INVESTIGATING LUNAR 2-DIMENSIONAL TOPOGRAPHIC PROPERTIES AT DIFFERENT SPATIAL SCALES USING LUNAR ORBITER LASER ALTIMETER DATA AND THE WAVELET LEADERS METHOD

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1. Introduction

- The roughness of planetary bodies is commonly studied to identify smooth surfaces that would be the best landing sites candidates or to identify the geophysical processes that shaped these bodies.
- The Wavelet Leaders Method (WLM) is a method that allows the characterization of surface roughness both spatially and in frequency, unlike most other approaches which focus on either the former or the latter. The roughness characterization can be done in 1D using either lines of latitude or lines of longitude of data to provide information on them, or in 2D using a local spatial analysis centered on each pixel, and thus providing a more thorough analysis.
- The WLM allows the identification of (1) scaling regimes, (2) the mono- or multifractal behavior of the surface, and (3) the value of the Hölder exponent for each pixel.
- The WLM has been rarely used in a planetary science context. It has been used to characterize the roughness of Mars in 1D and in 2D using Mars Orbiter Laser Altimeter (MOLA) gridded data in [1]. It has also been used in [2] to characterize the roughness of the Moon in 1D using the Lunar Orbiter laser Altimeter (LOLA) gridded data.

2. Objectives

2.1 Main objective:

- Use the WLM to study the roughness of the Moon in 2D using gridded topographic data from LOLA.

2.2 Secondary objectives:

- Identify the different scaling regimes present (*i.e.*, at which scales or spatial resolution changes in what governs topographic processes occur),
- Determine whether the data is monofractal or multifractal,
- Determine the value of the Hölder exponent for each pixel.

3. Data

- We used gridded topographic data from LOLA that has been projected into a simple cylindrical projection (PDS3, V1.05) at 1024 ppd (or ~30 m/pixel), which is the highest spatial resolution currently available for the whole Moon. We downloaded individual tiles of 15° in latitude by 30° in longitude to obtain data for the whole globe, for a total of 368,640 by 184,320 pixels.
- The WLM uses data of size 2^x as input, so we downsampled the global dataset to 2^{18} (262,144) pixels to 2^{17} (131,072) pixels. This corresponds to a spatial resolution of 728 ppd or ~41 m/pixel.
- Every other pixel and its neighbors are compared to four 2-dimensional filters (HH, LL, HL, LH) derived from the scaling (L, or low-pass) and wavelet (H, or high-pass) components of a 3rd order Daubechies wavelet.

