Advanced Computational Method in Engineering – ACOMEN
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Simulation of the highly non linear properties of bulk superconductors: finite element approach with a backward Euler method and a single time step

Gregory P. Lousberg[1,*], M. Ausloos[2], C. Geuzaine[1], P. Dular[1], P. Vanderbemden[1], and B. Vanderheyden[1]

[1]Department of Electrical Engineering and Computer Science, University of Liege, Belgium
[2]SUPRATECS (B5a), University of Liege, Belgium
[*]FRS – FNRS fellowship

Université de Liège

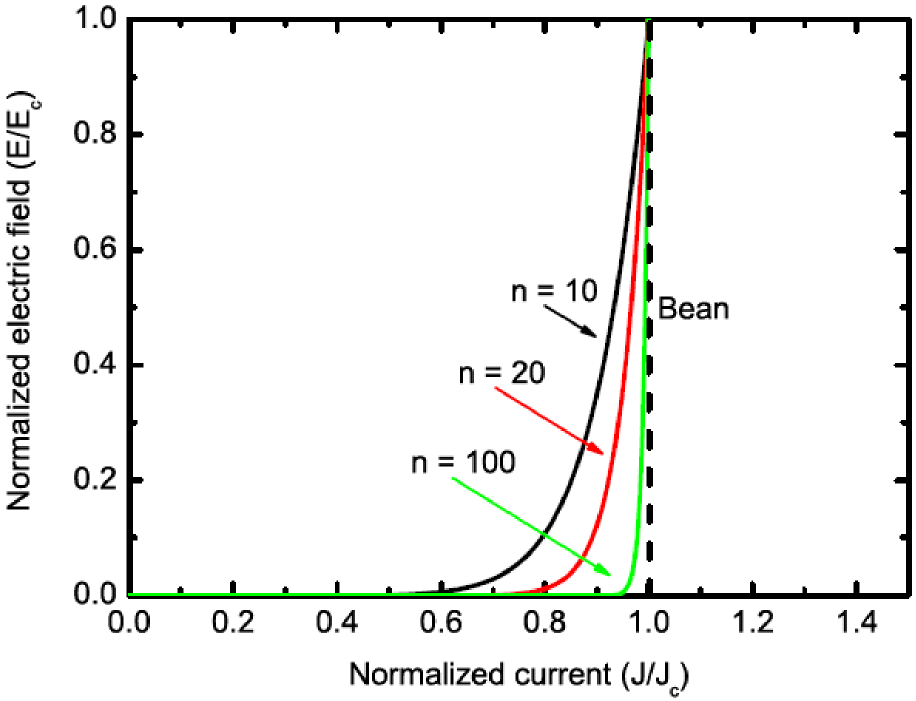
Breaking the gravity….

…with the magnetic levitation

Electrical resistivity of HTS is modelled by a non linear power law

$$E(J) = \rho(J)J = E_c \left(\frac{J}{J_c}\right)^n$$

n is the critical exponent
(*usually large !*)
J_c is the critical current density



Asymptotic behavior

- **n $\rightarrow \infty$: Bean model**

Analytical calculations available for comparison in specific geometries

Finite-element softwares are widely used for simulating HTS-based systems

Advantages

- **Many more geometries** can be treated
- **No extensive writing of numerical codes** is required
- **Treatment of non-linear problems** available in most commercial packages

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Drawbacks

- **Long calculation time** on fine meshing or in 3D geometry
- **Convergence problems** when *n* is **large**

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Proposed improvements

Single time step method implemented in an open-source solver, GetDP

- **Better control** of the algorithm parameters
- Used for simulating the **penetration of an external magnetic field** that varies linearly with time

Outline

- Finite-element formulation and implementation

  A- ϕ formulation

  Numerical resolution scheme

- Validation and comparison of the FEM model on simple geometries

A - ϕ formulation - Variables

The Maxwell equations are solved for two independent variables

- the **vector potential A**
- the **scalar potential** ϕ

defined as

$$\mathbf{B} = \mathbf{B}_{react} + \mathbf{B}_a = \text{curl } \mathbf{A} + \text{curl } \mathbf{A}_a$$

$$\mathbf{E} = - d\mathbf{A}/dt - d\mathbf{A}_a/dt - \text{grad } \phi$$

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Applied magnetic flux density (uniform)

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Induced magnetic flux density by the HTS

Applied magnetic flux density (uniform)

A - ϕ formulation - Variables

The Maxwell equations are solved for two independant variables

- the **vector potential A**
- the **scalar potential** ϕ

approximated by

Edge function (1st order)

$$\mathbf{A} = \sum_i a_i \mathbf{A}_i$$

Node function (1st order)

$$\phi = \sum_j b_j \phi_j$$

where $\boldsymbol{A}_i$ *and* ϕ_j *are known functions*

- *ensures continuity of the tangential component of* ***A***
- *ensures continuity of* ϕ

A - ϕ formulation – Equations and gauge condition

- The Maxwell equations are reduced to 2 equations

$$\begin{cases} \nabla \times \nabla \times \mathbf{A} = \mu_0 \sigma(\mathbf{A}, \phi) \left(-\dot{\mathbf{A}} - \dot{\mathbf{A}}_a - \nabla\phi\right) \\ \nabla \cdot \left\{\sigma(\mathbf{A}, \phi) \left(-\dot{\mathbf{A}} - \dot{\mathbf{A}}_a - \nabla\phi\right)\right\} = 0 \end{cases}$$

Ampere's law (rot H = J)

