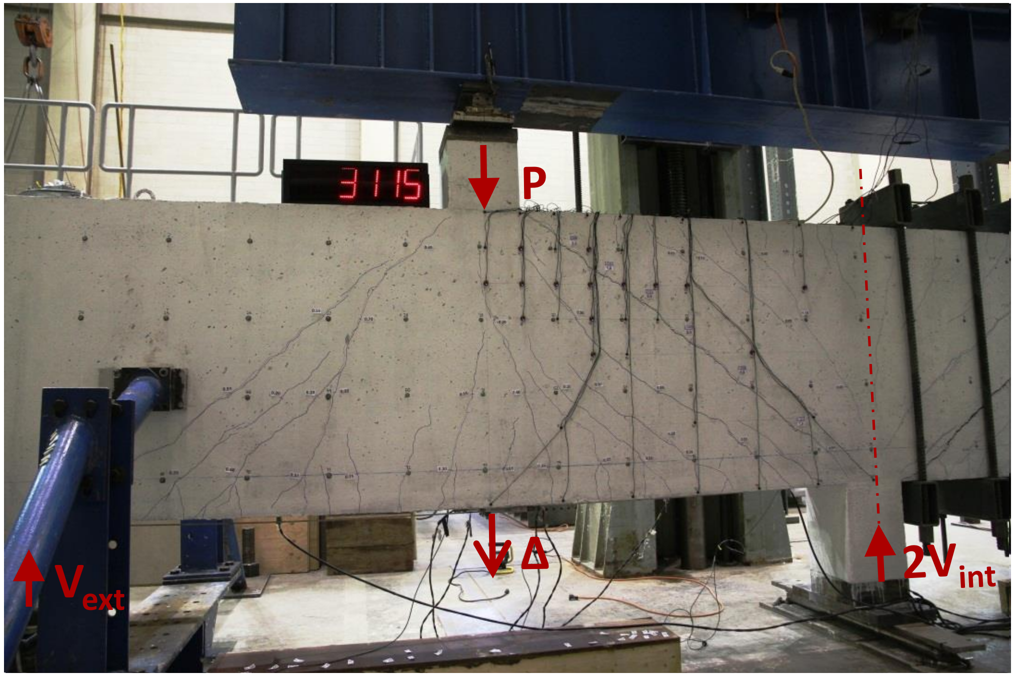
Macroelement for Complete Shear Behaviour of Deep Beams

Shear behaviour of deep beams

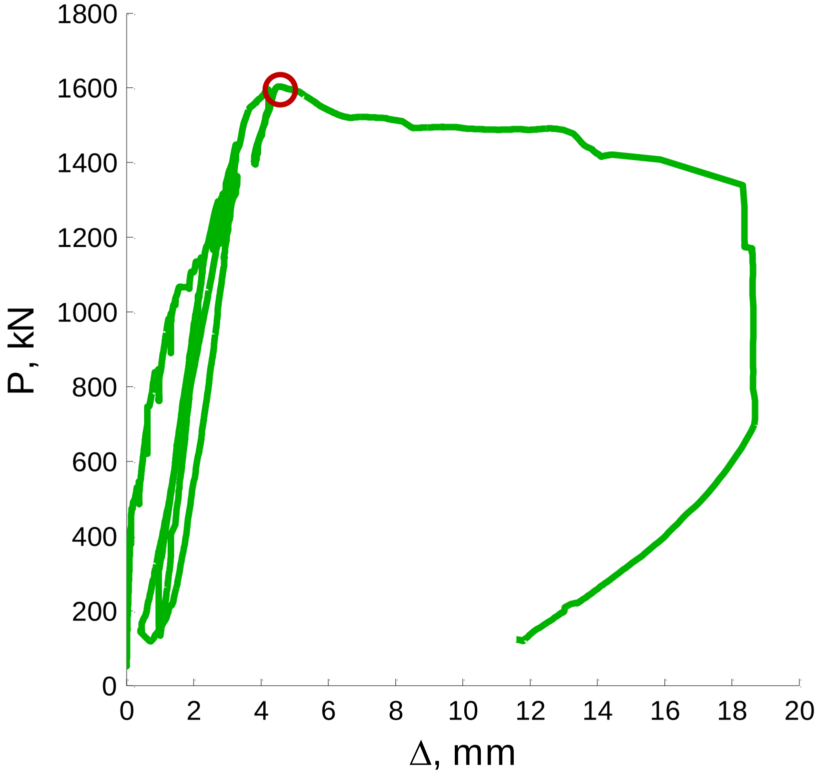


(tested by Mihaylov et al., 2015)

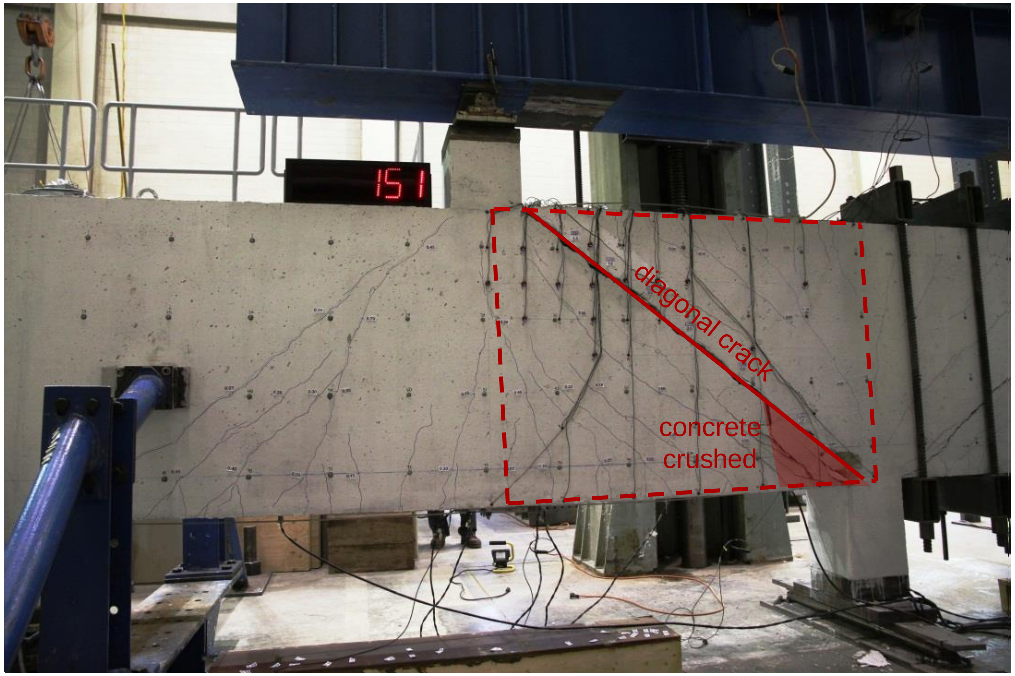
Shear behaviour of deep beams



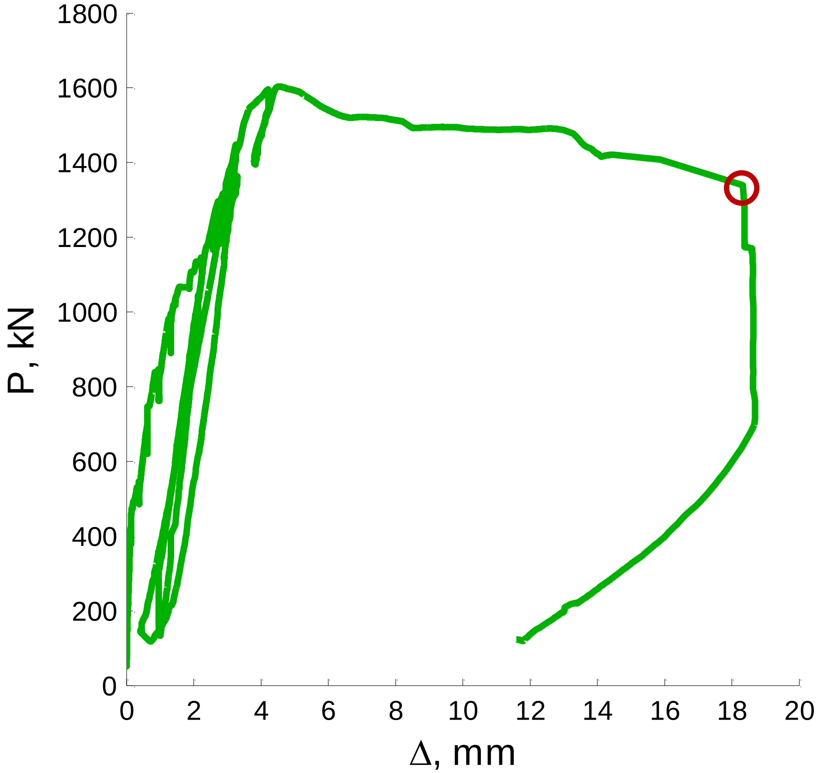
(tested by Mihaylov et al., 2015)



Shear behaviour of deep beams

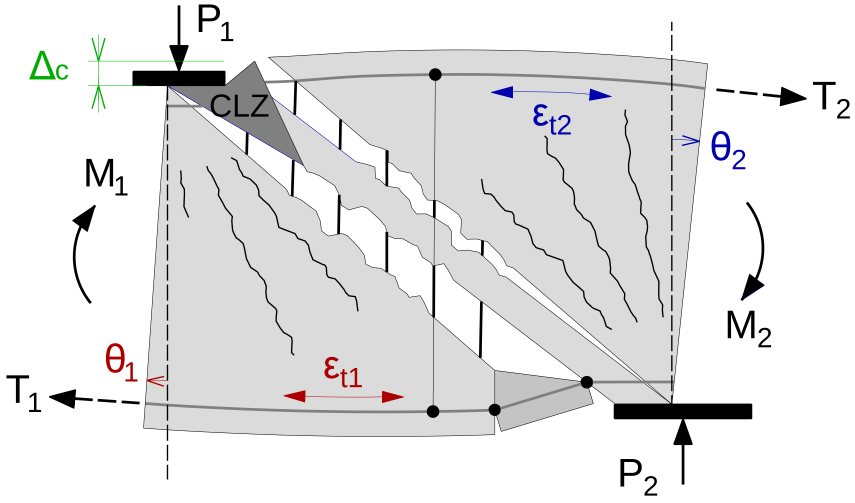


(tested by Mihaylov et al., 2015)



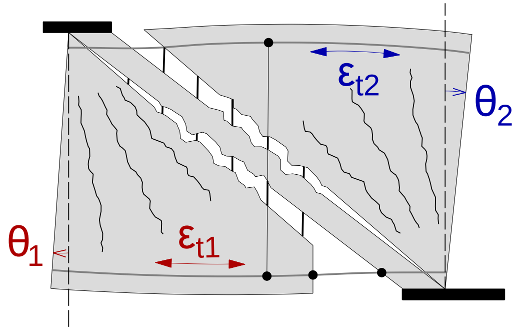
Three-parameter kinematic model for deep beams

Complete deformation pattern



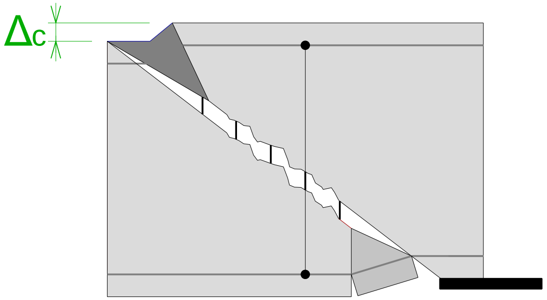
=

DOFs ε_{t1} and ε_{t2} (or θ_1 and θ_2)

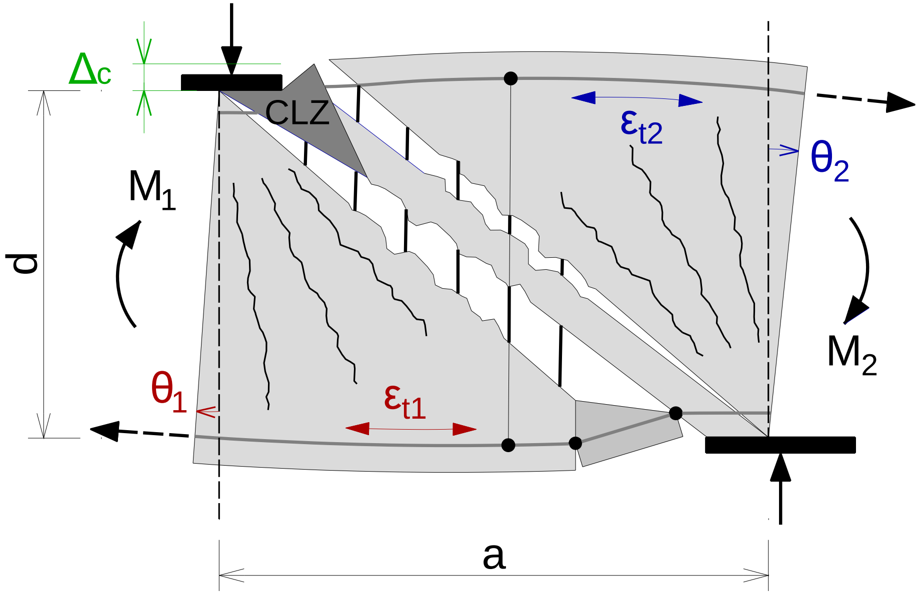


+

DOF Δ_c



Macroelement for deep beams

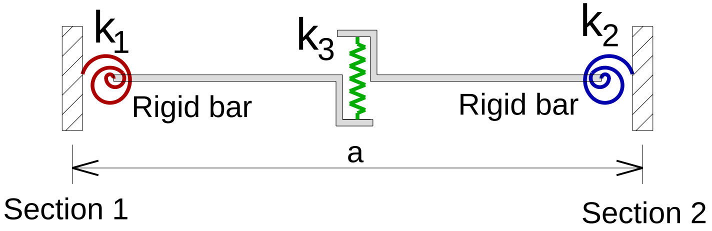


$$\theta_1 = \varepsilon_{t1}\, a / d$$

$$\theta_2 = \varepsilon_{t2}\, a / d$$

$$\mathbf{M_1 (\theta_1) + M_2 (\theta_2) = V (\Delta_c, \theta_1 ,\theta_2)\, a}$$

⇩



⇩

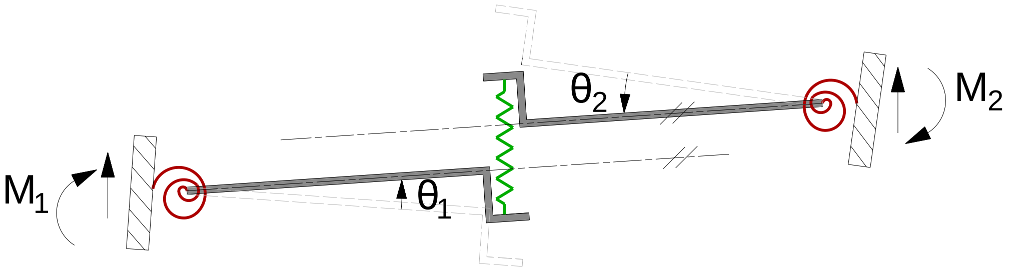
$$\mathbf{k_1 \theta_1 + k_2 \theta_2 = k_3 \Delta_c\, a}$$

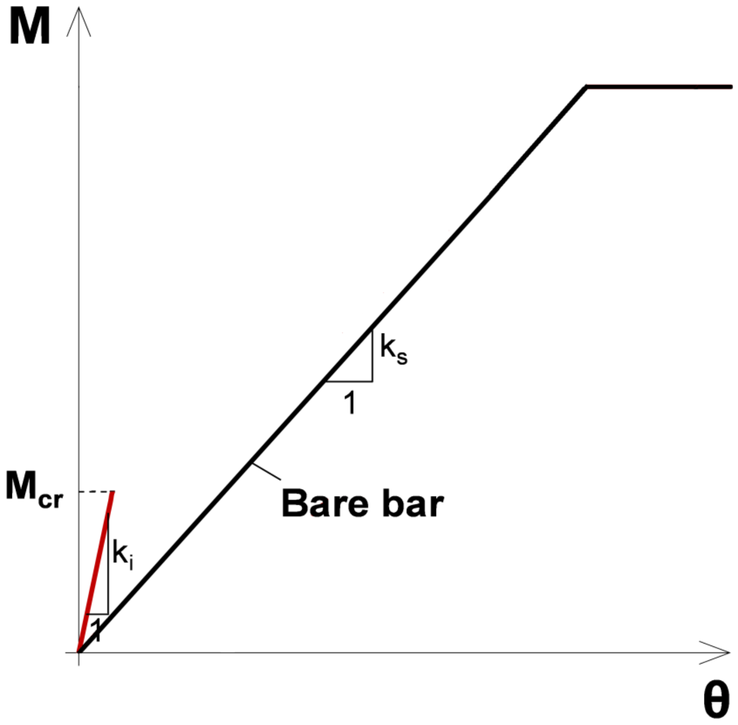
$$k_1 = M_1 / \theta_1$$

$$k_2 = M_2 / \theta_2$$

$$k_3 = V / \Delta_c$$

Constitutive relationship of rotational spring



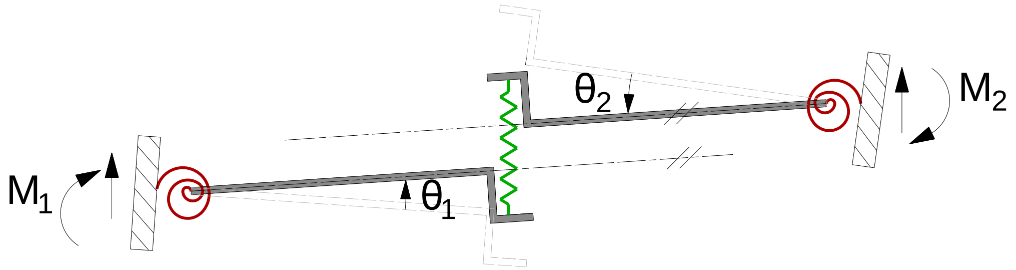


Timoshenko beam theory

$$k_i = \frac{6kAGEI}{12EI + kAGL^2}\left(\frac{4EI}{kAGL} + \frac{L}{3}\right)$$

