Discussion

E. Steen, T.K. Østvold, and S. Valsgård, Members

The authors present a comprehensive paper on the buckling strength of plates and a lot of information is gathered from relevant research taken place over the last decades. The paper is exclusively devoted to the strength assessment of unstiffened plating between longitudinal stiffeners and transverse frames in ship structures. Load interaction from bi-axial compression/tension loads combined with shear and lateral pressure, and effects from geometrical imperfections and residual stresses are of main interest. The less explored field of impact loading is also dealt with.

Five different topics are presented:

1. Modeling of geometrical imperfections and residual stresses.
2. Rotational restraints and torsional rigidity of stiffeners.
3. Ultimate strength design equation under combined bi-axial compression/tension, edge shear and lateral pressure loads.
4. Dynamic collapse strength characteristics, dynamic axial loads.
5. Dynamic collapse strength characteristics, dynamic slamming induced lateral pressure loads.

We have read the paper with great interest and we have some comments, as follows.

1. The authors show a significant knockdown effect from the residual stresses on the ideal elastic buckling stress. The knockdown effect is considered as a type of pre-stress effect for geometrically perfect flat plates, and it would have been interesting to see verifications of these values, using FE analysis. Another matter is whether it is of any interest to study the effect of residual stresses in this way, isolated from the simultaneous effect of the real geometrical imperfections and nonlinear post-buckling behavior. These elastic buckling stresses do not enter the ultimate strength assessment presented later in the paper and it would be interesting to hear the authors' meaning on the field of application for these parameters.
2. The authors document that the boundary conditions for plate element between stiffeners is found to vary between simply supported and clamped conditions, and a comprehensive study is presented on how the torsional restraints influences the ideal elastic buckling stress of the plating. We agree that this range of boundary conditions normally applies, but we have also observed that slender stiffeners, e.g., flat bar profiles prone to torsional buckling, may even destabilize the plating. Thus the plate and stiffener seen together as a unit may then even buckle at a nominal stress level below that for a simply supported plate. The fact that the stiffeners and girder members also have a buckling problem (in interaction with the plate) makes it difficult to assess the level of rotational restraint and it will be interesting to hear the authors' comments on this. It is not clear how the rotational restraint effects influence the ultimate strength assessment presented later in the paper.
3. The authors' ultimate strength procedure is separated from the buckling strength calculation procedure, in particular we cannot see that the rotational restraint effects influence the ultimate strength values. Is the buckling strength procedure only meant for serviceability type of criteria and thus not to be applied for ultimate design loading at all? The ultimate strength solution is formulated as an implicit solution of a limit state based on von Mises yield criterion in certain points along the supported edges. This is as sound approach as we see it. The authors document a significant knockdown effect from the presence of lateral pressure (Fig. 29) which we cannot see has been verified by nonlinear FE analysis. The opposite effect has been documented for the elastic buckling stress of rectangular plates ($a/b=3$) as shown in Fig. 24b. Have the authors considered the effect of boundary conditions in the assessment of the ultimate strength assessment, i.e., the fact that the lateral pressure will act over several bays in all directions? Presently classification societies consider the lateral pressure effect not to be critical for the in-plane capacity of plate-elements between stiffeners and girders and it is neglected in this respect. However, pressure effects are more important for the strength assessment of the stiffeners. We will be interested to hear if the authors have any comments on such a basic philosophy.

4, 5. The authors present the cases of impact loading and the results clearly illustrate the effect of dynamic effects on the capacity. This field is not as extensively explored as the case of static loads, but the results presented are interesting in a general context. For practical applications to ship structures, say to vessels with a large bow flare, studies like the present may prove to be useful for designing the plating such that not too large permanent deflections or rupture will occur. This is a very complex area and more studies are needed before dynamic effects can be utilized in the strength assessment of both ship skin and in the dimensioning of the internal supporting stiffener and frame structures.

We conclude that the paper contains a lot of interesting results and descriptions of new design methods. The philosophy is clearly a step in the direction of applying more advanced methods in the design of ship structures, a philosophy the present discussers fully support.

Philippe Rigo, Member

First of all, I would like to congratulate Professor J.K. Paik and the co-authors for the very thorough and extensive study carried out. In my discussion, I would like to make some comments and ask for additional information. These comments and questions are not based on criticisms but should be taken positively as being a guide for the authors for further improvement of this already valuable world-class contribution.