Continuity equations (div J = 0)

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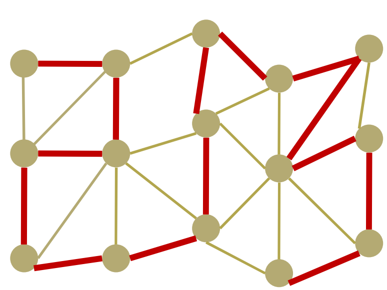
- A is not expressed in the Coulomb gauge, gauge condition : $\mathbf{A}.\mathbf{w} = 0$

Set of meshing edges that **connects** all the nodes **without closed contours**

Examples



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- Boundary conditions

$$\left.\begin{array}{l} \mathbf{A} = 0 \\ \phi = 0 \end{array}\right\} \text{ at infinity}$$

⟶ Use of **Jacobian transformation** for sending the outer surface of a spherical shell to infinity

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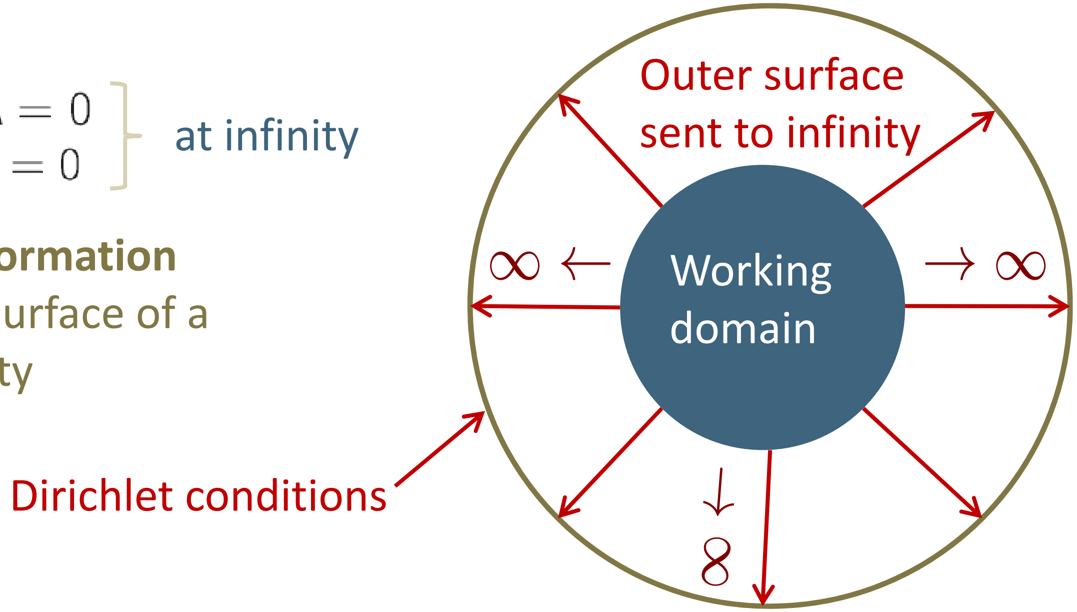

A - ϕ formulation – External field

- The Maxwell equations are reduced to 2 equations

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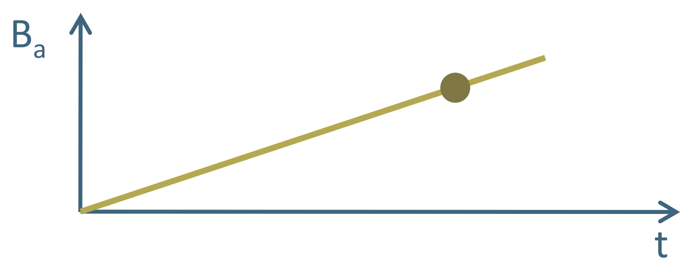
Ampere's law (rot H = J)

Continuity equations (div J = 0)

- A is not expressed in the Coulomb gauge, gauge condition : $\mathbf{A}.\mathbf{w} = 0$
- Boundary conditions $\left.\begin{matrix} \mathbf{A} = 0 \\ \phi = 0 \end{matrix}\right\}$ at infinity

Source field $\mathbf{A}_a$ corresponds to a **uniform magnetic flux density** $\mathbf{B}_a$

The source field is a temporal **ramp** with a **constant sweep rate** *(mT/s)*



Implicit time-resolution and non-linear Picard iteration

- The equations are solved with a **Galerkin residual minimization method**
- We use the **Backward Euler method** at each time step
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Solution (A, ϕ) @ $t - \Delta t$

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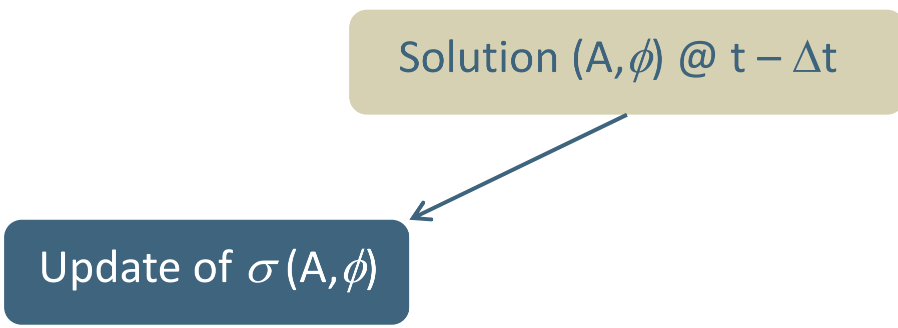