$$M_{cr} \approx N_{cr} \times 0.9d$$

Constitutive relationship of rotational spring



Parabola

$T_0 \approx 2N_{cr}$

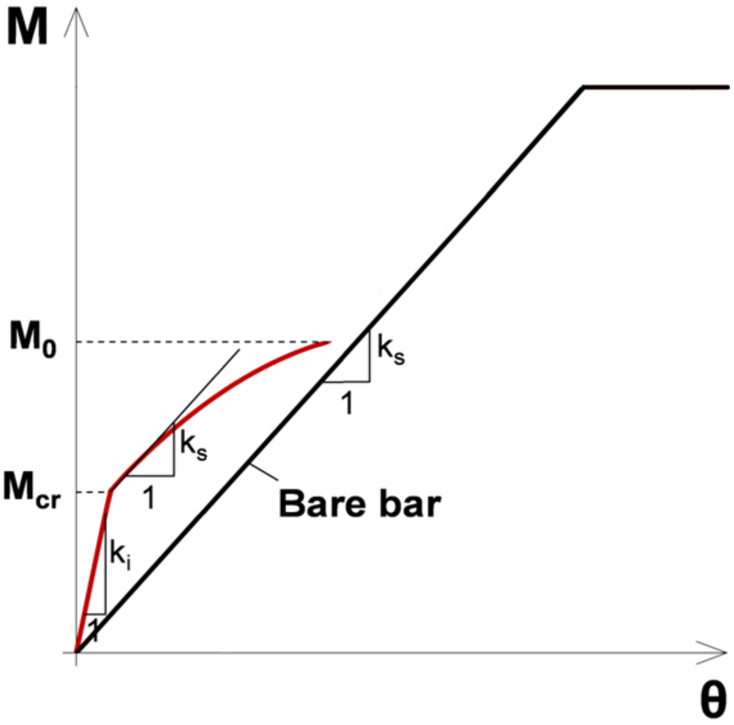
$M_0 \approx T_0 \times 0.9d$

N_{cr} - cracking force of zone

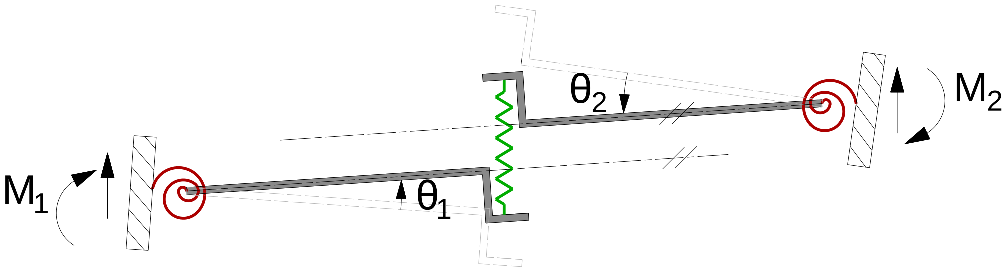
influenced by bottom

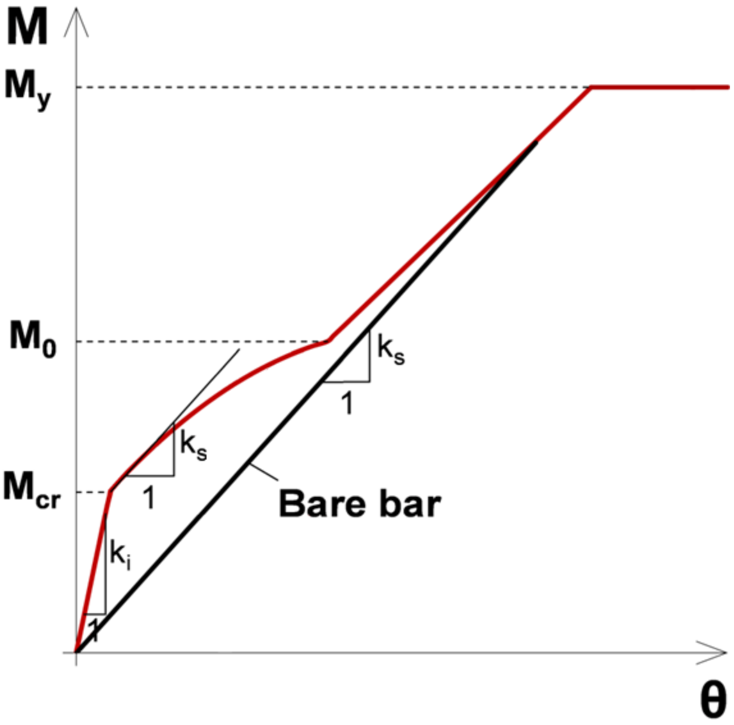
reinforcement.

$$N_{cr} = \left[A_{c,eff} + \left(E_s/E_c - 1\right)A_s\right]0.63\sqrt{f_c^{'}}$$



Constitutive relationship of rotational spring





$M \approx T\,(0.9d)$

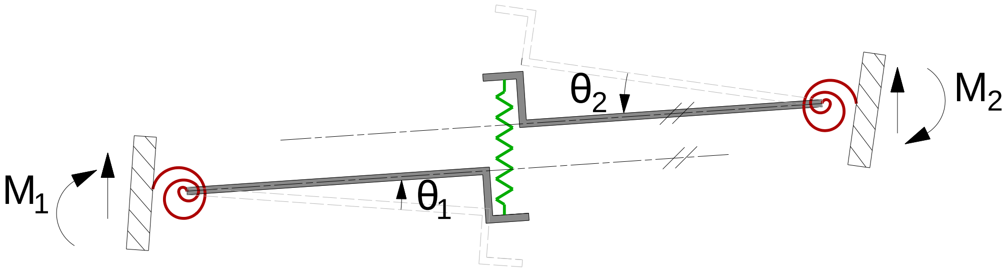
T is the tension in long. reinf.:

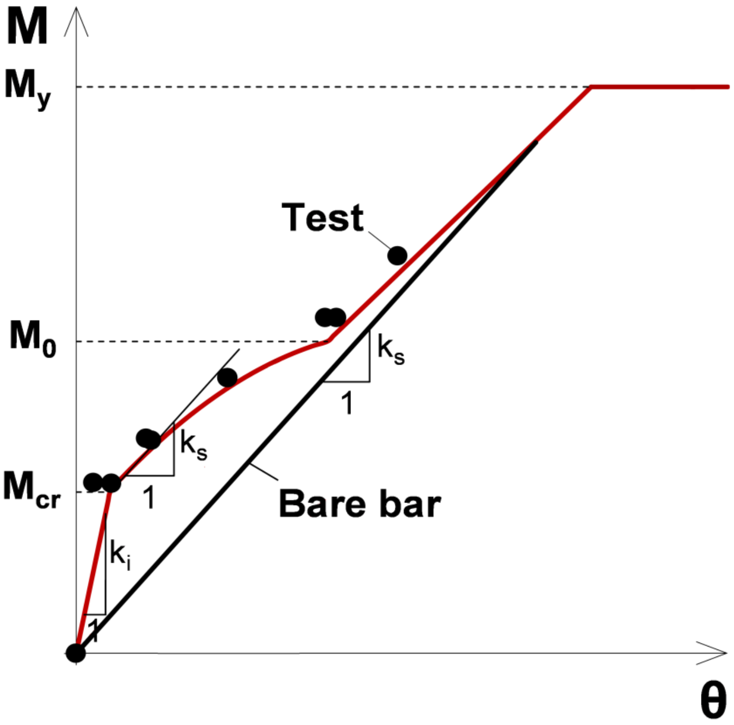
$$T = E_s A_s \varepsilon_t + \frac{0.33\sqrt{f_c}}{\sqrt{1+200\varepsilon_t}}$$

$$\leq A_s f_y$$

$$\varepsilon_t = \theta\, d \,/\, a$$

Constitutive relationship of rotational spring





M ≈ T (0.9d)

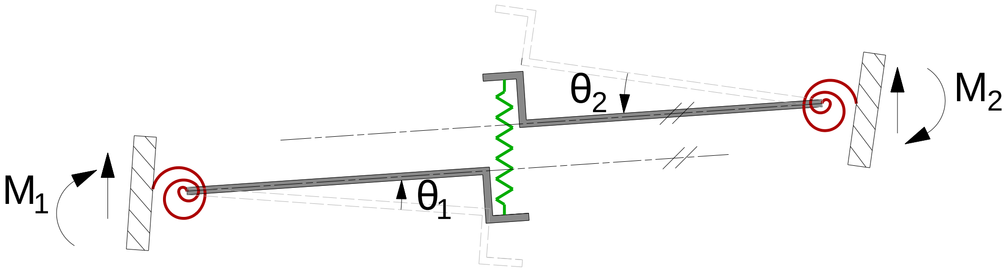
T is the tension in long. reinf.:

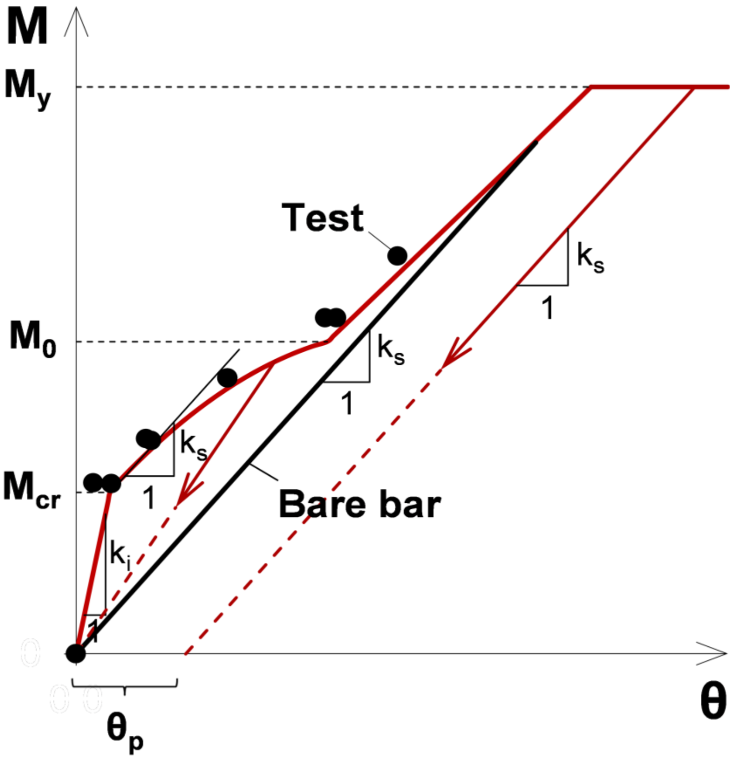
$$T = E_s A_s \varepsilon_t + \frac{0.33\sqrt{f_c}}{\sqrt{1+200\varepsilon_t}}$$

$$\leq A_s f_y$$

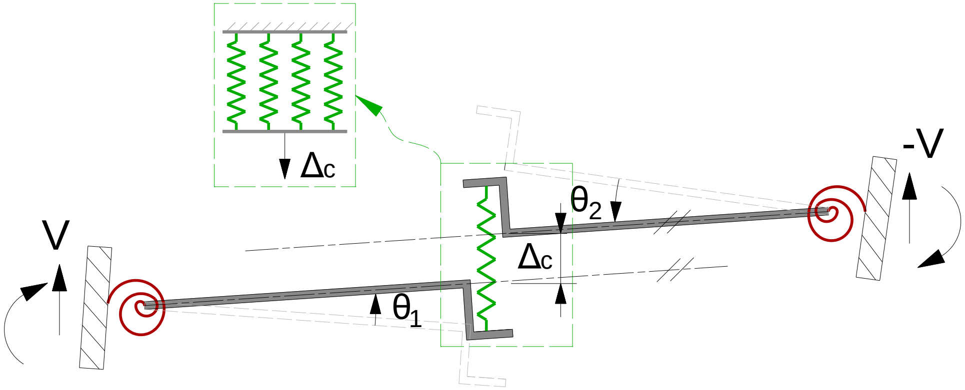
$$\varepsilon_t = \boldsymbol{\theta}\, d / a$$

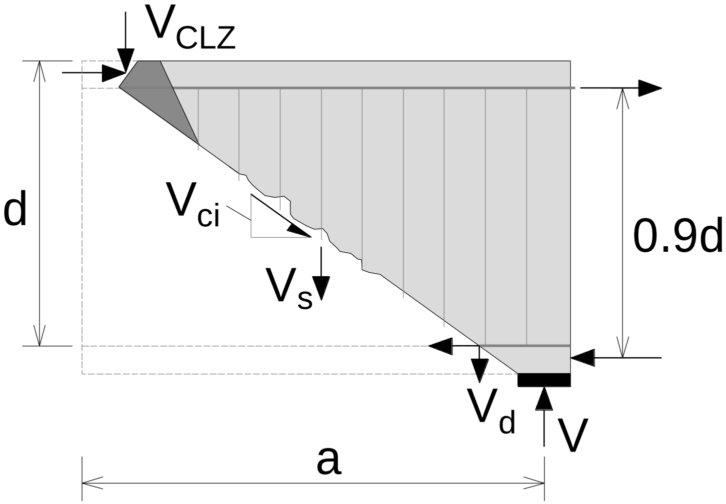
Constitutive relationship of rotational spring





Transverse springs (shear behaviour)





Shear components from:

V_{CLZ} — critical loading zone

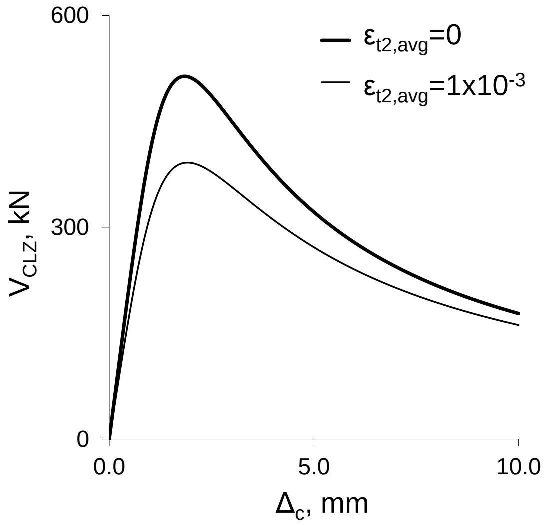
V_{ci} — aggregate interlock

V_s — stirrups

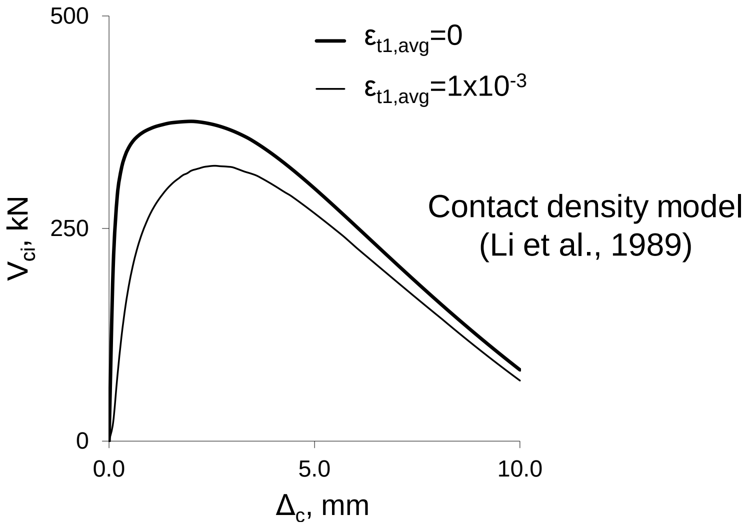
V_d — dowel action

Springs of the four shear mechanisms

- **Critical loading zone**

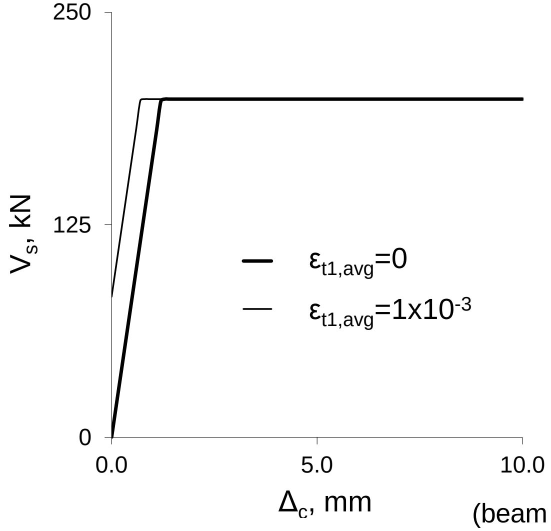


- **Aggregate interlock**

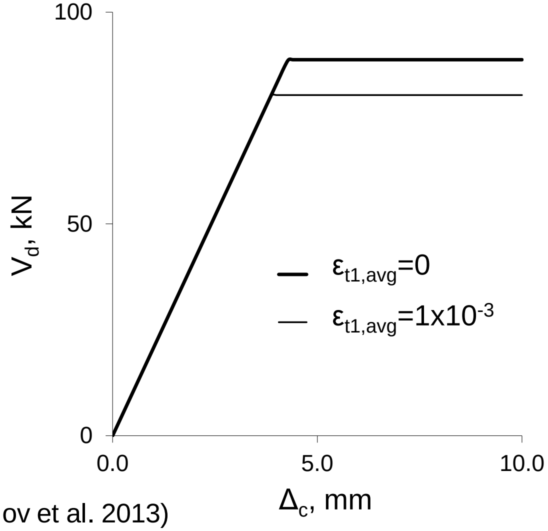


Contact density model
(Li et al., 1989)

- **Stirrups**

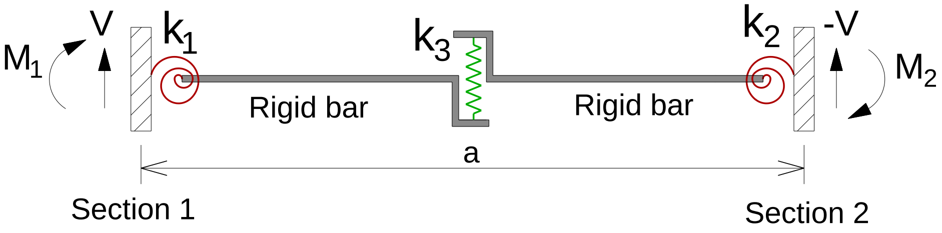


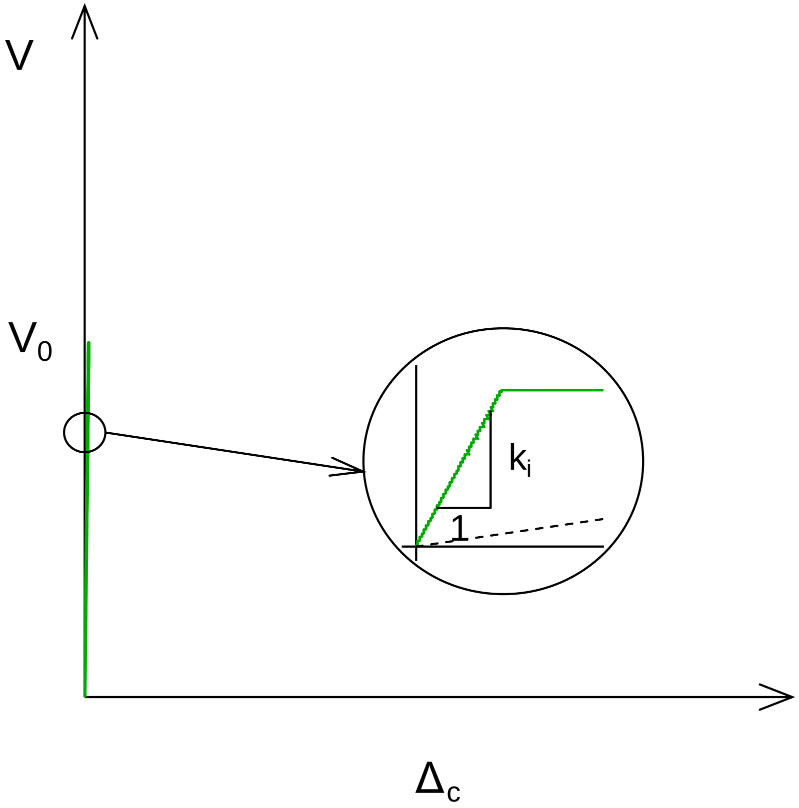
- **Dowel action**



(beam S1M tested by Mihaylov et al. 2013)

