

State complexity of the multiples of the Thue-Morse set

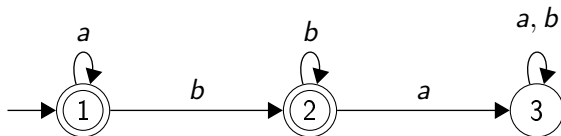
Adeline Massuir

Joint work with Émilie Charlier and Célia Cisternino

Comprehensible seminar

February 27th, 2019

Deterministic finite automaton (DFA) : $\mathcal{A} = (Q, q_0, F, \Sigma, \delta)$



Definition

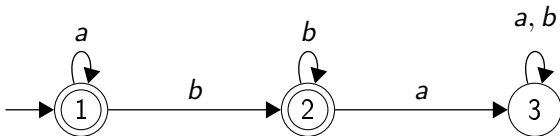
For each automaton $\mathcal{A} = (Q, q_0, F, \Sigma, \delta)$, the *language accepted by* \mathcal{A} is the set

$$L(\mathcal{A}) := \{w \in \Sigma^* : \delta(q_0, w) \in F\}.$$

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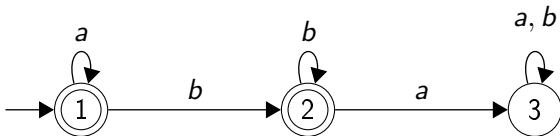


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Definition

A *regular language* is a language accepted by a DFA.

Given two regular languages L and M , is the language $L \cap M$ also regular?

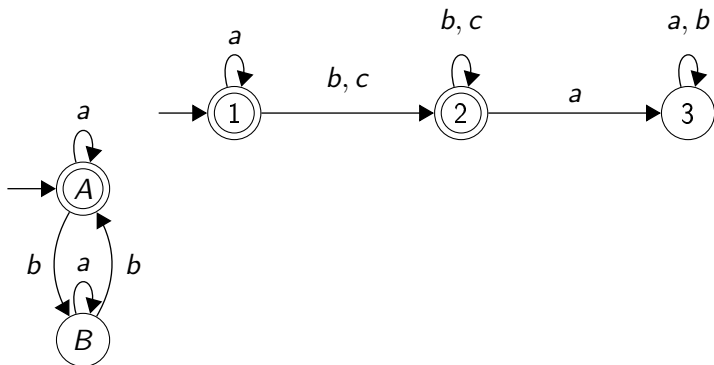
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Product of automata

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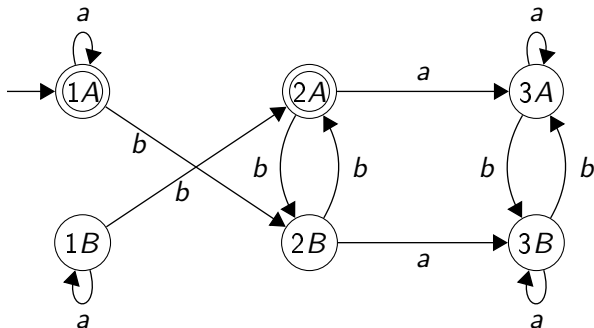
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An automaton $\mathcal{A} = (Q, q_0, F, \Sigma, \delta)$ is *complete* if $\forall q \in Q, \forall \sigma \in \Sigma,$

$$\delta(q, \sigma)$$

is defined.

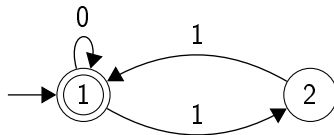
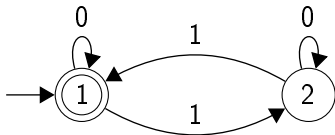
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Accessible automaton

Let $\mathcal{A} = (Q, q_0, F, \Sigma, \delta)$ be an automaton.