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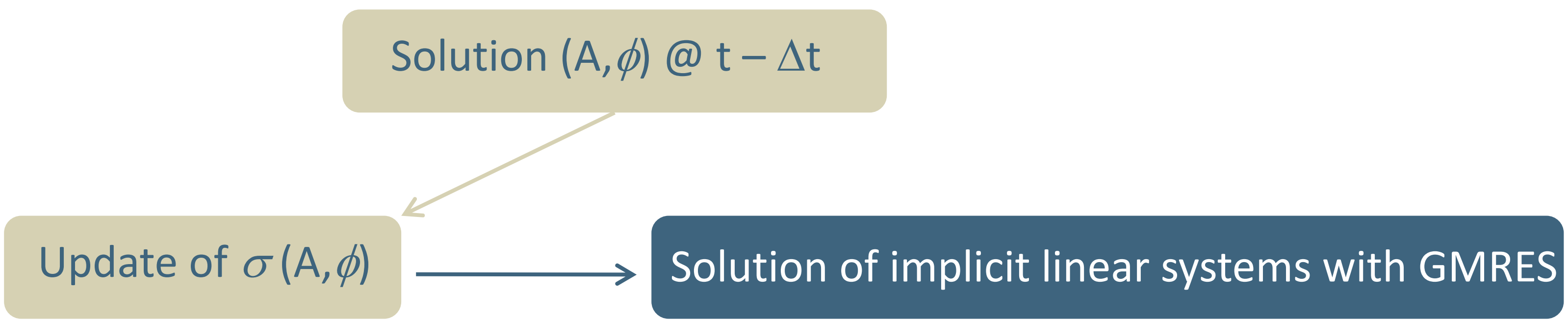

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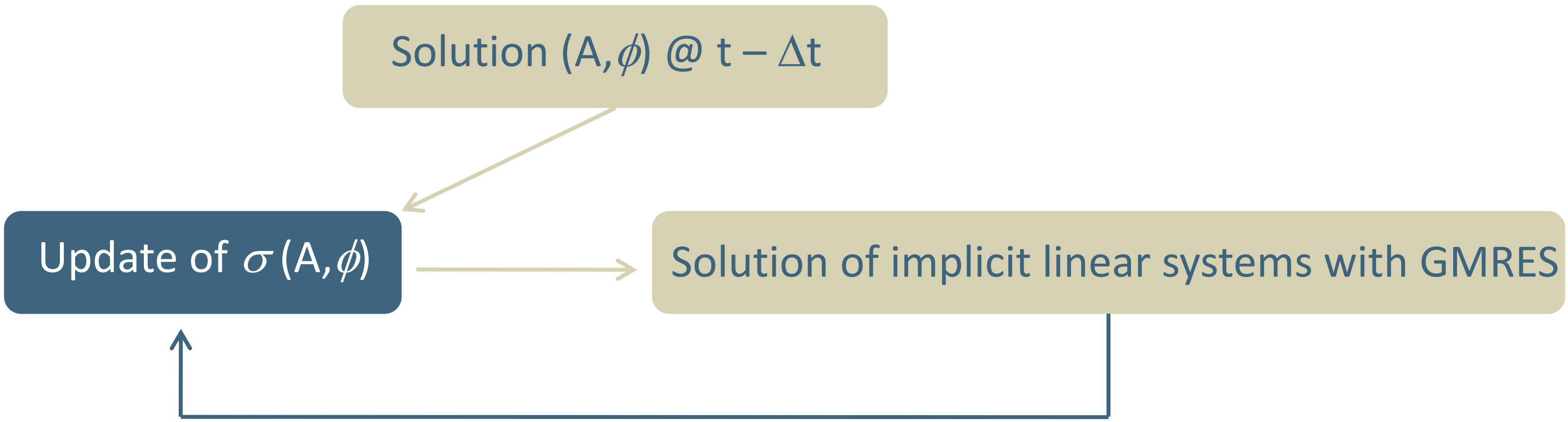

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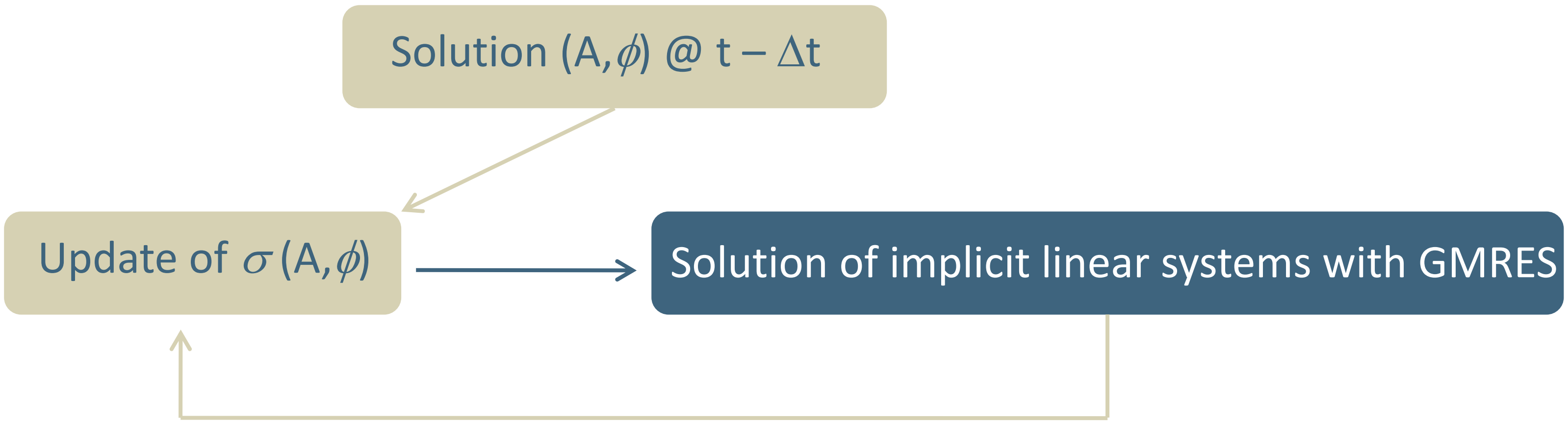

