Uncertainty Quantification of the Multi-centennial Response of the Antarctic Ice Sheet to Climate Change

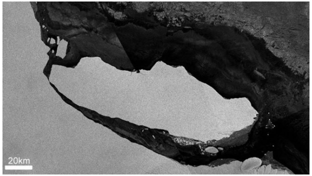
Kevin Bulthuis[1,2], M. Arnst[1], S. Sun[2] and F. Pattyn[2]

[1] Computational and Stochastic Modeling, Université de Liège, Belgium
[2]Laboratory of Glaciology, Université Libre de Bruxelles, Belgium

SIAM Conference on Mathematical & Computational Issues in the Geosciences Houston, USA March 11, 2019

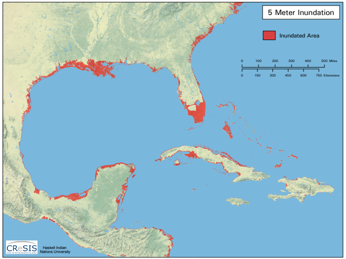
Motivation

- Predicting Antarctica's contribution to future sea-level rise in a warming world (∼200 million people at risk in coastal regions).
- Understanding and identifying the physical processes, feedbacks and instability mechanisms that govern Antarctica's response to climate change.
- Robust policy response strategies to tackle climate changes should rely on integrated risk and uncertainty assessment in climate change projections [IPCC, 2013].



Iceberg breakaway, Larsen C
[Sentinel 1, 2017]

Thwaites Glacier
[NASA, 2014]



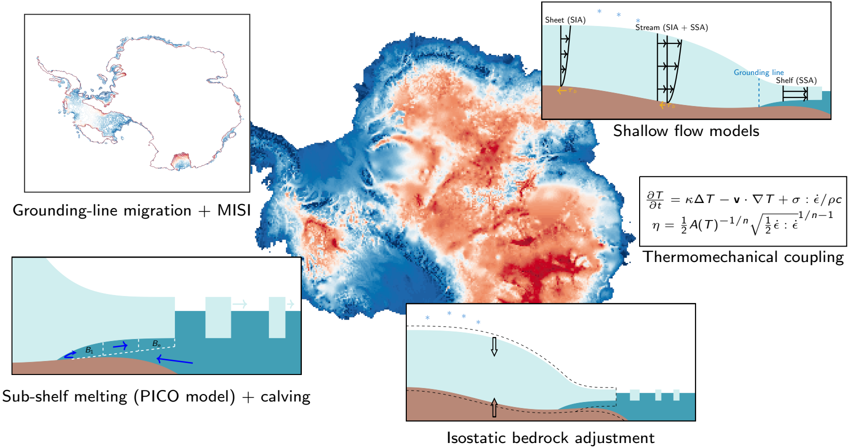
Sea-level rise map
[CReSIS, 2013]

Motivation: UQ in ice-sheet models

- Increased confidence in predictions requires assessment of uncertainties in ice-sheet models.
- Challenges:
  - Wide range of uncertainties: initial conditions, modelling errors, model parameters, climate forcing, basal friction conditions,...
  - Spatially non-homogeneous input and output fields.
  - Computational cost.
- This talk:
  - Assessment of the AIS response to uncertainties in key processes.
  - Methods for propagation of uncertainty and sensitivity analysis in ice-sheet models.
  - Local outputs vs global outputs.
  - Essential ice-sheet models: Representations of key processes through parameterisations and reduced-order models.

Outline

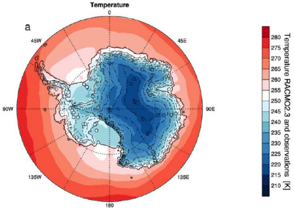
"Essential" ice-sheet models for UQ of multi-centennial response of AIS



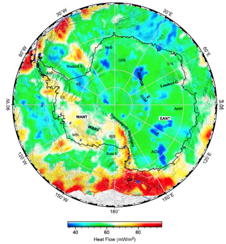
fast Elementary Thermomechanical Ice Sheet model (f.ETISh) [Pattyn, 2017].