Definition

A state $q \in Q$ is *accessible* if $\exists w \in \Sigma^*$ s.t.

$$\delta(q_0, w) = q.$$

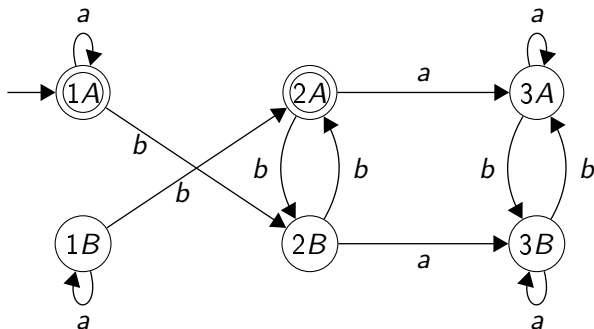
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Reduced automaton

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Two states $q, p \in Q$ are *distinguished* if $\exists w \in \Sigma^*$ s.t.

$(\delta(q, w) \in F \text{ and } \delta(p, w) \notin F)$ or $(\delta(q, w) \notin F \text{ and } \delta(p, w) \in F)$.

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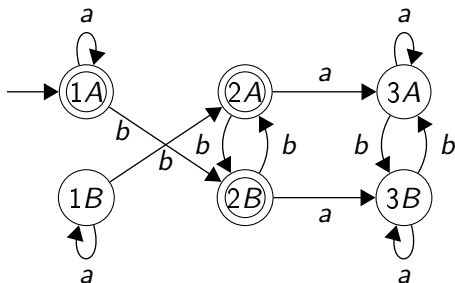
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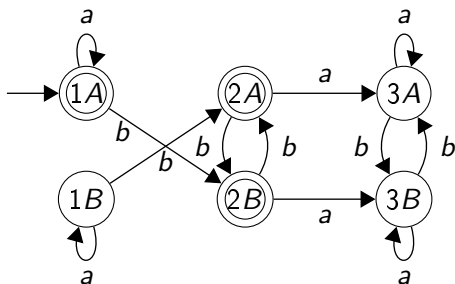
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Automaton with disjoint states

Let $\mathcal{A} = (Q, q_0, F, \Sigma, \delta)$ be an automaton.

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Two states $q, p \in Q$ are *disjoint* if $\forall w \in \Sigma^*$,

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The automaton \mathcal{A} *has disjoint states* if all its states are two by two disjoint.

Automaton with disjoint states

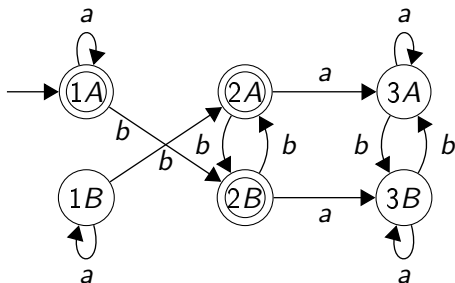
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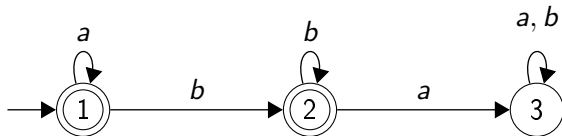
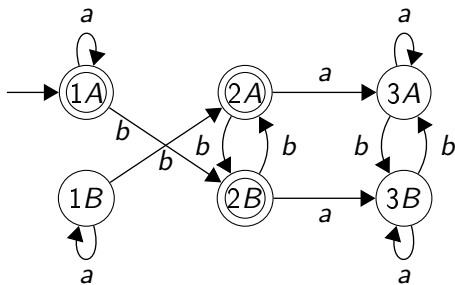
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Coaccessible + with disjoint states \Rightarrow reduced

Minimal automaton I



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One algorithm :

- 1 Eject non accessible states
- 2 Look for undistinguished states

Definition

The *state complexity* of a regular language is the number of states of its minimal automaton.

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The state complexity of a^*b^* is 3.

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Definition

Let $b \in \mathbb{N}_{\geq 2}$. A subset X of \mathbb{N} is *b-recognizable* if $\text{rep}_b(X)$ is regular.

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It is equivalent to work with $0^* \text{rep}_b(X)$.

