# Parameter inference and data modelling with deep learning 

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The probability of ending in bin $x$ corresponds to the total probability of all the paths $z$ from start to $x$.

$$
p(x \mid \theta)=\int p(x, z \mid \theta) d z=\binom{n}{x} \theta^{x}(1-\theta)^{n-x}
$$

What if we shift or remove some of the pins?

$$
\begin{aligned}
p(x \mid \theta) & =\underbrace{\int}_{\text {intractable! }} p(x, z \mid \theta) d z \\
& \neq\binom{ n}{x} \theta^{x}(1-\theta)^{n-x}
\end{aligned}
$$

Does this mean inference is no longer possible?

The Galton board is a metaphore of simulation-based science:

| Galton board device | $\rightarrow$ | Computer simulation |
| :---: | :--- | :---: |
| Parameters $\theta$ | $\rightarrow$ | Model parameters $\theta$ |
| Buckets $x$ | $\rightarrow$ | Observables $x$ |
| Random paths $z$ | $\rightarrow$ | Latent variables $z$ <br> (stochastic execution traces <br> through simulator) |

Inference in this context requires likelihood-free algorithms.


Prediction (simulation):

- Well-understood mechanistic model
- Simulator can generate samples


Observables
$\longrightarrow x$

Prediction (simulation):

- Well-understood mechanistic model
- Simulator can generate samples

Inference:

- Likelihood function $p(x \mid \theta)$ is intractable
- Goal: estimator $\hat{p}(x \mid \theta)$


