Perturbation finite element method for the analysis of earthing systems with vertical and horizontal rods

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Abstract — **This paper deals with the electrokinetic modeling of earthing systems by means of a sub-domain perturbation finite element technique. An axisymmetric problem is solved for each single grounding rod or cable. Its solution must then be corrected by taking into account the influence of the other rods and cables. The electric scalar potential is transferred from one problem to the other through projections between meshes. An inherently 3D problem can thus be solved as a succession of 2D sub-problems, which significantly speeds up the solution and enables to tackle complicated grounding systems.**

I. INTRODUCTION

Earthing systems aim at reducing the grounding resistance so as to protect low voltage equipment and personnel from the dangerous ground potential due to dissipation of fault currents or lightning discharge into the ground [1]. They generally comprise several vertical rods in parallel, interconnected by buried cables. Analytical formulas are since long available in the literature for simple configurations [2]. The analysis of more complicated arrangements demands most likely 3D numerical methods. The finite element (FE) method is well suited for tackling this kind of problem. However, it may become extremely expensive due to the required dense discretization in the vicinity of the rods [3].

The perturbation FE approach allows to overcome this drawback. It has already shown to be clearly advantageous in repetitive analyses [4]. This technique takes advantage of previous computations instead of solving a completely new FE problem for any variation of geometrical or physical characteristics. Further, different problem-adapted meshes are allowed and computational efficiency is clear due to the reduced size or each sub-problem.

In [5], a perturbation FE method was proposed for calculating the resistance in earthing systems consisting of vertical grounding rods. In this paper, the method is further developed for considering systems that comprise also horizontal cables connecting these vertical grounding.

Each vertical grounding rod or horizontal cable is defined in an independent axisymmetric domain and mesh and an electrokinetic sub-problem solved. Further, for each horizontal cable, its image must also be included as an additional axisymmetric sub-problem. The solution of each of these sub-problems must then be corrected and adapted to account for the effect of the other vertical rods, horizontal cables or their images. The electric scalar potential is transferred from one problem to the other through projections between their meshes. The successive solution of 2D axisymmetric sub-problems allows thus to solve a typically 3D problem.

II. ELECTROKINETIC MODELING

An electrokinetic problem p is defined in a domain $\Omega_p = \Omega_{c,p} \cup \Omega_{c,p}^C$ with conducting part $\Omega_{c,p}$, non-conducting part $\Omega_{c,p}^C$ and boundary $\Gamma_p = \Gamma_{e,p} \cup \Gamma_{j,p}$ (possibly at infinity). The equations, material relations and boundary conditions (BCs) characterizing the problem p in Ω_p are:

$$\operatorname{curl} \boldsymbol{e}_p = 0\,, \quad \operatorname{div} \boldsymbol{j}_p = 0\,, \quad \boldsymbol{j}_p = \sigma_p\, \boldsymbol{e}_p\,, \qquad (1\,\text{a-c})$$

$$\boldsymbol{n} \times \boldsymbol{e}|_{\Gamma_{e,p}} = 0\,, \quad \boldsymbol{n} \cdot \boldsymbol{j}|_{\Gamma_{j,p}} = 0\,, \qquad (1\,\text{d e})$$

with $\boldsymbol{e}_p$ the electric field, $\boldsymbol{j}_p$ the electric current density, σ_p the electric conductivity, $\boldsymbol{n}$ the unit normal exterior to Ω_p.

A. *Perturbation problems*

A modification of an initial problem due to a change of conductivity and/or an addition of sources in some regions leads to the perturbation of fields. In earthing systems, the perturbing regions are the additional rods/cables that influence the initial electric field distribution.

The perturbation FE method consists thus in determining the solution v_p, $\boldsymbol{e}_p$, $\boldsymbol{j}_p$ of P successive sub-problems $p = 1, \ldots, P$, the sum of which being the solution of the complete problem v, $\boldsymbol{e}$, $\boldsymbol{j}$. At the discrete level, independent meshes are used for all sub-problems p. Further, such a superposition of solutions allows each sub-problem to satisfy constraints and relations that are not shared by the complete problem. Consequently, as each sub-problem is generally perturbed by all the others, each solution v_p has to be calculated as a series of corrections, i.e. $v_p = v_{p,1} + v_{p,2} + \cdots$. The calculation of the corrections $v_{p,i}$ in a problem (p, i) is kept on till convergence up to a desired accuracy. Each correction $v_{p,i}$ must account for the influence of all the previous corrections $v_{q,j}$ of the other sub-problems, with $q = 1, \cdots, p-1, j = i$ and $q = p+1, \cdots, P, j = i-1$. Further, initial solutions $v_{p,0}$ are set to zero.