"Essential" ice-sheet model amenable to large-scale and long-term simulations and large-ensemble simulations

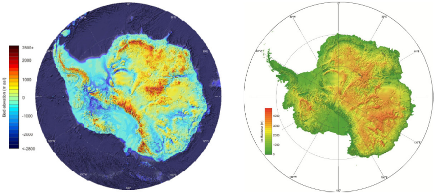
Ice-sheet model initialisation: Present-day geophysical datasets



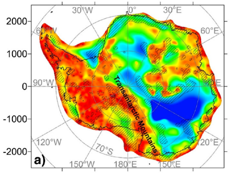
Atmospheric temperature
[Van Wessem et al., 2014]



Geophysical heat flux
[An et al., 2015]

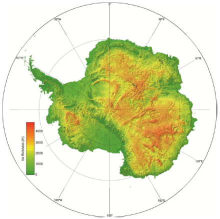


Bedrock elevation and ice thickness
[Fretwell et al., 2013]

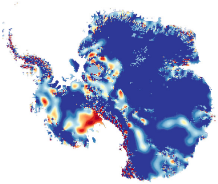


Effective lithosphere thickness
[Chen et al., 2018]

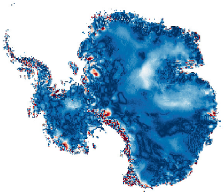
Ice-sheet model initialisation: Inversion of basal sliding conditions



Observed ice thickness h_{obs}



Basal sliding coefficient c_b^{opt}

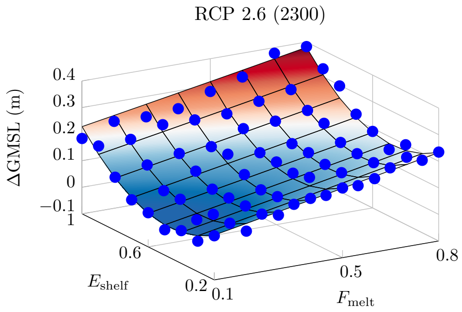


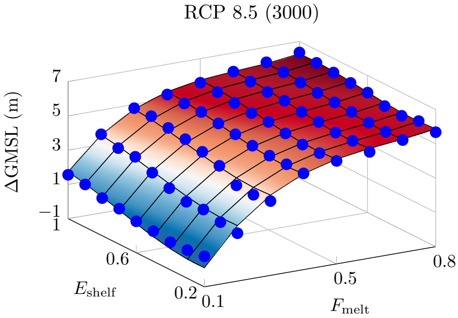
Model-data misfit $|h - h_{\text{obs}}|$

- The basal sliding coefficient is obtained by solving an inverse problem that seeks to match the observed present-day ice-sheet thickness while assuming that the ice sheet is in steady state.
- Fixed-point iteration algorithm: $c_b^{(i+1)} = c_b^{(i)} \times 10^{f(h-h_{\text{obs}})}$ where f is a "misfit" function that corrects the predicted basal sliding coefficient in order to match simulated steady-state ice thickness h with observations h_{obs} [Pollard et DeConto, 2012].
- Inversion of basal sliding conditions remains a key issue in numerical ice-sheet models.

Outline

Propagation of uncertainty and sensitivity analysis





- Quantity of Interest: Change in Global Mean Sea Level (ΔGMSL).
- Polynomial chaos expansions as substitutes for the f.ETISh model and the parameters-to-projection relationships (global outputs are expected to be smooth).
- For each RCP scenario and sliding law, we built a polynomial chaos expansion using 500 training points (1 forward simulation: ∼8h on the CÉCI clusters (F.R.S.-FNRS & Walloon Region)).
- Stochastic sensitivity analysis: Sobol indices evaluated with polynomial chaos expansions.