Theorem

Let $b \in \mathbb{N}_{\geq 2}$ and $m \in \mathbb{N}$. If $X \subseteq \mathbb{N}$ is *b-recognizable*, so is mX .

Theorem [Alexeev, 2004]

The state complexity of the language $0^* \text{rep}_b(m\mathbb{N})$ is

$$\min_{N \geq 0} \left\{ \frac{m}{\gcd(m, b^N)} + \sum_{n=0}^{N-1} \frac{b^n}{\gcd(b^n, m)} \right\}$$

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Definition

The Thue-Morse set is the set

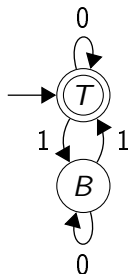
$$\mathcal{T} = \{n \in \mathbb{N} : |\text{rep}_2(n)|_1 \in 2\mathbb{N}\}.$$

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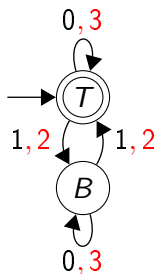


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Definition

Let $p, q \in \mathbb{N}_{\geq 2}$. We say that p and q are *multiplicatively independent* if

$$p^a = q^b \Rightarrow a = b = 0.$$

They are said *multiplicativement dependent* otherwise.

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They are said *multiplicatively dependent* otherwise.

Theorem [Cobham, 1969]

- Let b, b' two multiplicatively independent bases. A subset of \mathbb{N} is both b -recognizable and b' -recognizable iff it is a finite union of arithmetic progressions.
- Let b, b' two multiplicatively dependent bases. A subset of \mathbb{N} is b -recognizable iff it is b' -recognizable.

$$\mathcal{T} = \{n \in \mathbb{N} : |\text{rep}_2(n)|_1 \in 2\mathbb{N}\}$$

Theorem

Let $m \in \mathbb{N}$ and $p \in \mathbb{N}_{\geq 1}$.

Then the state complexity of the language $0^* \text{rep}_{2^p}(m\mathcal{T})$ is

$$2k + \left\lceil \frac{z}{p} \right\rceil$$

if $m = k2^z$ with k odd.

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$\mathcal{A}_{\mathcal{I}, 2^p} \times \mathcal{A}_{m, 2^p}$	$(0, 0)^* \text{rep}_{2^p} (\{(t, mt) : t \in \mathcal{I}\})$

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$\mathcal{A}_{\mathcal{I}, 2^p} \times \mathcal{A}_{m, 2^p}$	$(0, 0)^* \text{rep}_{2^p} (\{(t, mt) : t \in \mathcal{I}\})$
$\pi (\mathcal{A}_{\mathcal{I}, 2^p} \times \mathcal{A}_{m, 2^p})$	$0^* \text{rep}_{2^p} (m\mathcal{I})$

The automaton $\mathcal{A}_{\mathcal{T}, 2^p}$

$$(0, 0)^* \{ \text{rep}_{2^p}(t, n) : t \in \mathcal{T}, n \in \mathbb{N} \}$$

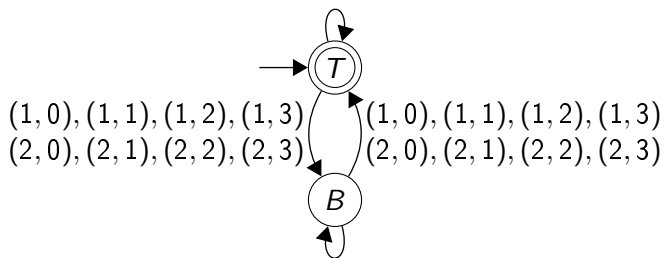
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States	T, B
Initial state	T
Final states	T
Alphabet	$\{0, \dots, 2^p - 1\}^2$
Transitions	$\delta_{\mathcal{A}_{\mathcal{T}, 2^p}}(X, (a, b)) = \begin{cases} X & \text{if } a \in \mathcal{T} \\ \bar{X} & \text{else.} \end{cases}$

The automaton $\mathcal{A}_{\mathcal{T},4}$

$(0, 0), (0, 1), (0, 2), (0, 3)$
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Lemma

The automaton $\mathcal{A}_{\mathcal{T}, 2^p}$

- is the minimal automaton of $(0, 0)^* \{\text{rep}_{2^p}(t, n) : t \in \mathcal{T}, n \in \mathbb{N}\}$,
- is complete,
- is trim,
- has disjoint states.