In our case, the added region $\Omega_{c,p}$ is a perfect conductor. This allows to determine the source of each perturbation problem (p, i) by taking into account that the total electric field must be zero in $\Omega_{c,p}$, i.e. $\boldsymbol{e}|_{\Omega_{c,p}} = 0$. This source can be written in terms of the electric scalar potential $v_{p,i}$ [5].

For vertical rods, each $\boldsymbol{j}_{p,i}$ verifies automatically BC (1 e) which also holds for the complete $\boldsymbol{j}$ (thanks to the principle of superposition). For orientations other than vertical, BC (1 e) is corrected, e.g. by applying image theory. For instance, when dealing with a horizontal cable, a fictitious additional cable is considered at a distance equal to twice the depth at which the real cable is buried.

B. *Weak Finite Element Formulation*

The electric scalar potential formulation of the electrokinetic problem p (1) is given by

$$(\sigma_p \operatorname{grad} v_p, \operatorname{grad} v')_{\Omega_p} + \langle \boldsymbol{n} \cdot \boldsymbol{j}_p, v' \rangle_{\Gamma_p} = 0\,, \qquad (2)$$

where $(\cdot,\cdot)_{\Omega_p}$ and $\langle \cdot,\cdot \rangle_{\Gamma_p}$ denote, respectively, a volume integral in Ω_p and a surface integral on Γ_p of the scalar product of their arguments.

This formulation is valid for any correction $v_{p,i}$ involved in the iterative process with the associated BC on $\Gamma_{c,p}$ strongly

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defined. Each solution $_{p,i}$ leads to a correction of the current and consequently of the resistance to ground of $\Omega_{c,p}$.

C. Projection of sources

Each grounding rod or cable (real or fictitious) is modeled by an axisymmetric and independent mesh. Consequently, each source scalar potential $v_{q,j}$ initially interpolated in the mesh of problem q has to be transferred to the mesh of problem p. This is done via a projection method [5]. Furthermore, as each mesh is defined in its own coordinate system, a geometrical transformation is required in the projection process. For a set of problems with vertical rods, this transformation is just a translation. When dealing with horizontal cables/rods, a rotation is also required. In case of a non-perfectly conducting perturbing region, the projection should be extended to the whole domain $\Omega_{c,p}$.

Note that when dealing with identical rods/cables, the working mesh is in fact the same for all of them.

III. APPLICATION EXAMPLE

First, we consider a cable (section 50 mm, length L between 1–25 m) buried at $h = 1$ m depth in a homogeneous soil (resistivity $\rho = 100\,\Omega$ m), parallel to the ground surface and subjected to a given voltage V. The following analytical expression is used to estimate the resistance to ground of such a case [2]:

$$R = \frac{\rho}{2\pi L}\left(\ln\frac{2L}{r} + \ln\frac{L+\sqrt{4h^2+L}}{2h} - 1 + \frac{2h-\sqrt{4h^2+L}}{L}\right) \quad (3)$$

with r the radius of the cable.

The values of the resistance to ground of a buried cable as a function of its length obtained with (3), the perturbation FE model and a classical 3D FE model are shown in Fig. 1 (up). The perturbation method allows to use a mesh which is much finer than the mesh of the 3D FE model. This is the reason why we choose (in all test cases) to compute a relative difference taking the perturbation result as reference. The relative difference is represented in Fig. 1 (down). The electric scalar potential distribution achieved with a 3D

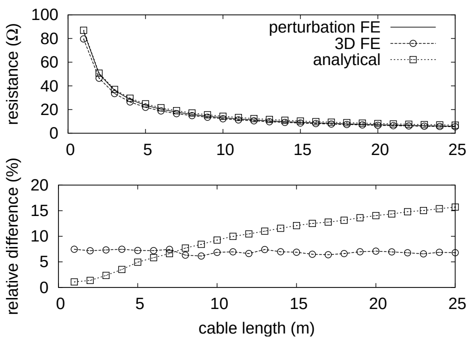

Fig. 1. Earth resistance versus length of the buried cable obtained by analytical formulas, the perturbation FE method and the 3D FE method. Relative difference (down)

model (1/4 of geometry) is depicted in Fig. 2 (left).