Random excursion sets and confidence regions

- Motivation: Uncertainty in ice-sheet models can trigger significant grounded-ice retreat under MISI $\Rightarrow$ Need to quantify grounded-ice retreat with uncertainty.
- We investigate uncertainty in grounded-ice retreat using confidence regions for random excursion sets.

Let the random field $BI = \{\rho_w b(\mathbf{s}; \boldsymbol{\Xi}) + \rho_i h(\mathbf{s}; \boldsymbol{\Xi}) : \mathbf{s} \in \Omega\}$ be the buoyancy imbalance. The grounded ice domain is defined as the following random positive excursion set

$$E_{0^+} : \mathcal{D}_{\boldsymbol{\Xi}} \to \Omega; \boldsymbol{\xi} \mapsto E_{0^+}(\boldsymbol{\xi}) = \{\mathbf{s} \in \Omega : BI(\mathbf{s}; \boldsymbol{\xi}) \geq 0\} .$$

Let $\mathcal{C}_{0^+}^{\mathcal{I}}(\alpha)$ be an open set in Ω. Then $\mathcal{C}_{0^+}^{\mathcal{I}}(\alpha)$ is an inner confidence region for E_{u^+} with inclusion probability at least α if

$$\mathbb{P}_{\boldsymbol{\Xi}}\left(\mathcal{C}_{0^+}^{\mathcal{I}}(\alpha) \subseteq E_{0^+}\right) \geq \alpha.$$

Computing confidence regions with a one-parameter family of sets

Let $\mathcal{M}_T^{\mathcal{I}}(\rho) = \{\mathbf{s} \in \Omega : T(\mathbf{s}) > \rho\}$ be a one-parameter family of sets indexed by a real number $\rho \in (0,1)$ and $T : \Omega \to [0,1]$ be a membership function. Then, an approximation for $\mathcal{C}_{0^+}^{\mathcal{I}}(\alpha)$ is given by

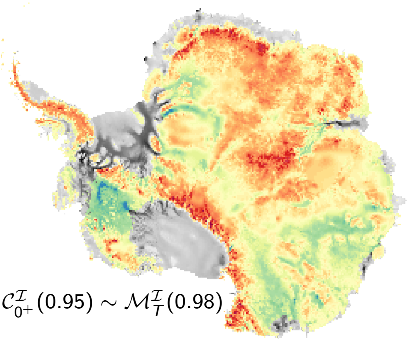
$$\rho^* = \inf_{\rho \in]0,1[} \rho \text{ s.t. } \mathbb{P}_{\mathbf{\Xi}}(\mathcal{M}_T^{\mathcal{I}}(\rho) \subseteq E_{0^+}(\mathbf{\Xi})) \geq \alpha.$$

This problem is equivalent to a quantile problem

$$\rho^* = \inf \{\rho \in]0,1[: c_{\Psi}(\rho) \geq \alpha\},$$

with c_{Ψ} the distribution function of the random variable

$$\psi(\mathbf{\Xi}) = \sup_{\mathbf{s} \in (E_{0^+}(\mathbf{\Xi}))^c} T(\mathbf{s}).$$



$\mathcal{C}_{0^+}^{\mathcal{I}}(0.95) \sim \mathcal{M}_T^{\mathcal{I}}(0.98)$

- Example for $T(\mathbf{s})$: $\mathbb{P}_{\mathbf{\Xi}}(BI(\mathbf{s}; \mathbf{\Xi}) \geq 0)$.
- Evaluate ρ^* with Monte Carlo sampling.