The automaton $\mathcal{A}_{m,b}$

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States	$0, \dots, m-1$
Initial state	0
Final states	0
Alphabet	$\{0, \dots, b-1\}^2$
Transitions	$\delta_{m,b}(i, (d, e)) = j \Leftrightarrow bi + e = md + j$

The automaton $\mathcal{A}_{m,b}$

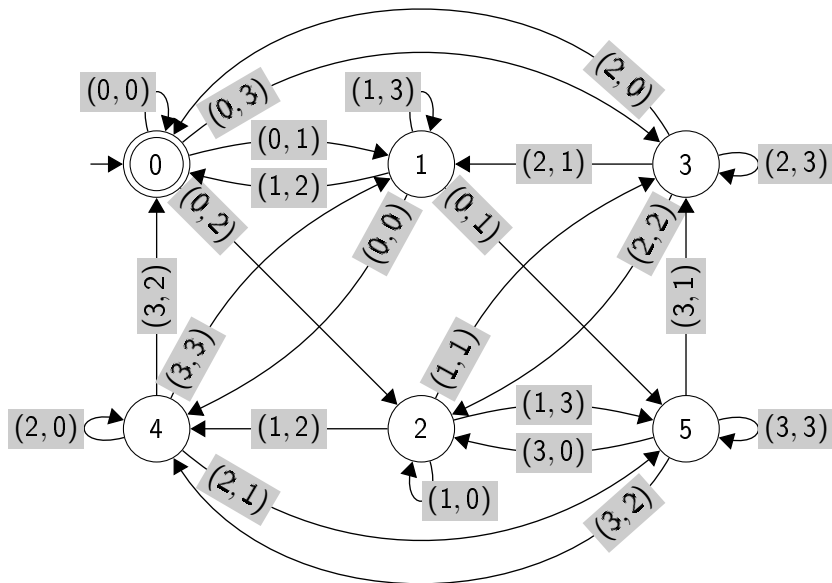
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For all $i, j \in \{0, \dots, m - 1\}$, for all $u, v \in \{0, \dots, b - 1\}^*$,

$$\delta_{m,b}(i, (u, v)) = j \Leftrightarrow b^{|(u,v)|} i + \text{val}_b(v) = m \text{val}_b(u) + j.$$

The automaton $\mathcal{A}_{6,4}$



Proposition

The automaton $\mathcal{A}_{m,b}$

- is accessible,
- is coaccessible,
- has disjoint states,
- is minimal,
- is trim.

The product automaton $\mathcal{A}_{m,2^p} \times \mathcal{A}_{\mathcal{T},2^p}$

$$(0,0)^* \{ \text{rep}_{2^p}(t, mt) : t \in \mathcal{T} \}$$

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States	$(0, T), \dots, (m-1, T), (0, B), \dots, (m-1, B)$
Initial state	$(0, T)$
Final states	$(0, T)$
Alphabet	$\{0, \dots, 2^p - 1\}^2$
Transitions	$\delta_{\mathcal{A}_{\mathcal{T},2^p}}((i, X), (u, v)) = (j, Y)$ $\Leftrightarrow 2^{p (u,v)}i + \text{val}_{2^p}(v) = m \text{val}_{2^p}(u) + j$ and $Y = \begin{cases} X & \text{if } \text{val}_{2^p}(u) \in \mathcal{T} \\ \bar{X} & \text{else.} \end{cases}$