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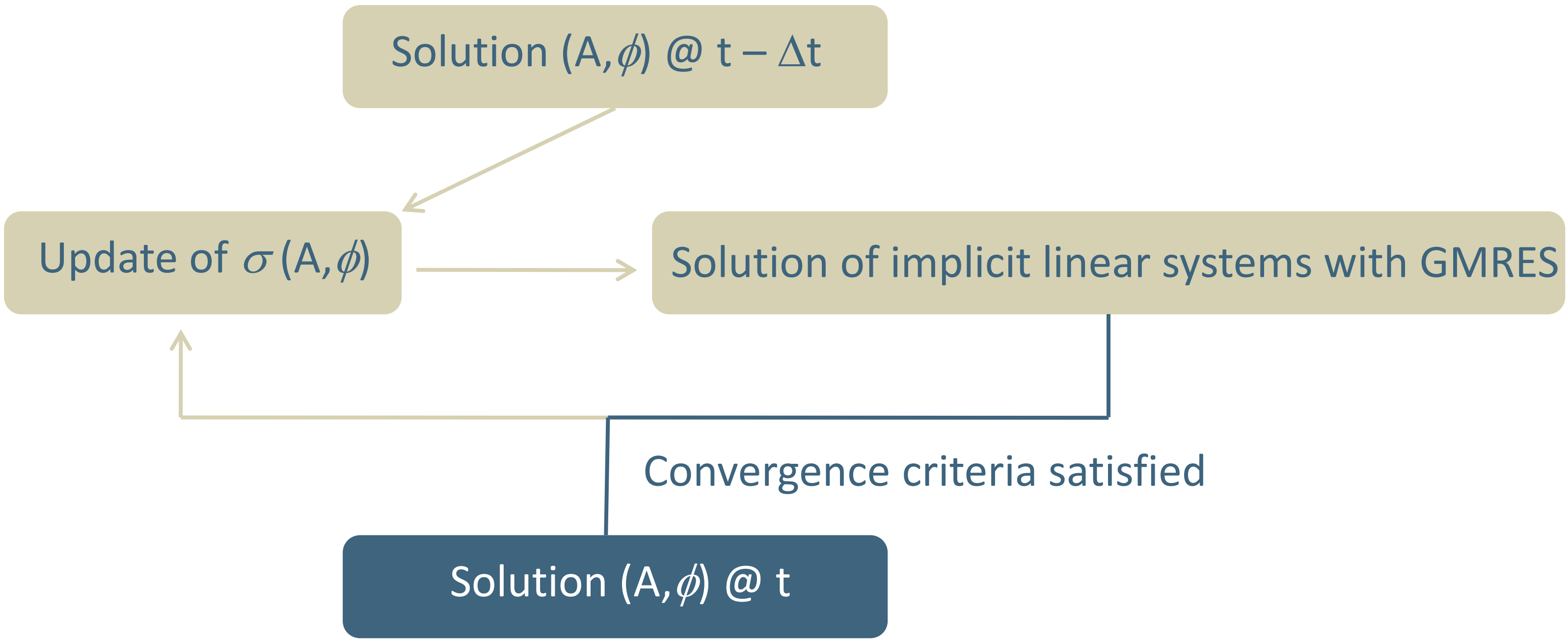


Outline

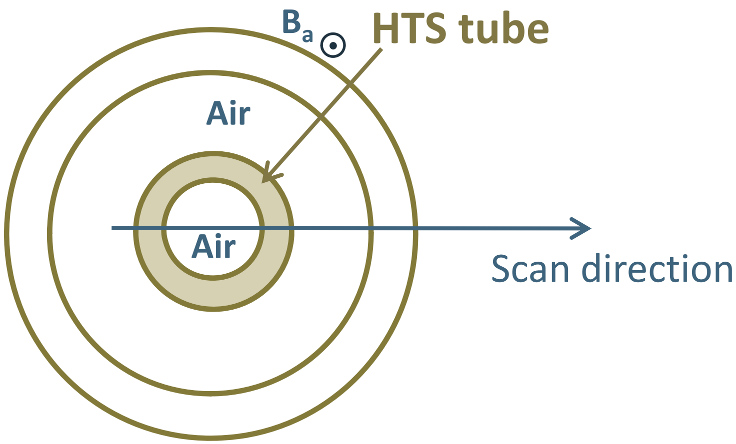
- Finite-element formulation and implementation
- Validation and comparison of the FEM model on simple geometries

  Magnetic field penetration in :

