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The Political Economy of Contributive Pensions in Developing Countries[1]

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Abstract

This paper sheds light on the role of public institutions as a way to reduce tax evasion through a close link between payroll taxation and pension benefits. We use a political economy model in which agents have the possibility to hide part of their earnings in order to avoid taxation and, where the public system is more efficient in providing annuitized pension benefits than the private sector. We show that in the absence of evasion costs, agents are indifferent to the tax rate level as they can always perfectly adapt compliance so as to face their preferred effective tax rate. There is unanimity in favour of the maximum tax rate and, the public pension system is found to be partially contributive in order to increase tax compliance and thus the resources collected. This, in turn, enables higher redistribution toward the worst-off agents. When evasion costs are introduced, perfect substitutability between compliance and taxation breaks down. At the majority-voting equilibrium, individuals at the bottom of the income distribution who are in favour of more redistribution, and those at the top who want to transfer more resources to the old age, form a coalition against middle-income agents, in favour of high tax rates. In addition to the previous tax base argument, the optimal level of the Bismarkian pillar is now chosen so as to account for political support.

Keywords: Ends-against-the-Middle Equilibrium, Majority Voting, Public Pensions, Tax compliance.
JEL codes: H55, D78, D91.

1 Introduction

Tax evasion is endemic in many countries, in particular in developing countries, which do not collect even half of what they should if taxpayers complied with the written letter of the law (Moore and Mascagni, 2014). For instance, according to the OECD, in sub-Saharan Africa, tax revenues correspond to less than a fifth of GDP and, at the same time, the cost of collection varies from 1% to 4% of that revenue.[1] Many developing countries still face tax shares of GDP below 15%, which corresponds to the threshold level that would be reasonable for ensuring government functioning.[2] In these countries, enforcement mechanisms are weak, tax-collecting authorities are held in low esteem, and courts may not enforce the law. In recent years, domestic revenue mobilization in developing countries has gained increasing prominence in the policy debate. This is due to several factors, including the potential benefits of taxation for state building, long-term independence from foreign assistance and the continuing acute financial needs of developing countries.

Plenty of specific solutions to how boosting compliance have been offered in the literature and public authorities have tried many more. Some of them consist in increasing and in focusing on better enforcement, in improving the collection and management of information, in reducing the costs of complying with the law, and in providing incentives for those who comply.[3] The solution we explore in this paper consists in establishing a close link between the amount of taxes paid and the payments obtained in return. After all, if that link were perfect, there would be no problem of compliance and the tax would play the same role as any market price. Yet, in many countries, this link is often not perfect as the government is expected to provide public goods and redistribute income. This is the avenue of research this paper pursues.

The objective of our paper is therefore to focus on payroll taxation and to study how a government who wishes to redistribute resources inside the economy should design the pension system when it faces contribution evasion and political restrictions on the tax rate. Here, individuals receive public pension benefits which depend (imperfectly) on their earlier tax payments. These benefits are essential to the individuals' welfare but have imperfect substitutes in the private sector. For instance, even in developed economies, the private annuity market is thin and faces important loading costs. It is therefore in the agents' own interest to contribute to the public pension system.[4]

[1] `http://oecdobserver.org/news/fullstory.php/aid/3510/Africa_s_tax_system:_A_survey.html`, last visited October 2018.

[2] For instance, Gaspar et al. (2016) show that a minimum tax to GDP ratio of 12.5% is needed to insure significant growth.

[3] Castro and Scartascini (2013) also explores the impact of the use of messages for affecting taxpayers' compliance by conducting a large-scale field experiment in Argentina. These messages operate on the beliefs and moral values that people attach to paying taxes.

We assume a two-period model in which individuals work in the first period and retire in the second one. In the first period, they choose how much income to report to fiscal authorities and pay taxes based on this reported amount. This, in turn, finances the public pension system. In the first part of the model, we assume that evading income is done at no cost so that there is a priori no reason not to do so if the government imperfectly observes income or lacks coercive measures to make individuals report it truthfully. Individuals further allocate disposable income between current consumption and private saving. The second period is uncertain as we assume that survival is random. If they survive, individuals use their private saving and the pension benefit for consumption. As in Casamatta et al. (2000a, b), the pension benefit depends on two components. The first component is proportional to *reported* labor income; the factor of proportionality is called the Bismarkian factor. The second one, the Beveridgian part, is common to all individuals and depends on average tax collection. This component allows for intra-generational redistribution. We also assume that there is a probability that the government defaults and is not able to pay the pensions for which agents previously contributed. If so, contributions are lost and disappear from the economy. The tax rate is decided by majority-voting while the Bismarkian factor is set at the constitutional level, behind the veil of ignorance, and prior to any votmajority-voting equilibriume on the tax rate. Following Rawls and its *Theory of Justice (1971)*, the social welfare criterion consists then in giving priority to the worst-off agent. In the last part of the paper, we extend our model to allow for the existence of (moral or psychological) costs of compliance and show how our results are modified.

Before going further, let us explain our choice of the political structure. First, one could have assumed that the Bismarkian factor, like the tax rate, is chosen by majority-voting. As it becomes clear later with the model, if the median voter has a low income and receives only the flat pension benefit, this does not change our results. One could also argue that the redistributivity of the pension system is a characteristic which results from countries specific traditions and culture regarding the role of public institutions. For this reason, it seems rather difficult to overturn it in the short run, contrary to the contribution rate which is more subject to fluctuations across time and to political cycles.[5] Second, one could have chosen a different social objective such as for instance, the Utilitarian one. We derive only the Rawlsian solution which is a special case of the Utilitarian one where the individual with minimum income is given priority. In this special case, the redistributive motive is maximum and our results are "biased" toward the highest level of redistribution, which may discourage compliance the most, at least for richer individuals.

crucial ingredients are 1) the inexistence of good private substitutes and 2) the existence of a link between benefits and individual contributions.

[5]For instance, Germany and France are traditionally more Bismarckian while the UK is traditionally more Beveridgean. For a discussion on the constitutional versus political choices of the tax rate and the Bismarkian factor, see Casamatta et

We obtain the following results. A necessary condition for agents to report some income is that the marginal return from public pensions is greater than that of the private annuity market. Even though the no-evasion-cost assumption "biases" our results toward minimum participation to the fiscal system, we find that agents comply with the tax system at a rate which is increasing in their income. Interestingly, choosing how much income to report or choosing the tax rate level are perfectly equivalent decisions: if the tax rate increases, agents perfectly adapt their compliance rate so as to pay the exact same amount of taxes. Hence, at the majority-voting equilibrium, there is unanimity in favour of the maximum tax rate. The reasons are twofold. First, because of perfect substitutability between taxation and compliance, the flat (Beveridgian) part of the pension benefit is increasing in the tax rate so that every agent wants the maximum of that benefit. Second, agents with high income prefer to use the more efficient pension system (rather than the private annuity market) to transfer resources to the old age. At the last constitutional stage, the level of redistribution is chosen so as to maximise the utility of the poorest agent and the Bismarkian factor is optimally set at a level that is higher than the marginal return from savings. The reason for a contributive pension system is therefore directly related to a tax base increase argument: a pension system with a contributory part makes the agents willing to report more income. This, in turn, increases the resources available for redistribution (through the flat pension benefit) and, as such it ensures the highest possible level of utility to the worst-off individual.

With a moral cost of evasion, our main results are preserved. Yet, the presence of an evasion cost breaks down perfect substitutability between the tax rate and the compliance rate so that the agent's preferred tax rate now depends on his income. Agents at the bottom of the income distribution, may partially comply with the tax system but are in favour of a positive tax rate which is decreasing in income. For them, the main reason for a pension system is to benefit from intra-generational redistribution. To the opposite, agents with higher income, who already fully comply, want a positive tax rate so as to transfer more resources to the second period. Their preferred tax rate is now increasing in income. Consequently, we obtain a ends-against-the-middle voting equilibrium, where poor agents form a coalition with high-income agents against middle-income agents, for implementing high tax rates. Finally, at the constitutional stage, the Bismarkian factor is likely to remain high. First, increasing the degree of contributiveness leads individuals to comply more, which in turn increases the tax base and income redistribution. Second, as in Casamatta et al. (2000a), a tighter link between contributions and pension benefits may increase the political support for the pension system.

All in all, independently of whether evasion entails individual costs, giving the possibility to individuals to participate in a contributory pension system increases compliance and the overall welfare of the society.

developed (for instance, annuity markets are absents and loading costs are high) and, the provision of pensions by the public may be more efficient.

Our paper can be related to at least two strands of the literature. First, there is a well-known theoretical literature on tax evasion and on how tax compliance varies with income, risk aversion, fiscal instruments, enforcement parameters and social norms (see Allingham and Sandmo, 1972; Yitzhaki, 1974; Cowell and Gordon, 1988; Gordon, 1989; Myles and Naylor, 1996). Also, related to our paper, Baumann et al. (2009) studies contribution evasion in unemployment insurance programs, in an Allingham and Sandmo (1972) framework.[6] These papers assume a probability of auditing and a penalty rate in case of detection and, sometimes a cost of non-compliance. To the contrary, we assume away auditing but model instead a moral cost of under-reporting as in Gordon (1989) as well as uncertainty in the provision of the pension benefits. Such a modelling is relevant in particular for countries where tax enforcement mechanisms are weak and the political risk is high.