Constitutive relationship of shear spring

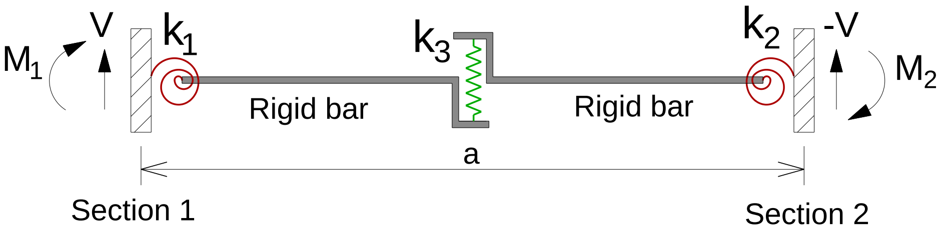


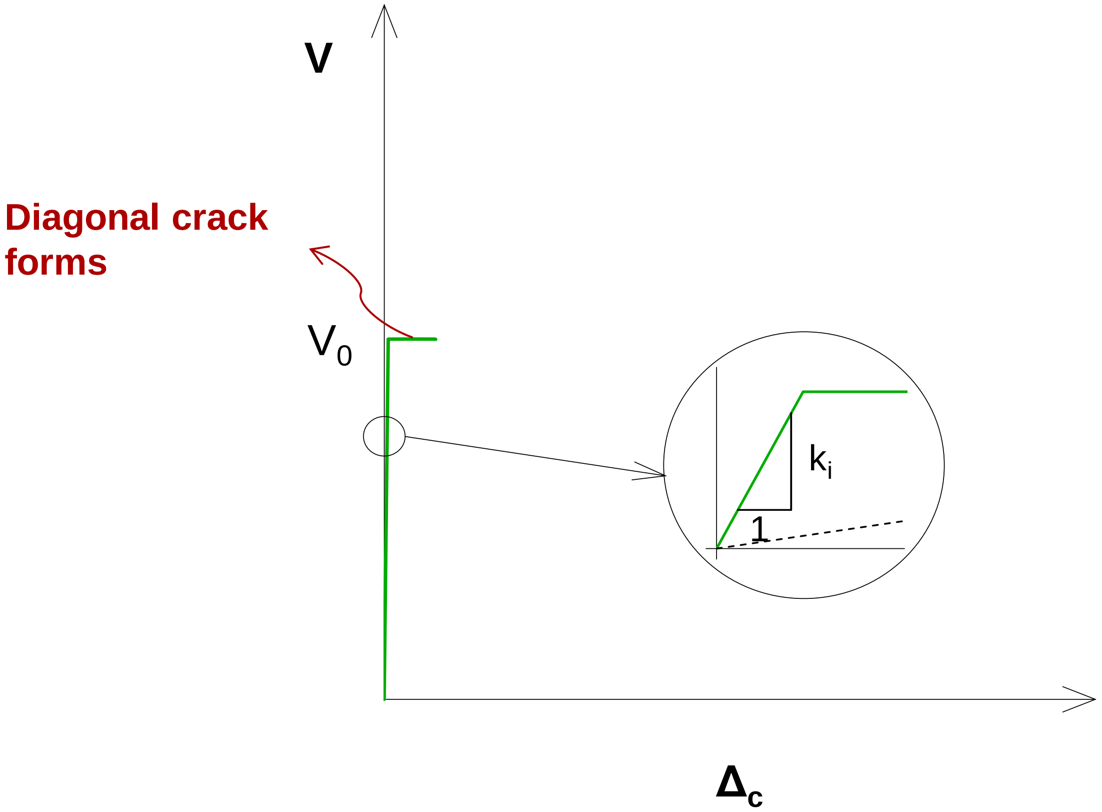


Timoshenko beam theory

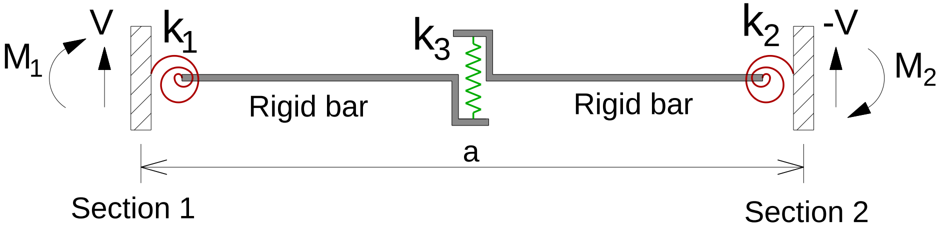
$$k_i = \frac{6kG_cAE_cI}{12E_cI + kG_cAa^2}\left(\frac{2}{a} + \frac{3kG_cAa}{6E_cI - kG_cAa^2}\right)$$

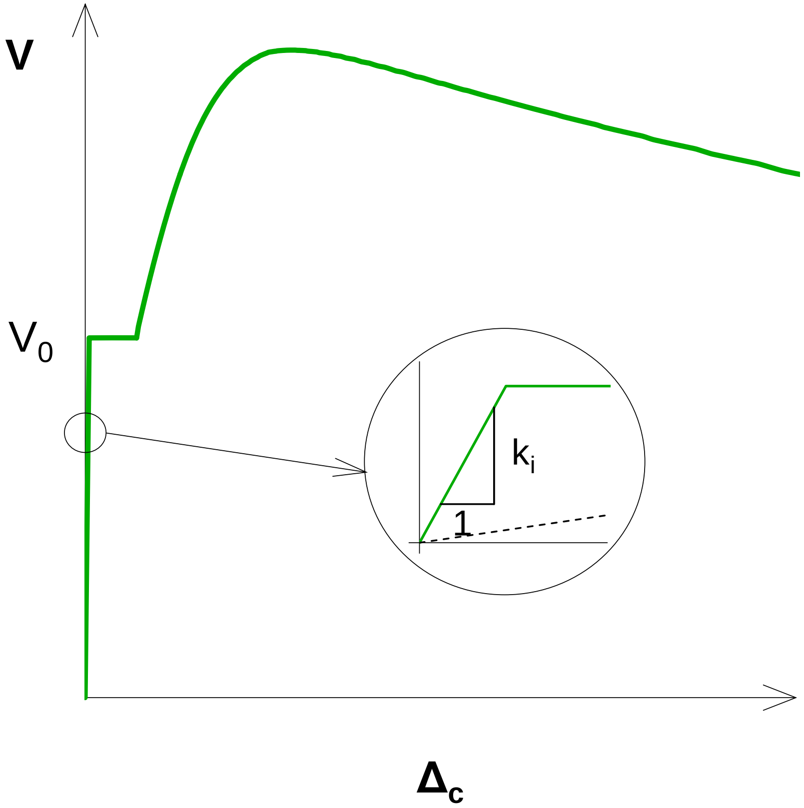
Constitutive relationship of shear spring





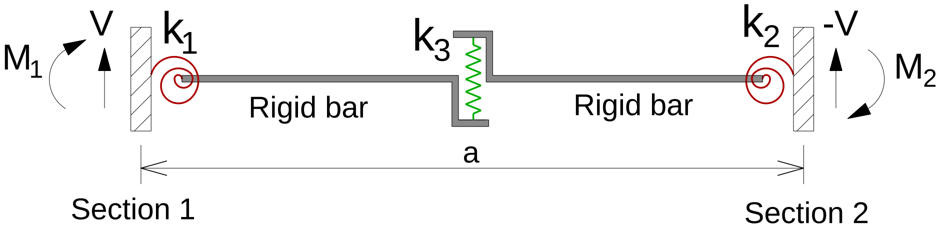
Constitutive relationship of shear spring

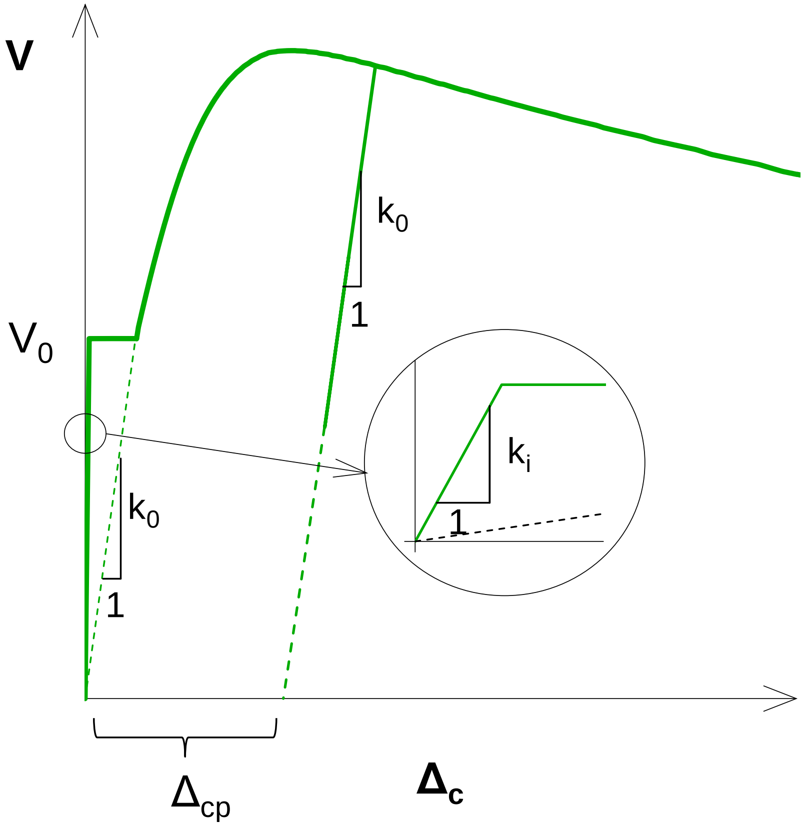




$$V = V_{CLZ} + V_s + V_{ci} + V_d$$

Constitutive relationship of shear spring

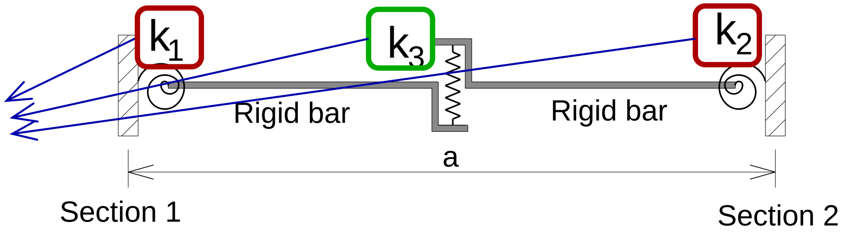




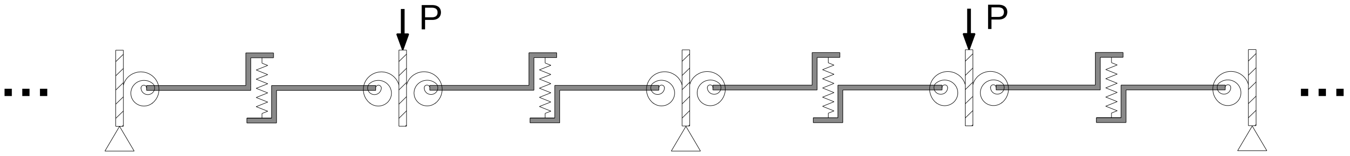
Solution procedure of macroelement

Structure

from previous converged step



$$[k]=\frac{1}{k_1+k_2+k_3a^2}\times\begin{bmatrix} k_1\left(k_2+k_3a^2\right) & -k_1k_3a & -k_1k_2 & k_1k_3a \\ -k_1k_3a & k_3\left(k_1+k_2\right) & -k_2k_3a & -k_3\left(k_1+k_2\right) \\ -k_1k_2 & -k_2k_3a & k_2\left(k_1+k_3a^2\right) & k_2k_3a \\ k_1k_3a & -k_3\left(k_1+k_2\right) & k_2k_3a & k_3\left(k_1+k_2\right) \end{bmatrix}$$

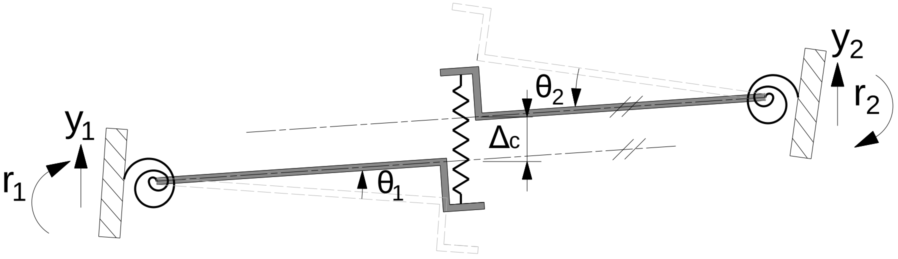


Global stiffness matrix [**K**]

Global linear analysis {Δ} = {P} \ [**K**]

Solution procedure of macroelement

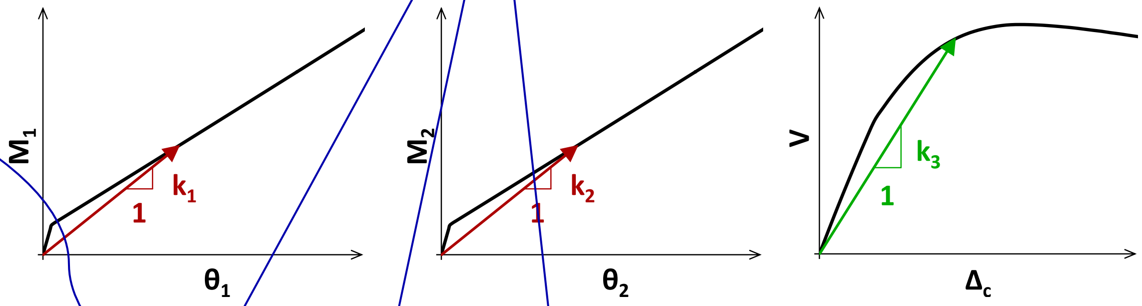
Element



from {Δ}

$$\begin{Bmatrix} \theta_1 \\ \theta_2 \\ \Delta_c \end{Bmatrix} = [T] \begin{Bmatrix} \varphi_1 \\ v_1 \\ \varphi_2 \\ v_2 \end{Bmatrix}$$

$$[T] = \frac{1}{k_1 + k_2 + k_3 a^2} \times \begin{bmatrix} (k_2 + k_3 a^2) & -k_3 a & -k_2 & k_3 a \\ -k_1 & -k_3 a & (k_1 + k_3 a^2) & k_3 a \\ k_1 a & -(k_1 + k_2) & k_2 a & (k_1 + k_2) \end{bmatrix}$$



Calculate M_1, M_2 and V from θ_1, θ_2 and Δ_c

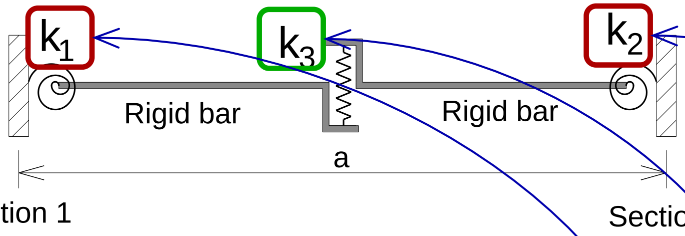
Update $\boxed{k_1} = M_1 / \theta_1$, $\boxed{k_2} = M_2 / \theta_2$, $\boxed{k_3} = V / \Delta_c$

Until convergence

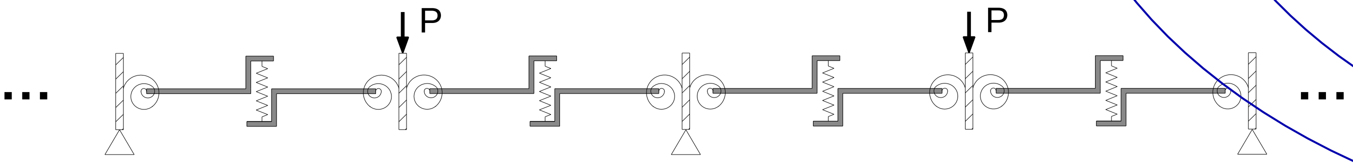
reached at element level

Solution procedure of macroelement

Structure



$$[k] = \frac{1}{k_1 + k_2 + k_3 a^2} \times \begin{bmatrix} k_1\left(k_2 + k_3 a^2\right) & -k_1 k_3 a & -k_1 k_2 & k_1 k_3 a \\ -k_1 k_3 a & k_3\left(k_1 + k_2\right) & -k_2 k_3 a & -k_3\left(k_1 + k_2\right) \\ -k_1 k_2 & -k_2 k_3 a & k_2\left(k_1 + k_3 a^2\right) & k_2 k_3 a \\ k_1 k_3 a & -k_3\left(k_1 + k_2\right) & k_2 k_3 a & k_3\left(k_1 + k_2\right) \end{bmatrix}$$



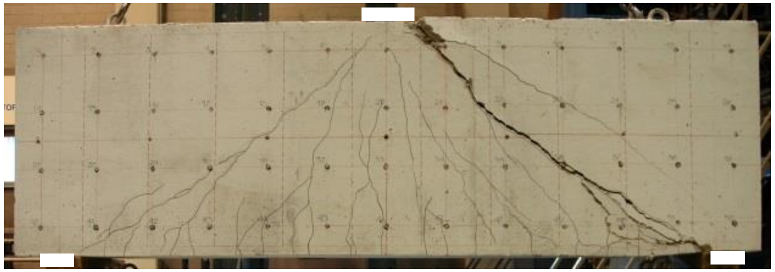
Global stiffness matrix [**K**]

Global linear analysis {Δ} = {P} \ [**K**]

Until convergence

reached at structural level

Complete shear response predicted with macroelement

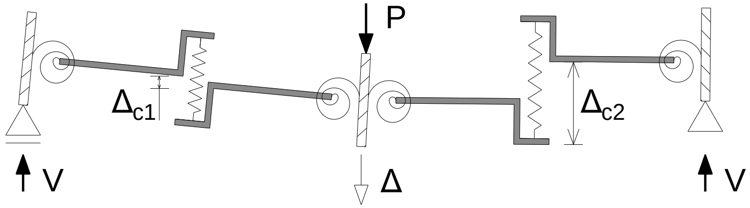


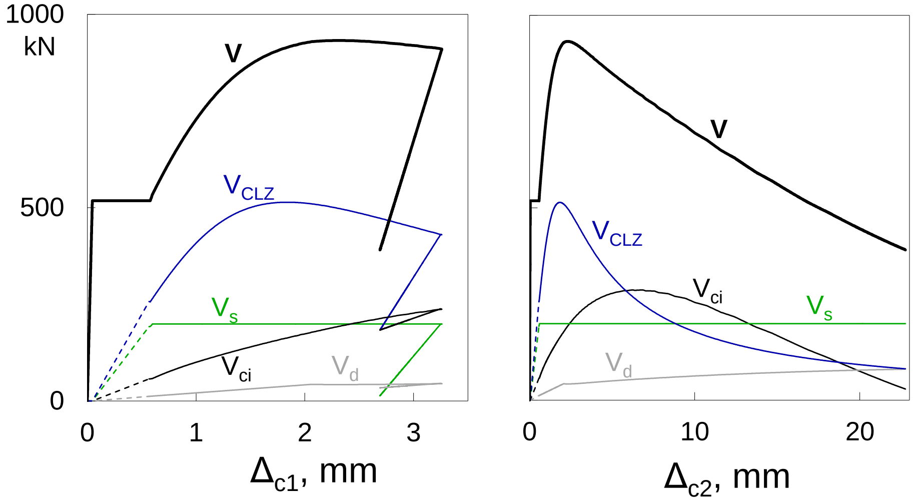
(S1M tested by Mihaylov et al. 2013)