  - a HTS tube of infinite extension
  - a HTS tube of finite height

Bean critical-state is solved with the E(J) model taking $n = 100$

HTS tube of infinite height



Bean model

linear decay of the magnetic flux density in the wall

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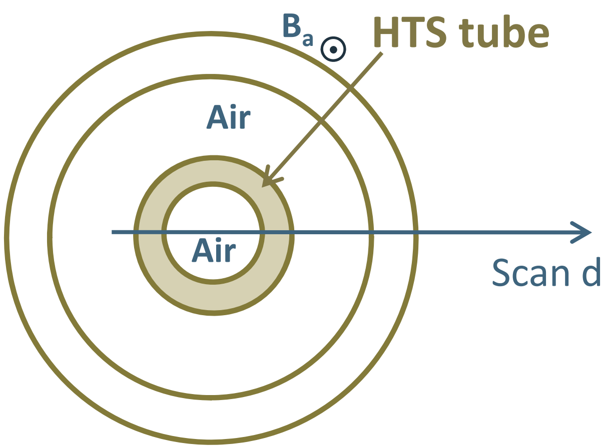
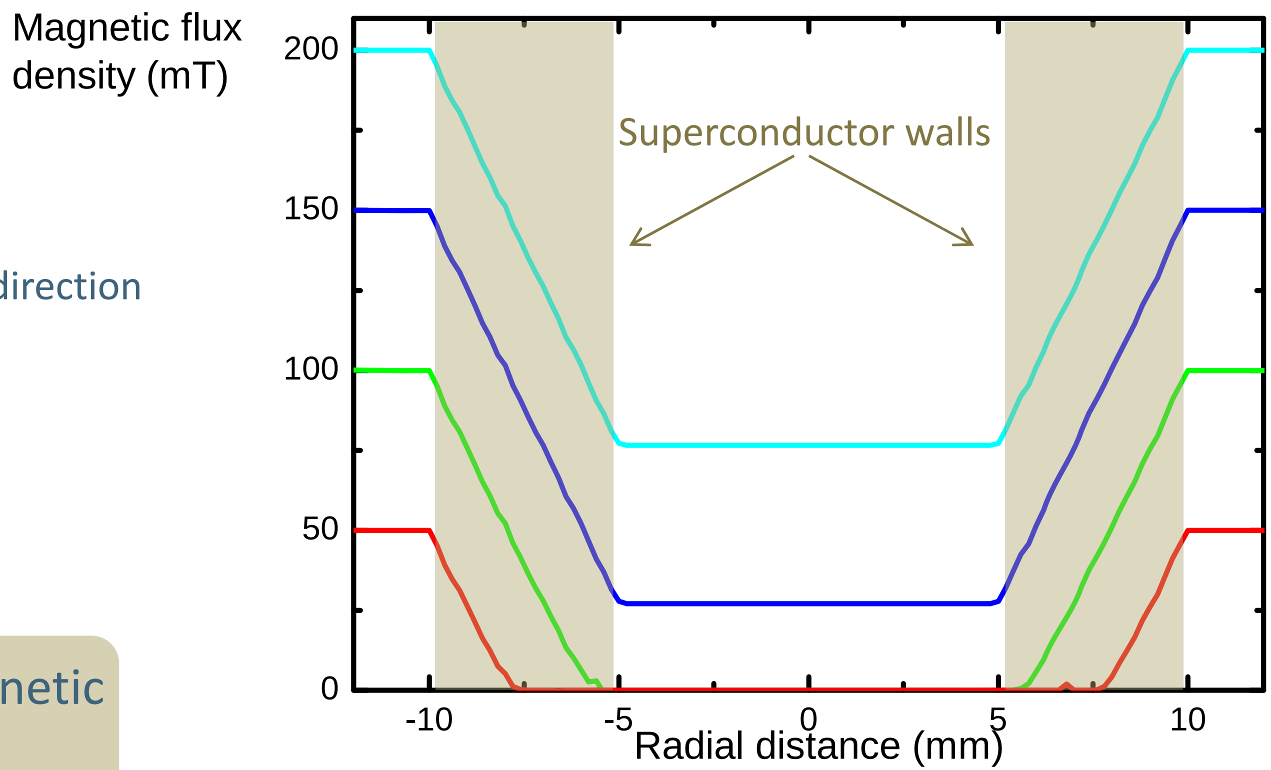


Magnetic flux penetration



Bean model

linear decay of the magnetic flux density in the wall

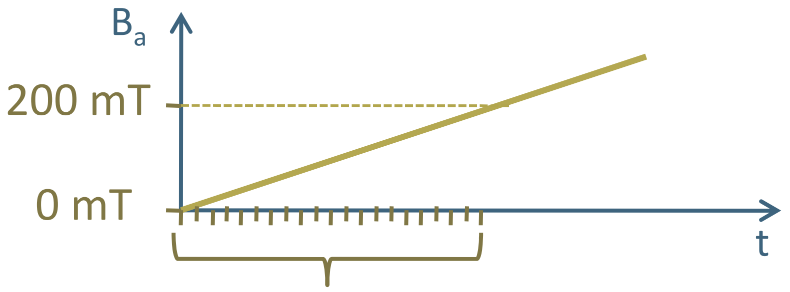
FEM results are consistent with the Bean model

Single time step method produces more accurate results in a smaller calculation time

Choice of the time step

For solving the problem with B_a = 200 mT with a sweep rate of 10 mT/s:

