Finite-Element Analysis of a Shielded Pulsed-Current Induction Heater – Experimental Validation of a Time-Domain Thin-Shell Approach

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***Abstract* — A time-domain extension of the classical frequency-domain thin-shell approach is used for the finite-element analysis of a shielded pulse-current induction heater. The time-domain interface conditions at the shell surface are expressed in terms of the average (zero-order) instantaneous flux and current density vectors in the shell, as well as in terms of a limited number of higher-order components. The three-dimensional thin-shell model is validated by comparing the numerical results with measurements performed on the heating device at different working frequencies.**

I. INTRODUCTION

Conducting pieces can be thermally treated by means of induction heaters that generate strong alternating magnetic fields and induce eddy currents in them. Traditionally the current source of these heating devices was sinusoidal. However, the use of pulsed currents becomes a very attractive alternative thanks to several interesting technological effects. Specifically, it allows to reduce the inductor dimensions and to achieve a more uniform warming [1].

The shielding of these devices is often crucial to mitigate the magnetic field in its environment and reduce the hazardous exposure of both the human operator and the electronic equipment. In practice, these shields are thin metallic sheets with holes to guarantee the accessibility of the heater (to guide control or power wires, to allow cooling...). Their numerical modelling becomes thus an essentially 3D task.

The finite element (FE) analysis of these magnetic shielding problems involving thin shells may suffer from both meshing difficulties and high computational cost. The well-known thin-shell approach allows to overcome these troubles, but it is most often restricted to linear and time-harmonic analyses [2, 3, 4].

Considering a pulsed current as heating source demands a time-domain model. In [5] a pure time-domain approach with the magnetic vector potential formulation is proposed. It is based on the use of orthogonal polynomial basis functions to account for the variation of the magnetic flux through the shell thickness.

This paper deals with the analysis of a shielded induction heater with a pulsed current. Numerical results obtained with a time-domain thin-shell approach are compared with measurements performed on an experimental setup.

II. MAGNETODYNAMIC FORMULATION

We consider a magnetodynamic problem in a bounded domain $\Omega = \Omega_c \cup \Omega_c^C \in \mathbb{R}^3$ with boundary Γ. The conductive and non-conductive parts of Ω are denoted by Ω_c and Ω_c^C. Source inductors constitute domain $\Omega_i \subset \Omega_c^C$ (Fig. 1).

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The Maxwell equations and constitutive laws governing the low-frequency eddy-current problems are

$$\operatorname{curl} h = j\,,\ \operatorname{div} b = 0\,,\ \operatorname{curl} e = -\partial_t\, b\,,\ b = \mu h\,,\ j = \sigma e\,, \tag{1 a-e}$$

where h is the magnetic field, b the magnetic flux density (or induction), e the electric field, j the electric current density, μ the permeability (reluctivity $\nu = 1/\mu$) and σ the conductivity (resistivity $\rho = 1/\sigma$).

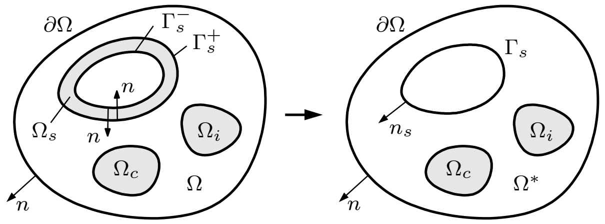

Fig. 1. Calculation domain Ω and reduction of the thin-shell domain Ω_s to the surface Γ_s

The $a-$formulation is obtained from the weak form of the Ampère law (1 a):

$$(\nu\operatorname{curl} a, \operatorname{curl} a')_\Omega + (\sigma\,\partial_t a, a')_{\Omega_c} + \langle n\times h, a'\rangle_\Gamma = (j_i, a')_{\Omega_i}\,, \tag{2}$$

where a is the magnetic vector potential; n is the outward unit normal vector on Γ; j_i is a prescribed current density; $(\cdot,\cdot)_\Omega$ and $\langle\cdot,\cdot\rangle_\Gamma$ denote a volume integral in Ω and a surface integral on Γ of the scalar product of their arguments.

The first step in the thin-shell approach consists in reducing the thin-shell volume $\Omega_s \subset \Omega_c$ (thickness d) to an average surface Γ_s situated halfway between the inner surface Γ_s^- and outer surface Γ_s^+ of Ω_s (outward normal n_s), as depicted in Fig. 1. Next the surface integral in (2) is modified on the basis of the 1-D thin-shell model described hereafter.

III. 1-D THIN-SHELL MODEL

In the 1-D model of the shell, only the variation of the magnetic field $h(z,t)$ and the magnetic induction $b(z,t)$ tangential to the boundary of the shell Γ_s is considered throughout the shell thickness. The 1-D eddy-current problem in the shell ($-d/2 \le z \le d/2$) is governed by:

$$\partial_z^2 h_t(z,t) = \sigma\,\partial_t b_t(z,t)\,, \tag{3}$$

with constitutive law $h_t(z,t) = \nu\, b_t(z,t)$. The associated boundary conditions on the upper $(+)$ and lower $(-)$ surfaces of the shell are given by $h_t^\pm(t) = h_t(\pm d/2, t)$.

The tangential induction $b_t(z,t)$ is expanded in terms of a set of orthogonal Legendre polynomials $\alpha_k(z)$, i.e. ,

$$b_t(z,t) = \sum\nolimits_{k=0}^{n} \alpha_k(z)\, b_k(t)\,, \tag{4}$$

with $|\alpha_k(\pm d/2)| = 1$.