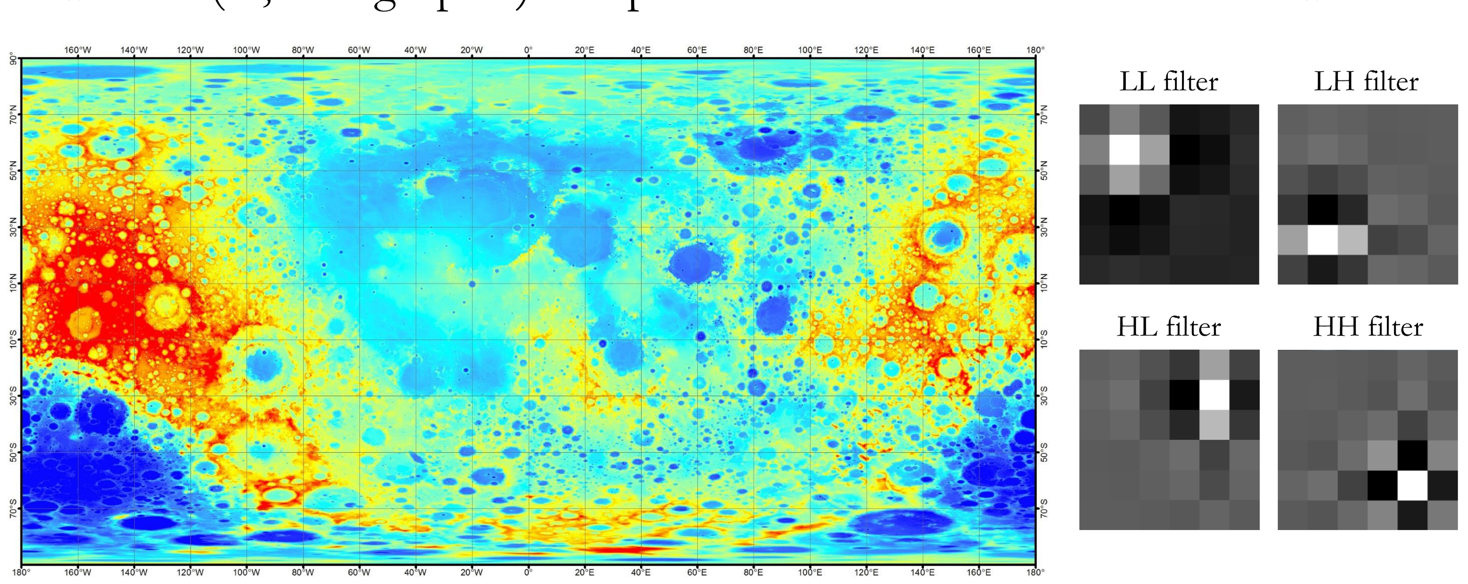


Figure 1. Every other pixel of the global gridded LOLA topographic dataset and its neighbors are compared to four 2-dimensional filters (HH, LL, HL, LH) derived from the scaling and wavelet components of a 3rd order Daubechies wavelet.

4. Method

4.1 Calculate the LL, HL, LH and HH wavelet coefficients at each scale

- We multiply every other pixel and its surrounding neighbors by the LL, LH, HL and HH wavelet filters. As only every other pixel are analyzed, the output (LL, LH, HL and HH wavelet coefficients) have half the size of the input (topographic) data. For the first scale (j), the topographic data is used as input. For the subsequent scales, the LL wavelet coefficients are used as input. This is done iteratively until there are only 2^3 by 2^2 pixels left.