Second, our paper can be related to the political economy literature on tax evasion. For instance, Borck (2004, 2009) study political economy models in which agents can evade income.[7] Appart from the questions raised (how stricter enforcement policies may actually increase tax evasion, or how redistribution is impacted by evasion), these papers differ from ours in several other respects. First, agents are risk neutral so that they either avoid all income or none of it. This seems rather restrictive as in general, evasion is not a "all or nothing decision". Second, it assumes that agents get a uniform transfer from their contributions while we model a contribution-related pension system. Third, the only motive for taxation is redistribution and there is no consumption smoothing motive as in our model. Traxler (2012) also looks at the efficiency effect of tax avoidance when the tax rate is chosen by a majority of taxpayers. The main finding is that the traditional inefficiency carried by majority voting decreases with the extent of tax avoidance. Alm et al. (1999) also briefly discuss a political economy model with risk-neutral agents where the presence of a social norm influences agents' tax compliance and thus, their voting behaviour over an increase in the tax rate, in the fine and in the probability of detection. This model is different from ours at least in three dimensions: agents either fully comply or not at all, they receive a lump sum benefit and, individuals' heterogeneity arises from differences in the psychological loss from not complying. Finally, Kopczuk (2001) studies an optimal income taxation scheme when agents have different tax avoidance behaviour, either because of different preferences for avoidance or because of different avoidance cost functions. This is different from our paper because first, it is normative and second, in our model agents

[6] These authors obtain, like us, that in order to stimulate truthful reporting, insurance programs should be made more contribution-related, even though this is not enough to eliminate evasion. However, we go further by modelling the political equilibrium and the possibility of government default.

[7] See also the survey by Borck (2007) on inequality, redistribution and taxation in political economy models.

differ in income only. In addition, none of of these models consider the possibility of government default as we do. To our knowledge, our paper is one of the few political economy papers which studies the interaction between tax compliance and the existence of a pension system and, how the latter helps reducing inefficiencies related to income under-reporting, even when evasion can be done at no cost.

The rest of the paper is organized as follows. The next section presents the model. Section 3 presents individuals' decisions in terms of compliance and private savings for given levels of the policy instruments. In Section 4, we derive individuals' preferred tax rate and the majority-voting equilibrium. In Section 5, we find the constitutional level of the Bismarkian factor. Section 6 extends our model to allow for compliance costs. The last section concludes.

2 The model

We assume a two-period model, with a mass one of individuals who face uncertain survival and no population growth. Agents have different productivity, y which is continuously distributed over $[y_{min}, y_{max}]$, with median productivity below the average, $y_m < \bar{y}$, and density function $f(y)$. In the first period, agents supply inelastically one unit of labour.[8] They consume, save on private markets and contribute to a public pension system. In the second period, say old-age, they are alive with a probability $0 \leq \pi \leq 1$. If alive, they are retired and consume their private accumulated savings as well as the public pension benefit. Agents however *perceive* that there is a positive probability $0 \leq \beta \leq 1$ that the government defaults and is not able to pay the pensions for which they previously contributed. This parameter aims at modelling what is often called the "time-bomb" of pensions, namely the predicted difficulty in paying for retirement pensions, due to a difference between pension obligations and the resources set aside to fund them.[9] In that case, agents believe their contributions are lost or that they are used for anything else which does not provide them direct utility. This assumption seems quite relevant in developing countries where governments are often held in low esteem.

Agents derive utility from consumption in each period. Without loss of generality, we assume that there are no pure time preferences. Individual preferences can be formulated as follows:

$$EU = u(c) + \pi\{(1-\beta)u(d_N) + \beta u(d_D)\}$$

where consumption in the first period is denoted by c while d_N (resp. d_D) represents the individual's

[8]Assuming endogenous labour supply would not change our results as, in our set-up, taxation would not generate any labour supply distortion. The reason is that, when agents choose how much to comply and if there is no compliance cost, the tax rate and the rate of compliance are perfect substitutes (see Section 3 below). Hence, the agents can always perfectly adapt compliance to a variation in the tax rate, thus preventing any distortion on the labour supply. Computations are available from the authors upon request.

[9]For instance, according to Gallup, one of the largest U.S. polling organization, in 2015, "51% of non-

consumption when the government does not default (resp. defaults) in the second period. Per period utility is such that $u'(.) > 0$, $u''(.) < 0$ and $u'(0) \to +\infty$. We also make the following assumption regarding the coefficient of relative risk aversion, $R_r(c) = -u''(c)c/u'(c)$:[10]

Assumption 1 *The coefficient of relative risk aversion is increasing in income and smaller than 1: $R_r(d) \leq R_r(c) \leq 1$ with $c \geq d$.*

In the first period of their life, agents work and pay a tax at rate τ on their *reported* income, $\tilde{y}$ so as to finance the public pension program. Implicit in our modelling is that the government only imperfectly observes the distribution of incomes in the society and/ or lacks coercive measures to make agents report income truthfully. For simplicity, we assume that reported income is simply equal to a fraction of the agent's true income, $\tilde{y} = \gamma y$ where $0 \leq \gamma \leq 1$ is the rate of compliance and it is privately chosen by the agent. In the baseline framework, we assume that evasion entails no (moral or financial) cost. We relax this assumption in Section 6. In the second period, individuals consume their savings as well as a pension benefit $P(\tau, \alpha; y)$ when the government does not default. First and second-period consumptions can thus be expressed as follows:

$$\begin{aligned} c &= y - \tau\tilde{y} - s = (1 - \tau\gamma)y - s \\ d_N &= P(\tau, \alpha; y) + \frac{\varepsilon}{\pi}s \\ d_D &= \frac{\varepsilon}{\pi}s \end{aligned}$$

The amount of savings s is assumed to be invested on the private annuity market and its return is $\varepsilon R/\pi$ where, for simplicity, the gross rate of savings return R is set to 1. The parameter $\varepsilon \in [\pi, 1]$ reflects both the weakness of the annuity market and the inefficiency of the financial institutions for individual savers.[11]

Since the current interest rate and the rate of population growth are null, the public pension system we model could be either funded or unfunded. The pension benefit is obtained from balancing the government's budget and it is thus equal to:

$$P(\tau, \alpha; y) = \frac{\tau}{\pi}\left(\alpha\tilde{y} + (1 - \alpha)E(\tilde{y})\right)$$

where $0 \leq \alpha \leq 1$ is the Bismarckian factor. It is constituted of two parts as in Casamatta et al. (2000a). The Bismarkian part $(\tau\alpha\tilde{y}/\pi)$ is directly linked to the agent's earlier contributions. The Beveridgian

[10]Karagyozova and Siegelman (2012) review the empirical literature about the estimation of relative risk aversion, and find that the assumption that $R_r(c) \leq 1$ is reasonable. For instance, Holt and Laury (2002) find that 64% of respondents have a coefficient comprised between 0.15 and 0.97. Chetty (2006) also finds, using estimates of wage and income elasticities, a mean value for the relative risk aversion coefficient equal to 0.71.

part, $b(\tau,\alpha) = \tau(1-\alpha)E(\tilde{y})/\pi$ is flat and depends on average reported earnings in the economy,

$$E(\tilde{y}) = E(\gamma(y)y) = \int_{y_{min}}^{y_{max}} \gamma(y)yf(y)dy$$

where $\gamma(y)$ is the compliance rate chosen by an agent with true income y. The uniform benefit $b(\tau,\alpha)$ constitutes the "redistributive part" of the pension benefit as individuals who report $\tilde{y} < E(\tilde{y})$ contribute less than what they actually obtain in the second period, $\tau\tilde{y} \leq \tau\left(\alpha\tilde{y} + (1-\alpha)E(\tilde{y})\right)$ thanks to that uniform part. This component makes intra-generational redistribution possible. For instance, in the extreme case where $\alpha = 0$, the system is said to be fully redistributive. Agents get a net benefit from the pension system equal to $\tau(E(\tilde{y}) - \tilde{y})$ which is positive (resp. negative) for $\tilde{y} < E(\tilde{y})$ (resp. $>$) and decreasing in $\tilde{y}$. On the contrary, if $\alpha = 1$, the pension system is fully contributive, agents receive exactly the amount they contributed to and there is no income redistribution.

The agent can thus transfer resources to the old age by contributing to the pension system (equivalently, by complying with the tax system) and / or investing in a private annuity market. The marginal return from private annuities is equal to ε/π while that of contributing to the pension system is α/π. At this stage, we make no specific assumption regarding the levels of α and ε.

The timing of the model is the following. First, the level of the Bismarkian factor, α, is set at the constitutional level behind the veil of ignorance. Agents then vote over the level of the tax rate, τ. Finally, given the tax rate chosen at the majority-voting equilibrium τ^V, and the constitutional level of the Bismarkian factor α^R, they make private decisions over compliance and savings, (γ, s). As usual in this type of problem, we proceed backward.