1. With regards to yielding limit state, use of equations (1) and (5) does not allow for consideration of the effect of the in-plane bending moment. Have the authors assessed the effect on the safety of neglecting these in-plane bending stresses?
2. The authors say that "*the Johnson-Ostenfeld equation has a tendency to underestimate the inelastic buckling strength.*" This statement is obvious, as this equation is only valid to account for the effect of plasticity of stocky plate and not to evaluate ultimate strength. Nevertheless, their analysis is interesting as:

- They point out the fact that the underestimation is particularly large with plating under shear load; and
- They provide a quantitative evaluation of this underestimation (about 5% for in plane compressive stress).

3. The authors mention that the rotational restraint parameter at long edges is normally in the range of 0.05 to 3.0. This is true for tankers with plating over 10 mm. For lightship, having thinner plates, lower values are usually obtained.

those found from equations by Cowper and Symonds (cited later by the authors), Reckling (1976) and Kaminski (1992) that quantify such strain-rate effects. Thus the recorded increase in strength may be entirely due to the strain-rate effect increase in yield strength rather than anything else.

Have the authors satisfied themselves that these dynamic test results can be nondimensionalized in the same way as similar quasi-static tests which is what the presented results seem to suggest. Frieze (1975) confirmed that under quasi-static conditions, identical nondimensional results are obtained for uniaxial compressed plates of different sizes and material properties (but with the same shape of material stress-strain curve) if they are normalized using (adopting the same symbols as the authors) β for slenderness, σ_{xav}/σ_o for stresses, σ_{rx}/σ_o for welding residual stresses and w/t for out-of-plane deflections where w is the magnitude at the center of the plate. However, with increasing axial loading rate, changes in buckling mode are expected, culminating in local buckling immediately under the point of load application. Perhaps the reported tests were not subject to loading rates that changed the buckling mode. Nevertheless, the question of nondimensionalization still arises. This is because the axial strain rate might be the same for two plates of similar proportions but different thickness, but the strain-rate in the extreme fibers of the thicker plate will be greater than those in the thinner plate. Consequently the effective yield stress in the extreme fiber of the thicker plate will be higher so it might be expected to demonstrate greater strength.

The authors examine dynamic lateral loading of plates from page 23 onwards. The reference by Biggs (1964) is well worth consulting to gain insight to this problem although Biggs concentrates on beams rather than plates. Nevertheless, a study of Biggs would give considerable appreciation into the effect of different forms of pulse impact loading. It might also discourage the authors from generalizing as they do at the end of the first complete paragraph on page 25 in relation to consequences of rectangular pulses of different durations which they then go on to exploit. When discussing such effects, it is extremely important to be aware of the natural frequency of the component in the mode into which it will deform when loaded.

Equation (38) appears to be a function of flexural hinge capacity only. Under the large deflections experienced by plates under lateral loading, membrane stresses will develop so that, along hinge lines, in-plane axial and shear stresses will also be present. Does the hinge line theory used in the authors' work include allowances for such axial and shear stresses? A proper analysis including such effects would also have to account for the normality rule of plastic flow in order to ensure the correct kinematics in the response.

In closing, and looking forward to the replies by the authors, again I commend the authors on their comprehensive and useful publication.

J. Roberts, Member

As requested at the AGM, this is a followup to the verbal discussion that I presented. I have read your paper in detail and consider it well presented with useful results. I was particularly encouraged that your references included one of the definitive aerospace works on the subject (Gerard and Becker, NACA Technical Note 3781 Handbook of Structural Stability Part I—Buckling of Flat Plates). I mention this because in many respects ship and aircraft structure are similar. Both make extensive use of stiffened panels where buckling is a concern.

One of the 'bibles' for aerospace structural engineering is: Bruhn, E.F., Analysis and Design of Flight Vehicles Structures Published and distributed by: Richard Jacobs Company, Indianapolis, Indiana 317-815-9400, Copyright 1973. Although 'Bruhn' is now somewhat dated it is still in use. There are several chapters on buckling. What makes 'Bruhn' of particular interest is that it references much of the earlier, definitive work such as that of Gerard and Becker. Aerospace addresses the issue of local buckling failures (e.g., crippling) that so far have not been applicable to ship structure. However, I suspect that the introduction of very thin high strength steel could result in local failures.

I have included a description of our experience. You will see that a good portion of our work is aerospace. We have witnessed opportunities for technology transfer between both industries—and in both directions. Also included is a paper that I gave at the FAST' 99 conference in Seattle, Washington. The paper gives an example of how aerospace technology might be adapted to high speed vessels. Clearly, there are differences between aerospace and ship structure including the important effects of initial deformation and residual stresses.

The issue that I raised was how can we manage nonlinear finite element analysis (FEA) in an appropriate engineering manner. Nonlinear FEA can be categorized into the following three general areas:

1. Geometric nonlinearity, for example: "tension only" cables or "surface-to-surface" contact.
2. Large strain, for example: secondary effects due to displacement such as the development of "membrane stresses" in a flat panel subject to transverse pressure load.
3. Material nonlinearity, or plasticity.

