Parameter inference and data modelling with deep learning

Flexible operation and advanced control workshop

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The probability of ending in bin x corresponds to the total probability of all the paths z from start to x.

$$p(x| heta) = \int p(x,z| heta) dz = inom{n}{x} heta^x (1- heta)^{n-x}$$

What if we shift or remove some of the pins?

$$p(x| heta) = \int\limits_{ ext{intractable!}} p(x,z| heta) dz
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eq egin{pmatrix} n \ x \end{pmatrix} egin{pmat$$

Does this mean inference is no longer possible?

The Galton board is a metaphore of simulation-based science:

Galton board device	\rightarrow	Computer simulation
Parameters $ heta$	\rightarrow	Model parameters $ heta$
Buckets x	\rightarrow	Observables x
Random paths z	\rightarrow	Latent variables <i>z</i> (stochastic execution traces through simulator)

Inference in this context requires likelihood-free algorithms.



Prediction (simulation):

- Well-understood mechanistic model
- Simulator can generate samples



Credits: Johann Brehmer

Applications



Particle physics







Cosmology



Epidemiology



Climatology





Computational topography



Astronomy

Particle physics



 $\mathcal{L}_{SM} = -\frac{1}{2} \partial_{\nu} g^a_{\mu} \partial_{\nu} g^a_{\mu} - g_s f^{abc} \partial_{\mu} g^a_{\nu} g^b_{\mu} g^c_{\nu} - \frac{1}{4} g^2_s f^{abc} f^{adc} g^b_{\mu} g^c_{\nu} g^d_{\mu} g^e_{\nu} - \partial_{\nu} W^+_{\mu} \partial_{\nu} W^-_{\mu} - \frac{1}{4} g^2_s g^b_{\mu} g^c_{\nu} g^d_{\mu} g^c_{\nu} - \frac{1}{4} g^2_s g^b_{\mu} g^c_{\nu} g^d_{\mu} g^c_{\nu} g^d_{\mu} g^c_{\nu} - \frac{1}{4} g^2_s g^b_{\mu} g^c_{\nu} g^d_{\mu} g^c_{\mu} g^d_{\mu} g^c_{\mu} g^d_{\mu} g^c_{\nu} g^d_{\mu} g^c_{\nu} g^d_{\mu} g$ $\begin{array}{l} \sum_{m=1}^{2N} W_{m} = \frac{2\pi^{2} \partial_{\mu}^{\mu} \partial_{\nu}^{\mu} \partial_{\mu}}{2} \sum_{m=1}^{2N} \frac{\partial_{\mu}^{\mu} \partial_{\mu}^{\mu} \partial_{\mu}^{\mu} \partial_{\mu}^{\mu}}{2} \sum_{m=1}^{2N} \frac{\partial_{\mu}^{\mu} \partial_{\mu}^{\mu} \partial_{\mu}^{\mu}}{2} \sum_{m=1}^{2N} \frac{\partial_{\mu}^{\mu} \partial_{\mu}^{\mu} \partial_{\mu}^{\mu}}{2} \sum_{m=1}^{2N} \frac{\partial_{\mu}^{\mu} \partial_{\mu}^{\mu} \partial_{\mu}^{\mu}}{2} \sum_{m=1}^{2N} \frac{\partial_{\mu}^{\mu} \partial_{\mu}^{\mu} \partial_{\mu}^{\mu}}{2} \\ W_{\mu}^{\mu} W_{\mu}^{\mu} - D_{\nu}^{\mu} (W_{\mu}^{\mu} \partial_{\nu} W_{\mu}^{\mu} - W_{\mu}^{\mu} \partial_{\nu} W_{\mu}^{\mu}) + Z_{\mu}^{\mu} (W_{\nu}^{\mu} \partial_{\mu} W_{\mu}^{\mu} - W_{\nu}^{\mu} \partial_{\nu} W_{\mu}^{\mu}) + D_{\mu}^{\mu} (W_{\nu}^{\mu} \partial_{\mu} W_{\mu}^{\mu} - W_{\nu}^{\mu} \partial_{\nu} W_{\mu}^{\mu}) + D_{\mu}^{\mu} (W_{\nu}^{\mu} \partial_{\mu} W_{\mu}^{\mu} - W_{\nu}^{\mu} \partial_{\nu} W_{\mu}^{\mu}) + D_{\mu}^{\mu} (W_{\nu}^{\mu} \partial_{\mu} W_{\mu}^{\mu} - W_{\nu}^{\mu} \partial_{\nu} W_{\mu}^{\mu}) + D_{\mu}^{\mu} (W_{\nu}^{\mu} \partial_{\mu} W_{\mu}^{\mu} - W_{\nu}^{\mu} \partial_{\nu} W_{\mu}^{\mu}) + D_{\mu}^{\mu} (W_{\nu}^{\mu} \partial_{\mu} W_{\mu}^{\mu} - W_{\nu}^{\mu} \partial_{\nu} W_{\mu}^{\mu}) + D_{\mu}^{\mu} (W_{\nu}^{\mu} \partial_{\mu} W_{\mu}^{\mu} - W_{\nu}^{\mu} \partial_{\nu} W_{\mu}^{\mu}) + D_{\mu}^{\mu} (W_{\nu}^{\mu} \partial_{\mu} W_{\mu}^{\mu} - W_{\nu}^{\mu} \partial_{\nu} W_{\mu}^{\mu}) + D_{\mu}^{\mu} (W_{\nu}^{\mu} \partial_{\mu} W_{\mu}^{\mu} - W_{\nu}^{\mu} \partial_{\mu} W_{\mu}^{\mu}) + D_{\mu}^{\mu} (W_{\nu}^{\mu} \partial_{\mu} W_{\mu}^{\mu} - W_{\mu}^{\mu} \partial_{\mu} W_{\mu}^{\mu}) + D_{\mu}^{\mu} (W_{\mu}^{\mu} \partial_{\mu} W_{\mu}^{\mu} - W_{\mu}^{\mu} \partial_{\mu} W_{\mu}^{\mu}) + D_{\mu}^{\mu} (W_{\mu}^{\mu} \partial_{\mu} W_{\mu}^{\mu} - W_{\mu}^{\mu} \partial_{\mu} W_{\mu}^{\mu}) + D_{\mu}^{\mu} (W_{\mu}^{\mu} \partial_{\mu} W_{\mu}^{\mu} - W_{\mu}^{\mu} \partial_{\mu} W_{\mu}^{\mu}) + D_{\mu}^{\mu} (W_{\mu}^{\mu} \partial_{\mu} W_{\mu}^{\mu} - W_{\mu}^{\mu} \partial_{\mu} W_{\mu}^{\mu}) + D_{\mu}^{\mu} (W_{\mu}^{\mu} \partial_{\mu} W_{\mu}^{\mu} - W_{\mu}^{\mu} \partial_{\mu} W_{\mu}^{\mu}) + D_{\mu}^{\mu} (W_{\mu}^{\mu} \partial_{\mu} W_{\mu}^{\mu}) + D_{\mu}^{\mu} ($ $igs_{w}(\partial_{\nu}A_{\mu}(W_{\mu}^{+}W_{\nu}^{-} - W_{\nu}^{+}W_{\mu}^{-}) - A_{\nu}(W_{\mu}^{+}\partial_{\nu}W_{\mu}^{-} - W_{\mu}^{-}\partial_{\nu}W_{\mu}^{+}) + A_{\mu}(W_{\nu}^{+}\partial_{\nu}W_{\mu}^{-} - W_{\mu}^{-}\partial_{\nu}W_{\mu}^{-}) + A_{\mu}(W_{\nu}^{+}\partial_{\nu}W_{\mu}^{-} - W_{\mu}^{-}) + A_{\mu}(W_{\mu}^{+}\partial_{\nu}W_{\mu}^{-} - W_{\mu}^{-}) + A_{\mu}(W_{\mu}^{+}\partial_{\nu}W_{\mu}^{-}) + A_{\mu}(W_{\mu}^{+}\partial_{\nu}W_{\mu}^{-}) + A_{\mu}(W_{\mu}^{+}\partial_{\nu}W_{\mu}^{-}) + A_{\mu}(W_{\mu}^{+}\partial_{\nu}W_{\mu}^{-}) + A_{\mu}(W_{\mu}^{+}\partial_{\nu}W_{\mu}^{-}) + A_{\mu}(W_{\mu}^{+}\partial_{\mu}W_{\mu}^{-}) + A_{\mu$ $W_{\nu}^{-}\partial_{\nu}W_{\mu}^{+})) - \frac{1}{2}g^{2}W_{\mu}^{+}W_{\nu}^{-}W_{\nu}^{+} + \frac{1}{2}g^{2}W_{\mu}^{+}W_{\nu}^{-}W_{\mu}^{+}W_{\nu}^{-} + g^{2}c_{w}^{2}(Z_{u}^{0}W_{\mu}^{+}Z_{\nu}^{0}W_{\nu}^{-} Z_{\mu}^{0}Z_{\mu}^{0}W_{\nu}^{+}W_{\nu}^{-}) + g^{2}s_{w}^{2}(A_{\mu}W_{\mu}^{+}A_{\nu}W_{\nu}^{-} - A_{\mu}A_{\mu}W_{\nu}^{+}W_{\nu}^{-}) + g^{2}s_{w}c_{w}(A_{\mu}Z_{\nu}^{0}(W_{\mu}^{+}W_{\nu}^{-} - A_{\nu}A_{\nu}W_{\nu}^{+}W_{\nu}^{-}))$ $W_{\nu}^{+}W_{\mu}^{-}) - 2A_{\mu}Z_{\mu}^{0}W_{\nu}^{+}W_{\nu}^{-}) - \frac{1}{2}\partial_{\mu}H\partial_{\mu}H - 2M^{2}\alpha_{h}H^{2} - \partial_{\mu}\phi^{+}\partial_{\mu}\phi^{-} - \frac{1}{2}\partial_{\mu}\phi^{0}\partial_{\mu}\phi^{0} - \frac$ $\beta_h \left(\frac{2M^2}{a^2} + \frac{2M}{a}H + \frac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-) \right) + \frac{2M^4}{a^2}\alpha_h \begin{array}{c} & \int g^{-g} & g \\ & g \alpha_h M \left(H^3 + H \phi^0 \phi^0 + 2 H \phi^+ \phi^- \right) \\ & \frac{1}{8} g^2 \alpha_h \left(H^4 + (\phi^0)^4 + 4 (\phi^+ \phi^-)^2 + 4 (\phi^0)^2 \phi^+ \phi^- + 4 H^2 \phi^+ \phi^- + 2 (\phi^0)^2 H^2 \right) \\ & - g M W^+_\mu W^-_\mu H - \frac{1}{2} g \frac{J^0_\mu}{Z^0_\mu} Z^0_\mu H - \end{array}$ $\frac{1}{2}ig\left(W_{\mu}^{+}(\phi^{0}\partial_{\mu}\phi^{-}-\phi^{-}\partial_{\mu}\phi^{0})-W_{\mu}^{\infty}(\phi^{0}\partial_{\mu}\phi^{+}-\phi^{+}\partial_{\mu}\phi^{0})\right)+$ $\frac{1}{2}g\left(W^+_{\mu}(H\partial_{\mu}\phi^- - \phi^-\partial_{\mu}H) + W^-_{\mu}(H\partial_{\mu}\phi^+ - \phi^+\partial_{\mu}H)\right) + \frac{1}{2}g\frac{1}{c_{-}}(Z^0_{\mu}(H\partial_{\mu}\phi^0 - \phi^0\partial_{\mu}H) +$ $M\left(\frac{1}{2}Z_{\mu}^{0}\partial_{\mu}\phi^{0}+W_{\mu}^{+}\partial_{\mu}\phi^{-}+W_{\mu}^{-}\partial_{\mu}\phi^{+}\right)-ig\frac{s_{w}^{2}}{2}MZ_{\mu}^{0}(W_{\mu}^{+}\phi^{-}-W_{\mu}^{-}\phi^{+})+igs_{w}MA_{\mu}(W_{\mu}^{+}\phi^{-}-W_{\mu}^{-}\phi^{+})$ $W^{-}_{\mu}\phi^{+}) - ig\frac{1-2c_{w}^{2}}{2c_{w}}Z^{0}_{\mu}(\phi^{+}\partial_{\mu}\phi^{-} - \phi^{-}\partial_{\mu}\phi^{+}) + igs_{w}A_{\mu}(\phi^{+}\partial_{\mu}\phi^{-} - \phi^{-}\partial_{\mu}\phi^{+}) - igs_$ $\frac{1}{4}g^2W^+_\mu W^-_\mu (H^2 + (\phi^0)^2 + 2\phi^+\phi^-) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(z^2_w - 