3 Individuals' decisions over compliance and savings

For given pension parameters (τ,α), the problem of an agent with income y consists in solving[12]

$$\max_{s,\gamma} EU(\gamma, s; y) = u((1-\gamma\tau)y - s) + \pi\{(1-\beta)u(\frac{\tau}{\pi}(\alpha\gamma y + (1-\alpha)E(\tilde{y})) + \frac{\varepsilon}{\pi}s) + \beta u(\frac{\varepsilon}{\pi}s)\} \quad (1)$$

First order conditions with respect to (s,γ) are:

$$\frac{\partial EU}{\partial s} = -u'(c) + \varepsilon\{(1-\beta)u'(d_N) + \beta u'(d_D)\} \leq 0 \quad (2)$$

$$\frac{\partial EU}{\partial \gamma} = \tau y[-u'(c) + \alpha(1-\beta)u'(d_N)] \leq 0 \quad (3)$$

Using (2), it is clear that $s^*(\tau,\alpha;y) > 0$ under the assumption that $u'(0) \to +\infty$. Private savings serve here as an insurance mechanism against government default. Substituting for (2), the FOC with respect to γ can be rewritten as follows:

$$\frac{\partial EU}{\partial \gamma} = \tau y[-\varepsilon\beta u'(d_D) + (1-\beta)(\alpha-\varepsilon)u'(d_N)] \leq 0. \quad (4)$$

Evaluating it in $\gamma = 0$, it is clear that a sufficient condition for $\gamma = 0$ is $\alpha \leq \varepsilon$. Indeed, if investing in public pensions is both risky and gives a lower return than private savings, the agent should better invest in the private annuity market only. For instance, if $\alpha = \varepsilon$, increasing compliance entails a marginal utility cost in the first period which is identical to that of private savings, but lower expected marginal utility (because of the default probability) in the second period so that agents prefer private savings over public pensions. Hence, the marginal return from pensions α, needs to be sufficiently larger than the marginal return from private savings ε, to compensate for the uncertainty of government payments and provide incentives to comply.[13]

For this reason, from now on, we focus on the case where $\alpha > \varepsilon$ and we obtain the following ranking of consumptions: $c > d_N > d_D$.[14] Using Cramer's rule and Assumption 1, we find that both the compliance rate, $\gamma^*(\tau, \varepsilon; y)$ and saving $s^*(\tau, \varepsilon; y)$ are increasing in income.[15] This also implies that reported income, $\tilde{y} = \gamma^*(\tau, \varepsilon; y)y$ increases with income. Indeed, as the agent becomes richer, both the (first-period) marginal cost of transferring resources to the old age and its (second-period) marginal benefit decrease; Assumption 1 guarantees however that marginal cost decreases more rapidly than marginal benefit so that a richer agent is willing to transfer more resources to the second period.

The variation of $\gamma^*(\tau, \alpha; y)$ with y therefore enables us to define two income thresholds, one below which agents do not comply, and one above which they fully comply. First, the income threshold below which agents choose not to report any income, $\hat{y}(\tau, \alpha)$, is defined by

$$u'(\hat{y} - \hat{s}) = \alpha(1-\beta)u'(\frac{\tau}{\pi}(1-\alpha)E(\tilde{y}) + \frac{\varepsilon}{\pi}\hat{s}) \tag{5}$$

with $\hat{s} = s^*(\tau, \alpha; \hat{y})$. Hence, agents with $y \leq \hat{y}(\tau, \alpha)$ choose $\gamma^* = 0$ while those with $y > \hat{y}(\tau, \alpha)$ choose a strictly positive γ^*.[16] Second, the income threshold, $\breve{y}(\tau, \alpha)$ above which agents fully comply with the fiscal system is defined by

$$u'(\breve{y}(1-\tau) - \breve{s}) = (1-\beta)\alpha u'(\frac{\tau}{\pi}(\alpha\breve{y} + (1-\alpha)E(\tilde{y})) + \frac{\varepsilon}{\pi}\breve{s}) \tag{6}$$

with $\breve{s} = s^*(\tau, \alpha; \breve{y})$. Agents with $y \geq \breve{y}(\tau, \alpha)$ report all their income, $\gamma^* = 1$ and for them, marginal benefit from compliance is greater than its marginal cost, $\alpha(1-\beta)u'(d_N) \geq u'(c)$. Finally, agents with $\hat{y}(\tau, \alpha) < y < \breve{y}(\tau, \alpha)$ report $0 < \gamma^*(\tau, \alpha; y) < 1$ which satisfies (3) with equality.

[13] Assuming that the pension system can never default ($\beta = 0$) would not drastically change our results. The only difference is that savings would now not always be strictly positive (there is no need for an insurance against government default anymore) so that one would have $s^*(\tau, \varepsilon; y) = 0, \gamma^*(\tau, \varepsilon; y) \geq 0 \, \forall y$ under $\alpha > \varepsilon$.

[14] We show in Section 5 that the case where $\varepsilon > \alpha^R$ (α^R being the constitutional level of α chosen by a Rawlsian government) never happens.

[15] Allingham and Sandmo (1972) obtain a similar result that the fraction of reported income increases with income under increasing relative risk-aversion.

[16] We abbreviate $\gamma^*(\tau, \alpha; y)$ and $s^*(\tau, \alpha; y)$ by γ^* and s^*, and do this also for other variables in the manuscript when

Hence, for given pension parameters (τ, α), agents comply only if they expect a sufficiently high marginal utility return from public pensions, which depends on their income level. If their income is below $\hat{y}(\tau, \alpha)$, the marginal utility cost of reporting income is higher than the marginal utility benefit obtained from previous contributions in the second period so that they choose $\gamma^*(\tau, \alpha; y) = 0$. On the contrary, if $y > \hat{y}(\tau, \alpha)$, marginal utility obtained from previous contributions is always higher than marginal cost of reporting income and $\gamma^*(\tau, \alpha; y) > 0$. If income is sufficiently important, above $\breve{y}(\tau, \alpha)$, agents would even want to report more than their full income so as to transfer more resources (with a higher marginal return α than that of private savings) toward the second period.

We summarize individuals' optima decisions in the following proposition and Figure 1 below graphes the variation of $\gamma^*(\tau, \alpha; y)$ as a function of y for given pension parameters (τ, α).[17]

Proposition 2 *Assume a population of agents with income $y \in [y_{min}, y_{max}]$ who have the possibility to save on a private annuity market and to choose how much income to report so as to finance a risky public pension system.*

- *If $\alpha \leq \varepsilon$, every agent prefers private savings over the public pension system, so that $\gamma^*(\tau, \alpha; y) = 0$ and $s^*(\tau, \alpha; y) > 0 \,\forall y$.*
- *If $\alpha > \varepsilon$, agents with income $y \leq \hat{y}(\tau, \alpha)$ do not report any income and receive $b(\tau, \alpha)$ at retirement. Agents with income $y > \hat{y}(\tau, \alpha)$ report a fraction $0 < \gamma^*(\tau, \alpha; y) \leq 1$ and receive $P(\tau, \alpha; y)$ at retirement. In addition, $s^*(\tau, \alpha; y) > 0$ for all agents.*

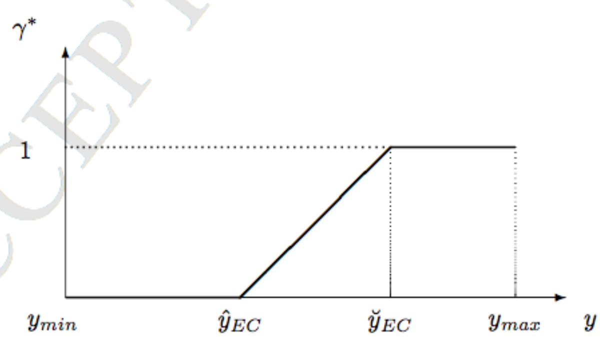


Figure 1: Individuals' compliance rate $\gamma^*(y)$ as a function of their income, y under $\alpha > \varepsilon$.

Let us now show that, for agents with income $\hat{y}(\tau, \alpha) < y < \breve{y}(\tau, \alpha)$, choosing γ for a given τ is equivalent to letting them choose their individually preferred *effective* tax rate level $t \in [0, 1]$ defined as $t = \gamma\tau$ with $t \leq \tau$. In order to understand this, we make a change of variable in the agent's utility function (1) so that it is now written as:

$$EU(t, s; y) = u((1-t)y - s) + \pi\{(1-\beta)u(\frac{1}{\pi}(\alpha ty + (1-\alpha)E(ty)) + \frac{\varepsilon}{\pi}s) + \beta u(\frac{\varepsilon}{\pi}s)\}$$

where τ disappears from the above expression and $E(ty)$ is considered as exogenous by the agent.[18] Maximizing $EU(t, s; y)$ with respect to t yields the following first-order condition:[19]

$$\frac{\partial EU(t, s; y)}{\partial t} = y[-u'(c) + \alpha(1-\beta)u'(d_N)] \leq 0 \tag{7}$$

This optimality condition, which gives the optimal level $t^*(\alpha; y)$, is the same as the optimality condition for $\gamma^*(\tau, \alpha; y)$. Hence, for an agent with income $\hat{y}(\tau, \alpha) < y < \breve{y}(\tau, \alpha)$, the problem where he chooses γ^* for a given τ is exactly equivalent to the problem where he chooses directly his preferred rate of contribution t^*, not having any choice on how much income to report. In addition, as it is clear from (7), $t^*(\alpha; y)$ is independent of τ for these agents. This means that for any tax rate $\tau > 0$, agents with $\hat{y}(\tau, \alpha) < y < \breve{y}(\tau, \alpha)$ always pay their preferred amount of effective taxation $t^*(\alpha; y)$ by adapting exactly how much they report, i.e. $\gamma^*(\tau, \alpha; y)$, to a variation in the level of the tax rate.