1. 20 time steps of 1s



System is solved @ t =1s, 2s, …, 20s

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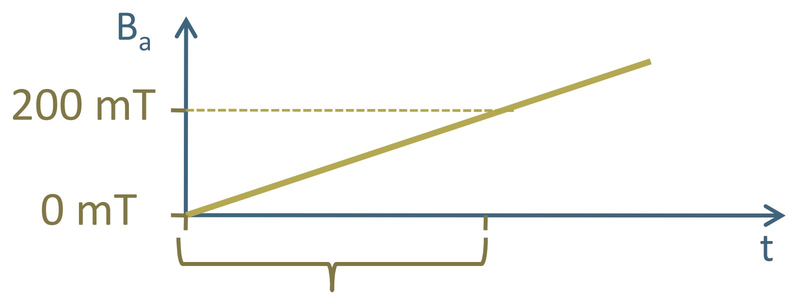
1. 20 time steps of 1s
2. 1 time step of 20 s → Single time step method



System is solved @ t = 20s

Single time step method produces more accurate results in a smaller calculation time

Magnetic field penetration of an external field increasing from 0 mT to 200 mT (10mT/s)

Comparison of two methods

1. **in a single simulation** with 20 time steps of 1s
2. **in 20 different simulations** with time steps of 1s, 2s, ..., 20s

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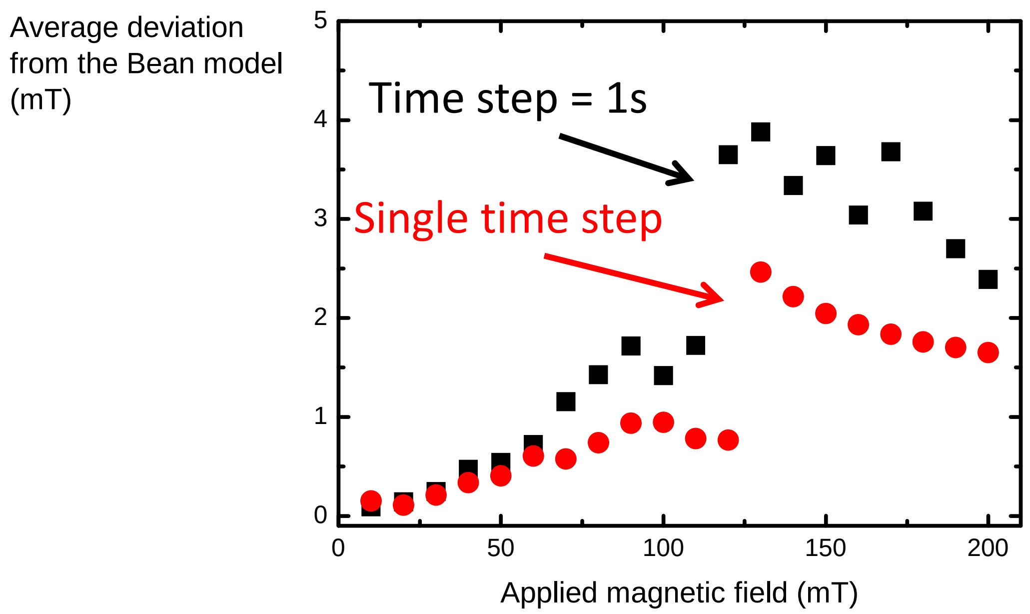
Analysis of the error



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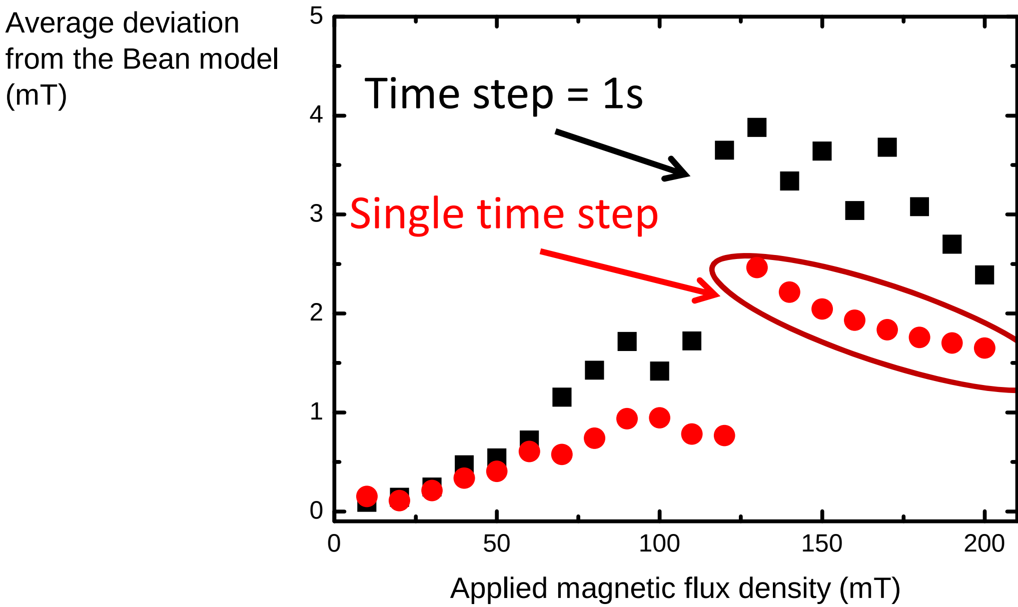