Outline

(1) Motivation

(2) Ice-sheet modeling

(3) UQ with the f.ETISh ice-sheet model: Methodology

(4) UQ with the f.ETISh ice-sheet model: Results

(5) Conclusion

Problem setting

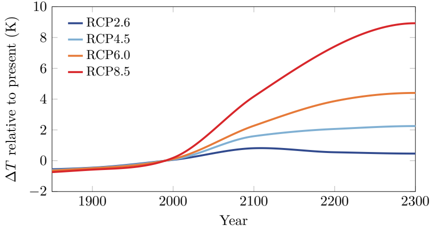
- Goal: Predicting the response of the AIS over the next millenium with quantified uncertainty.
- Set of representative scenarios of anthropogenic greenhouse gas emissions (RCP 2.6, RCP 4.5, RCP 6.0, RCP 8.5).
  $\Rightarrow$ Trajectory for change ΔT in background atmospheric temperature.
- ΔT acts as a forcing on
  - Temperature and precipitation

$$T = T_{\text{obs}} - \gamma(h - h_{\text{obs}}) + \Delta T,$$
$$P = P_{\text{obs}} \times 2^{\Delta T/\delta T}$$

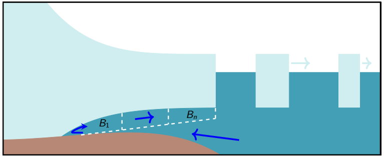
  - Surface melting
  - Ocean temperature and sub-shelf melting

$$To = To_{\text{obs}} + F_{\text{melt}} \Delta T.$$

- Set of sliding laws defined as characteristic cases of power-law friction $\mathbf{v}_b = -c_b \|\boldsymbol{\tau}_b\|^{m-1} \boldsymbol{\tau}_b$ with exponent $m = 1$ (linear), $m = 2$ (weakly nonlinear) and $m = 3$ (strongly nonlinear).



RCP scenarios [IPCC, 2013;Golledge et al., 2015]



Sub-shelf melting [Reese et al., 2018]

Uncertain parameters and characterisation of uncertainty

$$C_r = F_{\text{calv}} \times C_r^{\#}(h, \mathbf{v})$$

$$F_{\text{calv}} \sim \mathcal{U}(0.5, 1.5)$$

Uncertain calving factor

$$To = To_{\text{obs}} + F_{\text{melt}} \Delta T$$

$$F_{\text{melt}} \sim \mathcal{U}(0.1, 0.8)$$

Uncertain melt factor

$$\mathbf{D} = E_{\text{shelf}} \times A\left(\frac{\|\boldsymbol{\sigma}\|_F}{\sqrt{2}}\right)\boldsymbol{\sigma}$$

$$E_{\text{shelf}} \sim \mathcal{U}(0.2, 1)$$

Uncertain shelf enhancement factor

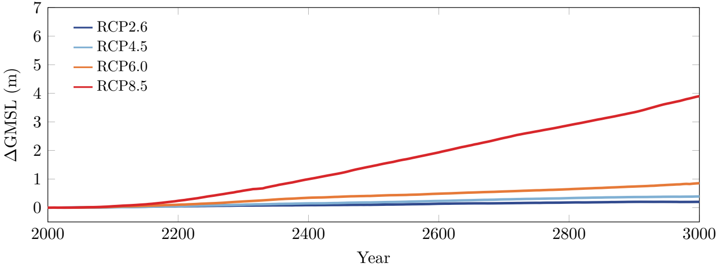
$$\frac{\partial b}{\partial t} = -\frac{1}{\tau}(b - b_{\text{eq}} + w_{\text{b}})$$

$$\tau_{\text{w}} \sim \mathcal{U}(1000, 3500) \text{ yrs}$$
$$\tau_{\text{e}} \sim \mathcal{U}(2500, 5000) \text{ yrs}$$

Uncertain bedrock relaxation times

- In essential ice-sheet models, key processes are represented through parameterisations and reduced-order models with free parameters.
- These are lumped representations of various sources of uncertainty. E.g. uncertainty in F_{melt} also entails uncertainties in the shifting of ocean currents, ice-ocean interactions, ...
- Ranges of uncertainty are determined from expert assessment.