## Applications



Particle physics


Epidemiology


Computational topography


Cosmology


## Particle physics

Parameters
$\theta$ $\qquad$


Observables
$\longrightarrow \quad x$
$\mathcal{L}_{S M}=-\frac{1}{2} \partial_{\nu} g_{\mu}^{a} \partial_{\nu} g_{\mu}^{a}-g_{s}{ }^{f a b c} \partial_{\mu} g_{\nu}^{a} g_{\mu}^{b} g_{\nu}^{c}-\frac{1}{4} g_{s}^{2} f^{a k c} f^{a d e} g_{\mu}^{b} g_{\nu}^{c} g_{\mu}^{d} g_{\nu}^{e}-\partial_{\nu} W_{\mu}^{+} \partial_{\nu} W_{-}^{-}$
 $\left.\left.W_{\nu}^{+} W_{\mu}^{-}\right)-Z_{\nu}^{( }\left(W_{\mu}^{+} \partial_{\nu} W_{\mu}^{-}-W_{\mu}^{-} \partial_{\nu} W_{\mu}^{+}\right)+Z_{\mu}^{0}\left(W_{\nu}^{+} \partial_{\nu} W_{\mu}^{-}-W_{\nu}^{-} \partial_{\nu} W_{\mu}^{+}\right)\right)-$
$s_{w}\left(\partial_{\nu} A_{\mu}\left(W_{+}^{+} W_{\nu}^{-}-W_{+}^{+} W_{-}^{-}\right)-A_{\nu}\left(W^{+} \partial_{\nu} W_{-}^{-}-W^{-} \partial_{\nu} W^{+}\right)+A_{\mu}\left(W^{+} \partial_{\nu} W^{-}\right.\right.$ $\left.\left.{ }^{2} S_{\nu}^{-} \partial_{\nu} W_{\mu}^{+}\right)\right)-\frac{1}{2} g^{2} W_{\mu}^{+} W_{\mu}^{-} W_{\nu}^{+} W_{\nu}^{-}+\frac{1}{2} g^{2} W_{\mu}^{+} W_{\nu}^{-} W_{\mu}^{+} W_{\nu}^{-}+g^{2} c_{w}^{2}\left(Z_{\mu}^{0} W_{\mu}^{+} Z_{\nu}^{0} W_{\nu}^{-}\right.$ $\left.Z_{\mu}^{0} Z_{\mu}^{\nu} W_{\nu}^{+} W_{\nu}^{-}\right)+g^{2} s_{w}^{\mu}\left(A_{\mu}^{\mu} W_{\mu}^{+} A_{\nu} W_{\nu}^{-}-A_{\mu} A_{\mu} W_{\nu}^{+} W_{\nu}^{-}\right)+g^{2} s_{w} c_{w}\left(A_{\mu} Z_{\nu}^{\circ}\left(W_{\mu}^{+} W_{\nu}^{\nu}\right.\right.$ $\left.\left.W_{\nu}^{+} W_{\mu}^{-}\right)-2 A_{\mu} Z_{\mu}^{0} W_{\nu}^{+} W_{\nu}^{-}\right)-\frac{1}{2} \partial_{\mu} H \partial_{\mu} H-2 M^{2} \alpha_{h} H^{2}-\partial_{\mu} \phi^{+} \partial_{\mu} \phi^{-}-\frac{1}{2} \partial_{\mu} \phi^{0} \partial_{\mu} \phi^{0}$
$\beta_{h}\left(\frac{2 M 2^{2}}{g^{2}}+\frac{2 M}{g} H+\frac{1}{2}\left(H^{2}+\phi^{0} \phi^{0}+2 \phi^{+} \phi^{-}\right)\right)+\frac{2 M \Lambda^{4}}{g^{2}} \alpha_{h}$
$g \alpha_{h} M\left(H^{3}+H \phi^{0} \phi^{0}+2 H \phi^{+} \phi^{-}\right.$
$g \alpha_{h} M\left(H^{3}+H \phi^{0} \phi^{0}+2 H \phi^{+} \phi^{-}\right)$
$+4\left(\phi^{+} \phi^{-}\right)^{2}+4\left(\phi^{0}\right)^{2} \phi^{+} \phi^{-}+4 H^{2}$
$4\left(\phi^{+} \phi^{-}\right)^{2}+4\left(\phi^{0}\right)^{2}+\phi^{-}+4 H^{2} \phi^{+} \phi^{-}+2\left(\phi^{0}\right)^{2} H^{2}$
$i g\left(W^{+}\left(\delta^{0} \partial_{\mu} \phi^{-}-\phi^{-} \partial_{\mu} \phi^{0}\right)-W^{-}\left(\delta^{0} \partial_{\mu} \phi^{+}\right.\right.$
 $\frac{1}{2} g\left(W_{\mu}^{+}\left(H \partial_{\mu} \phi^{-}-\phi^{-} \partial_{\mu} H\right)+W_{\mu}^{-}\left(H \partial_{\mu} \phi^{+}-\delta^{+} \partial_{\mu} H\right)+\frac{1}{2} g \frac{1}{\iota_{\mu}}\left(Z_{\mu}^{0}\left(H \partial_{\mu} \phi^{0}-\phi^{0} \partial_{\mu} H\right)+\right.\right.$ $M\left(\frac{1}{c_{w}} Z_{\mu}^{0} \partial_{\mu} \phi^{0}+W_{\mu}^{+} \partial_{\mu} \phi^{-}+W_{\mu}^{-} \partial_{\mu} \phi^{+}\right)-i g \frac{s_{w}^{2}}{c_{w}} M Z_{\mu}^{0}\left(W_{\mu}^{+} \phi^{-}-W_{\mu}^{-} \phi^{+}\right)+i g s_{w} M A_{\mu}\left(W_{\mu}^{+} \phi^{-}\right.$