As a consequence, the amount of resources collected, equal to

$$\tau E(\tilde{y}) = \int_{\hat{y}}^{\breve{y}} \tau\gamma^* y f(y) dy + \int_{\breve{y}}^{y_{max}} \tau y f(y) dy,$$

increases in τ since the first term on the RHS, $t^* = \tau\gamma^*$ is independent of τ and the second term depends linearly on τ. This implies that the flat pension benefit, $b(\tau, \alpha) = (1-\alpha)\tau E(\tilde{y})/\pi$ is increasing in τ.

Also, since $t^*(\alpha; y)$ is independent of τ, we can show that $d\gamma^*(\tau, \alpha; y)/d\tau < 0$ for agents with $\hat{y}(\tau, \alpha) < y < \breve{y}(\tau, \alpha)$.[20] There is not a clear consensus in the literature regarding the effect of increasing the tax rate on compliance. It depends on the design of the fiscal system and specifically, on whether penalties are applied on income evaded or on taxation evaded (see Yitzhaki, 1974 and Christiansen, 1980). Here, the reason is different: it is simply that in our framework, γ and τ are perfect substitutes.

We now turn to studying individuals' preference for the tax rate as well as the majority-voting equilibrium.

[18] When choosing t, each agent has no impact on the aggregate level of fiscal resources, $E(ty)$.

[19] The FOC with respect to s does not change.

[20] To see this, recall that

$$\frac{dt^*(\alpha; y)}{d\tau} = \gamma^*(\tau, \alpha; y) + \frac{d\gamma^*(\tau, \alpha; y)}{d\tau}\tau = 0$$

which implies that $d\gamma^*(\tau, \alpha; y)/d\tau < 0$.

4 Preferences over the tax rate and the majority-voting equilibrium

The indirect utility function of an agent with income y takes the following form:

$$\begin{aligned} V(\tau,\alpha;y) &= u((1-\gamma^*(\tau,\alpha;y)\tau)y - s^*(\tau,\alpha;y)) \\ &+ \pi\{(1-\beta)u(\frac{\tau}{\pi}(\alpha\gamma^*(\tau,\alpha;y)y+(1-\alpha)E(\tilde{y}))+\frac{\varepsilon}{\pi}s^*(\tau,\alpha;y))+\beta u(\frac{\varepsilon}{\pi}s^*(\tau,\alpha;y))\} \quad (8) \end{aligned}$$

where $0 \leq \gamma^*(\tau,\alpha;y) \leq 1$ and $s^*(\tau,\alpha;y) > 0$ when $\alpha > \varepsilon$. The first order condition with respect to τ,

$$\frac{\partial V(\tau,\alpha;y)}{\partial \tau} = \gamma^*(\tau,\alpha;y)y\big[\alpha(1-\beta)u'(d_N)-u'(c)\big]+\pi u'(d_N)(1-\beta)\frac{db(\tau,\alpha)}{d\tau} \quad (9)$$

is always positive for all $\tau \geq 0$. Indeed, the second term accounts for the impact of increasing the tax rate on the uniform pension benefit and it is always positive as we showed in the last section. It represents the impact of τ on the redistributive term of the pension benefit. The first term accounts for the impact of increasing the tax rate on the agent's utility, through contributions paid in the first period and the benefits linked to these contributions, obtained in the second period. For every agent with $y \leq \hat{y}(\tau,\alpha)$, $\gamma^*(\tau,\alpha;y) = 0$ so that τ has no direct impact on their utility and this term is nil. This term is also nil for agents with $\hat{y}(\tau,\alpha) < y \leq \breve{y}(\tau,\alpha)$ since for them, $\gamma^*(\tau,\alpha;y) > 0$ satisfies (3) with equality, and τ and γ are perfect substitutes from their point of view. For agents with $y > \breve{y}(\tau,\alpha)$, the first term is strictly positive since for them, $\gamma^* = 1$ and $\alpha(1-\beta)u'(d_N) - u'(c) > 0$. These agents would always want a higher tax rate so as to be able to transfer more resources toward old age, through the more generous public pension system (if they could, they would report even more than their entire income).

The majority-voting equilibrium under $\alpha > \varepsilon$ is then quite straightforward. For a fixed α, there is unanimity in favour of the maximum tax rate possible, $\tau^V \to 1$.[21] Agents with $y \leq \breve{y}(\tau,\alpha)$ want to obtain the maximum of the uniform pension benefit $b(\tau,\alpha)$ while agents with $y > \breve{y}(\tau,\alpha)$ want, in addition, to transfer additional resources to the second period. Note finally that even in the extreme cas where $\alpha = 1$ and $b(\tau,\alpha) = 0$, $\tau^V \to 1$. Indeed, agents with $y \leq \breve{y}(\tau,\alpha)$ would be indifferent to the level of τ but, still those with $y > \breve{y}(\tau,\alpha)$ would be in favour of the maximum tax rate to transfer resources to the old age.

Once the majority-voting tax rate is implemented, agents choose their rate of compliance. Those with $y \leq \hat{y}(\tau,\alpha)$ report nothing, $\gamma^*(\tau^V,\alpha;y) = t^*(\tau^V,\alpha;y) = 0$ and they obtain $b(\tau,\alpha)$ in the second period. Individuals with $\hat{y}(\tau,\alpha) < y \leq \breve{y}(\tau,\alpha)$, choose a positive level of compliance, $0 \leq \gamma^*(\tau^V,\alpha;y) \leq 1$ or equivalently $t^*(\tau^V,\alpha;y) = \tau^V\gamma^*(\tau^V,\alpha;y) > 0$ which increases in income. Finally, agents with $y > \breve{y}(\tau,\alpha)$ report their full income and $t^*(\tau^V,\alpha;y) = \tau^V > 0$. Agents with $y \geq \hat{y}(\tau,\alpha)$ obtain $P(\tau,\alpha;y)(> b(\tau,\alpha))$,

[21] Assuming endogenous labour supply in our model with endogenous compliance, would not change our result that

which is increasing in y. In the end, compliance to the tax system of agents with $y > \hat{y}(\tau, \alpha)$ benefits the entire population, even those agents who did not report any income, through the distribution of the lump sum benefit $b(\tau, \alpha)$.

Finally note that, for a given α, agents with $y \leq \breve{y}(\tau, \alpha)$, always obtain, at the majority-voting equilibrium, the highest possible level of utility given the policy environment, and it corresponds to the level obtained at their preferred allocation $(\gamma^*(\tau^*(\alpha; y), \alpha; y), s^*(\tau^*(\alpha; y), \alpha; y), \tau^*(\alpha; y))$.[22] To the contrary, agents with $y > \breve{y}(\tau, \alpha)$ obtain a lower level of utility since the fraction of income they can report is bounded at one.

Our results are summarized in the following proposition.

Proposition 3 *When $\alpha > \varepsilon$,*

- *Every agent is in favour of the maximum tax rate.*
- *At the majority-voting equilibrium, there is unanimity in favour of $\tau^V \to 1$.*
- *At the majority-voting equilibrium, for a given α, agents with $y \leq \breve{y}(\tau, \alpha)$ obtain the highest possible level of utility.*

5 Constitutional choice of the level of the Bismarkian factor

Like in Casamatta et al. (2000a), we assume that the Bismarkian factor, α is chosen at an "initial" constitutional stage where, behind the veil of ignorance, a (Rawlsian) social planner maximizes the utility of the agent with income y_{min}. Since, by definition, $y_{min} < \hat{y}(\tau^V, \alpha)$, the agent with minimum income does not comply and thus, receives at retirement only the flat pension benefit, $b(\tau^V, \alpha)$. The problem of the Rawlsian social planner then reduces to maximizing $b(\tau^V, \alpha) = \tau^V(1 - \alpha)E(\tilde{y})/\pi$, with τ^V being independent of α and $E(\tilde{y}) = E(\gamma^*(\tau^V, \alpha; y)y)$. Differentiating this expression with respect to α, we obtain:

$$\frac{db(\tau^V, \alpha)}{d\alpha} = (1 - \alpha)\frac{dE(\tilde{y})}{d\alpha} - E(\tilde{y}) \tag{10}$$

The optimal level of α is then determined by two (opposing) forces. On the one hand, the first term in equation (10) is related to incentives to comply with the tax system. If agents had no other choice than reporting their full income, this effect would be absent. When $dE(\tilde{y})/d\alpha > 0$, a higher Bismarkian factor makes agents report more income and, in turn, it increases the tax base, which benefits the least favoured through a higher uniform pension benefit. On the other hand, the second term in (10) accounts for the fact that increasing the degree of contributiveness implies less redistribution, which is to the detriment of low-income agents. Hence, this second (redistributive) term pushes toward a lower level of α.

We can show that the constitutional level of α is effectively greater than ε (i.e. the necessary condition to have $\gamma(\tau, \alpha; y) \geq 0 \; \forall y$ is satisfied, see Section 3). Indeed, we show in the appendix that $dE(\tilde{y})/d\alpha|_{\alpha=\varepsilon} > 0$ while $\gamma^* = 0 \; \forall y$ and $E(\tilde{y}) = 0$ at $\alpha = \varepsilon$. Hence, evaluating (10) in $\alpha = \varepsilon$, it is always strictly positive.[23] The optimal α^R is then above ε and maximizes $b(\tau^V, \alpha)$ such that:

$$(1 - \alpha^R)\frac{dE(\tilde{y})}{d\alpha} = E(\tilde{y}). \tag{11}$$

The reason why α is strictly greater than ε is related here to its indirect (positive) effect on $b(\tau, \alpha)$ through $E(\tilde{y})$ while, if compliance was fixed, α would be minimum and equal to ε. This reason (i.e. giving incentives to compliance) was absent from the existing literature on the political economy of pension systems (for instance, in models *à la* Casamatta et al. 2000a, b), which did not consider the possibility to evade contributions.