It is my opinion that 1 and 2 above have already been well addressed by FEA software. An exception might be excessive distortion of a finite element mesh that produces 'ill mannered' elements but at least the problem is clearly understood. It is the issue of material nonlinearity that needs careful thought as to how we can proceed with *engineering solutions*. I make the distinction between *engineering* and *science* because engineers need a more general (hence, less rigorous) approach to problem solving. One of the reasons this is a problem is the rapid advancement of engineering tools such as FEA. Nonlinear capability is now common within FEA software; consequently, engineers are making more use of it. I believe that we do so with only a limited understanding of what the analysis means. Engineering traditionally deals with simplistic models and this is fine as long as the model works for the intended application. For example, in a uniaxial stress condition, assuming a piece-wise linear *engineering* (as opposed to *true*) stress-strain curve is probably quite acceptable. I think you can easily see, however, that a three-dimensional state of stress is not so easily described. I don't believe that engineers in general receive adequate training in materials science to clearly understand multi-dimensional yielding within a volume. Obviously I cannot go into great detail on this complex subject but let's just consider a few simple points. First, there are actually three types of nonlinear material behavior:

1. Nonlinear elastic
2. Inelastic behavior (hysteresis)
3. Linear elastic—plastic with nonlinear strain hardening

Material nonlinearity studies (yielding) require the following to be addressed:

1. Strain decomposition to elastic and plastic parts
2. Yield criterion to define initiation of yielding (such as Tresca or von Mises)
3. Flow rule (to define growth of plastic strain)
4. Hardening rule

Strain decomposition is not as simple as expected. Material data are usually characterized in terms of engineering strain. However, plasticity usually deals with *Green's strain* or *Almansi strain* (as two examples). Two apparently common hardening rules are *isotropic* and *kinematic*. In isotropic hardening the yield surfaces are a uniform expansion of the original yield surface without translation. In kinematic hardening the yield surface translates without change in shape or size. We need to understand how the physical material behaves relative to various models and then understand what 'magic' the finite element software is performing. It would be natural to expect that we can test how realistic the FEA is to the physical material but this is apparently not quite that easy. Consider the points that you made regarding the sensitivity of some materials to strain rates. Let's ignore the more esoteric effects of varying strain rates, visco-elastic and visco-plastic behavior and consider only quasi-static analysis. The FEA software we use (Algor) offers two formulations for nonlinear analysis:

1. Total Lagrangian, formulated in terms of *2nd Piola-Kirchoff stresses* and *Green-Lagrangian strains*
2. Updated Lagrangian, formulated in terms of *Cauchy stresses* and *Almansi strains*

Don't be concerned if you are not familiar with the definition of the above stresses and strains as this only reinforces my point. Where does the practicing engineer get material data characterized in terms of such stresses and strains? I suspect that material data are one of the more important issues for material non-linear analysis. I hope that I am correctly interpreting Fig. 11 of your paper. If so, then I find it very interesting how the FEA results greatly differ with the Johnson-Ostenfeld equation. We have tried duplicating empirical results for panels loaded in shear using FEA but with poor results—even for elastic buckling. I suspect that a significant problem with shear loading is the correct balance of loads and boundary conditions. I certainly agree with your statement (just before the Concluding Remarks) "The strain hardening effect of material may vary with the magnitude of plate deflection. . . ." This statement is the essence of my discussion. Our main application for nonlinear FEA is service failure analysis. As opposed to design work, service failure analysis cannot rely on "conservative" assumptions. Instead, we are trying to duplicate—as exactly as possible—a material behavior. If you have any suggestions on how engineers can improve their use of nonlinear FEA then I would be happy to hear them.

Authors' Closure

First of all, the authors would like to thank all discussers for their valuable discussions and comments. The authors will try to reply for the discussers one by one.

Dr. Steen and his colleagues from the DnV raise a question about the influence of welding residual stress on the elastic buckling and ultimate strength stress. While some researchers (e.g., Ueda & Tall 1967) previously investigated the reduction characteristics of elastic and elastic-plastic compressive buckling strengths of plates due to welding residual stress in the loading direction, the added contribution of the present paper is to study the expected knockdown effect when residual stresses exist in the direction normal to the loading direction as well as in the loading direction. While the buckling strength formulations presented in our paper are based on analytical solutions to take into account the influence of residual stresses, comparisons with FE analysis are being planned. The buckling formulations including residual stress as a parameter of influence can be used as the basis of the design for serviceability limit state. Even though the elastic buckling stres[ses] are not directly involved in the proposed design procedure [for] the ultimate limit state, the post-weld initial imperfections (i. both initial deformations and residual stresses) are included [in] the ultimate strength assessment as parameters of influence.