$\left.W_{\mu}^{-} \phi^{+}\right)-i \frac{1-2 c_{w}^{2}}{2} Z_{\mu}^{0}\left(\phi^{+} \partial_{\mu} \phi^{-}-\phi^{-} \partial_{\mu} \phi^{+}\right)+i g s_{w} A_{\mu}\left(\phi^{+} \partial_{\mu} \phi^{-}-\phi^{-} \partial_{\mu} \phi^{+}\right)-$
$g^{2} W^{+} W^{-}\left(H^{2}+\left(\rho^{0}\right)^{2}+2 \phi^{+} \phi^{-}\right)-\frac{1}{2} q^{2} \frac{1}{2} Z^{0} Z^{0}\left(H^{2}+\left(\phi^{0}\right)^{2}+2\left(2 s^{2}-1\right)^{2} \phi^{+}{ }^{+}\right)$
$\frac{1}{4} g^{2} W_{\mu}^{+} W_{\mu}^{-}\left(H^{2}+\left(\phi^{\circ}\right)^{2}+2 \phi^{+} \phi^{-}\right)-\frac{1}{8} g^{2} \overline{1}_{\epsilon_{w}^{2}}^{\sigma_{\mu}^{0}} Z_{\mu}^{0}\left(H^{2}+\left(\phi^{0}\right)^{2}+2\left(2 s_{w}^{2}-1\right)^{2} \phi^{+} \phi^{-}\right)-$ $\frac{1}{2} g^{2} \frac{s_{\omega_{w}^{2}}^{2}}{c_{w}} Z_{\mu}^{0} \phi^{0}\left(W_{\mu}^{+} \phi^{-}+W_{\mu}^{-} \phi^{+}\right)-\frac{1}{2} i g^{2} \frac{s_{\omega}^{2}}{c_{w}} Z_{\mu}^{0} H\left(W_{\mu}^{+} \phi^{-}-W_{\mu}^{-} \phi^{+}\right)+\frac{1}{2} g^{2} s_{w} A_{\mu} \phi^{0}\left(W_{\mu}^{+} \phi^{-}+\right.$ $\left.W_{\mu}^{-} \phi^{+}\right)+\frac{1}{2} i g^{2} s_{w} A_{\mu} H\left(W_{\mu}^{+} \phi^{-}-W_{\mu}^{-} \phi^{+}\right)-g^{2} \frac{s_{w}}{c_{\mu}}\left(2 c_{w}^{2}-1\right) Z_{\mu}^{0} A_{\mu} \phi^{+} \phi^{-}$ $g^{2} s_{w}^{2} A_{\mu} A_{\mu} \phi^{+} \phi^{-}+\frac{1}{2} i g_{s} \lambda_{i j}^{a}\left(\tilde{q}_{i}^{\top} \gamma^{\mu} q_{j}^{\sigma}\right) g_{\mu}^{a}-\bar{e}^{\lambda}\left(\gamma \partial+m_{e}^{\lambda}\right) e^{\lambda}-\bar{\nu}^{\lambda}\left(\gamma \partial+m_{\nu}^{\lambda}\right) \nu^{\lambda}-\bar{u}_{j}^{\lambda}(\gamma \partial+$
$\left.m_{u}^{\lambda}\right) u_{j}^{\lambda}-d_{j}^{\lambda}\left(\gamma \partial+m_{d}^{\lambda}\right) d_{j}^{\lambda}+i g s_{w} A_{\mu}\left(-\left(\bar{e}^{\lambda} \gamma^{\mu} e^{\lambda}\right)+\frac{2}{3}\left(\bar{u}_{j}^{\lambda} \gamma^{\mu} u_{j}^{\lambda}\right)-\frac{1}{3}\left(d_{j}^{\lambda} \gamma^{\mu} d_{j}^{\lambda}\right)\right)+$
$\frac{i}{4 c_{c}} Z_{\mu}^{0}\left\{\left(\bar{\nu}^{\lambda} \gamma^{\mu}\left(1+\gamma^{5}\right) \nu^{\lambda}\right)+\left(\bar{e}^{\lambda} \gamma^{\mu}\left(4 s_{w}^{2}-1-\gamma^{5}\right) e^{\lambda}\right\}+\left(d_{j}^{\lambda} \gamma^{\mu}\left(\frac{4}{3} s_{w}^{2}-1-\gamma^{5}\right) d_{j}^{\lambda}\right)+\right.$
$\left.\left(\bar{u}_{j}^{\lambda} \gamma^{\mu}\left(1-\frac{\delta}{3} s_{u}^{2}+\gamma^{5}\right) u_{j}^{\lambda}\right)\right\}+\frac{i g}{2 \sqrt{2}} W_{\mu}^{+}\left(\left(\bar{\nu}^{\lambda} \gamma^{\mu}\left(1+\gamma^{5}\right) U^{\ell \epsilon p}{ }_{\lambda \kappa} e^{\kappa}\right)+\left(\bar{u}_{j}^{\lambda} \gamma^{\mu}\left(1+\gamma^{5}\right) C_{\lambda \kappa} d_{j}^{\kappa}\right)\right)+$
$\frac{i g}{2 \sqrt{2}} W_{\mu}^{-}\left(\left(\bar{c}^{\kappa} C^{l e}{ }_{k \lambda}{ }_{k} \gamma^{\mu}\left(1+\gamma^{5}\right) \nu^{\lambda}\right)+\left(\bar{d}_{j}^{k} C_{k \lambda}^{\dagger} \gamma^{\mu}\left(1+\gamma^{5}\right) u_{j}^{\lambda}\right)\right)+$