A public pension system with parameters (τ^V, α^R) will therefore provide the highest possible level of utility to agents with the lowest income since 1) at the majority-voting equilibrium, they obtain the highest possible level of utility for a given α and 2) α is here chosen so as to maximise their utility.[24] Our results are summarized in the following proposition.

Proposition 4 *At the constitutional stage,*

- *the optimal level of the Bismarkian factor is greater than the return from private savings:* $\alpha^R > \varepsilon$,
- *agents with income* y_{min} *obtain the highest possible level of utility.*

Finally, let us come back on the implications of modelling the choice of α through a constitutional stage assuming a Rawlsian planner, and show how a different political structure would have modified our results. First, α, like τ, could have instead been chosen by majority voting. Our results would not change, provided the median voter has income below $\hat{y}(\tau, \alpha)$ and thus, receives only $b(\tau, \alpha)$. In that case, he would choose α so as to maximise $b(\tau, \alpha)$ in the same way a Rawlsian planner does. If to the contrary, the median voter had an income above $\hat{y}(\tau, \alpha)$, he would report some positive fraction of income, $\gamma^*(\tau^V, \alpha; y_m) > 0$ and we would find that $\alpha^V > \alpha^R$ as a consequence the median voter willing to have contributions more tightly linked to the benefits received.[25] The pension system would then be even more contributive.

[23] It is also straightforward to see that evaluating the above expression in $\alpha = 1$, it is strictly negative.

[24] This solution (τ^V, α^R) corresponds to the one that would be chosen by a Rawlsian social planner, if he were choosing both the tax rate and the Bismarkian factor. Indeed, in our framework, he would also choose $\tau^R \rightarrow 1$ as $b(\tau, \alpha)$ is monotonically increasing in τ.

[25] To see this, differentiate the indirect utility function (8) with respect to α accounting for $\gamma^*(\tau^V, \alpha; y_m) > 0$. It is

Second, we assume a Rawlsian social objective, which can be viewed as a special case of the Utilitarian social welfare function where priority is given to the worst-off individual. In the (extreme) Rawlsian case, the redistributive motive is maximum and, in a sense, this "biases" our results toward the highest level of redistribution (i.e. the lowest level of α). In spite of this bias, we find that α should be strictly positive and greater than ε, demonstrating that pension benefits should be linked to previous contributions so as to incentivize tax compliance and in turn, foster redistribution.

6 Extension: introducing a cost of evasion

6.1 Compliance cost and individual decisions

In this section, we depart from our initial model to assume that, if the agent deviates from the social norm of reporting honestly income, he would bear a moral, psychological or reputation cost (See Gordon,1989; Traxler, 2010). Following Gordon (1989), we assume that an agent with income y now faces a linear utility cost of evasion of the following form:

$$C(\gamma, \tau; y) = \frac{\chi}{\beta} \tau y (1 - \gamma), \tag{12}$$

where $\chi \geq 0$ is a parameter representing the intensity of the moral cost supported by the agent and, $\tau y(1-\gamma)$ are the evaded payments resulting from non compliance. This cost is certain and supported in the first period, at the time the agent makes the decision to evade income, and it is increasing in evaded payments.[26] We also assume that it depends negatively on the probability of default of the pension system so as to account for the idea that the individual would be less reluctant to avoid taxation if he believed the government were less likely to default.

In the following, we assume that $\alpha > \varepsilon$ so as to make direct comparisons with our previous model with no evasion cost.[27] For given pension parameters (τ, α), problem (1) of an agent with income y is now modified, so as to account for the cost of not complying with the tax system, as follows:

$$\max_{s,\gamma} u(y(1-\gamma\tau) - s) + \pi\{(1-\beta)u(\frac{\tau}{\pi}[\alpha\gamma y + (1-\alpha)E(\tilde{y})] + \frac{\varepsilon s}{\pi}) + \beta u(\frac{\varepsilon s}{\pi})\} - \frac{\chi}{\beta}\tau y(1-\gamma) \tag{13}$$

First order conditions with respect to s and γ are:

$$\begin{aligned} \frac{\partial EU}{\partial s} &= -u'(c) + \varepsilon\{(1-\beta)u'(d_N) + \beta u'(d_D)\} = 0 && (14) \\ \frac{\partial EU}{\partial \gamma} &= y\tau[-u'(c) + \alpha(1-\beta)u'(d_N) + \frac{\chi}{\beta}] \leq 0. && (15) \end{aligned}$$

[26] This is different from Allingham and Sandmo (1972), Yitzhaki (1974), Borck (2004, 2009) and Traxler (2012) where agents face a probability of being audited and of paying expensive fines. Here, the moral cost of non compliance is certain, but uncertainty resides in whether the government is going to default or not.

[27] Note however that this is not anymore a necessary condition for $\alpha > 0$ since, depending on the level of χ, one could

The first condition is identical to our original model so that $s^*(\tau,\alpha;y) > 0$. Again, private savings are used as an insurance mechanism in case of government default in the second period. The second condition which defines $\gamma^*(\tau,\alpha;y)$ however differs from our original model by the term χ/β which accounts for the marginal utility cost of not complying with the tax system. In other words, there is now an additional marginal benefit to compliance, which corresponds to a decrease in marginal (moral) utility cost from not complying with the tax law.

Since the above two FOCs are similar to the ones of the baseline model (up to the constant term in the FOC for γ), the comparative statics of $s^*(\tau,\alpha;y) > 0$ and $0 < \gamma^*(\tau,\alpha;y) \leq 1$ are also similar. Therefore, $ds^*(\tau,\alpha;y)/dy > 0$ and $d\gamma^*(\tau,\alpha;y)/dy > 0$ so that high-income agents save more on private markets and comply more with the fiscal system than low-income ones.

As in Section 3, there exists a threshold income level, $\hat{y}_{EC}(\tau,\alpha)$ below which agents report no income. It is now implicitly defined by:

$$u'(\hat{y}_{EC} - \hat{s}_{EC}) = \alpha(1-\beta)u'(b(\tau,\alpha) + \frac{\varepsilon}{\pi}\hat{s}_{EC}) + \frac{\chi}{\beta} \tag{16}$$

with $\hat{s}_{EC} = s(\tau,\alpha;\hat{y}_{EC})$. Therefore, for agents with $y \leq \hat{y}_{EC}(\tau,\alpha)$, the marginal cost of reporting income is higher than its marginal benefit: $u'(c) > \alpha(1-\beta)u'(d_N) + \chi/\beta$ and $\gamma^* = 0$.

In the same way, we define $\breve{y}_{EC}(\tau,\alpha)$, the income threshold above which agents report full income and $\gamma^*(\tau,\alpha;y) = 1$. It is defined by (15) evaluated in $\gamma = 1$ such that

$$u'(\breve{y}_{EC}(1-\tau) - \breve{s}_{EC}) = (1-\beta)\alpha u'(\frac{\tau}{\pi}(\alpha\breve{y}_{EC} + (1-\alpha)E(\tilde{y})) + \frac{\varepsilon}{\pi}\breve{s}_{EC}) + \frac{\chi}{\beta} \tag{17}$$

with $\breve{s}_{EC} = s(\tau,\alpha;\breve{y}_{EC})$. Agents with $y > \breve{y}_{EC}(\tau,\alpha)$ would like to report even more than their full income, since they would like to use the public pension system to transfer more resources to the old age at a higher return than that of private savings. The constraint that γ^* cannot be above one implies that for them, $\alpha(1-\beta)u'(d_N) + \chi/\beta > u'(c)$.

Individuals decisions are summarized in the following proposition.

Proposition 5 *Assume a population of agents with income $y \in [y_{min}, y_{max}]$ who have the possibility to save on a private annuity market and to choose the fraction of income they wish to report so as to finance a risky public pension system. Assume also that evasion generates a moral cost and that $\alpha > \varepsilon$.*

- *Every agent chooses to save on private markets: $s^*(\tau,\alpha;y) > 0$ $\forall y$ and it is increasing in y.*

- *Agents with income $y \leq \hat{y}_{EC}(\tau,\alpha)$ do not report any income: $\gamma^*(\tau,\alpha;y) = 0$.*

- *Agents with income $\hat{y}_{EC}(\tau,\alpha) < y \leq \breve{y}_{EC}(\tau,\alpha)$ report a positive fraction of income, $0 < \gamma^*(\tau,\alpha;y) \leq$*

- *Agents with $y > \breve{y}_{EC}(\tau, \alpha)$ report all income, $\gamma^*(\tau, \alpha; y) = 1$.*

One important difference with respect to the previous section is that the introduction of a moral cost of evasion breaks up the perfect substitutability between the rate of compliance and the tax rate, from the point of view of the individual. To see this, let us rewrite the expected utility function of agents with income $\hat{y}_{EC}(\tau, \alpha) < y < \breve{y}_{EC}(\tau, \alpha)$ as a function of the effective tax rate $t = \gamma\tau$:

$$EU(t, s; y) = u((1-t)y - s) + \pi\{(1-\beta)u(\frac{1}{\pi}(\alpha t y + (1-\alpha)E(ty)) + \frac{\varepsilon}{\pi}s) + \beta u(\frac{\varepsilon}{\pi}s)\} - \frac{\chi y(\tau - t)}{\beta}$$

Even after the change in variable, the tax rate τ appears explicitly in the expected utility of the agent, through the cost of compliance so that the agent will not be able anymore to perfectly adapt his level of γ to a variation of τ so as to keep his expected utility constant.