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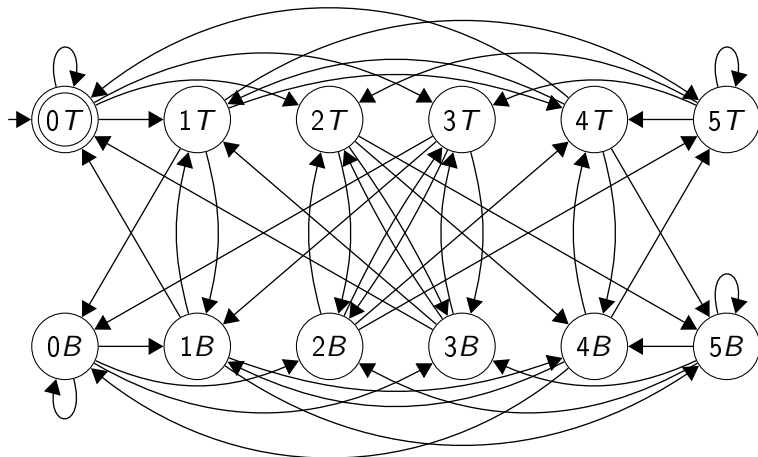
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Remark

If i, X, v are fixed, there exist unique j, Y, u such that we have a transition labeled by (u, v) from (i, X) to (j, Y) .

The automaton $\mathcal{A}_{6,4} \times \mathcal{A}_{\mathcal{T},4}$



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$$(0, T) \rightarrow (i, B) : \text{rep}_{2^p}(1, m + i)$$

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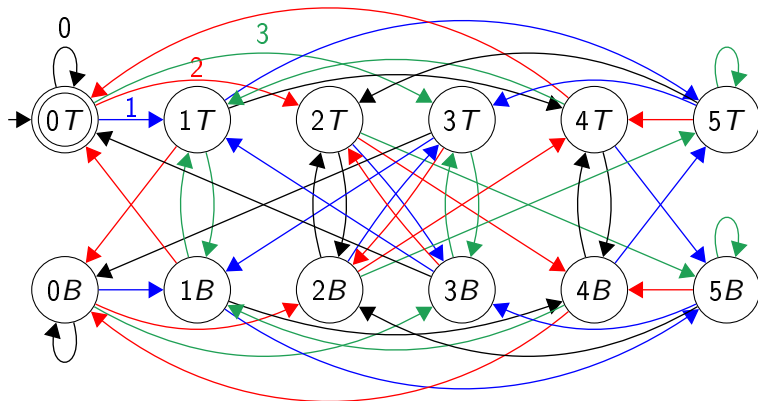
$$(0, B) \rightarrow (0, T) : \begin{array}{l} \text{rep}_{2^p}(1, m) \text{ if } p|z \\ \text{rep}_{2^p}(2^{p-r}, k2^{q+1}) \text{ if } z = qp + r \\ \text{with } q \in \mathbb{N} \text{ and } r \in \{1, \dots, p-1\}. \end{array}$$

The projected automaton $\pi(\mathcal{A}_{m,2^p} \times \mathcal{A}_{\mathcal{T},2^p})$

$$0^* \text{rep}_{2^p}(m\mathcal{T}) = 0^* \{\text{rep}_{2^p}(mt) : t \in \mathcal{T}\}$$

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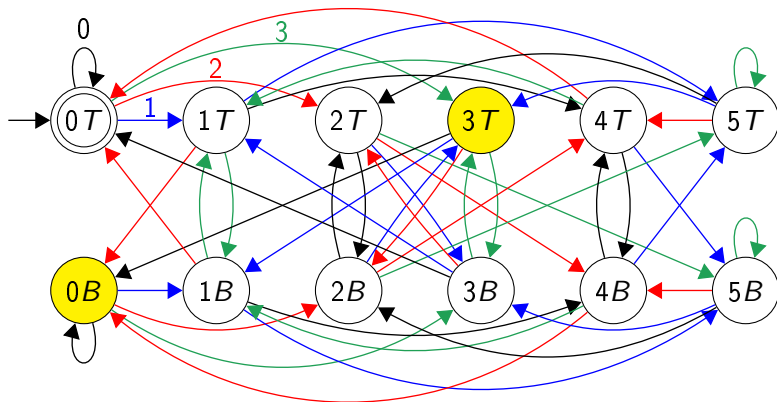
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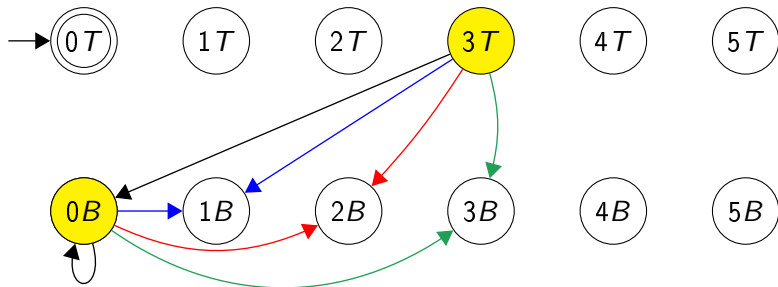
Proposition