Regarding the rotational restraints along the plate-stiffen[er] intersection, we in a general sense agree that for slender sti[ff]eners, including flat bar profiles prone to torsional buckling, t[he] torsional restraints may be small or even the stiffeners m[ay] destabilize the plating. However, our treatment is based on t[he] normal presumption that stiffeners and supporting membe[rs] have been properly designed so that their local instability w[ill] not occur prior to the failure of plating. When the stiffene[rs] are very weak, they can buckle together with plating as part [of] what may be called overall buckling. The design procedure f[or] the overall buckling of stiffened panels is outside the scope [of] the present paper. As Paik & Thayamballi (2000) previousl[y] suggested, however, the rotational restraint coefficients (i.e. ζ_L or ζ_s) may be approximately reduced to take into accoun[t] the effect of local buckling of stiffeners. In spite of the rota[tional] restraint effects on the elastic buckling, we do approxi[mate] that the ultimate strength may not be affected significantly. This is because if plasticity occurs earlier along the edges wher[e] the larger bending moments are developed, the rotational restraints at the yielded edges will then be lessened as the applied loads increase.

We agree that in our treatment, the ultimate strength procedure as presented is separated from the buckling strength calculation procedure, with the latter being considered as a basis of the serviceability limit state and the former being employed for the assessment of the ultimate limit state. Perhaps in the future, a more unified treatment might be possible.

Figure 24(b) shows the influence of lateral pressure on the elastic buckling strength of a rectangular plate with aspect ratio $a/b = 3$, indicating that lateral pressure can disturb occurrence of buckling in the elastic regime. On the other hand, Figure 29 represents the effect of lateral pressure on the plate ultimate strength when simply supported plate edges are presumed. In our study, the behavior of bare plate elements between stiffeners is being considered. For the design of stiffened panels under lateral pressure, the authors agree with the procedure of classification societies that stiffeners with associated plating will resist the lateral pressure loading. However, the authors point out that the effective breadth of associated plating can be reduced by a shear lag effect due to lateral pressure, a factor that is sometimes neglected.

Regarding Dr. Steen's comments about dynamic effects on the collapse behavior of ship plating, the authors fully agree that this subject is quite complicated to deal with, and that further studies are required in order to establish a more refined design procedure.

Regarding *Dr. Rigo*'s discussion to the influence of in-plane bending, the buckling based capacity formulations include in-plane bending as a parameter of influence, while the same is not considered for the ultimate strength based capacity formulations. The difficulty in considering in plane bending effects on ultimate strength arises because some part of in-plane bending results in axial tension while axial compression is applied by the other part. The tensile part works toward stabilizing the plate, and it is difficult to properly account for its effects until the ultimate strength is reached. It of course helps that in most real cases, in-plane bending effects tend to be small, but this is not always so, which is a fact that one needs to bear in mind.

The authors agree with Dr. Rigo's comment that the Johnson-Ostenfeld equation is intended to account for the effect of plasticity which is of course important for stocky plates.

Dr. Rigo had a question about applicable range of the rotational restraint parameters included in the elastic buckling formulations. We note in this regard that there are basically no range limitations assumed for the parameters; that is, they can have values that may range from zero to very large values. From a theory point of view, it would appear that the buckling and ultimate strength formulations presented in the paper can be directly applied to aluminum plates as well; verification studies in this regard are being planned.

Paik & Thayamballi (2000) suggested and provided calculations that appear to support that the buckling interaction relationship for simply supported plates can also be valid for plates with elastically restrained edges by using the corresponding buckling stress components. Figs. 19 and 20 show results for a limiting extreme case of plates with infinite rotational restrained edges, which might be a worse situation. The validity of the above suggestion appears to be confirmed by Figs. 19 and 20.

The authors agree with Dr. Rigo's characterization of the upper limit for the formulations of the buckling reduction factors due to the cut-outs presented in equation (21), which should cover the majority of practical cases. Related to this, equation (21) might be valid only for a square plate since the plate aspect ratio is of course an influential parameter on the buckling strength. Figures A.1–A.3 show the effects of the plate aspect ratio on the plate buckling strength varying the size of cut-outs and loading condition, as those obtained by ANSYS eigenvalue analysis, where all edges are simply supported and kept straight.

Dr. Rigo raises a question why the buckling strength in in-plane bending does not take into account the influence of residual stresses. This is because the strength formulations presented are concerned with the failure of plating primarily due to in-plane axial compressive loading, and thus equation (23) attempts to include the in-plane bending stress in the axial compressive buckling formulations as a parameter of influence. As described in the text of the paper, Appendix 1 defines the plate buckling coefficients for a variety of elastically restrained edge conditions.