We now turn to studying individual preferences for the tax rate as well as the majority-voting equilibrium.

6.2 Preference for the tax rate and the majority-voting equilibrium.

6.2.1 Individuals' preferred tax rate

The indirect utility function of an agent with income y writes:

$$\begin{aligned} V(\tau, \alpha; y) &= u(y(1 - \gamma^*(\tau, \alpha; y)\tau) - s^*(\tau, \alpha; y)) \\ &+ \pi\{(1-\beta)u(\frac{\tau}{\pi}[\alpha\gamma^*(\tau, \alpha; y)y + (1-\alpha)E(\tilde{y})] + \frac{\varepsilon s^*(\tau, \alpha; y)}{\pi}) + \beta u(\frac{\varepsilon s^*(\tau, \alpha; y)}{\pi})\} - \frac{\chi \tau y(1 - \gamma^*(\tau, \alpha; y))}{\beta} \end{aligned}$$

where $0 \leq \gamma^*(\tau, \alpha; y) \leq 1 \, \forall y$ and $s^*(\tau, \alpha; y) > 0$ have been characterized in the previous section.

In the following, we derive tax rate preferences of individuals depending on their income level. Differentiating $V(\tau, \alpha; y)$ with respect to τ, we obtain[28]

$$\begin{aligned} \frac{\partial V(\tau, \alpha; y)}{\partial \tau} &= -u'(c)y\gamma^*(\tau, \alpha; y) \\ &+ \pi(1-\beta)u'(d_N)\left[\frac{\alpha\gamma^*(\tau, \alpha; y)y}{\pi} + \frac{db(\tau, \alpha)}{d\tau}\right] - \frac{\chi y(1 - \gamma^*(\tau, \alpha; y))}{\beta} \leq 0. \end{aligned}$$

where we assume that we are on the increasing part of the Laffer curve and $db(\tau, \alpha)/d\tau > 0$. This expression differs from our baseline equation (9) by the last term in the above expression which accounts for the variation of the moral cost of evading income with respect to the tax rate. This term therefore decreases the preference for a positive tax rate, everything else equal, since the moral cost of evasion depends on evaded payments $\gamma^* y\tau$ which increase with the tax rate τ.

[28] We use the envelop theorem for $s^*(\tau, \alpha; y)$ and $\gamma^*(\tau, \alpha; y)$ when the solution is interior. When $\gamma^*(\tau, \alpha; y) = 0$ or 1, the envelop theorem does not apply, but obviously, $\gamma^*(\tau, \alpha; y)$ does not vary with τ so that the FOC with respect to τ is identical in all cases. We assume that the second-order condition holds.

Rearranging terms in the above expression such that

$$\frac{\partial V(\tau,\alpha;y)}{\partial\tau} = y\gamma^*(\tau,\alpha;y)[-u'(c)+(1-\beta)\alpha u'(d_N)+\frac{\chi}{\beta}]+\pi(1-\beta)u'(d_N)\frac{db(\tau,\alpha)}{d\tau}-\frac{\chi y}{\beta}\leq 0, \tag{18}$$

we now find individuals preferred tax rates depending on whether their income level is above or below $\hat{y}_{EC}(\tau,\alpha)$ and $\breve{y}_{EC}(\tau,\alpha)$. For agents with $y \leq \hat{y}_{EC}(\tau,\alpha)$ who only save on private markets and have $\gamma^*(\tau,\alpha)=0$, the above FOC simplifies to:

$$\frac{\partial V(\tau,\alpha;y)}{\partial\tau} = \pi(1-\beta)u'(d_N)\frac{db(\tau,\alpha)}{d\tau}-\frac{\chi y}{\beta}\leq 0 \tag{19}$$

where $d_N = b(\tau,\alpha)+\varepsilon s^*/\pi$ since, for them, the contributive part of the pension benefit is null. The first term represents the marginal utility obtained from second-period consumption while the second term represents the marginal moral cost following an increase in taxation when the agent evades all his income. Note that if $\chi=0$, there is no cost of evasion and we are back to the baseline scenario. The above expression is always positive so that every agent within this income interval prefers the maximum tax rate. To the opposite, for $\chi>0$, the individual's preferred tax rate is decreasing in income.[29] Hence, within this income interval $y\in[y_{min},\hat{y}_{EC}(\tau,\alpha)]$, agents with a smaller income prefer a higher tax rate than those with higher income. Agents at the very bottom of the distribution prefer a positive tax rate; in their case, the marginal utility from second-period consumption dominates the marginal cost of taxation (since their income is very low). But, as income increases within the interval $y\in[y_{min},\hat{y}_{EC}(\tau,\alpha)]$, marginal utility from second-period consumption d_N decreases (saving increase) while the marginal moral cost of evading increases so that the preferred tax rate, $\tau^*(\alpha;y)$ decreases.

Let us now derive the preferred tax rate of agents with income $y\in[\hat{y}_{EC}(\tau,\alpha),\breve{y}_{EC}(\tau,\alpha)]$ for whom $0<\gamma^*\leq 1$. Replacing for (15) in (18), we obtain the same expression as (19), except that $\gamma^*(\alpha,\tau)>0$ for these agents so that second-period consumption d_N now includes the contributive part of the pension benefit.[30] Similarly, $\tau^*(\alpha;y)$ is decreasing in y within the interval $y\in[\hat{y}_{EC}(\tau,\alpha),\breve{y}_{EC}(\tau,\alpha)]$, but the slope is steeper.[31]

Finally, agents with income above $\breve{y}_{EC}(\tau,\alpha)$ have $\gamma^*(\tau,\alpha;y)=1$. For them, (18) simplifies to

$$\frac{\partial V(\tau,\alpha;y)}{\partial\tau} = y[-u'(c)+u'(d_N)(1-\beta)\alpha]+u'(d_N)(1-\beta)\pi\frac{db(\tau,\alpha)}{d\tau}\leq 0 \tag{20}$$

This formula is similar to Casamatta et al. (2000b) who study the political economy of pension systems when evasion is not possible (i.e. $\gamma=1$ always). The above expression shows that increasing the tax rate

[29] Using the implicit function theorem on (19) and recalling that $ds^*(\tau,\alpha;y)/dy>0\forall y$, we find that $d\tau^*(\alpha;y)/dy<0$.
[30] It is straightforward to show that $\tau^*(y)$ is continuous in $\hat{y}_{EC}(\tau,\alpha)$.
[31] To see this, we apply the implicit function theorem on (19). Note that, beyond some income level and depending on the

creates on the one hand a direct utility cost in terms of reduced first-period consumption but, on the other hand, a marginal utility benefit, due to increased pension benefits in the second period, through both the contributive part (second term above) and the uniform part (third term). We show in Appendix B that the variation of $\tau^*(y)$ with y for these agents may now be ambiguous, in particular because the above expression includes an additional marginal utility benefit term, $u'(d_N)(1-\beta)\alpha y$ which increases in y under Assumption 1, while the first and third terms decrease in y.[32] Nonetheless, we prove that agents with income $y \to \breve{y}_{EC}^+$ prefer a higher tax rate than $\tau(\breve{y}_{EC})$, and assuming monotonicity in $\tau^*(\alpha; y)$ for $y > \breve{y}_{EC}$, we find that for these agents, the preferred tax rate is increasing in income. Hence, agents with higher incomes most-prefer higher values of the tax rate, which is in line with Casamatta et al. (2000b). Agents with income above $\breve{y}_{EC}(\tau, \alpha)$ would like to transfer more resources to the future than what they actually can: they already fully comply and cannot report more than their true income level. The best way to do so is through the pension system which provides a higher return than private savings ($\alpha > \varepsilon$). This explains why they are in favour of higher tax rates as their income increases.

6.2.2 Majority-voting equilibrium

One of the difficulties in characterizing the voting equilibrium resides in that individuals' levels of compliance shape their preference for the tax rate differently so that one cannot use directly the single-crossing condition established by Gans and Smart (1996).

Using the methodology developed by Epple and Romano (1996), we show in Appendix C that, for agents with $y \leq \breve{y}_{EC}(\tau, \alpha)$, the marginal rates of substitution between τ and $b(\tau, \alpha)$ are monotonically increasing in y. This is also the case for agents with $y > \breve{y}_{EC}(\tau, \alpha)$ provided that, for them, the condition that $R_r(d_D)\varepsilon_{d_D,y} < R_r(d_N)\varepsilon_{d_N,y}$, where $\varepsilon_{d,y}$ is the elasticity of second-period consumption to income, holds.[33] Therefore, when that condition is satisfied, every marginal rate of substitution between τ and b varies monotonically in the same direction (i.e. the single-crossing property is satisfied) so that a global Condorcet winner exists, $0 \leq \tau^V < 1$ and it corresponds to the tax rate preferred by a majority of agents.