Strongly satisfying (3), the magnetic field $h_t(z,t)$ can thus be written as

$$\begin{aligned} h_t(z,t) = {} & \frac{h_t^+(t) + h_t^-(t)}{2} + \frac{h_t^+(t) - h_t^-(t)}{d}\, z \\ & + \sigma d^2 \sum\nolimits_{k=0}^{n} \beta_k(z)\, \partial_t b_k(t)\,, \end{aligned} \tag{5}$$

where $d^2\, \partial_z^2\, \beta_k = \alpha_k(z)$ and $\beta_k(\pm d/2) = 0$.

Next, with a finite number of basis functions, the constitutive law $h(z,t) = \nu\, b(z,t)$ can be weakly imposed as:

$$\int_{-d/2}^{d/2} \alpha_k(z) \Big(h_t(z,t) - \nu\, b_t(z,t) \Big)\, \mathrm{d}z = 0\,, \tag{6}$$

which leads to $n+1$ differential equations ($k = 0, \ldots, n$) in terms of $b_0(t), \ldots, b_n(t)$, $h_t^+(t)$ and $h_t^-(t)$ [5].

For the FE implementation, the surface integral term in (2) is modified on the basis of this 1-D thin-shell model. The time-domain behavior of the thin shell is taken into account by introducing the tangential vector fields b_0, b_1, $\cdots$ b_n on the thin-shell surface Γ_s as unknowns [5].

IV. ANALYSIS OF THE INDUCTION HEATER

The induction heater comprises a pulsed-current excitation coil and a cylindrical perforated steel shield (190 mm high, 0.65 mm wide, $\sigma = 5.9\, 10^6$ S/m, $\mu_r = 372$). The shield has circular perforations of 76 mm diameter; two holes aligned in the axial direction and repeated periodically along the circumference. The distance between the holes in the axial and azimuthal directions is approximately the same. The workpiece is a cylindrical aluminium plate (radius = 191 mm, height = 10 mm, $\sigma = 3.7\, 10^7$ S/m,$\mu_r = 1$). The induction heating setup is shown in Fig. 2. The time-domain thin-shell approach is applied to the perforated shield.

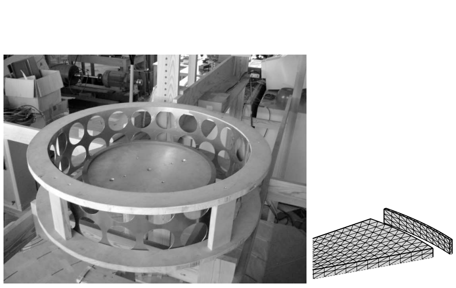

Fig. 2. Picture of the studied induction heating application (left). Detail of the 3D model (right)

The analytical expression of the pulsed current can be found in [1]. The linear amplifier used in our experimental setup clearly deforms the shape of the pulse when increasing the frequency (see Fig. 3). We take thus the measured current wave as input for the numerical computations at three different frequencies ($f = 100$ Hz, 1 kHz and 10 kHz). Note that the three curves in Fig. 3 are not in phase due to the lack of triggering when measuring. This phase displacement could be easily avoided though it would not influence the quality of the results.

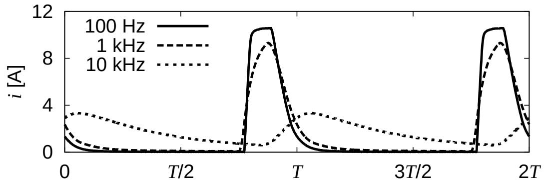

Fig. 3. Measured pulsed current at different frequencies: $f = 100$ Hz, 1 kHz and 10 kHz (period $T = 1/f$)

Simulation results are compared with the performed measurements. The vertical component of the magnetic flux density b at a point outside the shield in the symmetry plane (50 cm from the center of the device, 20 cm from the shield) is measured and compared to computation result given by the thin-shell approach. Three different frequencies are considered: $f = 100$ Hz, 1 kHz and 10 kHz. At 100 Hz, there is hardly any skin effect (uniform distribution of the eddy currents), so that the thin-shell method gives an excellent approximation with $n = 0$ (only one additional unknown on Γ_s: b_0). At 10 kHz, the skin effect is much more important. However, the thin-shell approximation gives a quite good approximation already with $n = 2$ (additional unknowns on Γ_s: b_0, b_1 and b_2). The numerical model shows a very good correlation with the measurements.

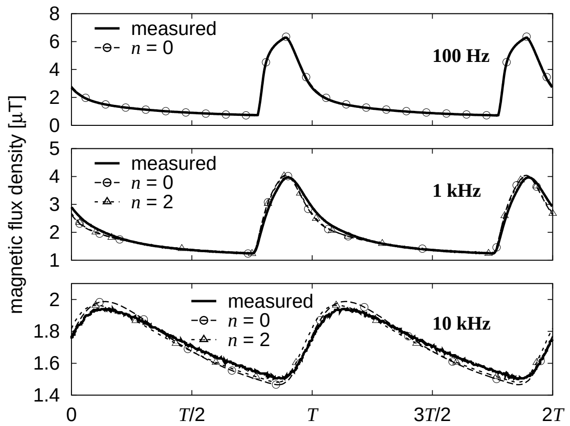

Fig. 4. Vertical component of magnetic flux density outside the shield at a distance of 50 cm from the center of the device

Further results and a discussion on the computational cost will be given in the full paper.

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