Since the influence of lateral pressure on the buckling and ultimate strength of plating under shear is normally small in most practical cases, the shear strength formulations presented do not at this stage include lateral pressure as a parameter of influence.

While the elastic buckling coefficients for plates with elastically restrained edge conditions and the ultimate shear strength formulations are derived by curve fitting based on a series of analytical or numerical computations, the primary strength formulations for axial loads are based on analytical solutions as those obtained by solving the nonlinear governing differential equations of large deflection plate theory. The validity of the developed strength formulations are to some extent undertaken by a comparison with either the nonlinear finite element solutions or the mechanical model tests in the paper.

As described in our reply to the discussion of Dr. Steen and his colleagues as well as in the text of the paper, the effects of either rotational restraints along the plate edges or in-plane bending are not considered in the plate ultimate strength formulations.

As indicated in the section "Buckling/Ultimate Strength Design Procedure," the present paper is concerned with the strength behavior of the bare plate element level alone where local buckling and collapse of plating between stiffeners is a primary failure mode in stiffened panels. The authors have separately developed the advanced ultimate strength formulations for ship stiffened panels and grillages, and plan to present them in near future.

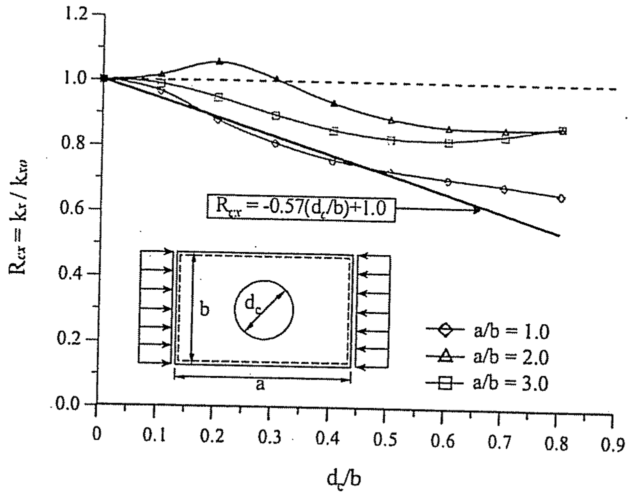

Figure A.1 Buckling reduction factor accounting for the effect of cut-outs under longitudinal compression

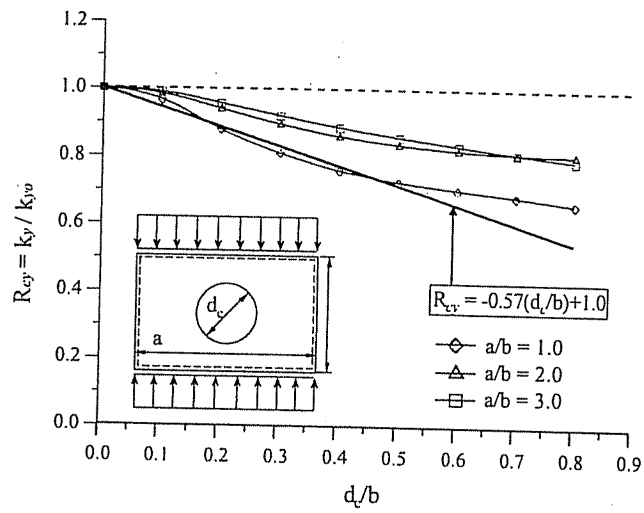

Figure A.2 Buckling reduction factor accounting for the effect of cut-outs under transverse compression

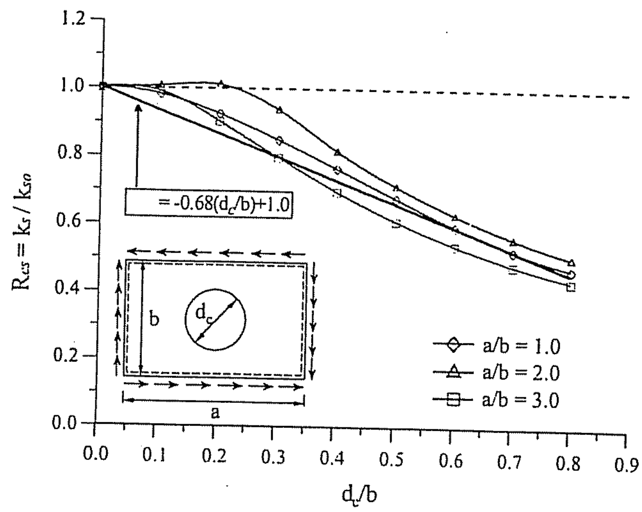

Figure A.3 Buckling reduction factor accounting for the effect of cut-outs under edge shear