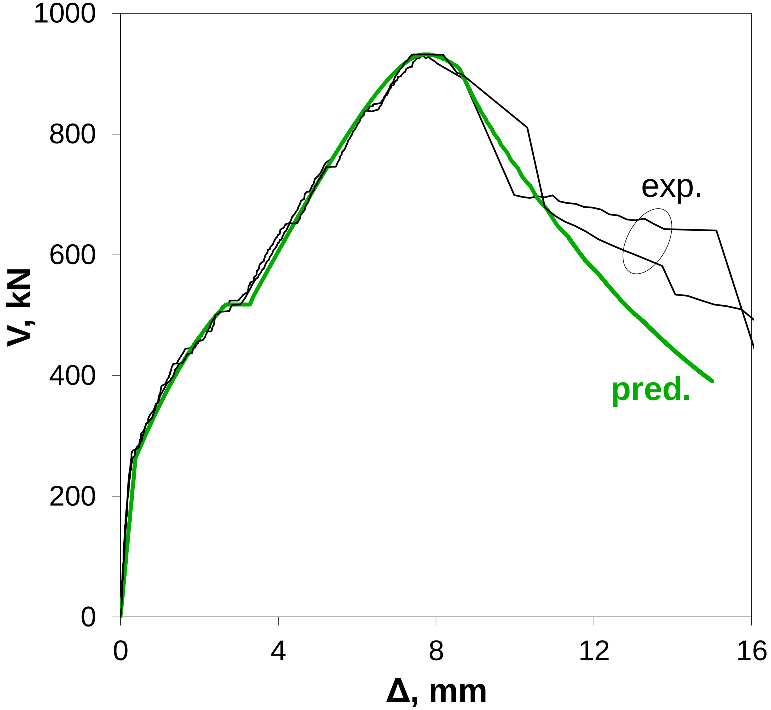
$a/d=1.55$ $d=1095mm$

$\rho_l=0.70\%$ $\rho_v=0.70\%$

$f_c=33.0MPa$

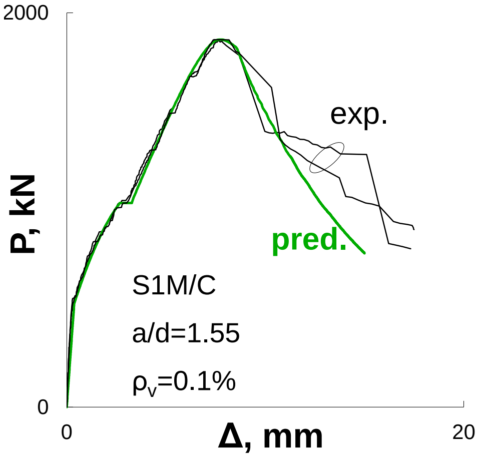


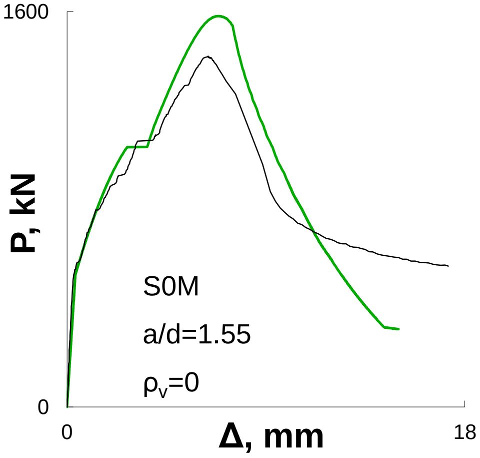


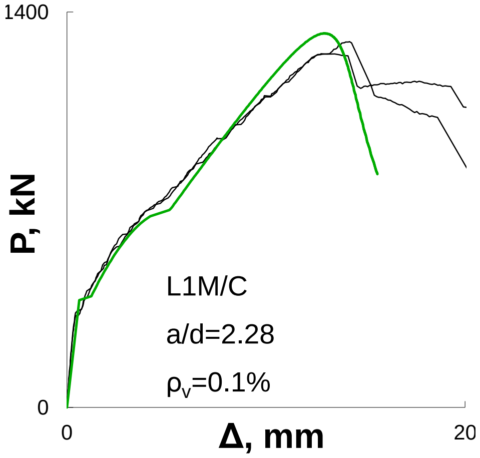


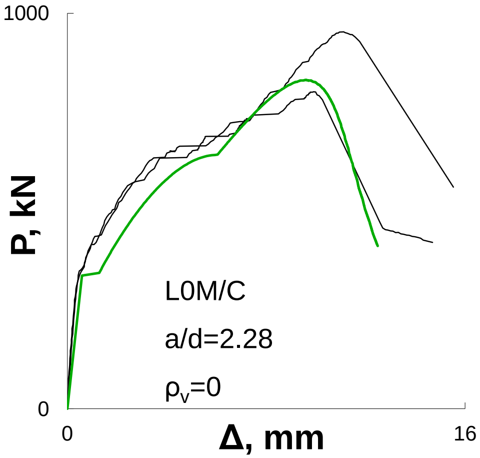
Complete shear response predicted with macroelement

Simply-supported deep beam

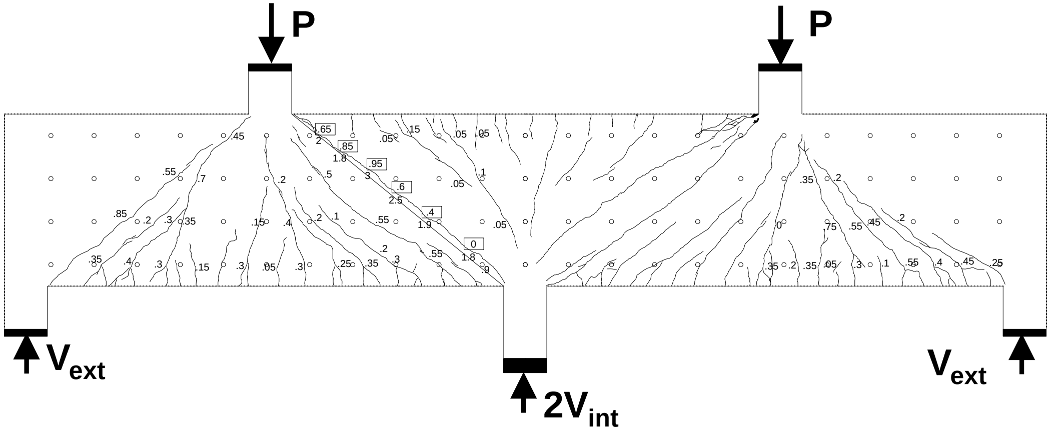


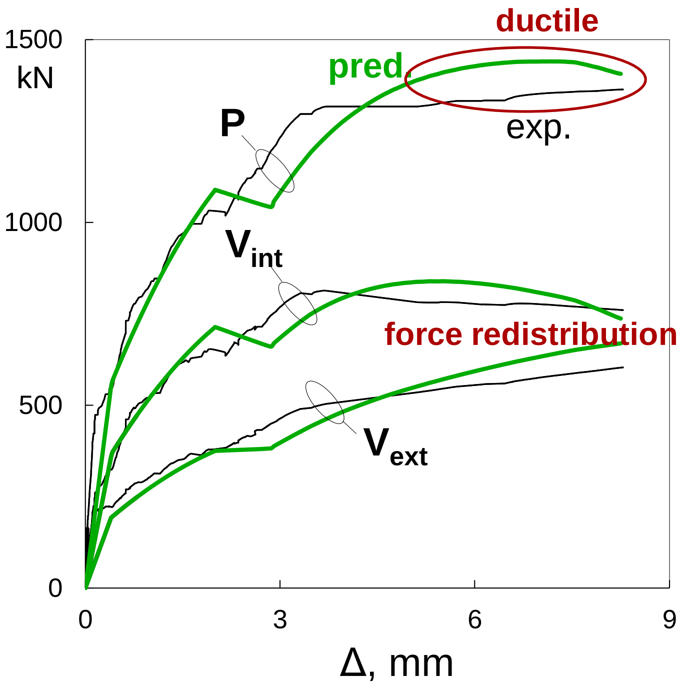




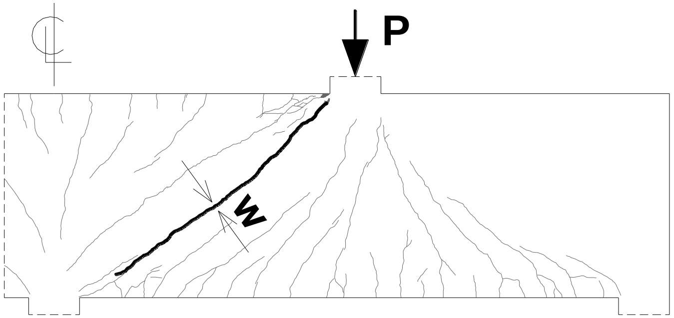


Complete shear response predicted with macroelement

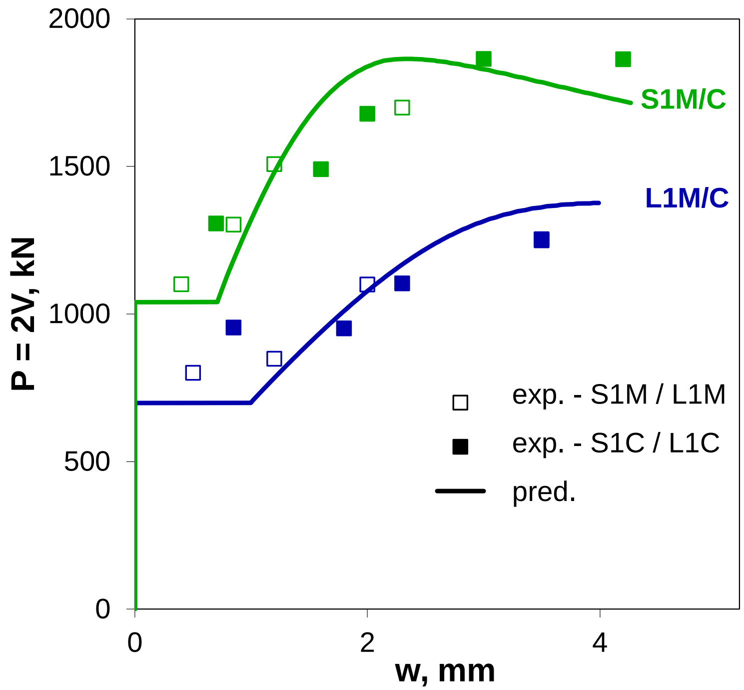




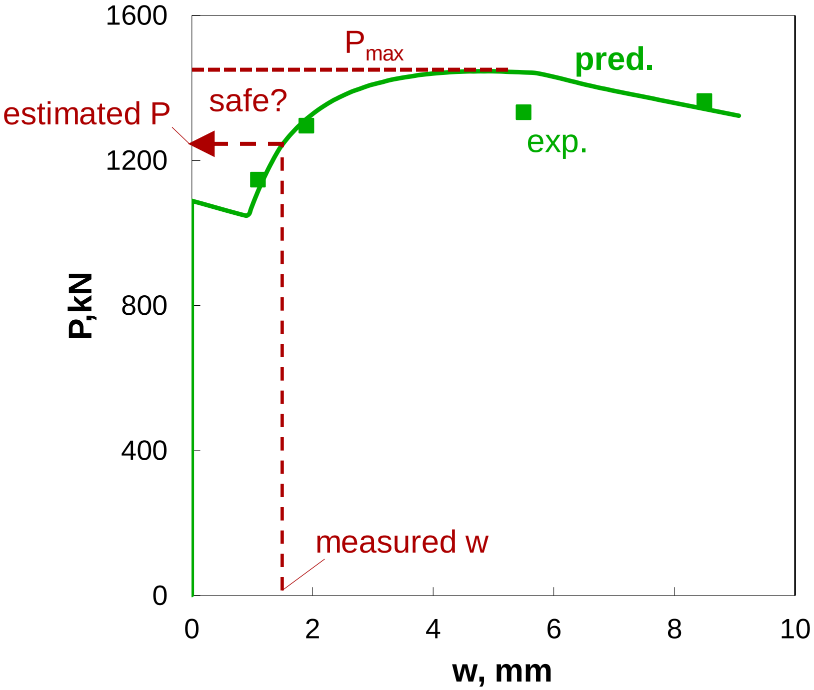
Crack widths predicted with macroelement



- **Simply-supported deep beam**



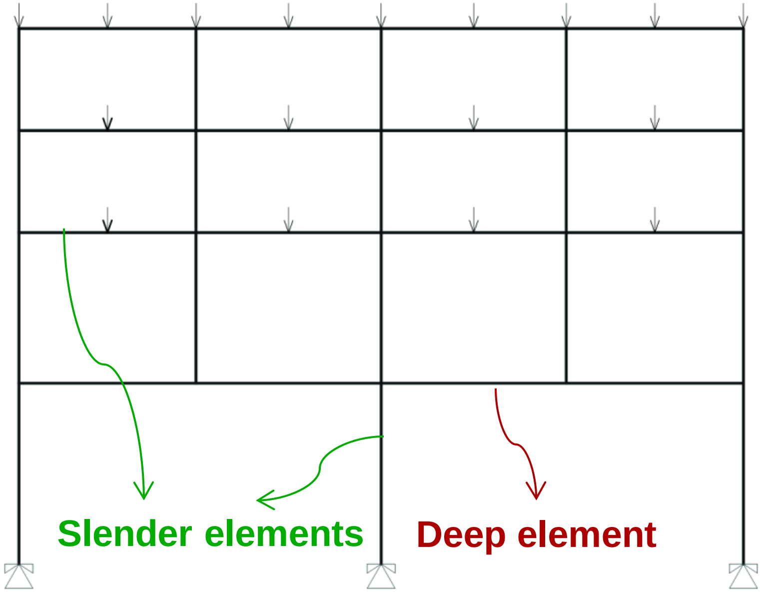
- **Continuous deep beam**



Mixed-type Modelling of Structures with Slender and Deep Beam Elements

Modelling of large structures with deep beams

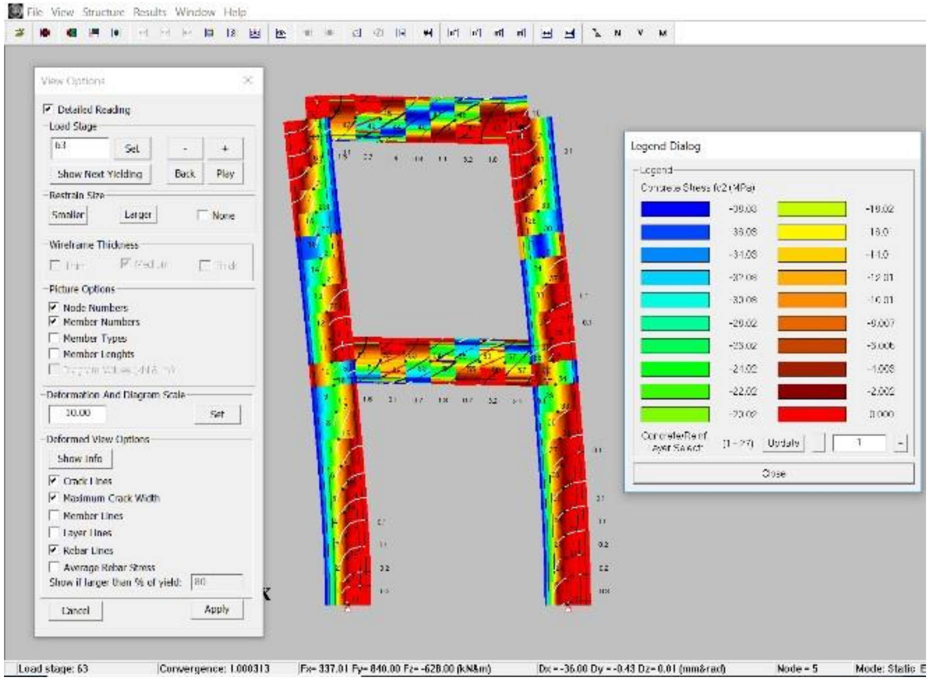
Model with 1D slender and deep elements



Slender elements

Deep element

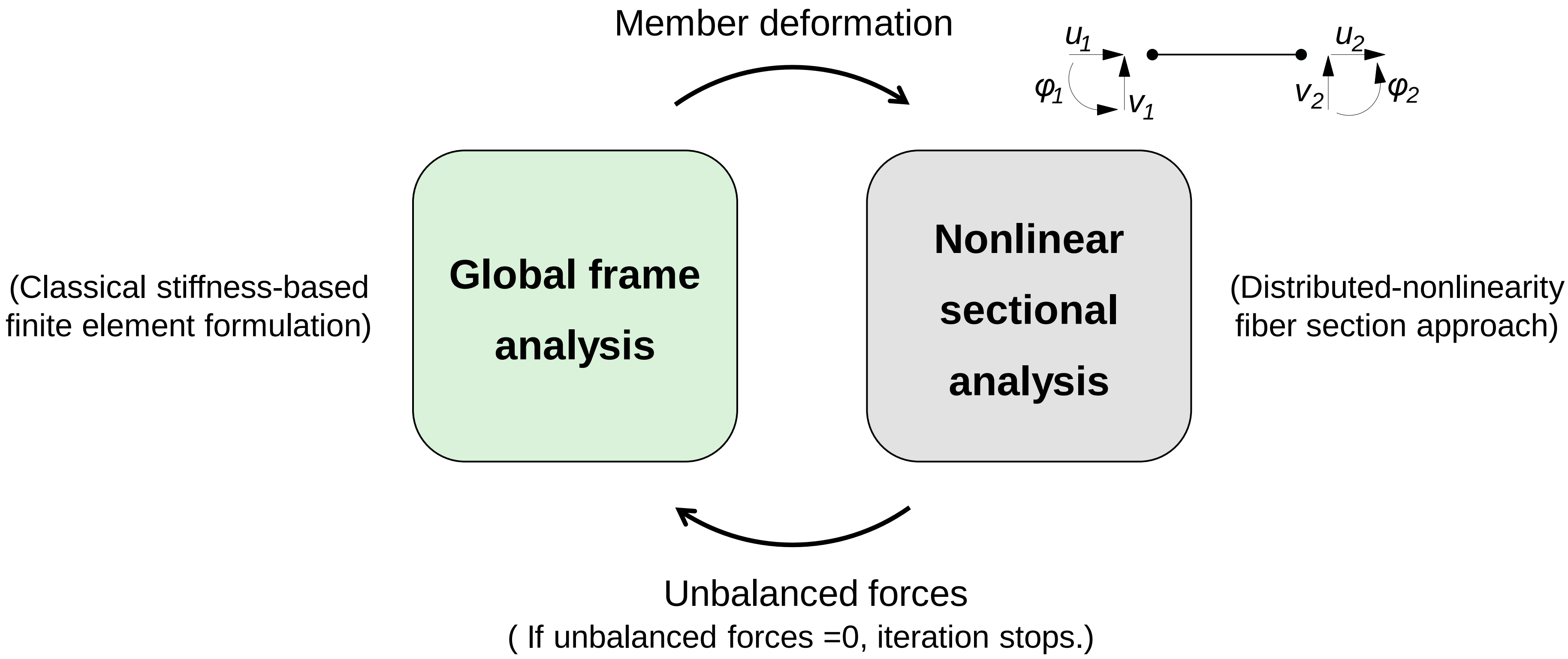
Existing FE program: VecTor5



(http://vectoranalysisgroup.com/vector5.html)

