A low-order analytical model to monitor tension in shallow cables

Margaux Geuzaine [a,b], Vincent Denoël [a]
[a] *Structural & Stochastic Dynamics, University of Liège, Belgium*
[b] *F.R.S.-FNRS, National Fund for Scientific Research, Belgium*

Contact: mgeuzaine@uliege.be

Abstract

In order to remotely monitor tension in a cable over time, an appropriate method is based on the natural frequencies of the cable measured by means of wireless accelerometers. In this paper, an existing method is extended to determine the tension in long or short shallow cables with small bending stiffness and arbitrary end conditions. First, a low-order analytical model is developed to compute the dynamic response of a given cable. Then, the unknown parameters such as the end conditions, the exact length of the cable and the tension in the cable are adjusted, using an iterative nonlinear least-square algorithm, until the computed natural frequencies and associated mode shapes match the measured ones. This procedure is finally validated with field measurements collected on long and short cables.

Keywords: bridge monitoring; cable; tension identification; system identification

1 Introduction

Recently, dramatic accidents, such as the collapse of Ponte Morandi in Genoa, draw attention to the monitoring of bridges. In this context and as Walloon bridges are aging, the Wallonia Public Service department has decided to launch a research project which aims at remotely keeping track of the time evolution of tension in cables. According to [1], the most suitable method to achieve this is to use the natural frequencies of the cables to identify their levels of tension. This technique is accurate, non-intrusive and only requires wireless accelerometers together with a communication system.

All cables studied in this project are highly tensioned, but their flexural rigidities are not always negligible, although small. Long cables behave like taut strings while short ones get closer to beams [2] and typical stay-cables or hangers lie somewhere in the continuum extending between these two well-known extremes. Apart from that, in some cases, the mass and the flexibility of the bottom anchorage of cables significantly influence the frequencies and thus have to be taken into account as well.

2 Low-order model

In order to describe the dynamic response of a cable, the low-order analytical model developed here contains 9 dimensional parameters, or equivalently 6 dimensionless numbers (see Figure 1 and Table 1).

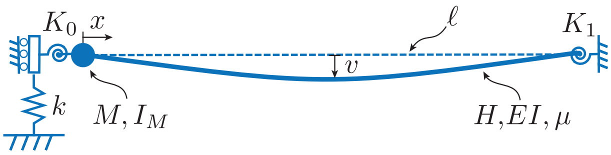
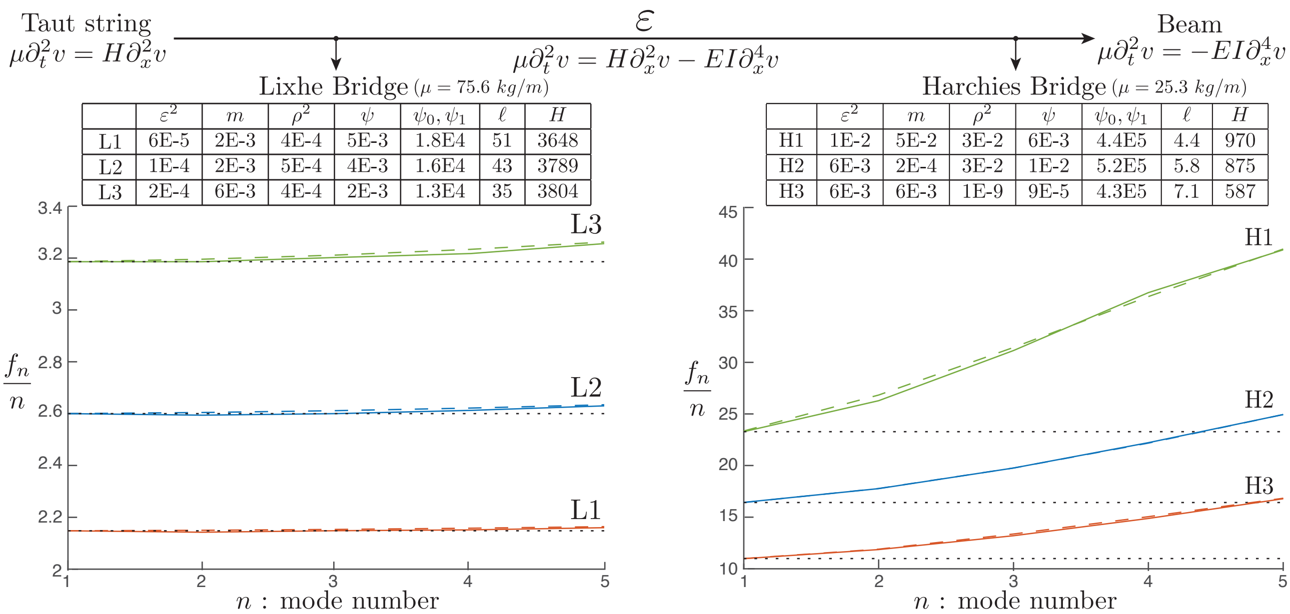