To analyze the progressive collapse behavior of ship hulls under vertical hull girder bending, the stress-strain relationship of plating in the pre-buckling, buckling, post-buckling, ultimate strength and post-ultimate regime are needed. While Ueda et al. (1984) and Paik (1995) present the stress-strain formulations of simply supported plates, the present paper develops the strength formulations for buckling and plastic collapse. In the following, the relationships between average stress and average strain of imperfect plates under predominantly longitudinal axial compressive loading are suggested, consistent with the developments presented in this paper. Note that the imperfect plates do not show a bifurcation buckling, and that the in-plane stiffness decreases from the beginning as the axial compressive stresses σ_{xav} increase. In this case, therefore, the stress-strain formulations can be divided for convenience into three regimes, namely pre-ultimate strength, ultimate strength and post-ultimate strength.

(1) Pre-ultimate strength regime

As long as the plate edges keep straight, the average strains can be obtained in terms of maximum membrane stresses which are a function of average axial stress σ_{xav} as well as post-weld initial imperfections, as follows

$$\varepsilon_{xav} = \frac{1}{E}\left(\sigma_{x\max} - \nu\sigma_{y\max}\right) \tag{A.1}$$

where

ε_{xav} = average strains in the direction

$\sigma_{x\max}$ = as defined in Appendix 6 of the present paper

$$\sigma_{y\max} = \sigma_{rcy} - \frac{\pi^2 E A_m (A_m + 2A_{om})}{8b^2}\cos\frac{2m\pi a_t}{a}$$

The above relationship can be rewritten in the incremental form, as follows

$$\Delta\varepsilon_{xav} = \frac{1}{E}\left(\frac{d\sigma_{x\max}}{d\sigma_{xav}} - \nu\frac{d\sigma_{y\max}}{d\sigma_{xav}}\right)\Delta\sigma_{xav} \tag{A.2}$$

(2) Ultimate strength regime

The plate reaches the ultimate strength when $\sigma_{xav} = \sigma_{xu}$, where σ_{xu} is obtained as the solution of equation (29) with regard to σ_{xav}.

(3) Post-ultimate strength regime

Paik (1995) derived an analytical expression of the average axial stress-strain relationship of the plate in the post-ultimate strength regime, as follows

$$\sigma_{xav} = \frac{1}{2}\left(1 + \frac{\sigma_{xE}}{E\varepsilon_{xav}}\right)\sigma_{xu} \tag{A.3}$$

where σ_{xE} = as defined in equation (17).

The incremental form of equation (A.3) can be given by

$$\Delta\sigma_{xav} = -\frac{\sigma_{xu}}{2}\frac{\sigma_{xE}}{E\varepsilon_{xav}^2}\Delta\varepsilon_{xav} \tag{A.4}$$

Figure A.4 demonstrates the validity of equations (A.1) to (A.4) by a comparison with the nonlinear finite element analysis in the case of square plating with initial deflection but without residual stresses. It is evident that the proposed stress-strain formulations fairly well correspond to the non-linear FEA. For combined loading, the stress-strain formulations can be derived by a method similar to that of the uniaxial load case.

Dr. Frieze has a comment about Figure 1. In this figure, the authors attempt to represent that thick plates will not buckle in the elastic regime while thin plates do.

Dr. Frieze raises a question about idealization of welding induced residual stress distribution for high tensile steel plates. According to the measurements of welding induced residual stresses in plating between stiffeners (Kmiecik 1970, Cheng et al. 1996), the residual stress distribution for mild steel plates is somewhat different from that for high tensile steel plates, as those indicated in Figure A5, while the maximum tensile residual stress well reaches the yield stress for both types of material. In evaluating the effect of residual stresses on the compressive buckling strength, a somewhat reduced tensile residual stress (e.g., 80% of the yield stress) has been used for idealization of the welding residual stress distribution for high tensile steel plates as long as the actual breadth of the tensile residual stress block is applied, as that indicated by the dotted line in Figure A.5b. Whether this type of approximation is appropriate in a general sense is worth further investigation, as there is no reason to think that residual stresses are a function of base material yield strength alone. We thank Dr. Frieze for pointing this out.

Regarding the comments on the straight edge condition of plating between stiffeners, the authors agree that the plate edges may not remain straight in some cases, specifically under large lateral pressure loading. In these cases, however, assuming that the stiffeners are slender, the plate will buckle together with stiffeners showing the overall buckling of the entire stiffened panel, and the behavior of the panel may be analyzed by a different approach, e.g., large deflection orthotropic plate theory. As long as the stiffeners are strong enough so that they do not fail prior to buckling of the plate, which is the case that the present study is concerned with, the plate will fail locally. In a continuous plated structure where such a hypothesis can be accepted, the edges of individual plate elements will remain almost straight due to the relative structural response to the adjacent plate elements even after the plate deflects.