- 1D fiber-based element for slender beams
- Distributed plasticity approach for shear behavior
- Excellent predictions for plane frames reported

Solution procedure of VecTor5



Member deformation

(Classical stiffness-based finite element formulation)

Global frame analysis

Nonlinear sectional analysis

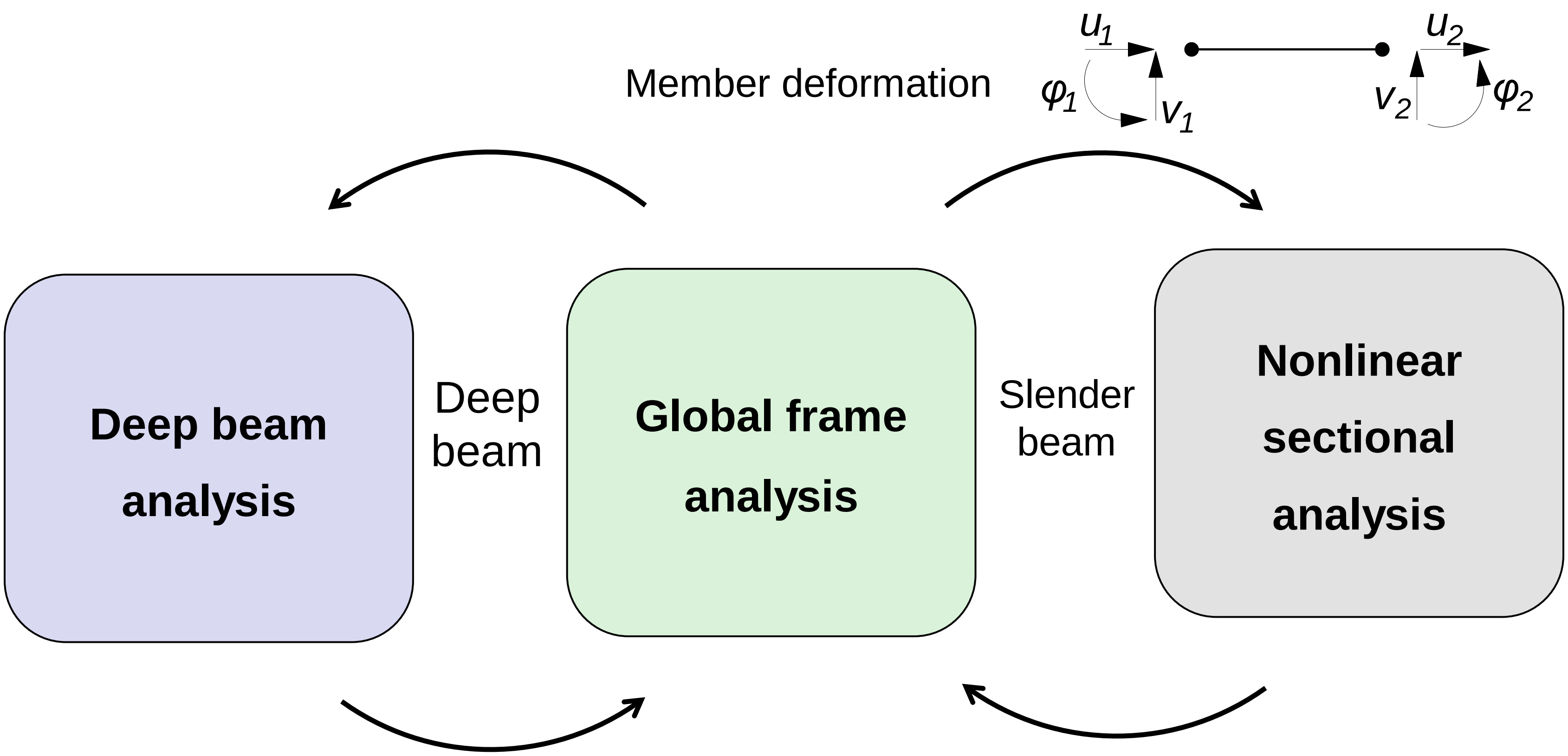
(Distributed-nonlinearity fiber section approach)

Unbalanced forces

(If unbalanced forces =0, iteration stops.)

Unbalanced forces = Global forces - Sectional forces

Solution procedure of modified VecTor5



Member deformation

Deep beam analysis

Deep beam

Global frame analysis

Slender beam

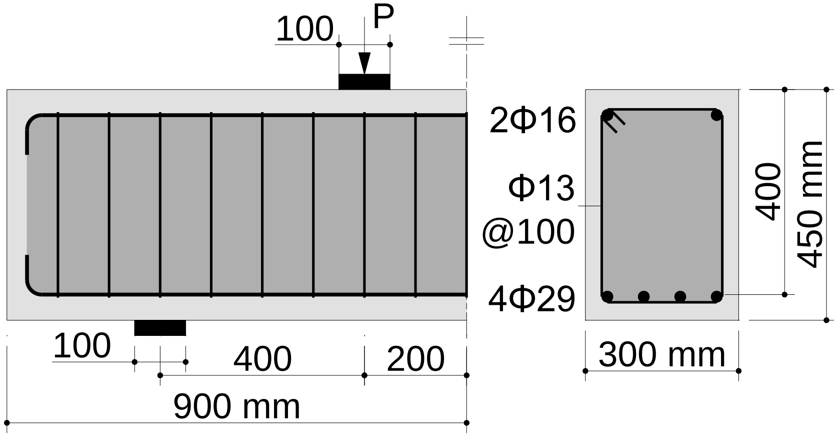
Nonlinear sectional analysis

Unbalanced forces

(If unbalanced forces =0, iteration stops.)

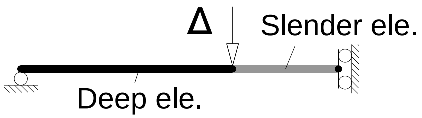
Unbalanced forces = Global forces - Sectional forces

Application to simply-supported deep beams

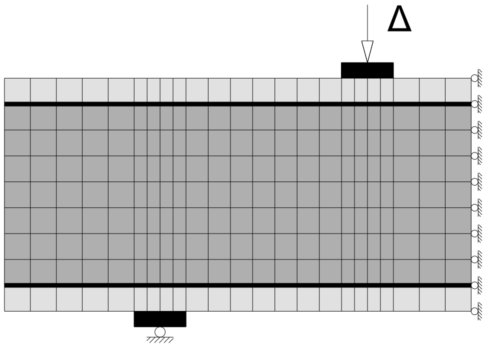


(Tested by Tanimura and Sato in 2005)

- **1D mixed-type model**

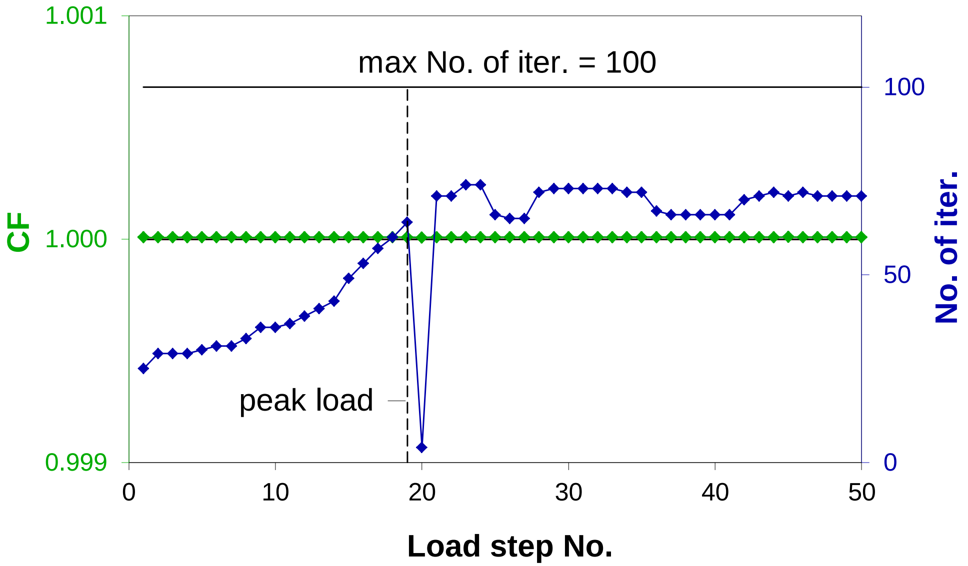


- **2D finite element model**

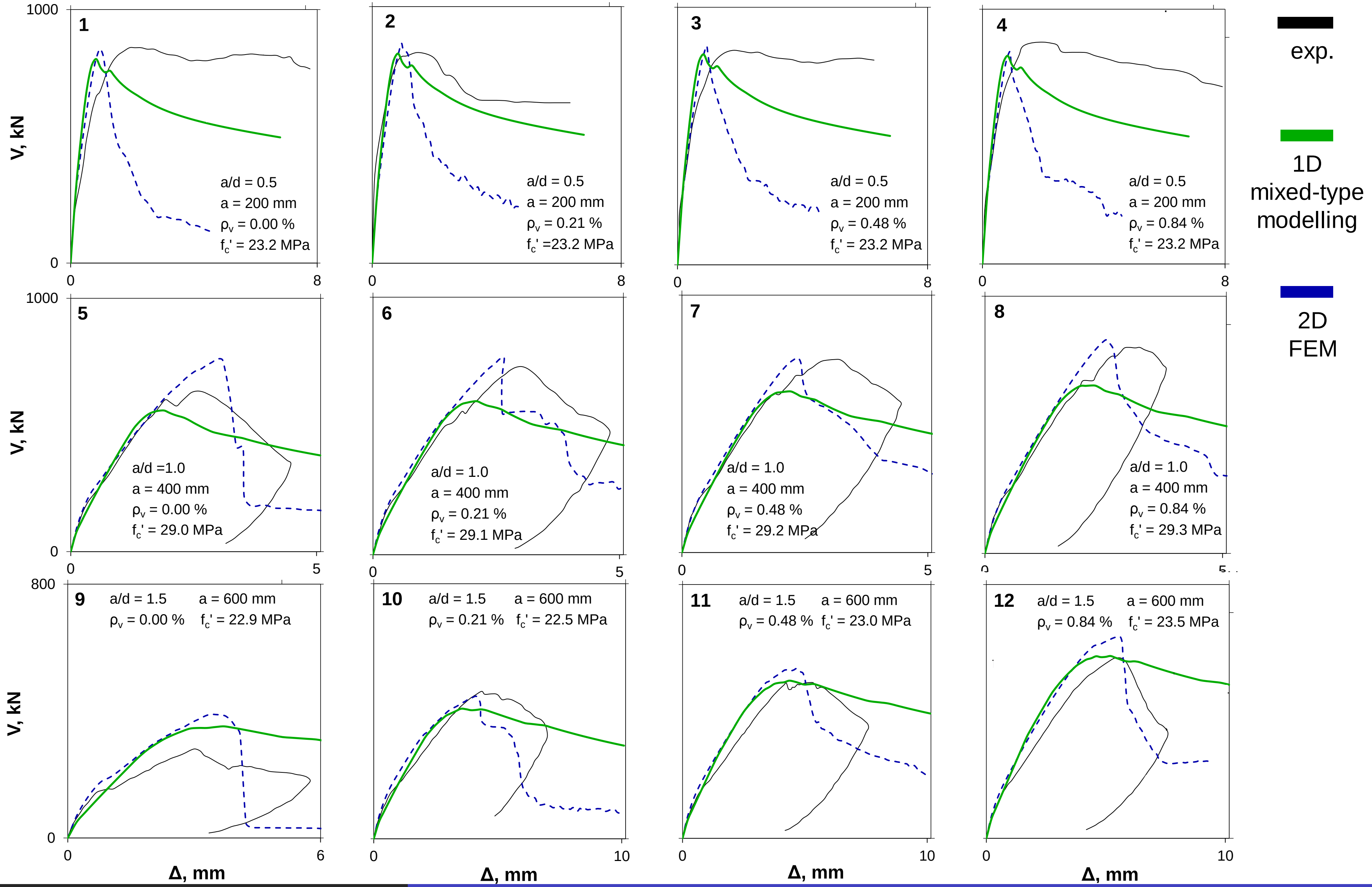


Convergence factor (CF):

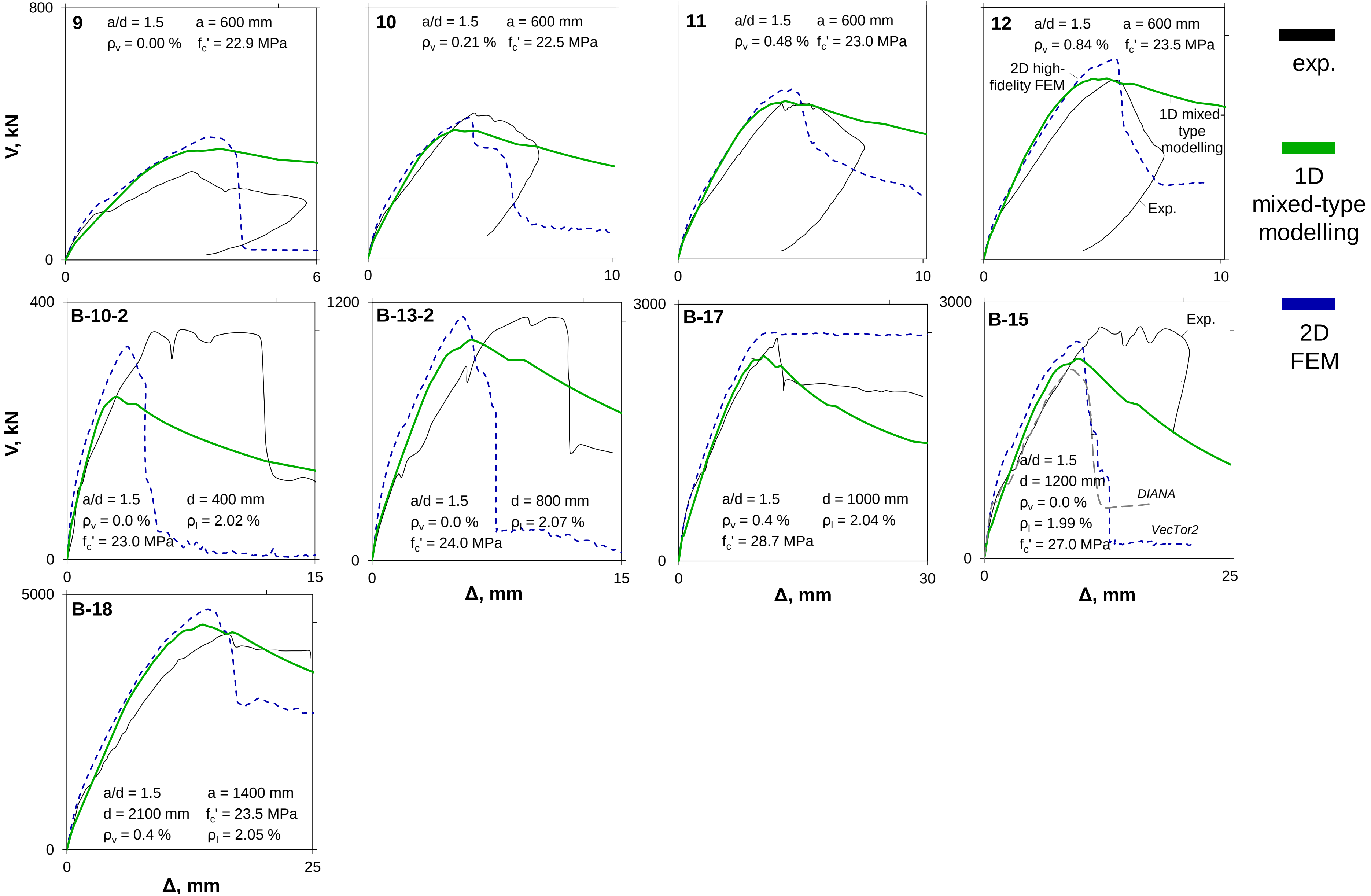
$$CF = 1 + \sqrt{\frac{1}{3\times n} \times \sum_{i=1}^{n}\left[\left(\frac{N_{ui}}{N_i}\right)^2 + \left(\frac{V_{ui}}{V_i}\right)^2 + \left(\frac{M_{ui}}{M_i}\right)^2\right]}$$