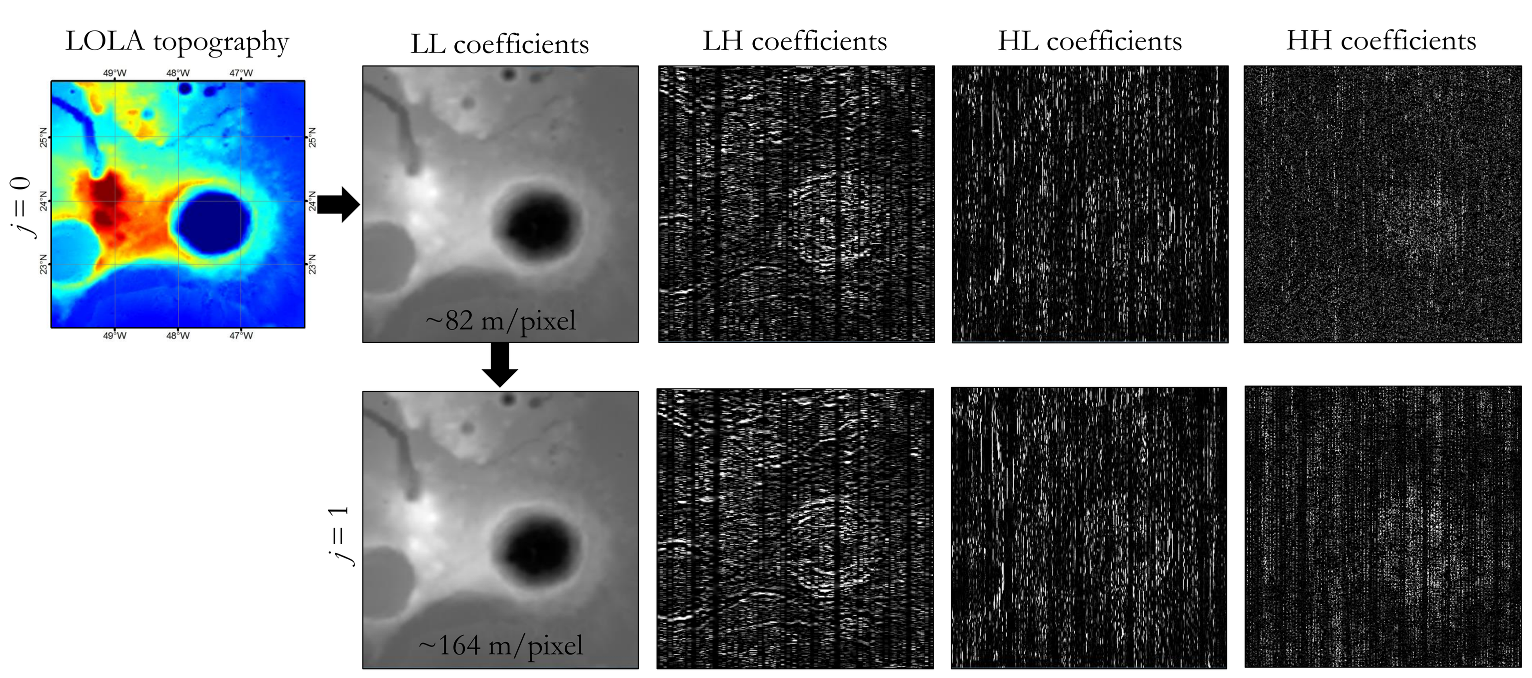


Figure 2. Comparing the topographic signal of pixel i and its neighbors at scale j (2^x pixels) to the LL, HL, LH and HH filters yields LL, HL, LH and HH wavelet coefficients at each scale until there are 2^3 by 2^2 pixels left (Aristarchus region shown).

4.2 Calculate the wavelet leader coefficients at each scale

- The wavelet leader coefficient for a pixel is the maximum absolute value between the LH, HL and HH wavelet coefficients of this given pixel, its 8 surrounding neighbors and the pixels in this dyadic cube (d_j) at all finer scales. The wavelet leader coefficients are calculated for each scale.

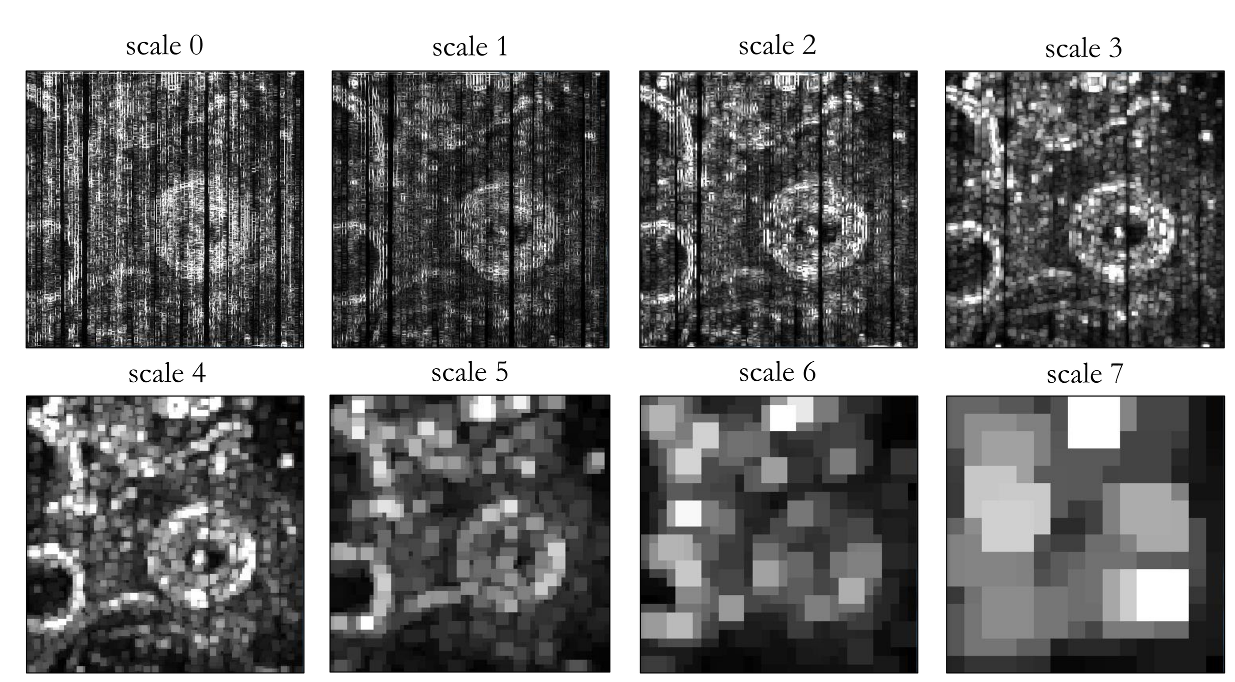
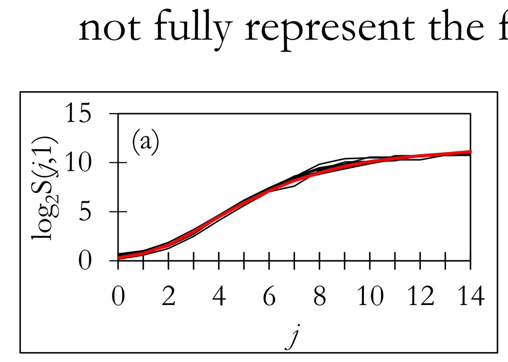