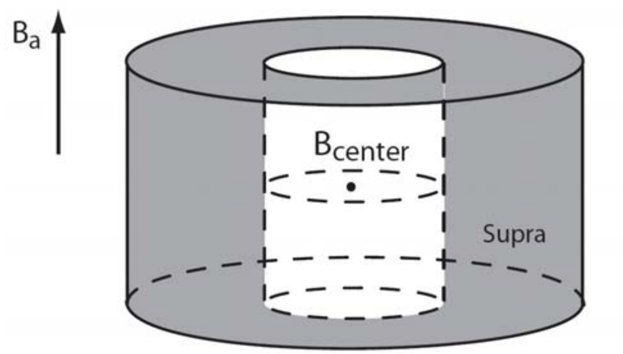
Calculation time

for solving B_a = 200 mT

20 time steps	2 days 3/4
Single time-step	20 min

3D FEM simulations on tubes with finite height are consistent with previous observations

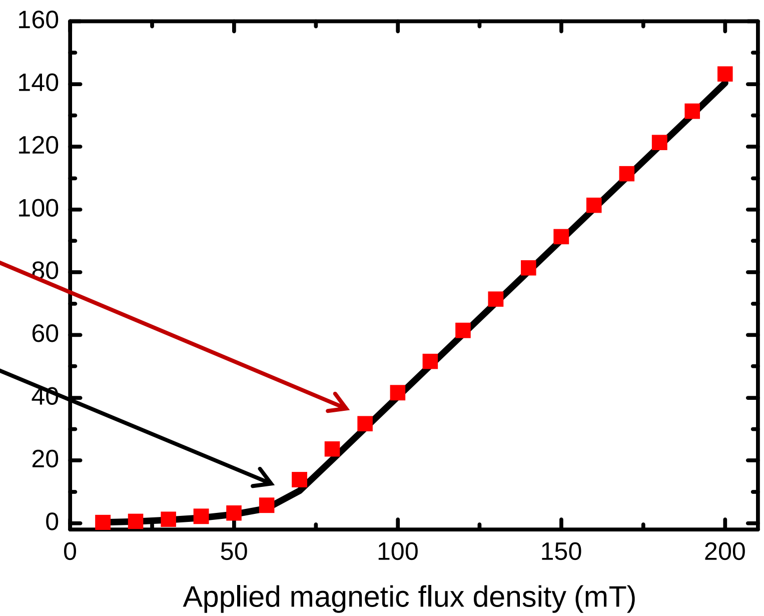
HTS tube of finite height



Simulation parameters

n = 100
B_a = 0 – 200 mT (10 mT/s)
Single time step method
3D geometry

Magnetic flux penetration



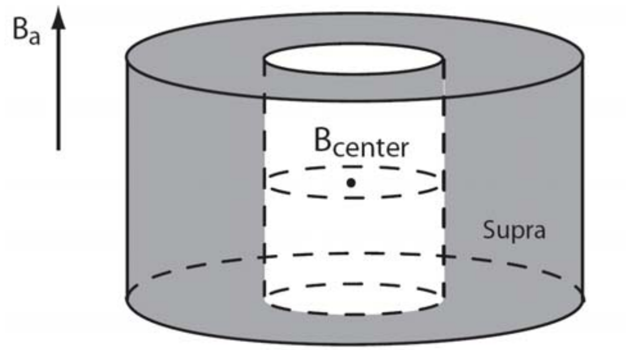
20 FEM simulations
with single time step method

Brandt method (*semi-analytical*)

multiple step method ($\Delta t = 5.10^{-4}$s)

Single time step method is more accurate with large critical exponent

HTS tube of finite height



Principle

- Applied magnetic flux density : 200 mT
- FEM single time step compared with Brandt method with the help of B_{center}

$$\text{Error} = \frac{B_{center,Brandt} - B_{center,FEM}}{B_{center,Brandt}}$$

Brandt (*semi-analytical*)

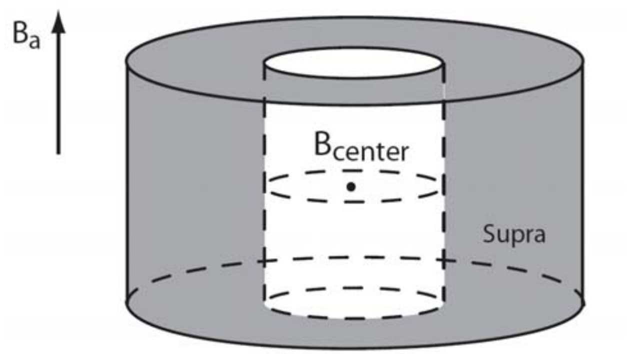
multiple time step method (Δt=5.10^{-4}s)

FEM

single time step method

Single time step method is more accurate with large critical exponent

HTS tube of finite height



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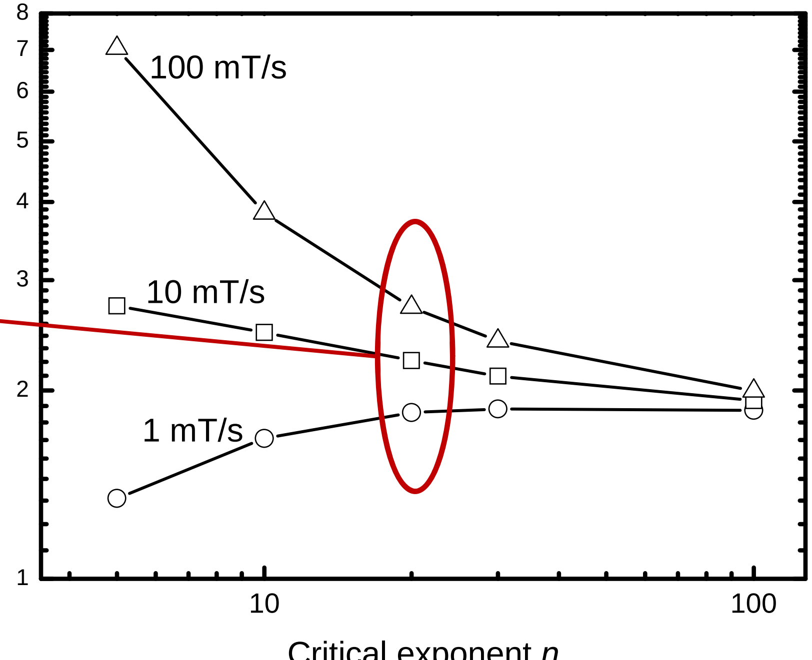
n ~ 20 : experimental value for the critical exponent