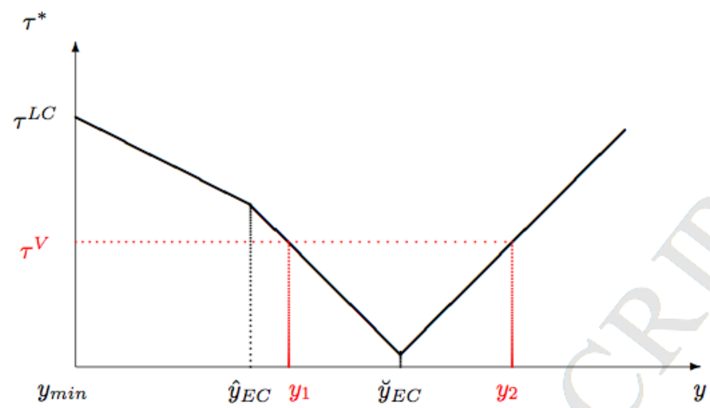
In order to understand better how the majority-voting equilibrium tax rate is obtained, we graph the individuals' preferred tax rate as a function of y in Figure 2.[34] As it is clear from that figure, we have a case of ends-against-the-middle equilibrium where low-income earners together with high-income earners oppose middle-income ones who would prefer smaller tax rates. Indeed, low-income earners are in favour

[32]Evaluating both (19) and (20) (where we substituted for (17)) in $\breve{y}_{EC}(\tau, \alpha)$, we obtain the same expressions so that $\tau^*(y)$ is continuous in $\breve{y}_{EC}(\tau, \alpha)$.

[33]Recall that under Assumption 1, $R_r(d_D) < R_r(d_N)$. If the above condition is not satisfied, a Condorcet winner may not exist and we may not have a stable voting equilibrium. For the reminder of the paper, we assume that this condition is satisfied so that a majority-voting equilibrium τ^V exists.

[34]Note that this graph is done for illustration purposes. Depending on the value of the parameters (for instance the value of χ), it may the case that around $\breve{y}_{EC}$, the preferred tax rate is zero (see footnote 31). Also, we may not have straight

of high tax rates so as to obtain a higher flat pension benefit. High-income earners also want high tax rates as they wish to transfer a greater proportion of their income to the second-period.



Note: τ^{LC} corresponds to the tax rate that maximises the Laffer curve.

Figure 2: Individuals' preferred tax rates $\tau^*(y)$ as a function of their income, y.

Therefore, looking at Figure 2, the voting equilibrium τ^V is such that, at this level, the population is divided in two halves. Half of the population would prefer a strictly higher tax rate, that is a mass $F(y_1)+(1-F(y_2))$ of low- and high-income agents, while the other half would prefer a lower tax rate than τ^V and is constituted of agents with middle income, with mass $F(y_2) - F(y_1)$.[35] The majority-voting equilibrium is formally defined in the following proposition:

Proposition 6 *Assume that $\alpha > \varepsilon$ and $R_r(d_D)\varepsilon_{d_D,y} < R_r(d_N)\varepsilon_{d_N,y}$ for individuals with $y > \breve{y}_{EC}(\tau, \alpha)$. The majority voting tax rate $\tau^V(\alpha)$ is such that*

$$\tau^V(\alpha) = \tau^*(\alpha; y_1) = \tau^*(\alpha; y_2)$$

where $\tau^*(\alpha; y_1)$ is the solution to (19) evaluated at y_1 and $\tau^*(\alpha; y_2)$ is the solution to (20) evaluated at y_2 and, where y_1 and y_2 satisfy jointly the following equation

$$F(y_2) - F(y_1) = 1/2.$$

6.3 Constitutional choice of the level of the Bismarkian factor

As in Section 5, the problem of the Rawlsian social planner consists in maximizing the utility of the individual with income y_{min} (recall that this agent does not report any income):

$$\begin{aligned}\max_{\varepsilon<\alpha\leq 1} V(\tau^V(\alpha), \alpha; y_{min}) &= u(y_{min} - s^*) \\ &+ \pi\{(1-\beta)u(\frac{\tau^V(\alpha)}{\pi}(1-\alpha)E(\tilde{y})) + \frac{\varepsilon}{\pi}s^*) + \beta u(\frac{\varepsilon}{\pi}s^*)\} - \frac{\chi\tau^V(\alpha)y}{\beta}\end{aligned}$$

where $\tau^V(\alpha)$ is defined in Proposition 6. Making use of the envelop theorem for $s^*(\tau^V, \alpha; y_{min})$, the first order condition of this problem is:

$$\frac{dV(\tau^V(\alpha), \alpha; y_{min})}{d\alpha} = \frac{\partial V(\tau^V(\alpha), \alpha; y_{min})}{\partial \alpha} + \frac{\partial V(\tau^V(\alpha), \alpha; y_{min})}{\partial \tau} \frac{d\tau^V(\alpha)}{d\alpha} \leq 0 \tag{21}$$

where

$$\frac{\partial V(\tau^V(\alpha), \alpha; y_{min})}{\partial \alpha} = (1-\beta)u'(d_N)\tau^V(\alpha)\left[(1-\alpha)\frac{dE(\tilde{y})}{d\alpha} - E(\tilde{y})\right] \tag{22}$$

is identical to equation (10) and represents the direct impact of increasing the Bismarkian factor on the utility of the poorest agent, through the flat pension benefit. Like in Section 5, evaluating equation (22) in $\alpha = \varepsilon$ and $\alpha = 1$, we find that the optimal level of α should lie within this interval.[36] This is the result of two opposite effects. On the one hand, a higher Bismarkian factor makes agents report more income, increases the tax base and therefore the uniform pension benefit. On the other hand, increasing the degree of contributiveness implies less redistribution, which is to the detriment of low-income agents and thus, pushes toward a lower level of α. Therefore, like in the absence of evasion costs and, contrary to standard political economy models of pension systems where agents have no other choice than reporting their true income, the fact that compliance depends on the Bismarkian factor, makes the social planner willing to have a higher α than if compliance was fixed. Again, linking contributions to pension benefits through a positive α is a way to increase compliance and, thus the utility of the worst-off agent.

The second term in (21) represents, like in Casamatta et al. (2000a), how the level of the Bismarkian factor affects the political support for the pension system, through $\tau^V(\alpha)$, and, in turn the utility of the poorest agent. This term was absent from the government problem in Section 5 as τ^V was independent of α. The expression $\partial V(\tau^V(\alpha), \alpha; y_{min})/\partial\tau$ is equal to (19) evaluated at $(\tau^V(\alpha), y_{min})$ and is positive. Indeed, as shown in Section 6.2.1, the preferred tax rate of the agents with income below $\hat{y}_{EC}(\tau, \alpha)$ is decreasing in income so that the agent with y_{min} prefers a higher tax rate than the one chosen at the majority-voting equilibrium. Finally, finding whether $\tau^V(\alpha)$ increases or decreases with α is rather difficult to prove as one would need to fully differentiate (19) evaluated in y_1 or (20) evaluated in y_2 with respect to α and to know the sign of $d^2E(\tilde{y})/d\tau d\alpha$.

Hence, to summarize this section, the level of α then depends on the combination of the different effects described above, i.e. redistribution, tax base and political support. There is a priori no reason to believe that it should be either minimum at $\alpha^R = \varepsilon$, or maximum (fully contributive) at $\alpha^R = 1$ so that a pension system should be made partly contributive.

[36] The proof is identical to that of Appendix A.

7 Conclusion

In standard models of income taxation, tax authorities observe the individuals' wage rate or at least total earnings. If tax evasion is possible, then an audit technology allows them to recoup a large chunk of tax revenue. Common knowledge of earnings and effective tax enforcement are in general characteristics of advanced economies. In many less advanced economies, informality and self-employment make it more difficult to enforce a workable tax system, which explains the low level of tax revenue observed there, particularly when the tax base is individual income. This paper shows that, even under those circumstances, public resources mobilization is possible and politically sustainable if taxes are at least partially related to benefits and if the government supplies services for which there are no (good) substitutes in private markets. This is precisely the case of retirement saving whose market returns are often not attractive because of the absence of annuity markets and of the high loading costs charged by financial institutions.

To do so, we study the political sustainability of a pension system that has three main features: it is financed by a payroll tax whose base is not observable; it provides benefits that are partly contribution-related and the marginal return of social contributions is higher than the financial interest rate. It appears that, even in the extreme case where evasion can be done at no cost, individuals are willing to report income. Interestingly, individuals are indifferent to the level of the tax rate as they can perfectly adapt their rate of compliance to any tax variation. As a result, there is unanimity in favour of the maximum tax rate possible and, the public pension system is found to be partially contributory in order to foster income reporting. A (partially) Bismarkian pension system therefore increases the tax base and the amount of resources that can be redistributed. Agents at the bottom of the income distribution obtain maximum utility.

When we introduce a compliance cost as in Gordon (1989), our main results are preserved. Even though perfect substitutability between the compliance rate and the tax rate breaks down so that now, individuals' preferred tax rate depends on income, we obtain that contributions and benefits should still be linked in order to foster compliance and to increase intra-generational redistribution. In addition, it may increase the political support for the pension system.

All in all, the policy implication of this paper is rather obvious. Linking (even imperfectly) individual contributions to individual benefits is enough to design a tax system with sufficient political support, to increase tax compliance and ultimately, to increase the welfare of the poorest individuals.