Dr. Frieze raises a question as to whether the torsional stiffness of a stiffener can be simply divided into two, to be allocated to each adjacent plating. As long as the plate deflection pattern is unsymmetrical with regard to a stiffener, the authors approximate that the contribution of the stiffener to the rotational restraints along the plate-stiffener intersection may approximately be equal for two adjacent plates. The validity of this approximation will be looked into in the future.

Dr. Frieze would like to see some comparisons of the proposed ultimate strength formulations with the mechanical model test results. While a limited amount of such effort has been made for plates under combined axial compression and lateral pressure as indicated in Fig. 29, we certainly feel that it is more worthwhile to perform the comparisons with nonlinear FEA, the FEA procedure being validated by the admittedly limited number of actual mechanical model tests.

In equation (35), the dimension of plate length is considered to be mm (or m) when the loading speed is in mm/sec (or m/sec) since the strain rate is in sec^{-1}. Also, this equation is valid for typical mild steel. It is realized that the strain rate effects for high tensile steel might be different from that for mild steel (Paik et al. 1999d, and others). There is usually a difficulty to theoretically estimate the strain rates of the structural components making up ship hulls, even though a monitoring system may be used to measure the strain rates at individual locations. While the Cowper-Symonds type equations represent

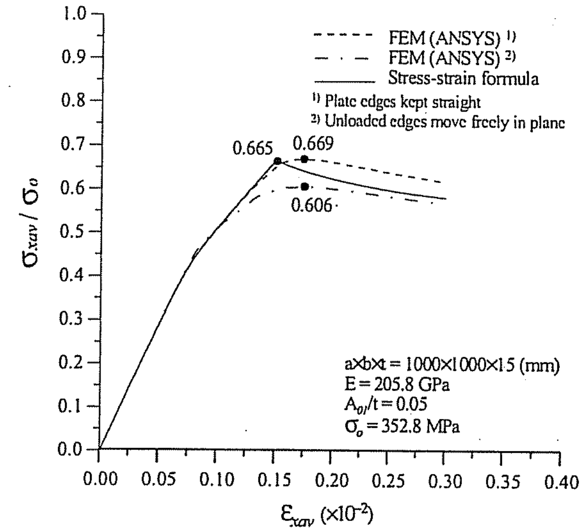

Figure A.4 The average stress-strain curves for a simply supported square plating under axial compressive loads

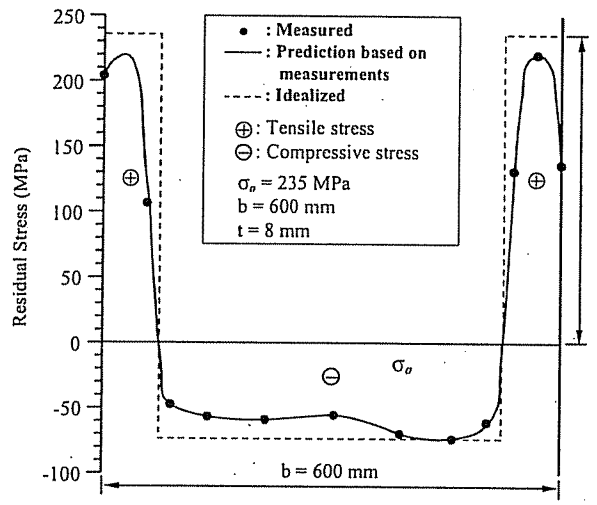

Figure A.5(a) Idealization of residual stress distribution in the mild steel plate between stiffeners based on the measurements of Kmiecik (1970)

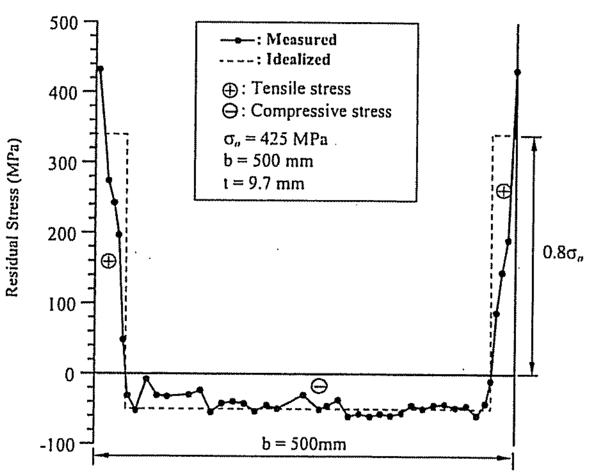

Figure A.5(b) Idealization of residual stress distribution in the high tensile steel plate between stiffeners based on the measurements of Cheng et al. (1996)

the effect of strain rates on the dynamic yield stress, equations (35) or (36) attempt to investigate the strain rate effects on the plate ultimate compressive stress.