$\frac{\lambda_{i}^{2}}{2 M \sqrt{2}} \phi^{+}\left(-m_{e}^{\kappa}\left(\bar{\nu}^{\lambda} U^{\ell \ell p_{\lambda \kappa}}\left(1-\gamma^{5}\right) e^{\kappa}\right)+m_{\nu}^{\lambda}\left(\bar{\nu}^{\lambda} U^{l \ell p} \lambda_{\lambda \kappa}\left(1+\gamma^{5}\right) e^{\kappa}\right)+\right.$
$\frac{i g}{2 M \sqrt{2}} \phi^{-}\left(m_{e}^{\lambda}\left(\bar{e}^{\lambda} U^{l e p_{\lambda k} \dagger}\left(1+\gamma^{5}\right) \nu^{\kappa}\right)-m_{\nu}^{\kappa}\left(\bar{e}^{\lambda} U^{l e p_{\lambda k}^{\dagger}}{ }_{\lambda k}\left(1-\gamma^{5}\right) \nu^{k}\right)-\frac{g}{2} \frac{m_{\lambda}^{\lambda}}{M} H\left(\bar{\nu}^{\lambda} \nu^{\lambda}\right)-\right.$ $\frac{g}{2} \frac{m_{c}^{2}}{M} H\left(\bar{e}^{\lambda} e^{\lambda}\right)+\frac{i q}{2} \frac{m_{\dot{s}}^{\lambda}}{M} \phi^{0}\left(\bar{\nu}^{\lambda} \gamma^{5} \nu^{\lambda}\right)-\frac{i q}{2} \frac{m_{s}^{M}}{M} \phi^{0}\left(\bar{e}^{\lambda} \gamma^{5} e^{\lambda}\right)-\frac{1}{4} \bar{\nu}_{\lambda} M_{\lambda \kappa}^{R}\left(1-\gamma_{5}\right) \hat{\nu}_{\kappa}-$ $\frac{1}{4} \overline{\bar{\nu}_{\lambda} M_{\lambda \kappa}^{R}\left(1-\gamma_{5}\right) \hat{\nu}_{\kappa}}+\frac{i g}{2 M \sqrt{2}} \phi^{+}\left(-m_{d}^{\kappa}\left(\bar{u}_{j}^{\lambda} C_{\lambda \kappa}\left(1-\gamma^{5}\right) d_{j}^{\kappa}\right)+m_{u}^{\lambda}\left(\bar{u}_{j}^{\lambda} C_{\lambda \kappa}\left(1+\gamma^{5}\right) d_{j}^{\kappa}\right)+\right.$
$\frac{i g}{2 M \sqrt{2}} \phi^{-}\left(m_{d}^{\lambda}\left(\bar{d}_{j}^{\lambda} C_{\lambda_{k}}^{\dagger}\left(1+\gamma^{5}\right) u_{j}^{\kappa}\right)-m_{n}^{\kappa}\left(\vec{d}_{j} C_{\lambda_{k}}^{\dagger}\left(1-\gamma^{5}\right) u_{j}^{\kappa}\right)-\frac{q}{2} \frac{m_{\vec{\lambda}}^{\lambda}}{M} H\left(\bar{u}_{j}^{\lambda} u_{j}^{\lambda}\right)-\right.$
 $X^{+}\left(\partial^{2}-M^{2}\right) X^{+}+X^{-}\left(\partial^{2}-M^{2}\right) X^{-}+X^{0}\left(\partial^{2}-\frac{M 2}{c_{2}^{2}}\right) X^{0}+Y \partial^{2} Y+i g c_{w} W_{\mu}^{+}\left(\partial_{\mu} X^{0} X^{-}\right.$
$\left.\partial_{\mu} \bar{X}^{+} X^{0}\right)+i g s_{w} W_{\mu}^{+}\left(\partial_{\mu} \bar{Y} X^{-}-\partial_{\mu} \bar{X}^{+} Y\right)+i g c_{w} W_{\mu}^{-}\left(\partial_{\mu} \bar{X}^{-} X^{0}\right.$
$\left.\partial_{\mu} \bar{X}^{0} X^{+}\right)+i g s_{w} W_{\mu}^{-}\left(\partial_{\mu} X^{-} Y-\partial_{\mu} Y X^{+}\right)+i g c_{w} Z_{\mu}^{0}\left(\partial_{\mu} \bar{X}^{+} X^{+}\right.$
$\left.\partial_{\mu} \bar{X}^{-} X^{-}\right)+i g s_{w} A_{\mu}\left(\partial_{\mu} \bar{X}^{+} X^{\dagger}\right.$
$\left.\partial_{\mu} \overline{X^{-}} X^{-}\right)-\frac{1}{2} g M\left(\bar{X}^{+} X^{+} H+\bar{X}-X^{-} H+\frac{1}{c_{2}^{2}} \bar{X}^{0} X^{0} H\right)+\frac{1-2 c_{c}^{2}}{2_{c \omega}} i g M\left(\bar{X}^{+} X^{0} \phi^{+}-\bar{X}^{-} X^{0} \phi^{-}\right)+$
$\frac{1}{2 c_{w}} i g M\left(\bar{X}^{0} X^{-} \phi^{+}-\bar{X}^{0} X^{+} \phi^{-}\right)+i g M s_{w}\left(\bar{X}^{0} X^{-} \phi^{+}-\bar{X}^{0} X^{+} \phi^{-}\right)+$ $\frac{1}{2} i g M\left(\bar{X}^{+} X^{+} \phi^{0}-\bar{X}^{-} X^{-} \phi^{0}\right)$