1)^2\phi^+) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(z^2_w - 1)^2\phi^+) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu (H^2 + (\phi^0)^2 + 2(z^2_w - 1)^2\phi^+) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu (H^2 + (\phi^0)^2 + 2(z^2_w - 1)^2\phi^+) - \frac{1}{8}g^2\frac{1}{c^2}Z^0_\mu (H^2 + (\phi^0)^2 + 2(z^2_w - 1)^2\phi^+) - \frac{1}{6}g^2\frac{1}{c^2}Z^0_\mu (H^2 + 2(z^2_w - 1)^2\phi^+) - \frac{1}{6}g$ $\frac{1}{2}g^{2}\frac{s_{w}^{2}}{c_{w}}Z_{\mu}^{0}\phi^{0}(W_{\mu}^{+}\phi^{-}+W_{\mu}^{-}\phi^{+}) - \frac{1}{2}ig^{2}\frac{s_{w}^{2}}{c_{w}}Z_{\mu}^{0}H(W_{\mu}^{+}\phi^{-}-W_{\mu}^{-}\phi^{+}) + \frac{1}{2}g^{2}s_{w}A_{\mu}\phi^{0}(W_{\mu}^{+}\phi^{-}+W_{\mu}^{-}\phi^{+}) + \frac{1}{2}g^{2}s_{w}A_{\mu}\phi^{0}(W_{\mu}^{+}\phi^{-}+W_{\mu}^{-}\phi^{-}) + \frac{1}{2}g^{2}s_{w}A_{\mu}\phi^{-}) + \frac{1}{2}g^{2}s_{w}A_{\mu}\phi^{0}(W_{\mu}^{+}\phi^{-}+W_{\mu}^{-}\phi^{-}) + \frac{1}{2}g^{2}s_{w}A_{\mu}\phi^{0}(W_{\mu}^{+}\phi^{-}) + \frac{1}{2}g^{2}s_{w}A_{\mu}\phi^{-}) + \frac{1}{2}g^{2}s_{w}A_{\mu}\phi^{0}(W_{\mu}^{+}\phi^{-}) + \frac{1}{2}g^{2}s_{w}A_{\mu}\phi^{-})$ $W_{\mu}^{-}\phi^{+}) + \frac{1}{2}ig^{2}s_{w}A_{\mu}H(W_{\mu}^{+}\phi^{-}-W_{\mu}^{-}\phi^{+}) - g^{2}\frac{s_{w}}{c_{-}}(2c_{w}^{2}-1)Z_{\mu}^{0}A_{\mu}\phi^{+}\phi^{-}$ $g^2 s_w^2 A_\mu A_\mu \phi^+ \phi^- + \frac{1}{2} i g_s \lambda_{ij}^a (\bar{q}_i^\sigma \gamma^\mu q_j^\sigma) g_\mu^a - \bar{e}^\lambda (\gamma \partial + m_e^\lambda) e^\lambda - \bar{\nu}^\lambda (\gamma \partial + m_\nu^\lambda) \nu^\lambda - \bar{u}_j^\lambda (\gamma \partial + m_\mu^\lambda) \bar{\nu}^\lambda - \bar{u}_j^\lambda (\gamma \partial + m_\mu^\lambda)$ $m_u^{\lambda} u_j^{\lambda} - \bar{d}_j^{\lambda} (\gamma \partial + m_d^{\lambda}) d_j^{\lambda} + i g s_w A_{\mu} \left(-(\bar{e}^{\lambda} \gamma^{\mu} e^{\lambda}) + \frac{2}{3} (\bar{u}_j^{\lambda} \gamma^{\mu} u_j^{\lambda}) - \frac{1}{3} (\bar{d}_j^{\lambda} \gamma^{\mu} d_j^{\lambda}) \right) +$ $\frac{m_{w}^{2}}{4c_{v}}Z_{\mu}^{0}\{(\bar{\nu}^{\lambda}\gamma^{\mu}(1+\gamma^{5})\nu^{\lambda})+(\bar{e}^{\lambda}\gamma^{\mu}(4s_{w}^{2}-1-\gamma^{5})e^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}^{\lambda}\gamma^{\mu}(\frac{1}{4}s_{w}^{2}-1-\gamma^{5})d_{j}^{\lambda})+(\bar{d}$ $(\bar{u}_{j}^{\lambda}\bar{\gamma}^{\mu}(1-\frac{8}{3}s_{w}^{2}+\gamma^{5})u_{j}^{\lambda})\}+\frac{ig}{2\sqrt{2}}W_{\mu}^{+}\left((\bar{\nu}^{\lambda}\bar{\gamma}^{\mu}(1+\gamma^{5})U^{lep}_{\lambda\kappa}e^{\kappa})+(\bar{u}_{j}^{\lambda}\bar{\gamma}^{\mu}(1+\gamma^{5})C_{\lambda\kappa}d_{i}^{\kappa})\right)+$ $\frac{ig}{2\sqrt{2}}W^{-}_{\mu}\left((\bar{e}^{\kappa}U^{lep^{\dagger}}_{\kappa\lambda}\gamma^{\mu}(1+\gamma^{5})\nu^{\lambda})+(\bar{d}^{\kappa}_{i}C^{\dagger}_{\kappa\lambda}\gamma^{\mu}(1+\gamma^{5})u^{\lambda}_{i})\right)+$ $\frac{ig}{2M\sqrt{2}}\phi^{+}\left(-m_{e}^{\kappa}(\bar{\nu}^{\lambda}U^{lep}{}_{\lambda\kappa}(1-\gamma^{5})e^{\kappa})+m_{\nu}^{\lambda}(\bar{\nu}^{\lambda}U^{lep}{}_{\lambda\kappa}(1+\gamma^{5})e^{\kappa}\right)+$ $\frac{ig}{2M\sqrt{2}}\phi^{-}\left(m_{e}^{\lambda}(\bar{e}^{\lambda}U^{lep}_{\lambda\kappa}^{\dagger}(1+\gamma^{5})\nu^{\kappa})-m_{\nu}^{\kappa}(\bar{e}^{\lambda}U^{lep}_{\lambda\kappa}^{\dagger}(1-\gamma^{5})\nu^{\kappa}\right)-\frac{g\,m_{\nu}^{\lambda}}{2\,M}H(\bar{\nu}^{\lambda}\nu^{\lambda}) \frac{g}{2}\frac{m_{\epsilon}^{\lambda}}{M}H(\bar{e}^{\lambda}e^{\lambda}) + \frac{ig}{2}\frac{m_{\epsilon}^{\lambda}}{M}\phi^{0}(\bar{\nu}^{\lambda}\gamma^{5}\nu^{\lambda}) - \frac{ig}{2}\frac{m_{\epsilon}^{\lambda}}{M}\phi^{0}(\bar{e}^{\lambda}\gamma^{5}e^{\lambda}) - \frac{1}{4}\bar{\nu}_{\lambda}\frac{M_{\lambda\kappa}^{R}}{M_{\lambda\kappa}^{2}}(1-\gamma_{5})\hat{\nu}_{\kappa} - \frac{ig}{2}\frac{m_{\kappa}^{\lambda}}{M}\phi^{0}(\bar{e}^{\lambda}\gamma^{5}e^{\lambda}) - \frac{1}{4}\bar{\nu}_{\lambda}\frac{M_{\lambda\kappa}^{R}}{M}\phi^{0}(\bar{e}^{\lambda}\gamma^{5}e^{\lambda}) - \frac{1}{4}\bar{\nu}_{\lambda}\frac{M_{\lambda\kappa}^{R}}{M}\phi^{0}(\bar{e}^{\lambda}$ $\frac{1}{4}\overline{\nu_{\lambda}} \frac{M_{\lambda\kappa}^{R}(1-\gamma_{5})\hat{\nu}_{\kappa}}{M_{\lambda\kappa}^{2}(1-\gamma_{5})\hat{\nu}_{\kappa}} + \frac{ig}{2M\sqrt{2}}\phi^{+}\left(-m_{d}^{\kappa}(\bar{u}_{j}^{\lambda}C_{\lambda\kappa}(1-\gamma^{5})d_{j}^{\kappa}) + m_{u}^{\lambda}(\bar{u}_{j}^{\lambda}C_{\lambda\kappa}(1+\gamma^{5})d_{j}^{\kappa})\right) + \frac{ig}{2M\sqrt{2}}\phi^{+}\left(-m_{d}^{\kappa}(\bar{u}_{j}^{\lambda}C_{\lambda\kappa}(1-\gamma^{5})d_{j}^{\kappa}) + m_{u}^{\lambda}(\bar{u}_{j}^{\lambda}C_{\lambda\kappa}(1+\gamma^{5})d_{j}^{\kappa})\right) + \frac{ig}{2M\sqrt{2}}\phi^{+}\left(-m_{d}^{\kappa}(\bar{u}_{j}^{\lambda}C_{\lambda\kappa}(1-\gamma^{5})d_{j}^{\kappa}) + m_{u}^{\lambda}(\bar{u}_{j}^{\lambda}C_{\lambda\kappa}(1-\gamma^{5})d_{j}^{\kappa})\right) + \frac{ig}{2M\sqrt{2}}\phi^{+}\left(-m_{d}^{\kappa}(\bar{u}_{j}^{\lambda}C_{\lambda\kappa}(1-\gamma^{5})d_{j}^{\kappa}\right) + \frac{ig}{2M\sqrt{2}}\phi^{+}\left(-m_{d}^{\kappa}(1-\gamma^{5})d_{j}^{\kappa}\right) + \frac{ig}{2M\sqrt{2}}\phi^{+}\left(-m_{d}^{\kappa}(1-\gamma^{5})d_{j}^{\kappa}\right) + \frac{ig}{2M\sqrt{2}}\phi^{+}\left(-m_{d}^{\kappa}(1-\gamma^{5})d_{j}^{\kappa}\right) + \frac{ig}{2M\sqrt{$ $\frac{ig}{2M\sqrt{2}}\phi^{-}\left(m_{d}^{\lambda}(\bar{d}_{j}^{\lambda}C_{\lambda\kappa}^{\dagger}(1+\gamma^{5})u_{j}^{\kappa})-m_{u}^{\kappa}(\bar{d}_{j}^{\lambda}C_{\lambda\kappa}^{\dagger}(1-\gamma^{5})u_{j}^{\kappa})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{j}^{\lambda})-\frac{g}{2}\frac{m_{u}^{\lambda}}{M}H(\bar{u}_{j}^{\lambda}u_{\lambda$ $\frac{g}{2}\frac{m_d^2}{M}H(\bar{d}_i^{\lambda}d_i^{\lambda}) + \frac{ig}{2}\frac{m_d^{\lambda}}{M}\phi^0(\bar{u}_i^{\lambda}\gamma^5 u_i^{\lambda}) - \frac{ig}{2}\frac{m_d^{\lambda}}{M}\phi^0(\bar{d}_i^{\lambda}\gamma^5 d_i^{\lambda}) + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_{\mu}\bar{G}^a G^b g_{\mu}^c +$ $\bar{X}^{+}(\partial^{2} - M^{2})X^{+} + \bar{X}^{-}(\partial^{2} - M^{2})X^{-} + \bar{X}^{0}(\partial^{2} - \frac{M^{2}}{c^{2}})X^{0} + \bar{Y}\partial^{2}Y + igc_{w}W^{+}_{\mu}(\partial_{\mu}\bar{X}^{0}X^{-} - \frac{M^{2}}{c^{2}})X^{0} + \bar{Y}\partial^{2}X^{-} + \frac{M^{2}}{c^{2}})X^{0} + \bar{Y}\partial^{2}X^{-} + \frac{M^{2}}{c^{2}}X^{0} + \frac{M^{2$ $\partial_{\mu}\bar{X}^{+}X^{0}$)+ $igs_{w}W^{+}_{\mu}(\partial_{\mu}\bar{Y}X^{-}-\partial_{\mu}\bar{X}^{+}\bar{Y})$ + $igc_{w}W^{-}_{\mu}(\partial_{\mu}\bar{X}^{-}X^{0} \partial_{\mu}\bar{X}^{0}X^{+})+igs_{w}W^{-}_{\mu}(\partial_{\mu}\bar{X}^{-}Y-\partial_{\mu}\bar{Y}X^{+})+igc_{w}Z^{0}_{\mu}(\partial_{\mu}\bar{X}^{+}X^{+} \partial_{\mu}\bar{X}^{-}X^{-})+igs_{w}A_{\mu}(\partial_{\mu}\bar{X}^{+}X^{+} \partial_{\mu}\bar{X}^{-}X^{-}) - \frac{1}{2}gM\left(\bar{X}^{+}X^{+}H + \bar{X}^{-}X^{-}H + \frac{1}{c^{2}}\bar{X}^{0}X^{0}H\right) + \frac{1-2c_{w}^{2}}{2c_{w}}igM\left(\bar{X}^{+}X^{0}\phi^{+} - \bar{X}^{-}X^{0}\phi^{-}\right) + \frac{1}{c^{2}}\bar{X}^{0}\chi^{0}H$ $\frac{1}{2c_{-}}igM(\bar{X}^{0}X^{-}\phi^{+}-\bar{X}^{0}X^{+}\phi^{-})+igMs_{w}(\bar{X}^{0}X^{-}\phi^{+}-\bar{X}^{0}X^{+}\phi^{-})+$ $\frac{1}{2}igM(\bar{X}^+X^+\phi^0 - \bar{X}^-\bar{X}^-\phi^0)$.