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A Variation of s and γ with α

Replacing for equation (3) in equation (2), we rewrite the FOC with respect to s as follows:

$$\frac{\partial EU}{\partial s} = \frac{\varepsilon - \alpha}{\alpha} u'(c) + \beta \varepsilon u'(d_D) = 0$$

Differentiating this expression with respect to α and evaluating it in $\alpha \to \varepsilon$, we obtain that $\partial EU^2/\partial s \partial \alpha|_{\alpha \to \varepsilon} = -\varepsilon u'(c)/\alpha^2 < 0$ so that using the implicit function theorem, $ds/d\alpha|_{\alpha \to \varepsilon} < 0$.

Using equation (2), we rewrite (3) as follows:

$$\frac{\partial EU}{\partial \gamma} = \frac{\alpha - \varepsilon}{\varepsilon} u'(c) - \beta \alpha u'(d_D) = 0$$

Differentiating it with respect to α and evaluating it in $\alpha \to \varepsilon$, we obtain

$$\frac{\partial EU^2}{\partial \gamma \partial \alpha} = [\frac{u'(c)}{\varepsilon} - \beta u'(d_D)] - \frac{\beta \varepsilon \alpha}{\pi} u''(d_D) \frac{ds}{d\alpha}|_{\alpha \to \varepsilon}$$

Using equation (2), the first term above is positive while the second one is negative. Assuming that second-order effects are small (i.e. the second term) and applying the implicit function theorem, we can therefore conclude that $d\gamma/d\alpha|_{\alpha \to \varepsilon} > 0$. This implies that $dE(\tilde{y})/d\alpha|_{\alpha \to \varepsilon} > 0$.

B Preferences over τ of individuals with $y > \breve{y}_{EC}(\tau, \alpha)$

Differentiating (20) with respect to y, we obtain after rearranging terms, the following expression:

$$\begin{aligned} \frac{\partial^2 V(\tau, \alpha; y)}{\partial \tau \partial y} &= -u'(c)[1 - R_r(c)\frac{dc}{dy}\frac{y}{c}] + \alpha(1-\beta)u'(d_N)[1 - R_r(d_N)\frac{dd_N}{dy}\frac{y}{d_N}] \\ &+ (1-\beta)u''(d_N)\pi \frac{db(\tau, \alpha)}{d\tau}\frac{dd_N}{dy} \end{aligned}$$

which may be positive or negative, so that the preferred tax rate for agents with $y > \breve{y}_{EC}$ may be increasing or decreasing in y.

Let us define $\breve{\tau}$ as the preferred tax rate of an agent with income $\breve{y}_{EC}$. It is implicitly defined by equation (19) evaluated in $\breve{y}_{EC}$ and thus, such that

$$\frac{\chi \breve{y}_{EC}}{\beta} = (1-\beta)\pi u'(\breve{d}_N)\frac{db(\breve{\tau}, \alpha)}{d\tau}.$$

Evaluating (20) in $\breve{\tau}$, we obtain that for $y \to \breve{y}_{EC}^{+}$,

$$\frac{\partial V}{\partial \tau}|_{\breve{\tau}, y \to \breve{y}_{EC}^{+}} = y[u'(d_N)(1-\beta)\alpha + \frac{\chi}{\beta} - u'(c)] + \frac{\chi}{\beta}(\breve{y}_{EC} - y)$$

where the last term tends to 0. As for agents with $y > \breve{y}_{EC}^{+}$, γ^* is constrained to 1 and $u'(d_N)(1-\beta)\alpha + \chi/\beta - u'(c) > 0$ so that the above expression is strictly positive. Therefore, agents with $y > \breve{y}_{EC}^{+}$ prefer a strictly greater tax rate than $\breve{\tau}$. Assuming monotonicity of $\tau^*(\alpha; y)$ on the interval $[\breve{y}_{EC}(\tau, \alpha), y_{max}]$,

C Single crossing condition with evasion costs

Let us define the intermediate indirect utility function of an agent with income y as

$$\begin{aligned} I(\tau,\alpha;y) &= u(y(1-\gamma^*(\tau,\alpha;y)\tau) - s^*(\tau,\alpha;y)) \\ &+ \pi\{(1-\beta)u(\frac{\tau}{\pi}\alpha\gamma^*(\tau,\alpha;y)y + b(\tau,\alpha) + \frac{\varepsilon s^*(\tau,\alpha;y)}{\pi}) + \beta u(\frac{\varepsilon s^*(\tau,\alpha;y)}{\pi})\} - \frac{\chi\tau y(1-\gamma^*(\tau,\alpha;y))}{\beta} \end{aligned}$$

where $b(\tau,\alpha) = \tau(1-\alpha)E(\tilde{y})/\pi$. The marginal rate of substitution between τ and b has the following general form

$$MRS(\tau,b) = -\frac{\partial I/\partial\tau}{\partial I/\partial b} = -\frac{\gamma^*(\tau,\alpha;y)y[-u'(c) + \alpha(1-\beta)u'(d_N)] - \frac{\chi y(1-\gamma^*(\tau,\alpha;y))}{\beta}}{\pi(1-\beta)u'(d_N)} \ \forall y.$$

For agents $y \leq \hat{y}_{EC}(\tau,\alpha)$, it simplifies to

$$MRS(\tau,b)|_{y\leq\hat{y}_{EC}(\tau,\alpha)} = \frac{\chi y}{\pi\beta(1-\beta)u'(d_N)} > 0 \tag{23}$$

where we replaced for $\gamma^* = 0$. Since $dd_N/dy > 0$, $MRS(\tau,b)$ is increasing in y for agents $y \leq \hat{y}_{EC}(\tau,\alpha)$.

For agents with $\hat{y}_{EC}(\tau,\alpha) < y \leq \breve{y}_{EC}(\tau,\alpha)$, $MRS(\tau,b)$ reduces to the same expression as in (23). The only difference resides in the expression of d_N which now includes the contributive part of pension benefit, $\tau\alpha\gamma^* y/\pi$. As before, we find that $MRS(\tau,b)$ in increasing in y.

Note also that for given (τ,b), the $MRS(\tau,b)$ of agents with $\hat{y}_{EC}(\tau,\alpha) < y \leq \breve{y}_{EC}(\tau,\alpha)$ is bigger than $MRS(\tau,b)$ of agents with $y \leq \hat{y}_{EC}(\tau,\alpha)$. Hence in the space (τ,b), the slope of the indifference curve of agents with $\hat{y}_{EC}(\tau,\alpha) < y \leq \breve{y}_{EC}(\tau,\alpha)$ is steeper than that of agents with $y \leq \hat{y}_{EC}(\tau,\alpha)$

For agents with $y > \breve{y}_{EC}(\tau,\alpha)$, we have

$$MRS(\tau,b) = -\frac{\partial I/\partial\tau}{\partial I/\partial b} = -\frac{y[-u'(c) + \alpha(1-\beta)u'(d_N)]}{\pi(1-\beta)u'(d_N)} \tag{24}$$

First note that condition (20) can be rewritten as

$$\frac{\partial V(\tau,\alpha;y)}{\partial\tau} = \frac{\partial I}{\partial\tau} + u'(d_N)(1-\beta)\pi\frac{db(\tau,\alpha)}{d\tau} \leq 0 \tag{25}$$

so that $\partial I/\partial\tau < 0$ so as to have an interior solution for $\tau^*(\alpha;y)$. This also implies that $MRS(\tau,b) > 0$ of agents with $y > \hat{y}_{EC}(\tau,\alpha)$. In addition, since $\tau^*(\alpha;y)$ is increasing in y, we have, using the implicit function theorem on the above condition that necessarily $\partial^2 I/\partial\tau\partial y > 0$ (since the last term in eq. 25 decreases with y). We also have that $\partial^2 I/\partial b\partial y = \pi(1-\beta)u''(d_N)dd_N/dy < 0$.

Let us now see how the marginal rate of substitution varies with y:

$$\frac{\partial MRS(\tau,b)}{\partial y} = -\frac{\frac{\partial^2 I}{\partial\tau\partial y}\frac{\partial I}{\partial b} - \frac{\partial^2 I}{\partial b\partial y}\frac{\partial I}{\partial\tau}}{(\frac{\partial I}{\partial b})^2} \tag{26}$$

where

$$\begin{aligned}\frac{\partial I}{\partial \tau} \quad &= y[-u'(c) + \alpha(1-\beta)u'(d_N)] = y[-u'(c) + \frac{\alpha}{\pi}\frac{\partial I}{\partial b}] \\ &\Rightarrow \frac{\partial I^2}{\partial \tau \partial y} = y[-u''(c)\frac{dc}{dy} + \frac{\alpha}{\pi}\frac{\partial^2 I}{\partial b \partial y}] + \frac{\partial I}{\partial \tau}\frac{1}{y}\end{aligned}$$

Replacing for these expressions in the numerator of (26) and rearranging terms, we find that a sufficient condition for $MRS(\tau, b)$ to be monotonically increasing in y is

$$-\frac{u''(c)}{u'(c)}\frac{dc}{dy} < -\frac{u''(d_N)}{u'(d_N)}\frac{dd_N}{dy}$$

Fully differentiating (14) with respect to y and replacing into the above inequality, we find, after rearranging terms, that the above condition is equivalent to $R_r(d_D)\varepsilon_{d_D,y} < R_r(d_N)\varepsilon_{d_N,y}$ with $\varepsilon_{d_i,y}$, the elasticity of consumption in state $i = \{D, N\}$ to income. Under Assumption 1, $R_r(d_D) < R_r(d_N)$ since $d_D < d_N$.