## Energy?



## Likelihood-free inference algorithms


$x+\frac{2}{x}=$
Encoder/Decoder


ReLu
BatchNorm


Concat



$$
-\frac{0}{8}
$$

LSTM
:\%84
250
CTC


Attention



Capsule Nets


Mixture of Experts
Neural Collaborative Filtering


Block Sparse LSTM


Can we harness deep learning for inference and generation?


Neural networks are

- function approximators with a gazillion of parameters,
- tuned with stochastic gradient descent

$$
\theta_{t+1}=\theta_{t}-\gamma \hat{\nabla}_{\theta} \mathcal{L}\left(\theta_{t}\right)
$$

- are flexible enough to be structured by domain knowledge.

Treat the simulator as a black box


Treat the simulator as a black box


Learn to control the simulator

## Make use of the inner structure



Probabilistic programming

## The physicist's way

The Neyman-Pearson lemma states that the likelihood ratio

$$
r\left(x \mid \theta_{0}, \theta_{1}\right)=\frac{p\left(x \mid \theta_{0}\right)}{p\left(x \mid \theta_{1}\right)}
$$

is the most powerful test statistic to discriminate between a null hypothesis $\theta_{0}$ and an alternative $\theta_{1}$.

IX. On the Problem of the most Efficient Tests of Statistical Hypotheses.

By J. Neyman, Nencki Institute, Soc. Sci. Lit. Varsoviensis, and Lecturer at the Central College of Agriculture, Warsaw, and E. S. Pearson, Department of Applied Statistics, University College, London.
(Communicated by K. Pearson, F.R.S.)
(Received August 31, 1932.—Read November 10, 1932.)

Contents.

[^0]

Define a projection function $s: \mathcal{X} \rightarrow \mathbb{R}$ mapping observables $x$ to a summary statistics $x^{\prime}=s(x)$.

Then, approximate the likelihood $p(x \mid \theta)$ as

$$
p(x \mid \theta) \approx \hat{p}(x \mid \theta)=p\left(x^{\prime} \mid \theta\right)
$$

From this it comes

$$
\frac{p\left(x \mid \theta_{0}\right)}{p\left(x \mid \theta_{1}\right)} \approx \frac{\hat{p}\left(x \mid \theta_{0}\right)}{\hat{p}\left(x \mid \theta_{1}\right)}=\hat{r}\left(x \mid \theta_{0}, \theta_{1}\right)
$$



This methodology has worked great for physicists for the last 20-30 years, but ...