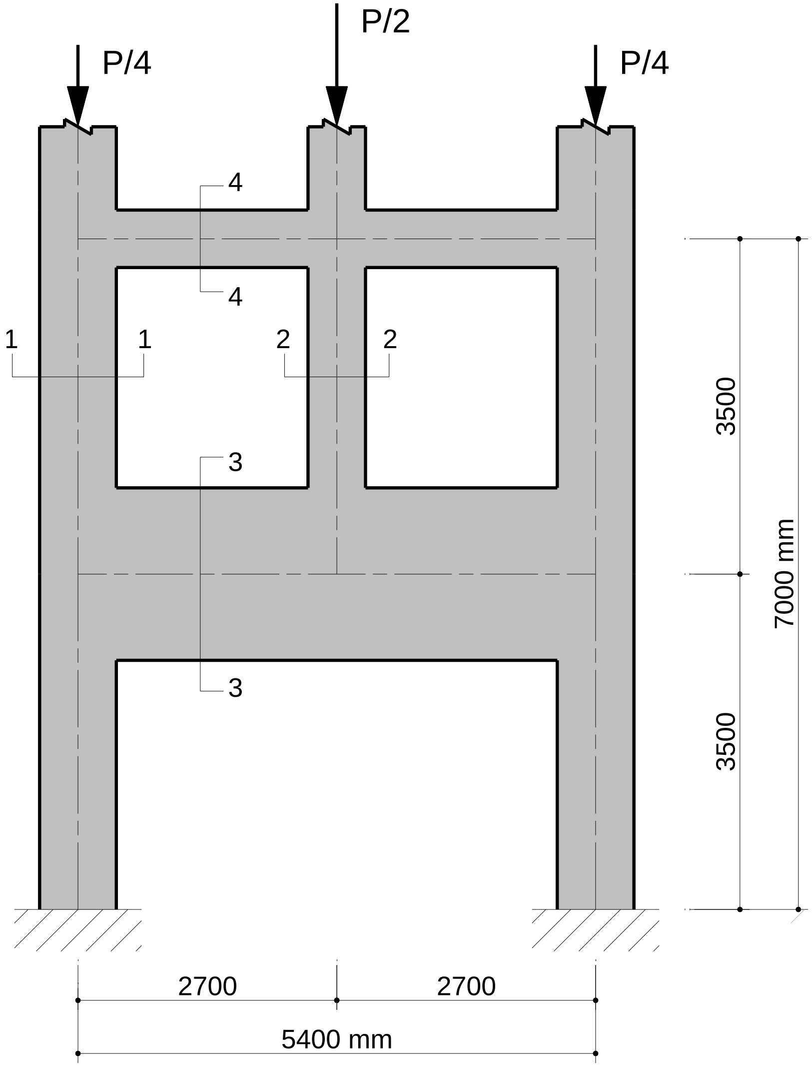
Prediction of entire shear response



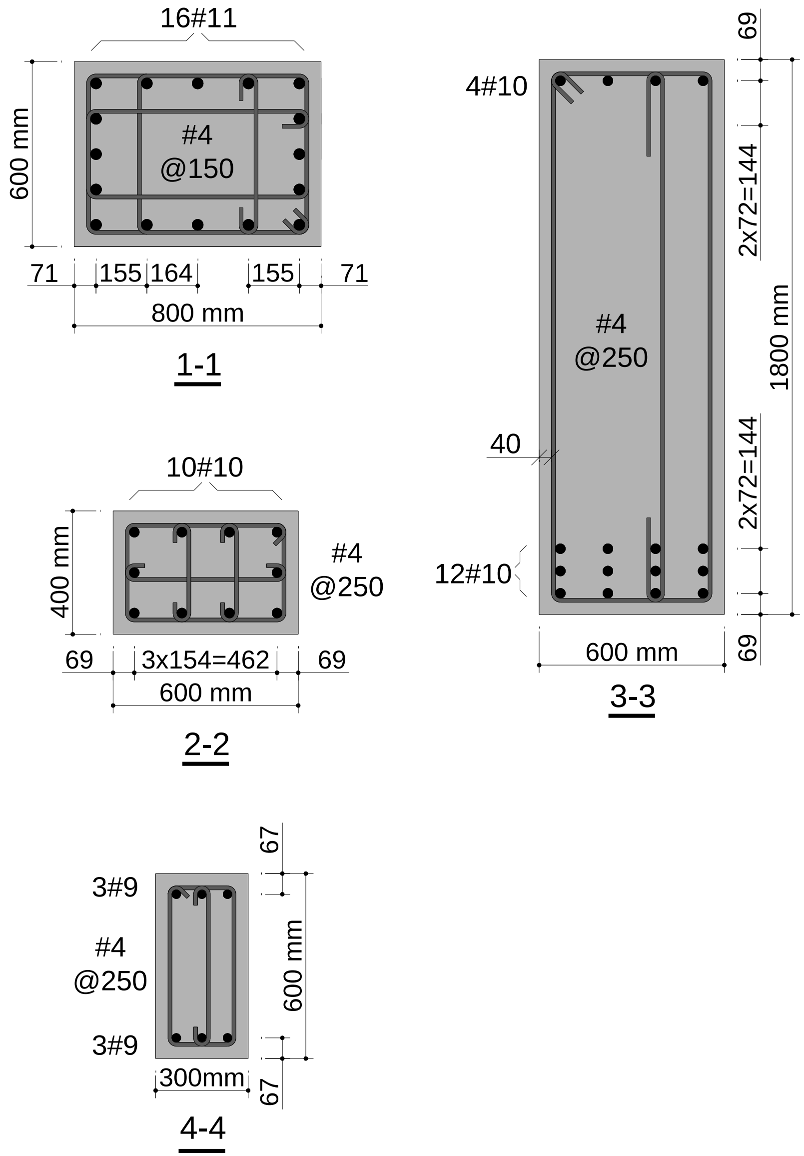
Prediction of entire shear response



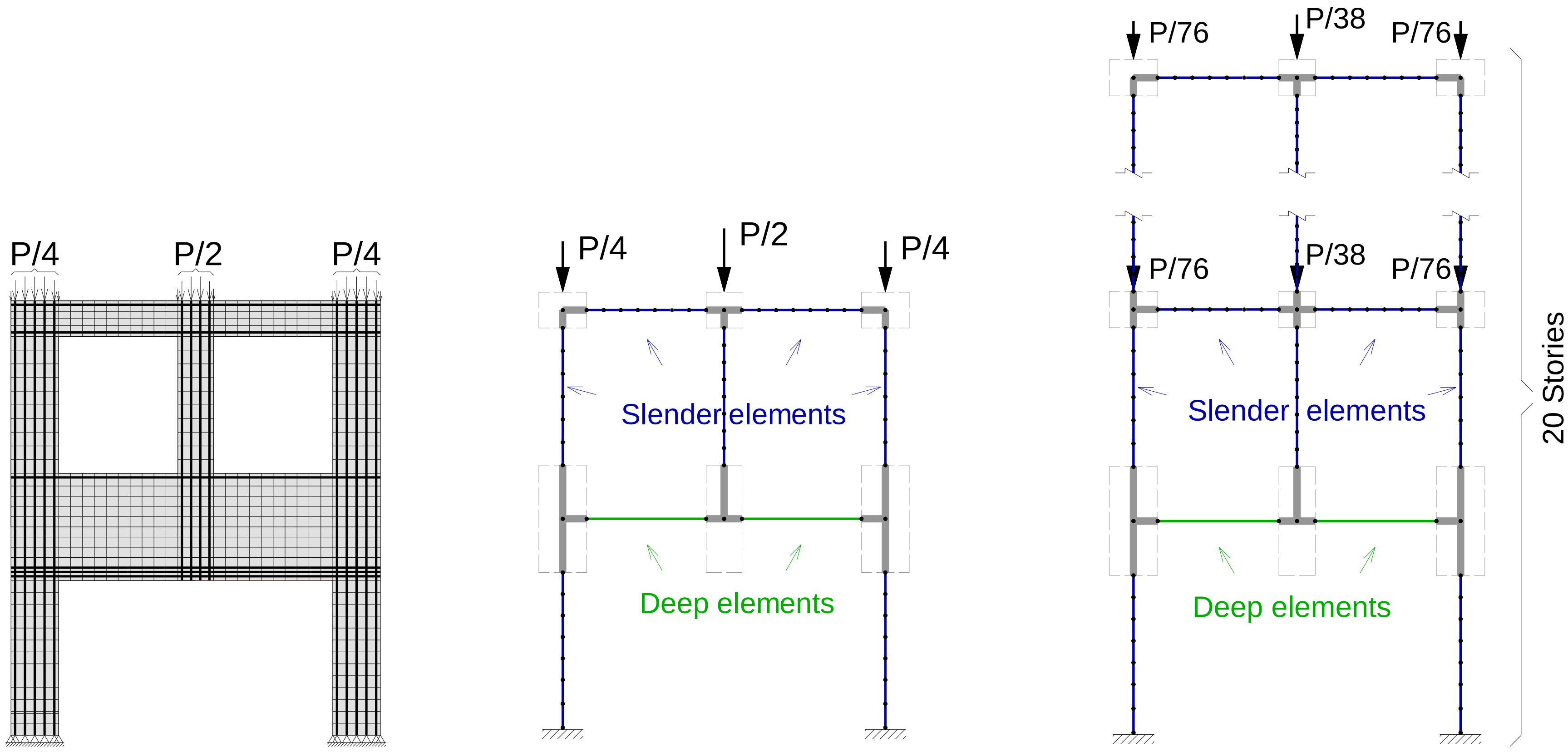
Modelling a 20-storey frame



(according to ACI 318)



Three modelling strategies



2D FEM

2-storey

1D mixed-type modelling

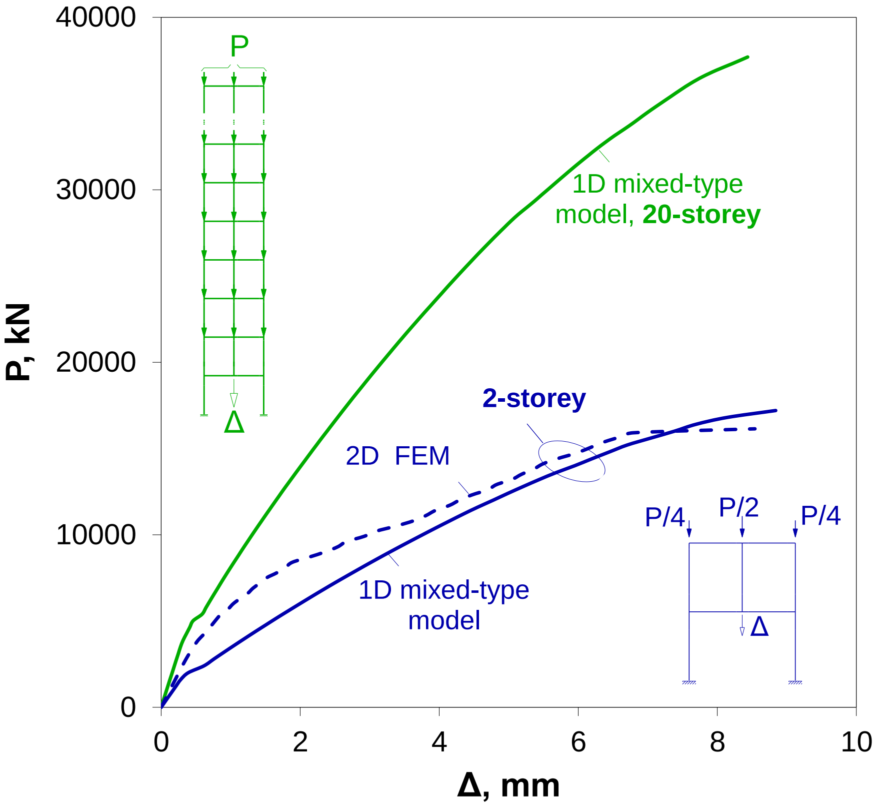
2-storey

1D mixed-type modelling

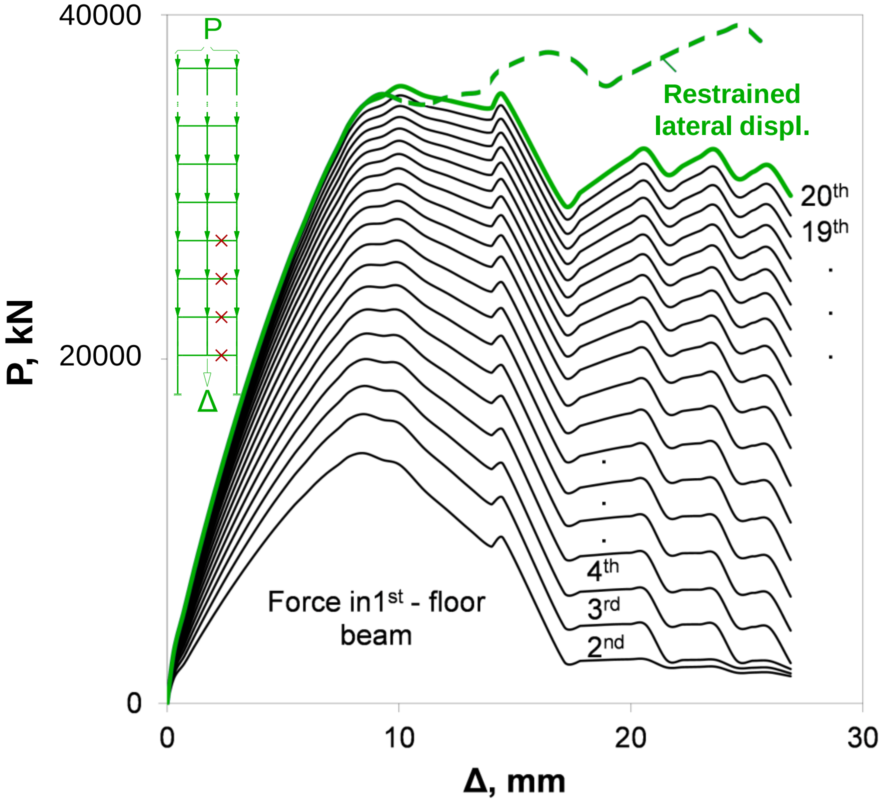
20-storey

Prediction of loading response

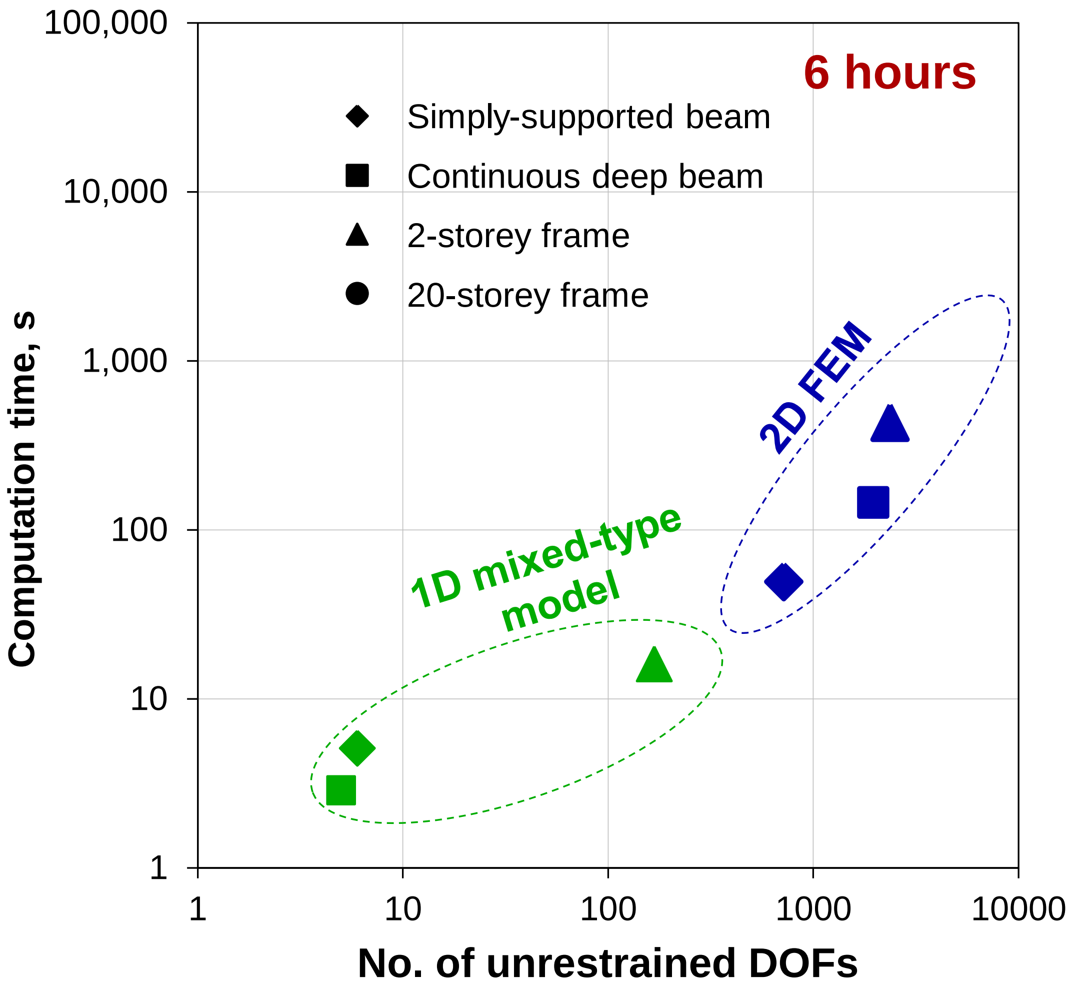
- **Load-disp. relationship**



- **Response in each storey of 20-storey frame**



Efficiency of studied modelling strategies



- **40 load steps**
- **Office desktop**

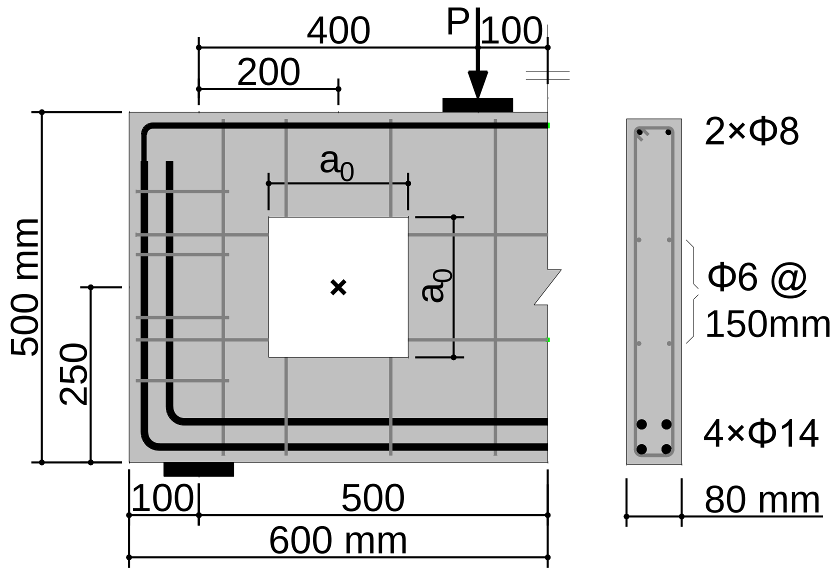
3.4 GHz quad-core processor

16 GB of RAM

Shear Strength of RC Deep Beams with Web Openings

Tests of deep beams with web openings (El-Maaddawy and Sherif, 2005)