Nominal projections



$F_{\text{calv}} = 1$

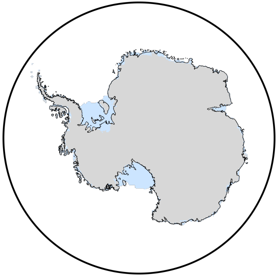
$F_{\text{melt}} = 0.3$

$E_{\text{shelf}} = 0.3$

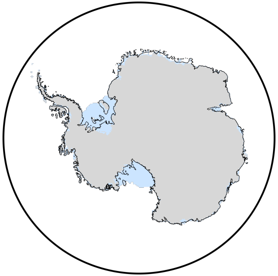
$\tau_{\text{w}} = \tau_{\text{e}} = 3000$ yrs

$m = 2$

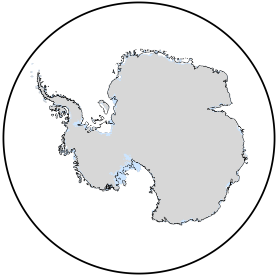
RCP 2.6



RCP 4.5

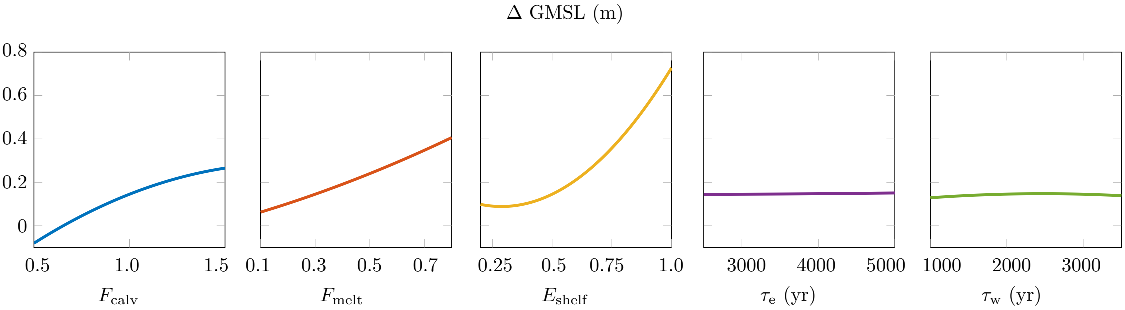


RCP 6.0



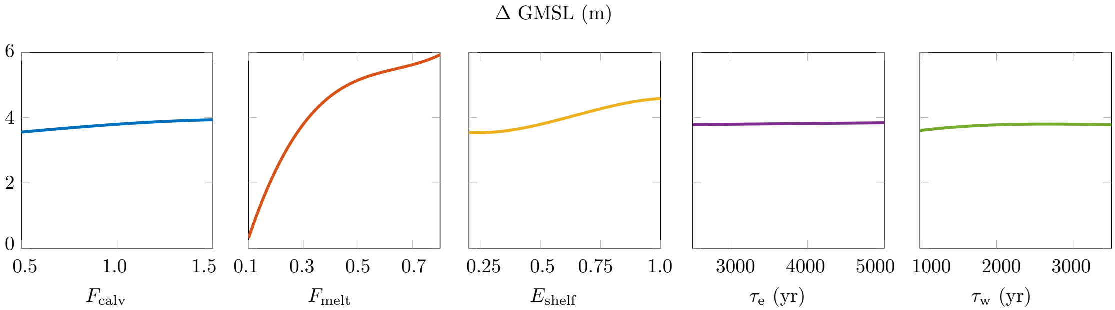
RCP 8.5

Parameters-to-projection relationship (RCP 2.6, $m = 2$, $t = 3000$ yr)