In the automaton $\pi(\mathcal{A}_{m,2^p} \times \mathcal{A}_{\mathcal{T},2^p})$, the states (i, T) and (i, B) are disjoint for all $i \in \{0, \dots, m-1\}$.

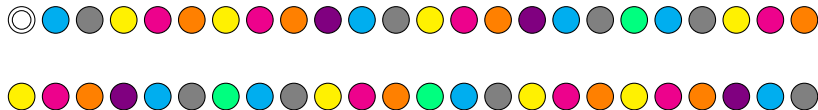
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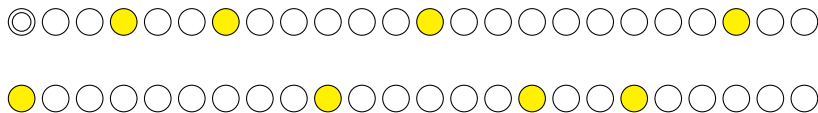
The automaton $\pi(\mathcal{A}_{6,4} \times \mathcal{A}_{\mathcal{T},4})$



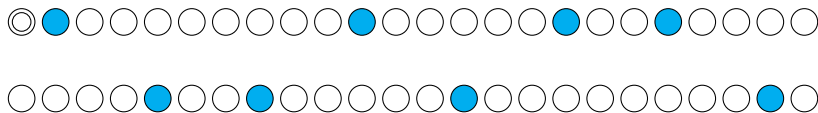
The automaton $\pi(\mathcal{A}_{24,4} \times \mathcal{A}_{\mathcal{T},4})$



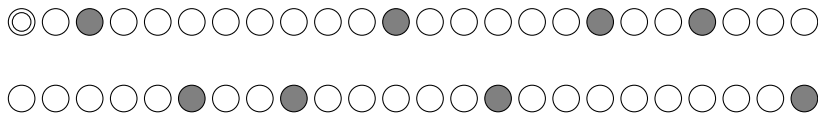
The automaton $\pi(\mathcal{A}_{24,4} \times \mathcal{A}_{\mathcal{T},4})$



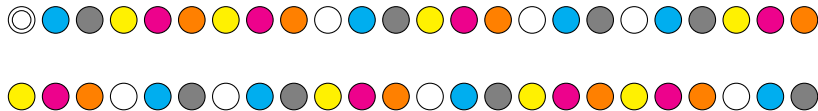
The automaton $\pi(\mathcal{A}_{24,4} \times \mathcal{A}_{\mathcal{T},4})$



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Recall

For $m \in \mathbb{N}$, $p \in \mathbb{N}_{\geq 1}$, we write $m = k2^z$ with z odd.