Likelihood-free inference algorithms











Can we harness deep learning for inference and generation?



Neural networks are

- function approximators with a gazillion of parameters,
- tuned with stochastic gradient descent

 $heta_{t+1} = heta_t - \gamma \hat{
abla}_ heta \mathcal{L}(heta_t),$

• are flexible enough to be structured by domain knowledge.



Histograms of observables Neural density (ratio) estimation

Adversarial variational optimization





Probabilistic programming

Treat the simulator as a black box

Make use of the inner structure





Histograms of observables Neural density (ratio) estimation



Mining gold from implicit models



Adversarial variational optimization

Probabilistic programming

The physicist's way

The Neyman-Pearson lemma states that the likelihood ratio

$$r(x| heta_0, heta_1) = rac{p(x| heta_0)}{p(x| heta_1)}$$



is the most powerful test statistic to discriminate between a null hypothesis θ_0 and an alternative θ_1 .

IX. <i>0</i>	n the Problem of the most Efficient Tests of Statistical Hypotheses.
By J. N Cent App	EYMAN, Nencki Institute, Soc. Sci. Lit. Varsoviensis, and Lecturer at the ral College of Agriculture, Warsaw, and E. S. PEARSON, Department of lied Statistics, University College, London.
	(Communicated by K. PEARSON, F.R.S.)
	(Received August 31, 1932.—Read November 10, 1932.)
	CONTENTS



Define a projection function $s:\mathcal{X} o\mathbb{R}$ mapping observables x to a summary statistics x'=s(x).

Then, approximate the likelihood p(x| heta) as

$$p(x| heta) pprox \hat{p}(x| heta) = p(x'| heta).$$

From this it comes

$$rac{p(x| heta_0)}{p(x| heta_1)}pproxrac{\hat{p}(x| heta_0)}{\hat{p}(x| heta_1)}=\hat{r}(x| heta_0, heta_1).$$



This methodology has worked great for physicists for the last 20-30 years, but ...

- Choosing the projection *s* is difficult and problem-dependent.
- Often there is no single good variable: compressing to any x' loses information.
- Ideally: analyse high-dimensional x', including all correlations.

Unfortunately, filling high-dimensional histograms is not tractable.



Who you gonna call? Machine learning!

Refs: Bolognesi et al, 2012 (arXiv:1208.4018)





Key insights

- The likelihood ratio is often sufficient for inference.
- Evaluating the likelihood ratio does not require evaluating the individual likelihoods.
- Supervised learning indirectly estimates likelihood ratios.

Supervised learning provides a way to automatically construct *s*:

- Let us consider a binary classifier \hat{s} (e.g., a neural network) trained to distinguish $x \sim p(x| heta_0)$ from $x \sim p(x| heta_1).$
- \hat{s} is trained by minimizing the cross-entropy loss

$$egin{aligned} L_{XE}[\hat{s}] &= -\mathbb{E}_{p(x| heta)\pi(heta)}[1(heta= heta_0)\log\hat{s}(x)+\ 1(heta= heta_1)\log(1-\hat{s}(x))] \end{aligned}$$

The solution \hat{s} found after training approximates the optimal classifier

$$\hat{s}(x)pprox s^*(x)=rac{p(x| heta_1)}{p(x| heta_0)+p(x| heta_1)}.$$

Therefore,

$$r(x| heta_0, heta_1)pprox \hat{r}\left(x| heta_0, heta_1
ight)=rac{1-\hat{s}(x)}{\hat{s}(x)}$$

That is, supervised classification is equivalent to likelihood ratio estimation and can therefore be used for MLE inference.



Histograms of observables Neural density (ratio) estimation Mining gold from implicit models



Adversarial variational optimization

Probabilistic programming

Mining gold from simulators



 $p(x|\theta)$ is usually intractable.

What about $p(x, z|\theta)$?

As the trajectory $z_1, ..., z_T$ and the observable x are emitted, it is often possible:

- to calculate the joint likelihood p(x,z| heta);
- to calculate the joint likelihood ratio $r(x,z| heta_0, heta_1);$
- to calculate the joint score $t(x, z | \theta_0) = \nabla_{\theta} \log p(x, z | \theta) \big|_{\theta_0}$.

We call this process mining gold from your simulator!

Observe that the joint likelihood ratios

 $r(x,z| heta_0, heta_1)=rac{p(x,z| heta_0)}{p(x,z| heta_1)}$

are scattered around $r(x| heta_0, heta_1)$.

Can we use them to approximate $r(x| heta_0, heta_1)$?



Let us define

$$L_r = \mathbb{E}_{p(x,z| heta_1)}\left[(r(x,z| heta_0, heta_1) - \hat{r}(x))^2
ight].$$

Via calculus of variations, we find that this functional is minimized by

$$egin{aligned} r^*(x) &= rac{1}{p(x| heta_1)} \int p(x,z| heta_1) rac{p(x,z| heta_0)}{p(x,z| heta_1)} dz \ &= rac{p(x| heta_0)}{p(x| heta_1)} \ &= r(x| heta_0, heta_1). \end{aligned}$$



How does one find r^* ?

$$r^*(x| heta_0, heta_1) = rg\min_{\hat{r}} L_r[\hat{r}]$$

Minimizing functionals is exactly what machine learning does. In our case,

- \hat{r} are neural networks (or the parameters thereof);
- L_r is the loss function;
- minimization is carried out using stochastic gradient descent from the data extracted from the simulator.