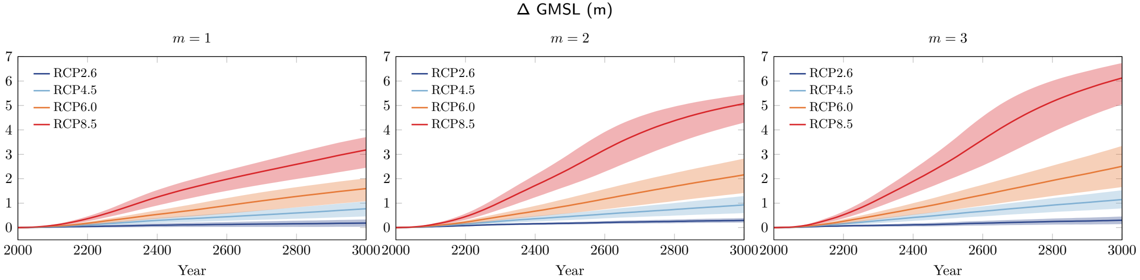
- Nonlinear response with respect to the calving and shelf enhancement factors.
- Linear response with respect to the melt factor.
- The bedrock relaxation times do not contribute significantly to the uncertainty in the projections.

Parameters-to-projection relationship (RCP 8.5, $m = 2$, $t = 3000$ yr)



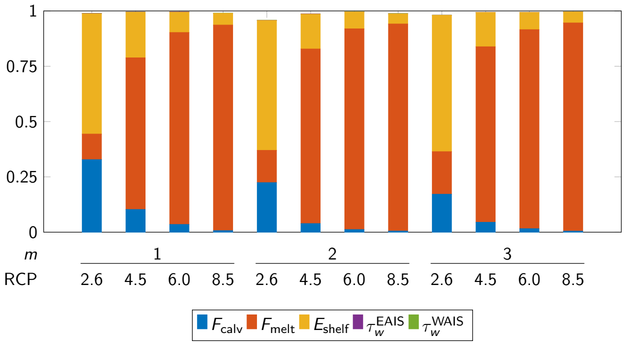
- Nonlinear response with respect with respect to the melt factor. Increasing sub-shelf melting leads to the collapse of the West Antarctic ice sheet. Once the WAIS is disintegrated, a plateau in the response function is reached until marine basins in East Antarctica are activated.
- In the high emission scenario RCP 8.5, the AIS response is controlled by sub-shelf melting.
- The bedrock relaxation times do not contribute significantly to the uncertainty in the projections.

Projections of ΔGMSL with quantified uncertainty



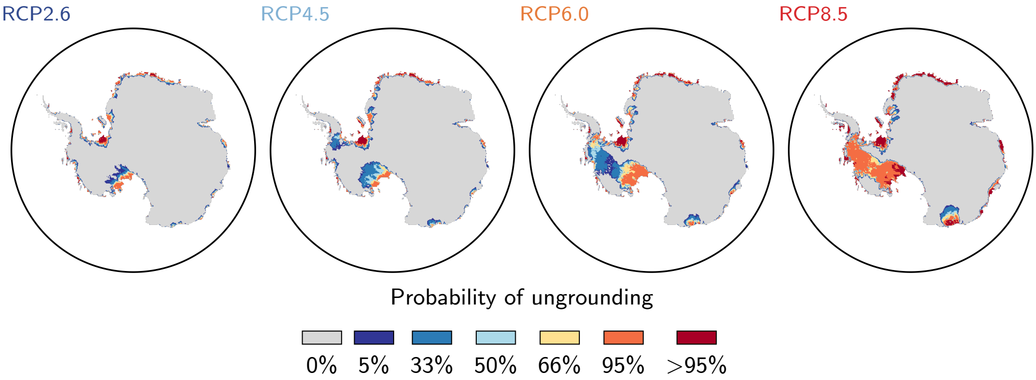
- The AIS contribution to sea level remains limited on short-time scales (2100).
- More nonlinear sliding conditions favour a more significant ice loss.
- The strongly mitigated RCP 2.6 scenario prevents any significant contribution to sea level.
- These results suggest that in warmer scenarios and on longer multi-centennial time scales, the AIS contribution to sea level and the impact of uncertainties on its projection may be significant.