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Definition

For all $j \in \{1, \dots, k-1\}$, we set

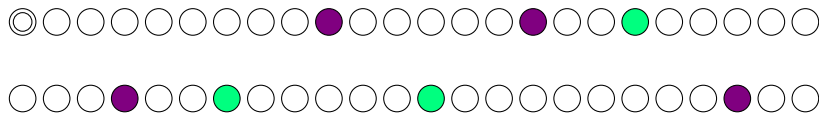
$$[(j, T)] := \{(j + k\ell, T_\ell) : 0 \leq \ell \leq 2^z - 1\}$$

$$[(j, B)] := \{(j + k\ell, \overline{T}_\ell) : 0 \leq \ell \leq 2^z - 1\}.$$

We also set

$$[(0, T)] := \{(0, T)\} \text{ and } [(0, B)] := \{(k\ell, \overline{T}_\ell) : 0 \leq \ell \leq 2^z - 1\}.$$

The automaton $\pi(\mathcal{A}_{24,4} \times \mathcal{A}_{\mathcal{T},4})$



Definition

For all $\alpha \in \{0, \dots, z-1\}$, we set

$$C_\alpha := \{(k2^{z-\alpha-1} + k2^{z-\alpha}l, \overline{T}_l) : 0 \leq l \leq 2^\alpha - 1\}.$$

For all $\beta \in \{0, \dots, \lceil \frac{z}{p} \rceil - 2\}$, we set

$$\Gamma_\beta := \bigcup_{\alpha \in \{\beta p, \dots, (\beta+1)p-1\}} C_\alpha.$$

We also set

$$\Gamma_{\lceil \frac{z}{p} \rceil - 1} := \bigcup_{\alpha \in \{(\lceil \frac{z}{p} \rceil - 1)p, \dots, z-1\}} C_\alpha.$$

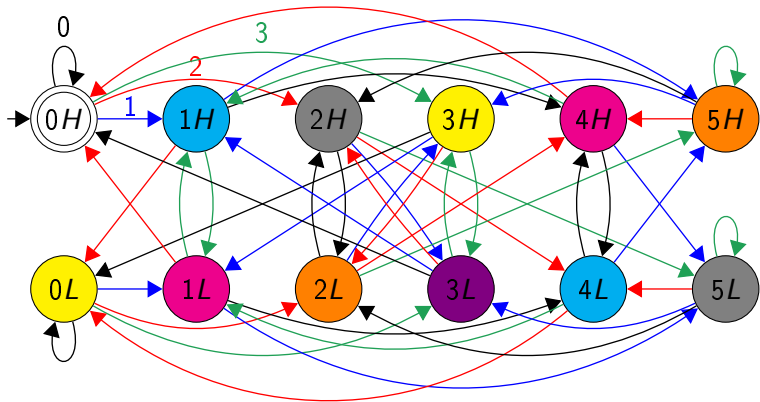
We can build a new automaton

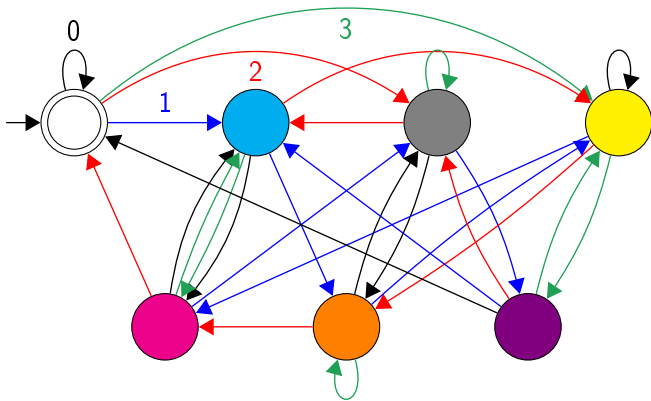
We can build a new automaton which is

- accessible

We can build a new automaton which is

- accessible
- reduced.





Theorem

Let $m \in \mathbb{N}$ and $p \in \mathbb{N}_{\geq 1}$. Then the state complexity of the language $0^* \text{rep}_{2^p}(m\mathcal{I})$ is equal to

$$2k + \left\lceil \frac{z}{p} \right\rceil$$

if $m = k2^z$ with k odd.

Theorem

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$$2 \times 3 + \left\lceil \frac{1}{2} \right\rceil = 7$$

What about the language

$$0^* \text{rep}_{2^p}(m\mathcal{T} + r)$$

where $r \in \{0, \dots, m-1\}$?