Figure 3. Calculating the wavelet leader coefficients at each spatial scale (Aristarchus region shown).

4.3 Scaling regimes, fractal behavior and Hölder exponent values

- Scaling regimes: we plotted $\log_2 S(j,1)$ versus j (Fig. 4a) for 400 random pixels, and calculated the absolute values of the curvature of these curves. The highest curvature values represent the likeliest scale breaks. S is the structure function (Eq. 1), j the scale and q is the order of S [1].
- Fractal behavior: we plotted $\eta(q)$ (Eq. 2) versus q (for q = -1.5 to 1.5) for each scaling regime, and calculated the correlation r between η and its linear regression (Fig. 4b). The data is monofractal if r>0.97, multifractal if r≤0.97.
- Hölder exponent: if the data is monofractal, the slope of $\eta(q)$ versus q coincides with the Hölder exponent and characterizes its irregularity. If the data is multifractal, the slope gives the dominant Hölder exponent but does not fully represent the fractal properties of the signal.



$$S(j,q) = 2^{-j} \sum_{\lambda} d_{\lambda}^{q} \quad (Eq\ 1)$$

$$\eta(q) = \lim_{j \to +\infty} \frac{\log S(j,q)}{\log 2^{-j}} \quad (Eq\ 2)$$

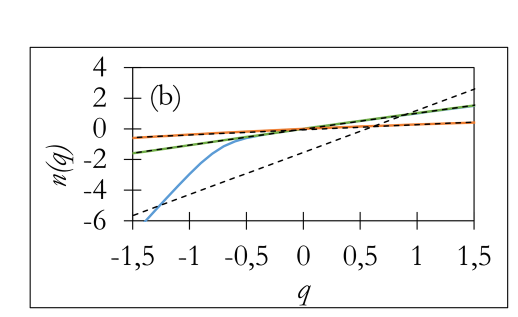


Figure 4. (a) $\log_2 S(j,q)$ versus j for 400 random pixels (where q = 1), the red curve represents the average value. (b) $\eta(q)$ versus q for the three scaling regimes identified (solid lines) identified and their linear regression (dashed lines).

5. Preliminary results

5.1 Scaling regimes

- We studied the absolute value of the curvature of $\log_2 S(j,1)$ versus j for 200 random pixels distributed in the highlands and 200 in the maria.
- For the pixels in the highlands, we observe, that scale breaks occur most often at j=3, around j=8 and around j=12 (Fig. 5a).
- For the pixels in the maria, the scale breaks occur most often around j=9 (Fig. 5b).
- Thus, we consider that globally, at the discrete scales we analyzed, three scale breaks are observed at j=3, 8 and 12, which correspond to spatial resolution of ~667 m, ~21 km and ~341 km per pixel.
- The smallest scaling regime is consistent with [3] who found that within the baselines they investigated (~17 m to ~2.7 km), competing surface processes mostly occurs near 1 km.