Single time step method is valid



Conclusion

- Implementation of a finite-element formulation in GetDP with high non linearity
- A-ϕ formulation in 3D geometry for calculating the magnetic field penetration
- Single time step method in the case of linearly time varying excitation is fast and accurate

Outline

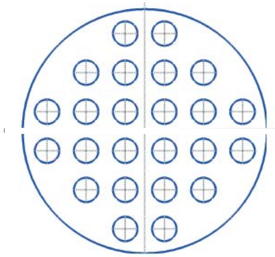
- Finite-element formulation and implementation
- Validation and comparison of the FEM model on simple geometries
- Optimization of the magnetic properties of drilled samples

    Influence of the lattice types

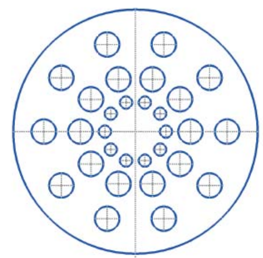
Polar triangular lattice wins …

HTS tube of infinite height

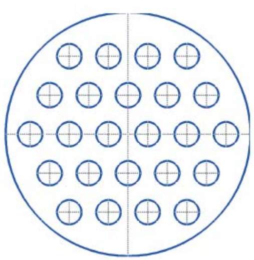
Squared



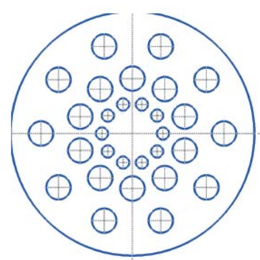
Polar squared

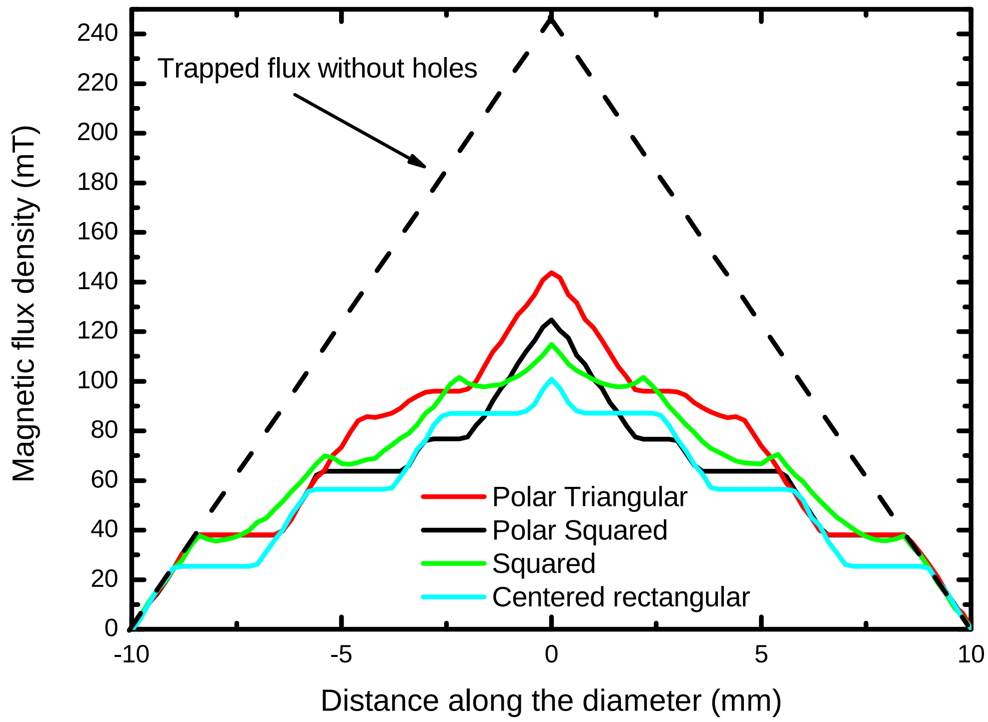


Centered rectangular



Polar triangular





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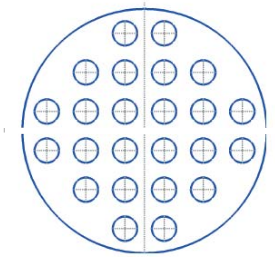

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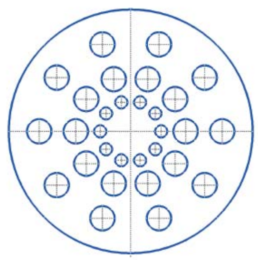

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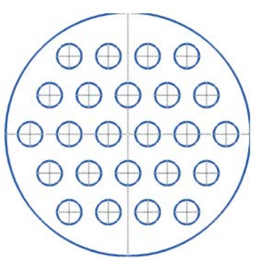

Polar triangular