Sensitivity analysis: Sobol indices ($t = 3000$ yr)



- In the strongly mitigated RCP 2.6 scenario, the dominant source of uncertainty is the uncertainty in the ice-shelf rheology followed by those in the calving rate and sub-shelf melting.
- The contribution of the uncertainty in sub-shelf melting to the uncertainty in the projections becomes more and more the dominant source of uncertainty as the scenario gets warmer.
- The bedrock relaxation times do not contribute significantly to the uncertainty in the projections.

Confidence regions for grounded ice ($t = 3000$ yr and $m = 2$)



- RCP 2.6: No significant retreat of the grounding line.
- RCP 6.0, RCP 8.5: Risk of complete collapse of the West Antarctic ice sheet.
- In warmer scenarios, a significant retreat of the AIS is triggered by accelerated retreat of marine drainage basins.

Outline

Conclusion

- UQ for ice-sheet models:
  - Global outputs (e.g. ΔGMSL) smooth out local non-smooth response. Use of MC sampling, surrogate models, sensitivity analysis;
  - Local outputs (e.g. ice thickness) are spatially non-homogeneous and potentially non-smooth responses. Use of MC sampling and confidence regions.
- Impact of parametric uncertainty on AIS projections:
  - The significance of the response of the AIS is controlled by the sensitivity, the response time and the vulnerability of marine drainage basins. The threshold for instability can be reached through various combinations of the parameters;
  - RCP 2.6: Projections are robust under parametric uncertainty (no collapse of AIS);
  - RCP 4.5, 6.0: Projections are sensitive to parametric uncertainty;
  - RCP 8.5: Projections are robust under parametric uncertainty (complete collapse of WAIS).
- Future perspectives:
  - Confidence regions with surrogate models;
  - Stability analysis under stochastic perturbations.

Uncertainty Quantification of the Multi-centennial Response of the Antarctic Ice Sheet to Climate Change

Kevin Bulthuis[1,2], M. Arnst[1], S. Sun[2] and F. Pattyn[2]

[1] Computational and Stochastic Modeling, Université de Liège, Belgium
[2]Laboratory of Glaciology, Université Libre de Bruxelles, Belgium

SIAM Conference on Mathematical & Computational Issues in the Geosciences Houston, USA March 11, 2019

References

- M. An et al. Temperature, lithosphere-asthenosphere boundary, and heat flux beneath the Antarctic Plate inferred from seismic velocities. Journal of Geophysical Research: Earth Surface, 2015.
- D. Bolin and F. Lindgren. Excursion and contour uncertainty regions for latent Gaussian models. Journal of the Royal Statistical Society: Series B, 2015.
- K. Bulthuis et al. Uncertainty quantification of the multi-centennial response of the Antarctic Ice Sheet to climate change. The Cryosphere Discussions, 2018.
- B. Chen et al. Variations of the effective elastic thickness reveal tectonic fragmentation of the Antarctic lithosphere. Tectonophysics, 2018.
- P. Fretwell. Bedmap2: improved ice bed, surface and thickness datasets for Antarctica. The Cryosphere, 2013.
- IPCC. Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change. Cambridge University Press, 2013.
- F. Pattyn. Sea-level response to melting of Antarctic ice shelves on multi-centennial time scales with the fast Elementary Thermomechanical Ice Sheet model (f.ETISh v1.0). The Cryosphere, 2017.
- D. Pollard and R. DeConto. A simple inverse method for the distribution of basal sliding coefficients under ice sheets, applied to Antarctica. The Cryosphere, 2012.
- R. Reese et al. Antarctic sub-shelf melt rates via PICO. The Cryosphere, 2018.
- J. Van Wessem et al. Improved representation of East Antarctic surface mass balance in a regional atmospheric climate model. Journal of Glaciology, 2014.

Acknowledgement

The first author, Kevin Bulthuis, would like to acknowledge the Belgian National Fund for Scientific Research (F.R.S.-FNRS) for its financial support (F.R.S.-FNRS Research Fellowship).

fnrs
FREEDOM TO RESEARCH