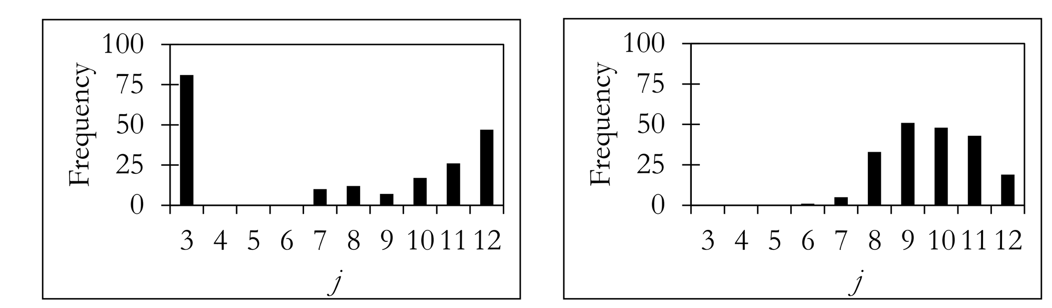


Figure 5. Histogram of the local maxima in the absolute values of the curvature of $\log_2 S(j,1)$ versus j for (a) 200 random pixels located in the highlands, and (b) 200 random pixels located in the maria. Local maximas represent the likeliest scale breaks.

- The three scaling regimes for which we calculate the Hölder exponents are thus j=3-7 (~667 m–11 km), j=8-11 (~21-171 km), and j=12-14 (341-1365 km). Interestingly these scaling regimes occur near the transition from simple to complex crater diameter (~15-20 km) [4], and the transition from complex crater to basin diameter (~140 km) [4].
- We hypothesize that the smallest scaling regime (j=3-7) is characterized by the formation of simple craters, the intermediate (j=8-11) by the formation of complex craters, and the largest (j=12-14) by the formation of impact basins.

5.2 Fractal behavior and value of the Hölder exponent

- Preliminary results are available for the Aristarchus region for the smallest and intermediate scaling regimes.
- For all pixels in both scaling regimes, the correlation r between η and its linear regression is >0.97, suggesting that the surface exhibits a monofractal behavior in this region and at these scales.
- At the smallest scaling regime, the Hölder exponents vary between 2.6-2.8, and at the intermediate scaling regime between 1.1-1.9 (Fig. 6).

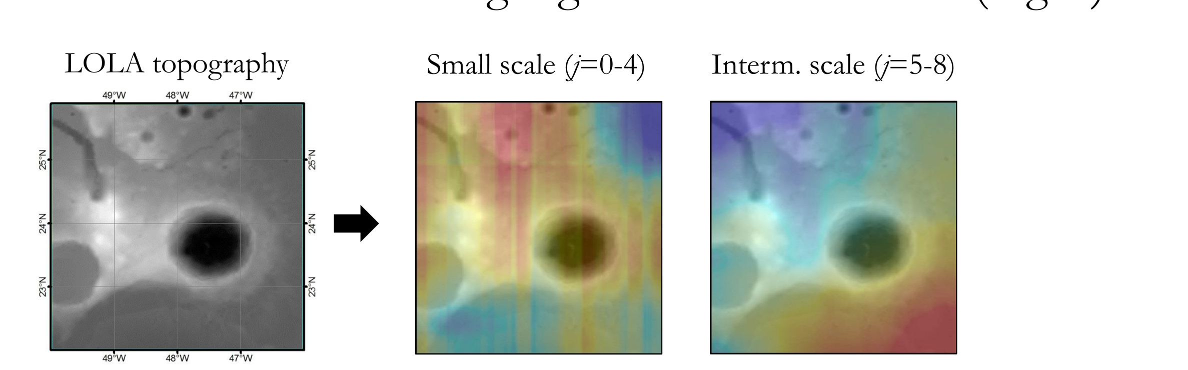


Figure 6. Maps of the LOLA topography and the resulting Hölder exponents values at the smallest scaling regime (from 2.6 to 2.8, blue to red) and at the intermediate scaling regime (from 1.2 to 1.8, blue to red). The Hölder exponents data is shown with transparency over the LOLA topography.

6. Upcoming work

The calculation of the fractal behavior and the Hölder exponent values for the three scaling regimes identified and the whole Moon are underway.

References: [1] Deliège A. et al. (2017), *PSS*, *136*, 46-58. [2] Lemelin M. et al. (2018), LPSC 49, abstract #1021. [3] Rosenburg M. A. et al. (2011) *JGR*, *116*, E02001. [4] Melosh, H. J. (2011) Planetary Surface Processes, Cambridge Planetary Science, 500p.