- Choosing the projection $s$ is difficult and problem-dependent.
- Often there is no single good variable: compressing to any $x^{\prime}$ loses information.
- Ideally: analyse high-dimensional $x^{\prime}$, including all correlations.

Unfortunately, filling high-dimensional histograms is not tractable.


Who you gonna call? Machine learning!

## CARL



## Key insights

- The likelihood ratio is often sufficient for inference.
- Evaluating the likelihood ratio does not require evaluating the individual likelihoods.
- Supervised learning indirectly estimates likelihood ratios.

Supervised learning provides a way to automatically construct $s$ :

- Let us consider a binary classifier $\hat{s}$ (e.g., a neural network) trained to distinguish $x \sim p\left(x \mid \theta_{0}\right)$ from $x \sim p\left(x \mid \theta_{1}\right)$.
- $\hat{s}$ is trained by minimizing the cross-entropy loss

$$
\begin{aligned}
L_{X E}[\hat{s}]=-\mathbb{E}_{p(x \mid \theta) \pi(\theta)}[1(\theta & \left.=\theta_{0}\right) \log \hat{s}(x)+ \\
1(\theta & \left.\left.=\theta_{1}\right) \log (1-\hat{s}(x))\right]
\end{aligned}
$$

The solution $\hat{s}$ found after training approximates the optimal classifier

$$
\hat{s}(x) \approx s^{*}(x)=\frac{p\left(x \mid \theta_{1}\right)}{p\left(x \mid \theta_{0}\right)+p\left(x \mid \theta_{1}\right)}
$$

Therefore,

$$
r\left(x \mid \theta_{0}, \theta_{1}\right) \approx \hat{r}\left(x \mid \theta_{0}, \theta_{1}\right)=\frac{1-\hat{s}(x)}{\hat{s}(x)}
$$

That is, supervised classification is equivalent to likelihood ratio estimation and can therefore be used for MLE inference.

Treat the simulator as a black box

Learn a proxy for inference


Histograms of observables
Neural density (ratio) estimation

## Learn to control

 the simulatorAdversarial variational optimization

Make use of the inner structure


Mining gold from implicit models


Probabilistic programming

## Mining gold from simulators


$p(x \mid \theta)$ is usually intractable.
What about $p(x, z \mid \theta)$ ?

As the trajectory $z_{1}, \ldots, z_{T}$ and the observable $x$ are emitted, it is often possible:

- to calculate the joint likelihood $p(x, z \mid \theta)$;
- to calculate the joint likelihood ratio $r\left(x, z \mid \theta_{0}, \theta_{1}\right)$;
- to calculate the joint score $t\left(x, z \mid \theta_{0}\right)=\left.\nabla_{\theta} \log p(x, z \mid \theta)\right|_{\theta_{0}}$.

We call this process mining gold from your simulator!

Observe that the joint likelihood ratios

$$
r\left(x, z \mid \theta_{0}, \theta_{1}\right)=\frac{p\left(x, z \mid \theta_{0}\right)}{p\left(x, z \mid \theta_{1}\right)}
$$

are scattered around $r\left(x \mid \theta_{0}, \theta_{1}\right)$.
Can we use them to approximate $r\left(x \mid \theta_{0}, \theta_{1}\right)$ ?

Let us define

$$
L_{r}=\mathbb{E}_{p\left(x, z \mid \theta_{1}\right)}\left[\left(r\left(x, z \mid \theta_{0}, \theta_{1}\right)-\hat{r}(x)\right)^{2}\right]
$$

Via calculus of variations, we find that this functional is minimized by

$$
\begin{aligned}
r^{*}(x) & =\frac{1}{p\left(x \mid \theta_{1}\right)} \int p\left(x, z \mid \theta_{1}\right) \frac{p\left(x, z \mid \theta_{0}\right)}{p\left(x, z \mid \theta_{1}\right)} d z \\
& =\frac{p\left(x \mid \theta_{0}\right)}{p\left(x \mid \theta_{1}\right)} \\
& =r\left(x \mid \theta_{0}, \theta_{1}\right)
\end{aligned}
$$



How does one find $r^{*}$ ?

$$
r^{*}\left(x \mid \theta_{0}, \theta_{1}\right)=\arg \min _{\hat{r}} L_{r}[\hat{r}]
$$

Minimizing functionals is exactly what machine learning does. In our case,

- $\hat{r}$ are neural networks (or the parameters thereof);
- $L_{r}$ is the loss function;
- minimization is carried out using stochastic gradient descent from the data extracted from the simulator.