Our second test case comprises two vertical rods (radius $r = 1.25$ cm, length L between 1–10 m, distance $d = 2\,L$) and a horizontal buried cable at a depth of $h = 1$ m. As no analytical formula is available, results of the perturbation method are just compared to those obtained by a 3D FE model. The electric scalar potential distributions given

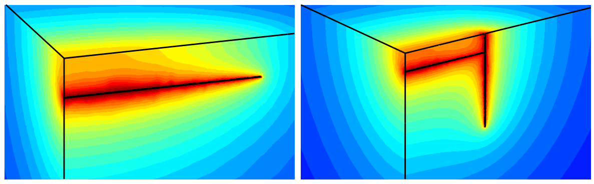

Fig. 2. Detail of the electric potential distribution calculated with a 3D FE model (1/4 of geometry): a 10 m long buried cable (left); two 5 m long rods and a 10 m long buried cable (right)

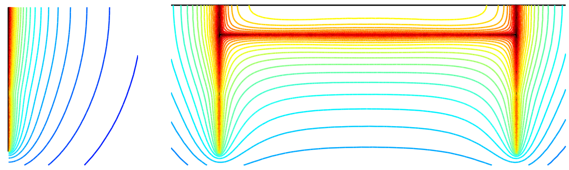

Fig. 3. Electric potential distributions given by the perturbation FE model: two 5 m long rods and a 10 m long buried cable. Axisymmetric result for vertical rod (left), projection of all results on a common mesh (right)

by the perturbation method in the three independent axisymmetric models have been projected on a common mesh and depicted in Fig. 3 (right). The axisymmetric result for a single vertical rod is shown as well in Fig. 3 (left). The 3D FE result is shown in Fig. 2 (right). The values of the resistance to ground as a function of the length of the rods obtained with the perturbation FE model and a classical 3D FE model are represented in Fig. 4 (up). The relative difference is also given in Fig. 4 (down).

The use of the perturbation FE method allows to speed up the resolution of the problem by roughly a factor 100. Further details and discussion on the results and computational cost will be given in the extended paper.

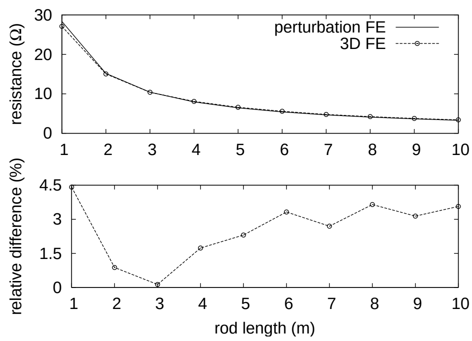

Fig. 4. Earth resistance versus length of the vertical rods obtained by the perturbation FE method and the 3D FE method. Relative difference (down)

REFERENCES

[1] IEEE Standards Board, *ANSI/IEEE Std 80-1986: An American National Standard IEEE Guide for Safety in AC Substations Grounding*. IEEE, Inc, 1986.

[2] G. F. Tagg, *Earth resistances*. George Newnes Ltd, London, Great Britain, 1964.

[3] M. Lorentzou, A. Kladas, and N. Hatziargyriou, "Finite element modelling of grounding systems considering electrode geometry effects," *IEEE Transactions on Magnetics*, vol. 35, no. 3, pp. 1757–1760, 1999.

[4] R. V. Sabariego and P. Dular, "A perturbation approach for the modelling of eddy current nondestructive testing problems with differential probes," *IEEE Transactions on Magnetics*, vol. 43, no. 4, pp. 1289–1292, April 2007.

[5] R. V. Sabariego, M. V. Ferreira da Luz, J. P. Nzuru Nsekere, P. Kuo-Peng, J. L. Lilien, and P. Dular, "Perturbation finite element method for the analysis of earthing systems with vertical rods," in *Proceedings of MOMAG2008*, Florianopolis, Santa Catarina, Brazil, September 7–10, 2008.