- **Opening at centre**

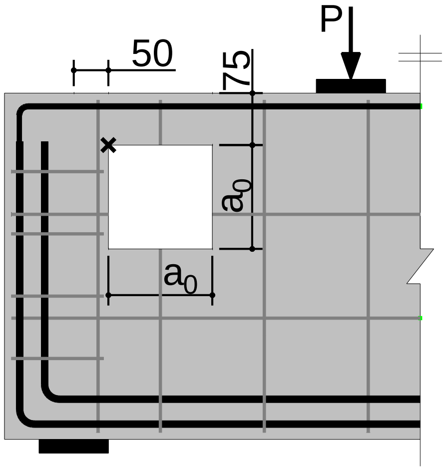


f_c = 21.0 MPa

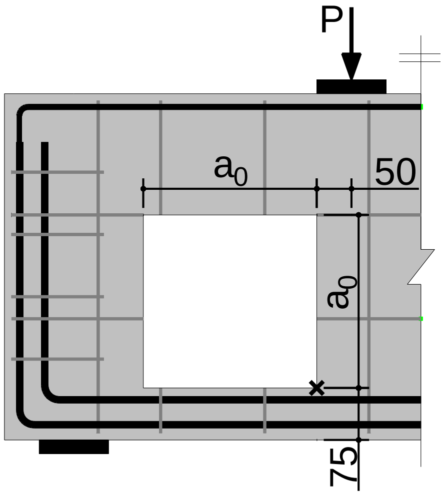
f_y = 420 MPa

f_{yv} = 300 MPa

- **Opening at top near support**

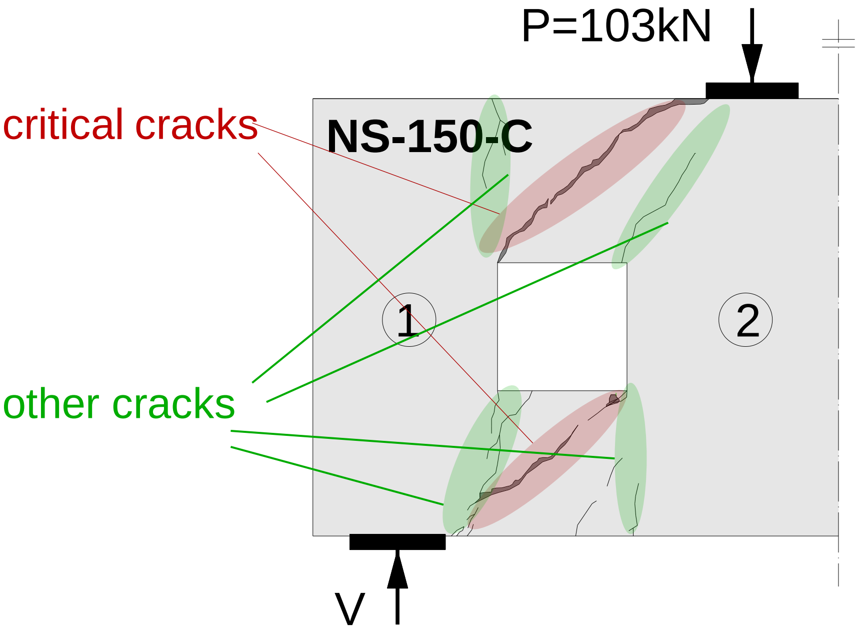


- **Opening at bottom near load**



Two typical failure modes

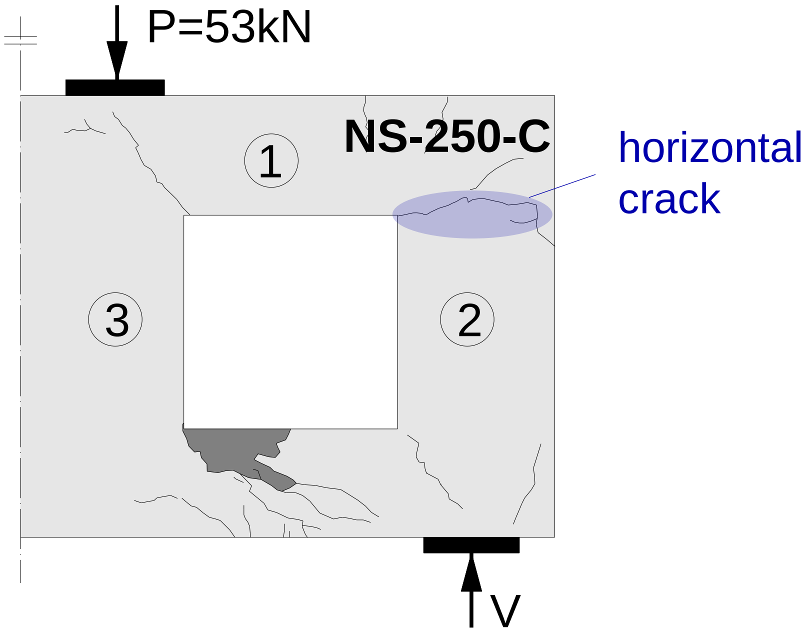
- **Small opening**



critical cracks

other cracks

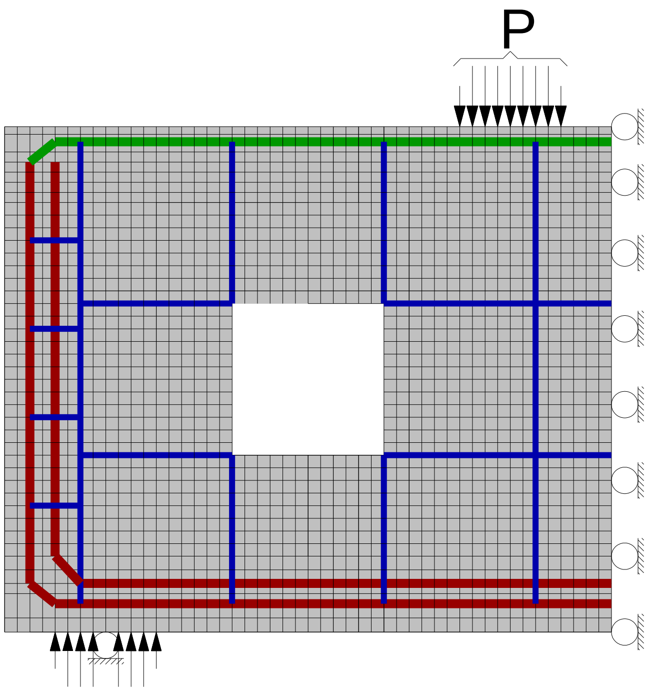
- **Large opening**



horizontal crack

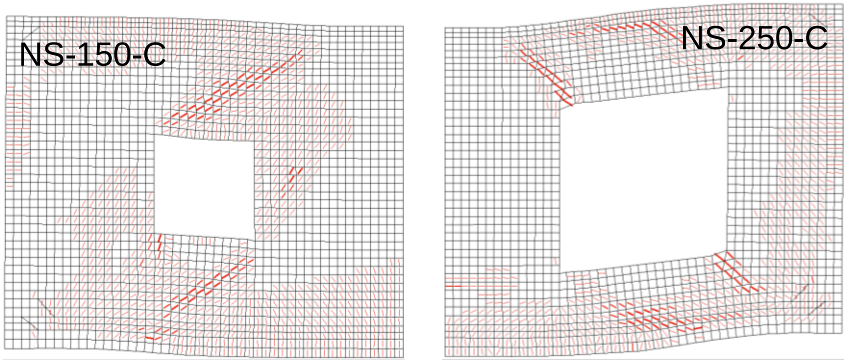
Deep beams with web openings studied by FEM

- **FEM model of beam NS-150-C**

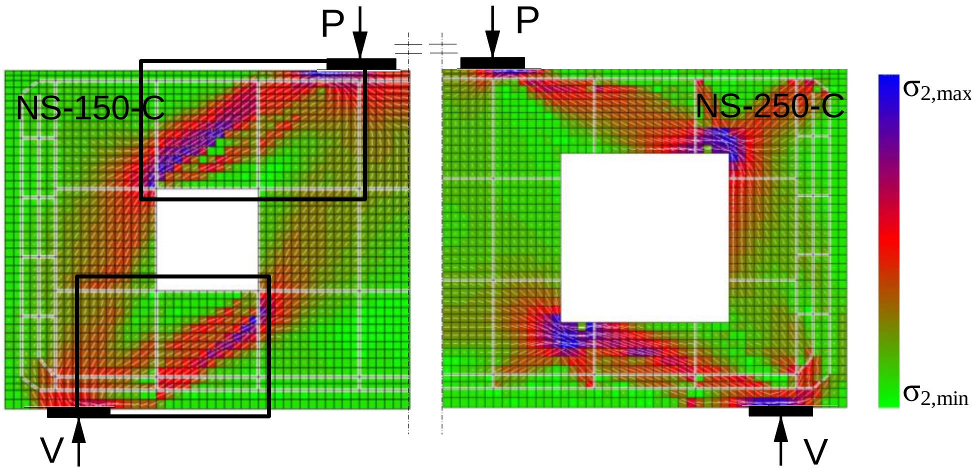


(With programme VecTor2)

- **Deformation (×10) and crack pattern**

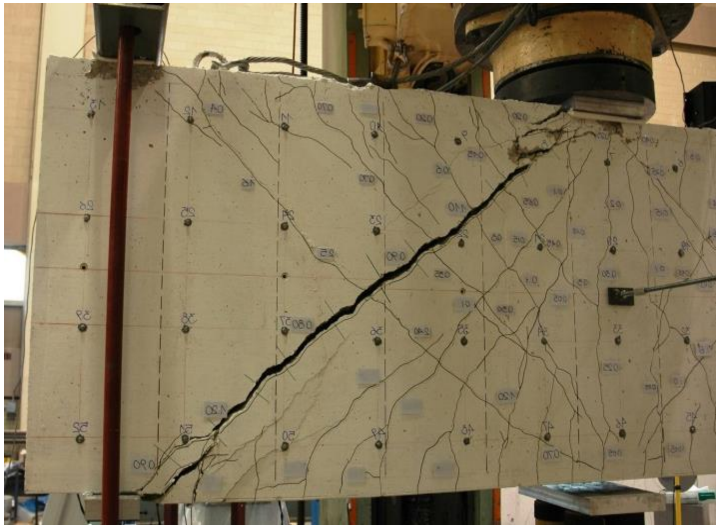


- **Principle compressive stress**

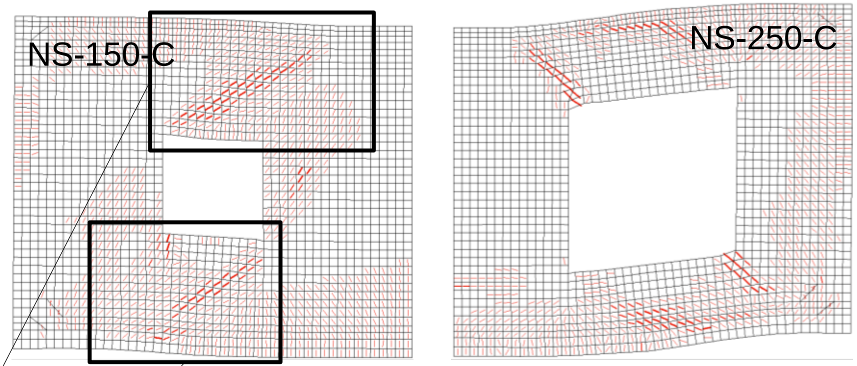


Deep beams with web openings studied by FEM

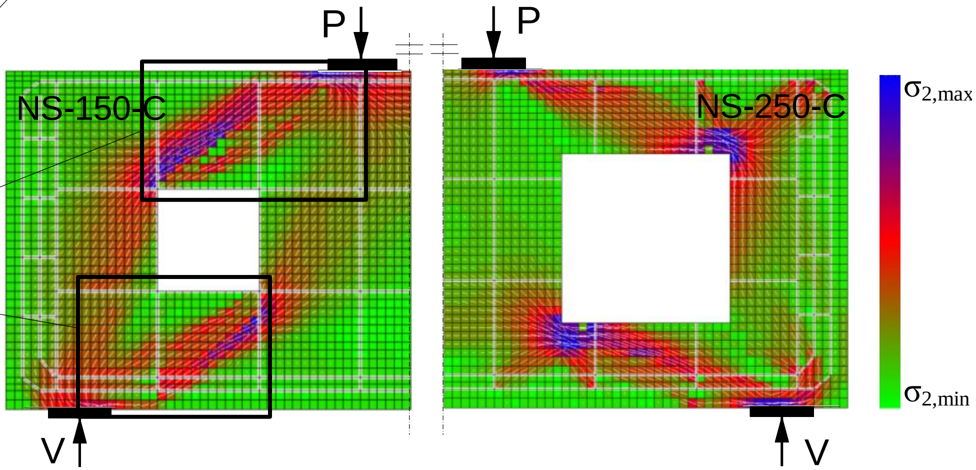
- **Crack pattern of solid deep beams**



- **Deformation (×10) and crack pattern**



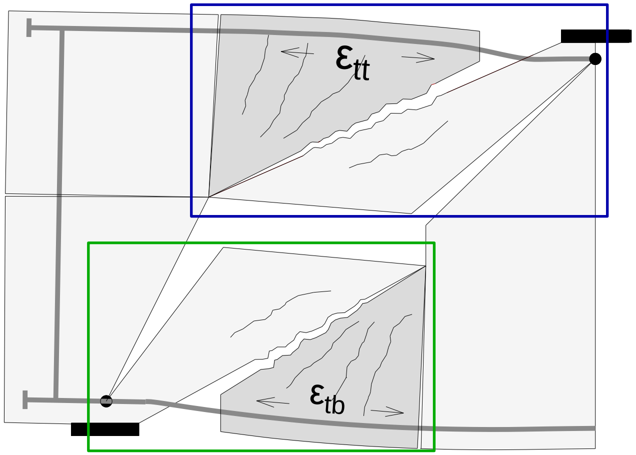
- **Principle compressive stress**



Similar to solid deep beams

Kinematics of deep beams with openings

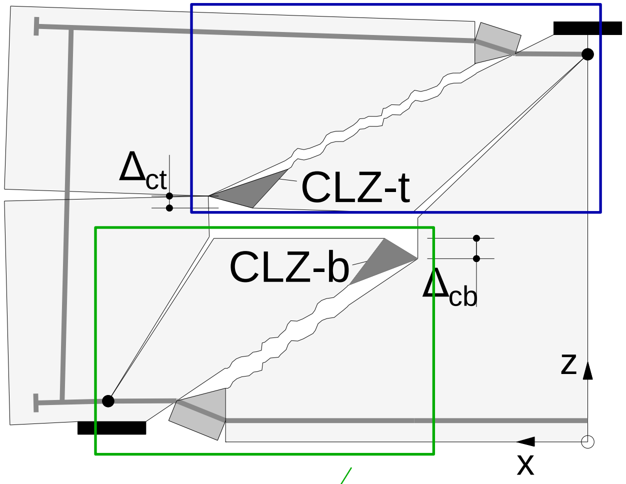
- **DOF ε_{tb} and ε_{tt}**
- **DOF Δ_{cb} and Δ_{ct}**