Similarly, we can mine the simulator to extract the joint score

$$
t\left(x, z \mid \theta_{0}\right)=\left.\nabla_{\theta} \log p(x, z \mid \theta)\right|_{\theta_{0}}
$$

which indicates how much more or less likely $x, z$ would be if one changed $\theta_{0}$.


We define

$$
L_{t}=\mathbb{E}_{p\left(x, z \mid \theta_{0}\right)}\left[\left(t\left(x, z \mid \theta_{0}\right)-\hat{t}(x)\right)^{2}\right]
$$

which can be shown to be minimized by $t^{*}(x)=t\left(x \mid \theta_{0}\right)$.

## RAsCAL

$$
L_{R A S C A L}=L_{r}+L_{t}
$$



## RAsCAL

$$
L_{R A S C A L}=L_{r}+L_{t}
$$




## Treat the simulator as a black box

Learn a proxy for inference


Histograms of observables Neural density (ratio) estimation


Adversarial variational optimization

Make use of the inner structure


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Probabilistic programming

## Generative adversarial networks




Odena et al 2016

Miyato et al 2017

Zhang et al 2018

Brock et al 2018


Figure 2. Uncurated set of images produced by our style-based generator (config F) with the FFHQ dataset. Here we used a variation of the truncation trick [5,29] with $\psi=0.7$ for resolutions $4^{2}-32^{2}$. Please see the accompanying video for more results.

Karras et al, 2018.

## AVO



Replace $g$ with an actual scientific simulator!

## Key insights

- Replace the generative network with a non-differentiable forward simulator $g(\mathbf{z} ; \theta)$.
- Let the neural network critic figure out how to adjust the simulator parameters.
- Combine with variational optimization to bypass the non-differentiability by optimizing upper bounds of the adversarial objectives

$$
\begin{aligned}
U_{d}(\phi) & =\mathbb{E}_{\theta \sim q(\theta ; \psi)}\left[\mathcal{L}_{d}(\phi)\right] \\
U_{g}(\psi) & =\mathbb{E}_{\theta \sim q(\theta ; \psi)}\left[\mathcal{L}_{g}(\theta)\right]
\end{aligned}
$$

respectively over $\phi$ and $\psi$.


Samples for $\theta=0$ (top) vs.
samples for $\theta=0.81$ (bottom).

## Treat the simulator as a black box

Learn a proxy for inference

Learn to control the simulator

Histograms of observables Neural density (ratio) estimation


Mining gold from implicit models


Probabilistic programming

## Probabilistic programming



CS

## Probabilistic programming




CS


Statistics

## Probabilistic programming



CS
Probabilistic Programming Statistics

## Probabilistic programming



Inference


CS Probabilistic Programming Statistics


## Key insights

Let a neural network take full control of the internals of the simulation program by hijacking all calls to the random number generator.

(defquery captcha
[image num-chars tol]
(let [[wh] (size image)
; ; sample random characters num-chars (sample
(poisson num-chars)) chars (repeatedly num-chars sample-char)]
; compare rendering to true image
(map (fn [y z]
(observe (normal $z$ tol) y))
(reduce-dim image)
(reduce-dim (render chars wh)))
; predict captcha text
\{: text
(map :symbol (sort-by :x chars))\}))

How to break captchas with probabilistic programming


## X

## y

event \& detector simulators ATLAS detector output


Probabilistic programming hooked to particle physics simulators (work in progress)

(a) Prior execution $p(\mathbf{x})$.

(b) Posterior execution $p(\mathbf{x} \mid \mathbf{y})$ conditioned on a given calorimeter observation $\mathbf{y}$.

## Summary

## Summary

- Much of modern science is based on "likelihood-free" simulations.
- Recent (and older) developments from machine learning offer solutions for likelihood-free inference, including:
- Supervised learning
- Neural networks trained with augmented data
- Adversarial training
- Probabilistic programming



## Collaborators



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The end.


[^0]:    I. Introductory Page
    II. Outline of General Theory

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