Similarly, we can mine the simulator to extract the joint score

$$t(x,z| heta_0) =
abla_ heta \log p(x,z| heta)igert_{ heta_0},$$

which indicates how much more or less likely x, z would be if one changed θ_0 .



We define

$$L_t = \mathbb{E}_{p(x,z| heta_0)}\left[(t(x,z| heta_0) - \,\hat{t}\,(x))^2
ight],$$

which can be shown to be minimized by $t^*(x) = t(x| heta_0).$



$L_{RASCAL} = L_r + L_t$





$L_{RASCAL} = L_r + L_t$







Histograms of observables Neural density (ratio) estimation Mining gold from implicit models

pyprob + PyTorch ((Python)



Probabilistic programming

Generative adversarial networks



 $egin{aligned} \mathcal{L}_d(\phi) &= \mathbb{E}_{\mathbf{x} \sim p_r(\mathbf{x})} \left[-\log(d(\mathbf{x};\phi))
ight] + \mathbb{E}_{\mathbf{z} \sim p(\mathbf{z})} \left[-\log(1 - d(g(\mathbf{z}; heta);\phi))
ight] \ \mathcal{L}_g(heta) &= \mathbb{E}_{\mathbf{z} \sim p(\mathbf{z})} \left[\log(1 - d(g(\mathbf{z}; heta);\phi))
ight] \end{aligned}$



Odena et al 2016

Miyato et al 2017

Zhang et al 2018

Brock et al 2018



Figure 2. Uncurated set of images produced by our style-based generator (config F) with the FFHQ dataset. Here we used a variation of the truncation trick [5, 29] with $\psi = 0.7$ for resolutions $4^2 - 32^2$. Please see the accompanying video for more results.

Karras et al, 2018.





Replace g with an actual scientific simulator!

Key insights

- Replace the generative network with a non-differentiable forward simulator $g(\mathbf{z}; \theta)$.
- Let the neural network critic figure out how to adjust the simulator parameters.
- Combine with variational optimization to bypass the non-differentiability by optimizing upper bounds of the adversarial objectives

$$egin{aligned} U_d(\phi) &= \mathbb{E}_{ heta \sim q(heta;\psi)} \left[\mathcal{L}_d(\phi)
ight] \ U_g(\psi) &= \mathbb{E}_{ heta \sim q(heta;\psi)} \left[\mathcal{L}_g(heta)
ight] \end{aligned}$$

respectively over ϕ and ψ .





Samples for heta=0 (top) vs. samples for heta=0.81 (bottom).



Histograms of observables Neural density (ratio) estimation Mining gold from implicit models



Probabilistic programming











CS

Statistics

ML:

Algorithms &

Applications

Probabilistic Programming

PL: Compilers, Semantics, Transformations STATS:

Inference & Theory







CS Probabilistic Programming Statistics

Probabilistic programming





CS Probabilistic Programming Statistics



Key insights

Let a neural network take full control of the internals of the simulation program by hijacking all calls to the random number generator.

Refs: Le et al, 2016 (arXiv:1610.09900); Baydin et al, 2018 (arXiv:1807.07706)



(**defquery** captcha [image num-chars tol] (let [[w h] (size image) ;; sample random characters num-chars (sample (poisson num-chars)) chars (repeatedly num-chars sample-char)] ;; compare rendering to true image (map (fn [y z] (observe (normal z tol) y)) (reduce-dim image) (reduce-dim (render chars w h))) ;; predict captcha text {:text (map :symbol (sort-by :x chars))}))

How to break captchas with probabilistic programming





Probabilistic programming hooked to particle physics simulators (work in progress)



(b) Posterior execution $p(\mathbf{x}|\mathbf{y})$ conditioned on a given calorimeter observation \mathbf{y} .



Summary

- Much of modern science is based on "likelihood-free" simulations.
- Recent (and older) developments from machine learning offer solutions for likelihood-free inference, including:
 - Supervised learning
 - Neural networks trained with augmented data
 - Adversarial training
 - Probabilistic programming





pyprob + PyTorch (Python)

SHERPA (C++





Collaborators



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The end.