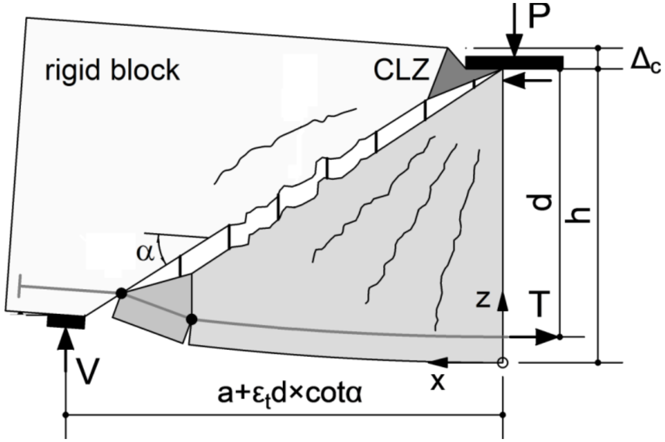
top sub shear span

+

bottom sub shear span

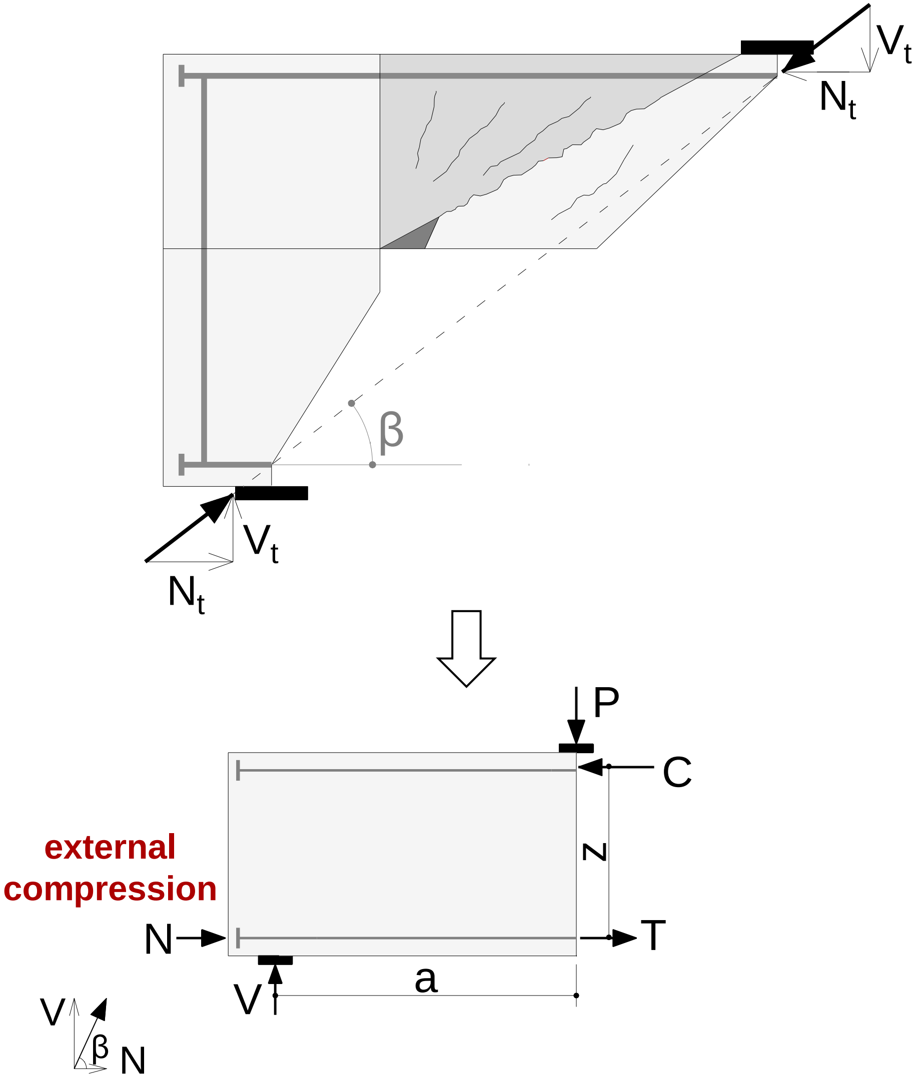


- **2PKT for solid deep beam**

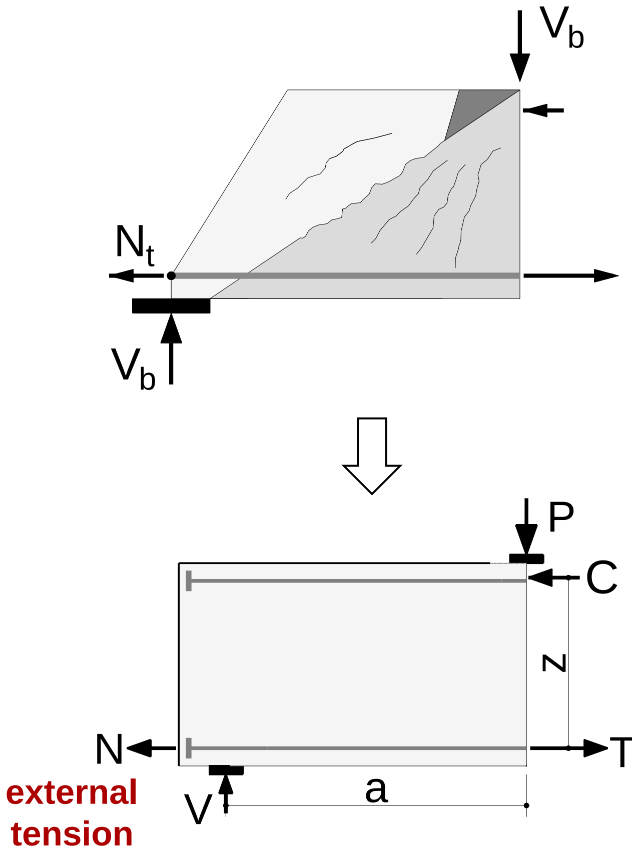


Equilibrium of forces in deep beams with openings

- **Top L-shaped region**

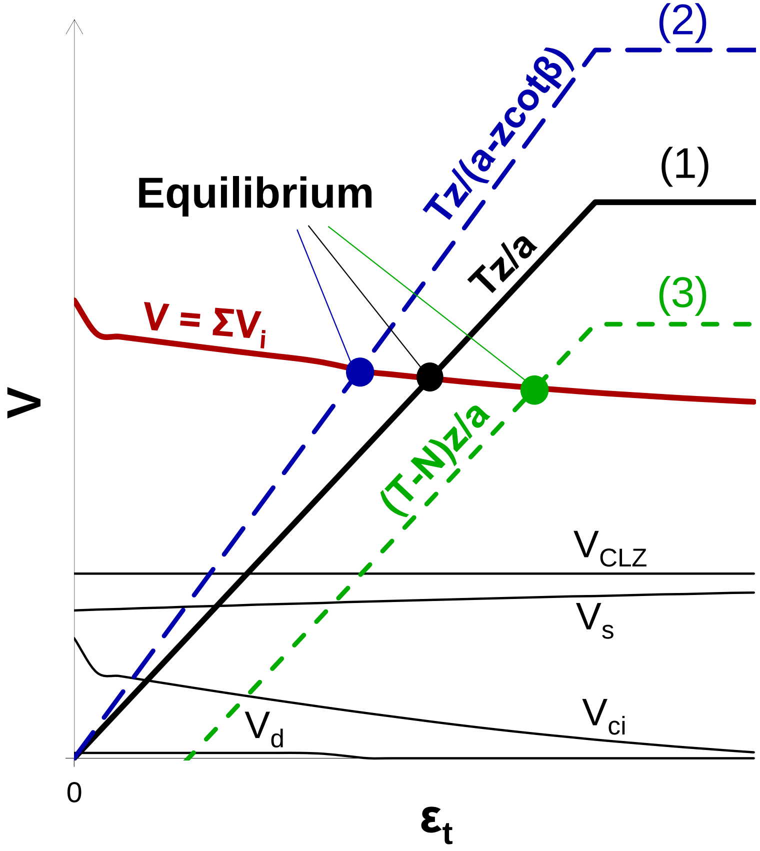


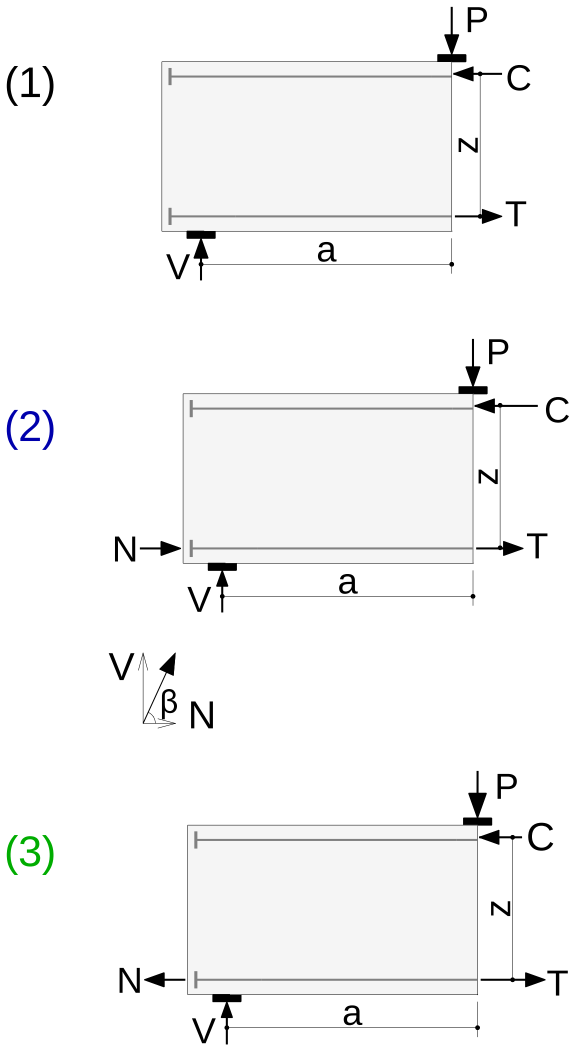
- **Bottom region**



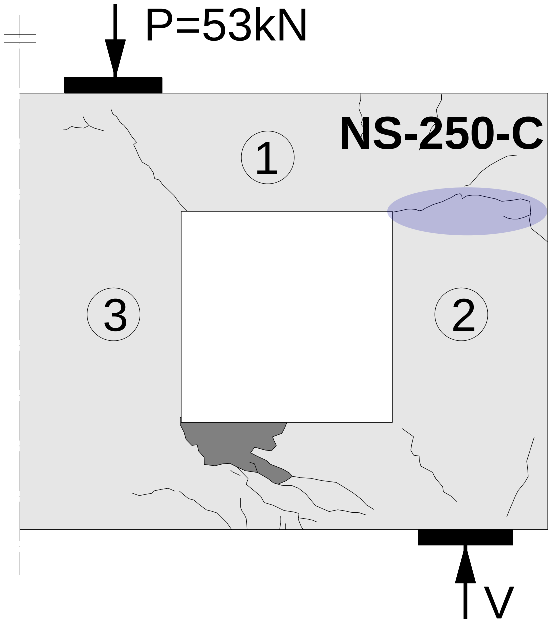
Solution procedure for 2PKT in other load cases

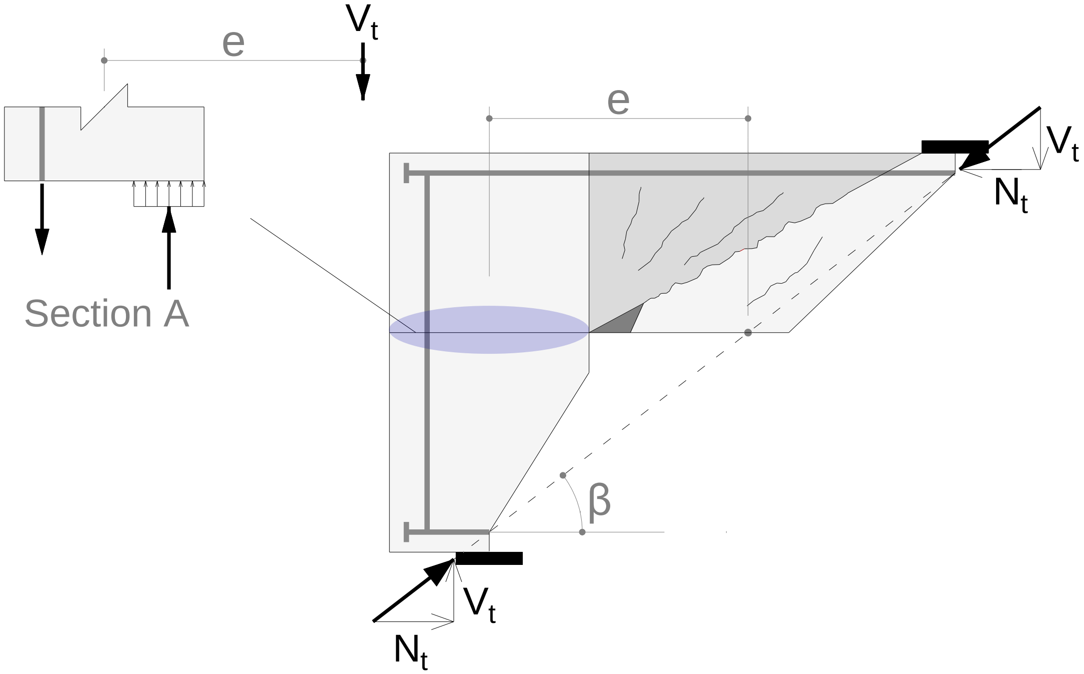
Solution procedure under given Δ_c





Failure along a crack

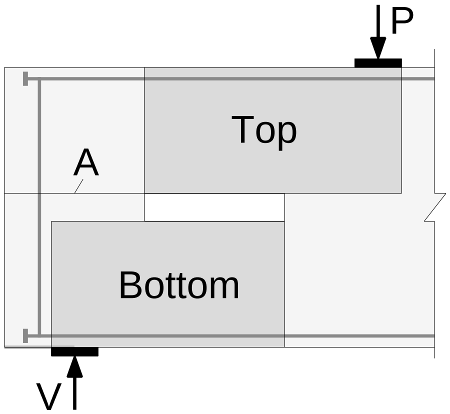


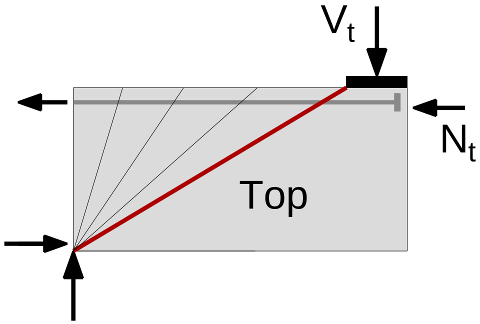


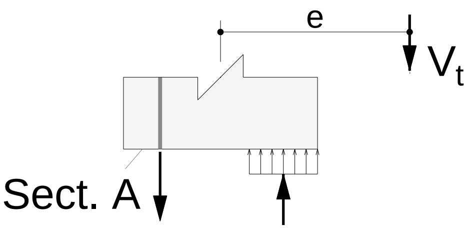
Need to consider the horizontal crack

- **Section A under M-N interaction**
- **V_t.e ≤ M_u of section A**

Calculation procedure



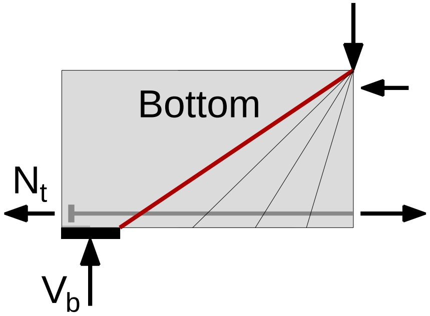




Step 1
Isolate two shear spans from the deep beam

Step 2
Calculate the shear strength of top shear span V_t

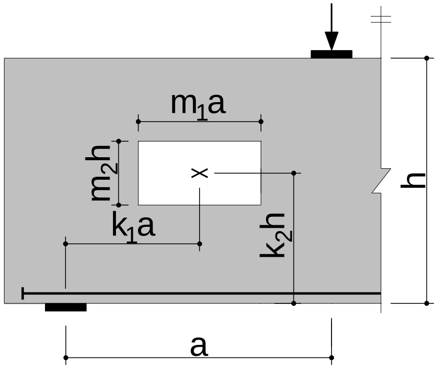
Step 3
Check section “A” and limit V_t if needed

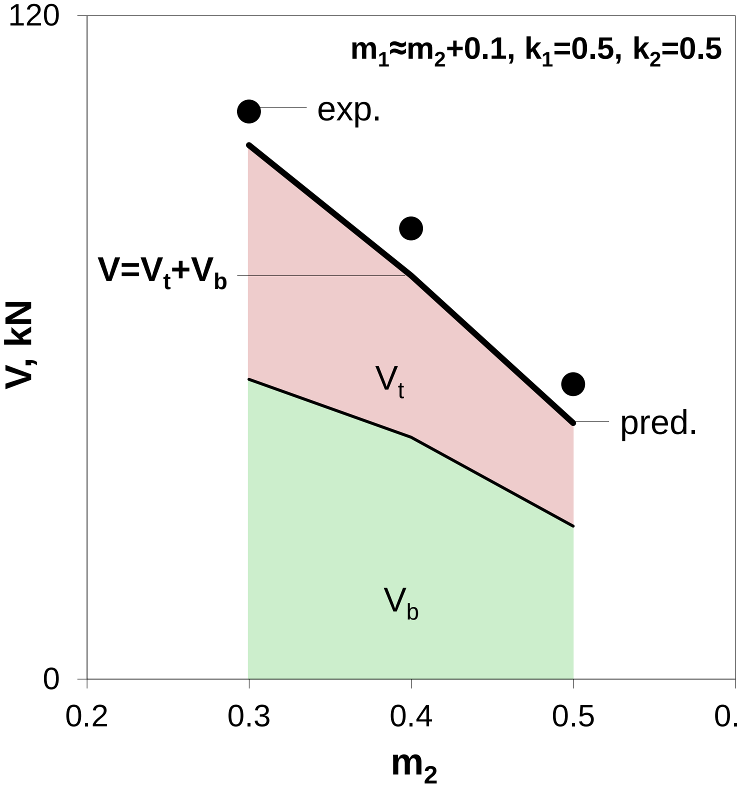


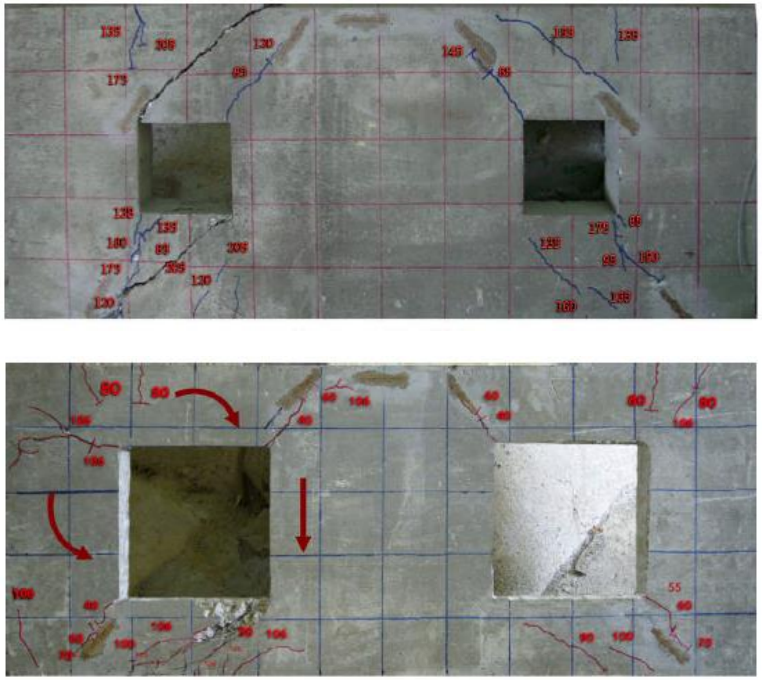
Step 4
Calculate the shear strength of bottom shear span V_b

Step 5
Obtain shear strength of the deep beam $V=V_t+V_b$

Parametric study: opening size

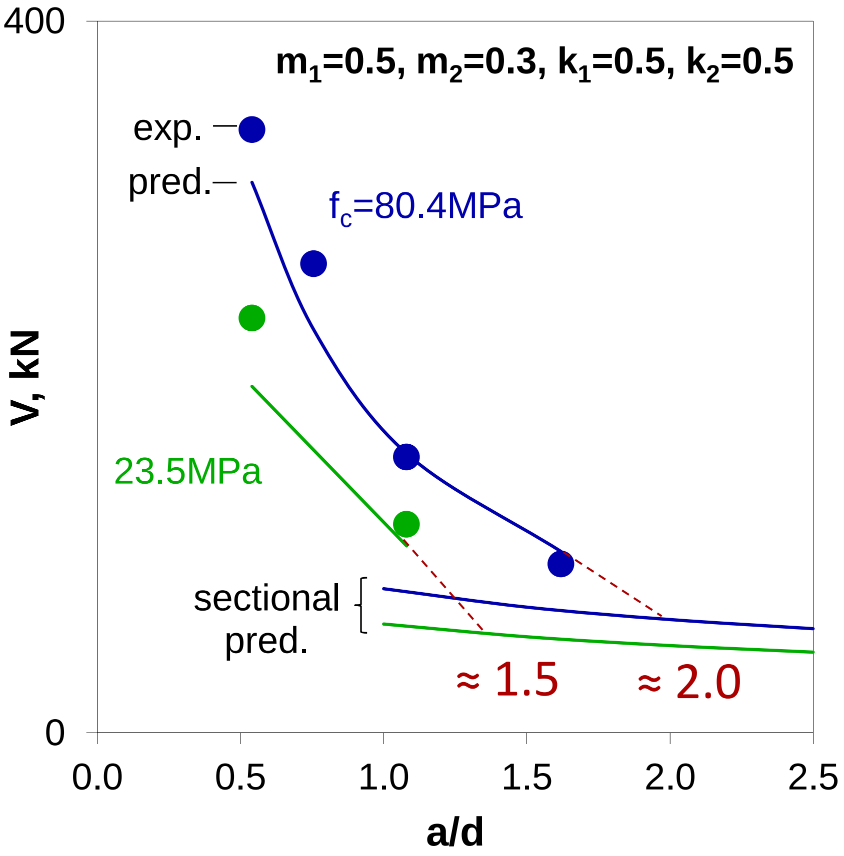


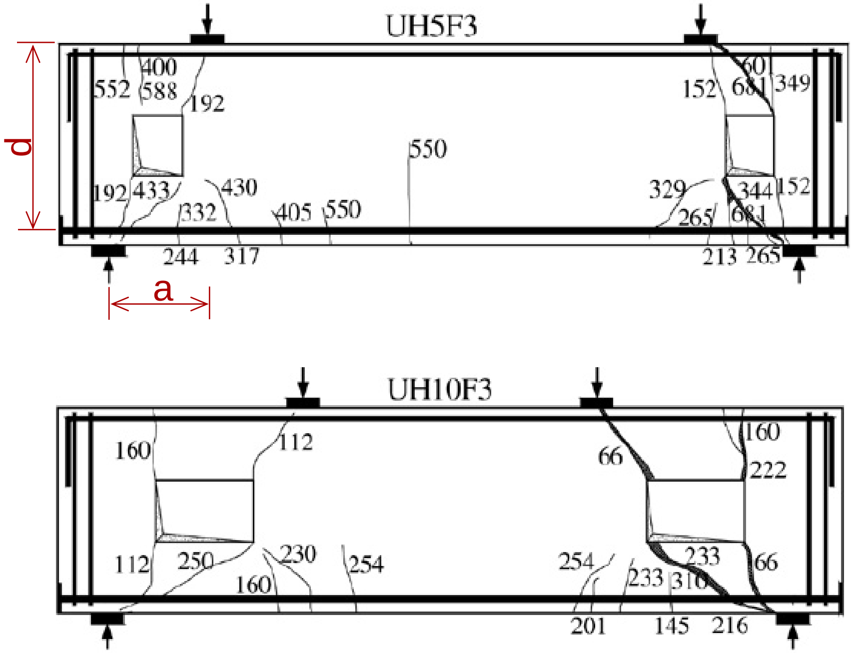




(tested by El-Maaddawy and Sherif, 2005)

Parametric study: a/d ratio





(tested by Yang et al., 2006)

V_{exp}/V_{pred} of 27 deep beams with openings:

Avg=1.03, COV=9.3%.

Conclusions and Future work

Summary and Conclusions

- **Adequate models** for shear strength of deep beams identified
- **Efficient 1D macroelement** formulated
- **Complete shear response** well predicted
- **Mixed-type modelling** framework proposed
- **Complex structures** under extreme loading analysed
- Kinematic model proposed for deep **beams with openings**

Future work

- Shear failures after flexural yielding → ductility
- Effect of axial force in macroelement → columns and shear walls
- Entire behaviour of RC deep beams with web openings
- Extend applicability of 3PKT, e.g. under cyclic loading
- Simplified 3PKT for design codes

Thank you for your attention.