Figure 1. Cable model with specific end conditions.

Table 1. Parameters of the low-order cable model.

H	[kN]	Cable tension, parallel to the chord
ℓ	[m]	Length between anchorages
μ	[kg/m]	Mass per unit length of the cable
EI	[kN.m^2]	Flexural rigidity of the cable
M	[kg]	Mass of left anchorage device (bottom)
I_M	[kg.m^2]	Rotational inertia of M
k	[N/m]	Transverse stiffness at left end (bottom)
K_0	[Nm]	Rotational stiffness at left end (bottom)
K_1	[Nm]	Rotational stiffness at right end (top)

$$\varepsilon^2 = \frac{EI}{H\ell^2};\ m = \frac{M}{\mu\ell};\ \rho^2 = \frac{I_M}{M\,\ell^2};\ \psi = \frac{H}{k\ell};\ \psi_0 = \frac{H\ell}{K_0};\ \psi_1 = \frac{H\ell}{K_1}$$



	ε^2	m	ρ^2	ψ	ψ_0, ψ_1	ℓ	H
L1	6E-5	2E-3	4E-4	5E-3	1.8E4	51	3648
L2	1E-4	2E-3	5E-4	4E-3	1.6E4	43	3789
L3	2E-4	6E-3	4E-4	2E-3	1.3E4	35	3804

	ε^2	m	ρ^2	ψ	ψ_0, ψ_1	ℓ	H
H1	1E-2	5E-2	3E-2	6E-3	4.4E5	4.4	970
H2	6E-3	2E-4	3E-2	1E-2	5.2E5	5.8	875
H3	6E-3	6E-3	1E-9	9E-5	4.3E5	7.1	587

Figure 2. Validation of the identification procedure where f_n [Hz] is the frequency associated to mode n. (solid line = measured, dashed line = computed, dotted line = taut string)

3 Identification procedure

A direct analysis of the model is carried out to understand how the model parameters respectively affect the dynamic response of cables. In practice, they are not known from the outset, except μ and, in some cases, ℓ.

Therefore, the first step is to precisely measure the dynamic response of cables to identify their dimensionless parameters based on natural frequencies and possibly their lengths using mode shapes. This step is a specificity of the identification method we propose and makes it significantly different from existing methods which usually disregard mode shapes.

Once parameters are identified, cables are equipped with wireless accelerometers that provide daily measurements of natural frequencies needed to follow the evolution of the tension in cables over time.

4 Validation

As shown on Figure 2, the procedure has been validated with natural frequencies measured on long stay-cables from Lixhe Bridge and on short hangers from Harchies Bridge (Wallonia, Belgium).

When cables are shorter, the parameter ε^2 related to flexural rigidity rises, resulting in an increasing difference between cable frequencies and taut string frequencies. Obviously, this difference also depends on the mode observed as higher ones are characterized by larger curvatures.

Several additional examples will be detailed in the presentation.

5 Conclusions

A low-order analytical model of shallow cables with small bending stiffness and specific end conditions has been presented. It is exploited by a two-step identification procedure which has been defined and validated on cables of any length. First, model parameters are identified using natural frequencies and modes shapes. Second, tension is adjusted based on natural frequencies measured remotely.

Acknowledgments

The project is funded by the Wallonia Public Service. It is carried out by the University of Liège and the company V2i (www.v2i.be).

References

[1] P. Demars and D. Hardy, “Mesure des efforts dans les structures à câbles”, Annales des travaux publics de Belgique, vol. 6, 1985.

[2] E. de Sá Caetano, Cable vibrations in cable-stayed bridges, IABSE-AIPC-IVBH, 2007.