The authors appreciate the comments and insights of Dr. Frieze on the nondimensionalization for the dynamic collapse test results. As previously noted, equations (35) or (36) represent the dynamic (strain rate) effects on the ultimate compressive strength and the authors approximate that the dynamic ultimate strength normalized by the quasi-static ultimate strength can be approximately predicted as a function of only strain rates, i.e., without the geometric and material properties which will be included in the calculations of the latter (i.e., the quasi-static ultimate strength). However, we agree that more data are needed to study the degree of approximation involved in the particular case of ultimate strength design of ship plating under dynamic in-plane loading.

Regarding collapse of the plating under dynamic lateral loading, the authors appreciate Dr. Frieze for the information on the effect of different forms of pulse impact loading. As that shown in Fig.39, various types of pressure pulse history can usually be considered. However, our conclusion based on some numerical computations is that idealization of impact pulse with the rectangular type may be appropriate for practical purposes.

Equation (38) based on the hinge line theory may not properly consider the effects of membrane stresses which are neglected, specifically when the plate lateral deflection is large. Hence we agree that the treatment only approximately takes into account the plate large deflection effects.

We thank *Mr. Roberts* for discussing the difficulties on the finite element analysis of material nonlinearity. He raises a question as to why there is a large difference between FEA and the Johnson-Ostenfeld formula in Fig. 11. This is because the FEA has in Fig. 11 calculated the plastic collapse strength of plating while the Johnson-Ostenfeld formula is based on initial yielding. Regarding the difficulty for plate shear strength analysis, we got the FE solutions applying the displacement control method when all edges are simply supported and kept straight. Mr. Roberts questions how engineers can improve their use of nonlinear FEA. This is a tough question while the FE technology has rapidly advanced in the last decade. Certainly the FE solutions significantly depend on how to model the object structures. A basic response to this question is that considering both accuracy and efficiency of FEA the FE modeling should be made so that all primary parameters affecting both geometric and material nonlinear behavior of the structures are more precisely and effectively handled.

Finally, the authors would again like to thank the various discussers for their time and effort, which have helped clarify the design oriented technology for ship plated structures against buckling and plastic collapse developed in this paper. In closing, the authors would like to thank the SNAME Papers Committee members for their guidance related to this paper, and the Society itself for having given the authors the opportunity to present this work.

Additional references

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On Advanced Buckling and Ultimate Strength Design of Ship Plating

Jeom Kee Paik, Member, Pusan National University, Pusan, Korea, **Anil Kumar Thayamballi,** Life Member, Chevron Shipping, San Ramon, California, **Ge Wang,** Member, American Bureau of Shipping, Houston, and **Bong Ju Kim,** Student Member, Pusan National University

ABSTRACT

This paper is a summary of recent research and development in areas related to advanced buckling and ultimate strength design of ship plating, jointly undertaken by the American Bureau of Shipping and the Pusan National University. The behavior of ship plating normally depends on a variety of influential factors, namely geometric / material properties, loading characteristics, initial imperfections, boundary conditions and deterioration arising from corrosion, fatigue cracking and accidental dents. In achieving a more advanced buckling and ultimate strength design of ship plating, we are still confronted with a number of problem areas to be more completely solved and these would need methods that are more sophisticated than most existing simplified approaches. In this regard, this paper focuses on the following five subjects which have been studied by the authors theoretically, numerically and experimentally: mathematical modeling of fabrication related imperfections (i.e., initial deflections and residual stresses), characteristics of the plate buckling with elastically restrained edge conditions, capacity equations based on buckling and ultimate strength under combined loads including biaxial loads, edge shear and lateral pressure, collapse strength characteristics under axial compressive dynamic loads, and design equation of the plate capacity under impact lateral pressure loading. Useful results, important insights and conclusions developed from the studies are summarized and recommendations are made with respect to both technologically improved design procedures, and also needed future research.

NOMENCLATURE

a = plate length

b = plate breadth

b_{fx}, b_{fy} = flange breadth of longitudinals or transverses

D = $\dfrac{Et^3}{12(1-\nu^2)}$

E = Young's modulus

h_{wx}, h_{wy} = web height of longitudinals or transverses

p = average net lateral pressure

t = plate thickness

t_{fx}, t_{fy} = flange thickness of longitudinals or transverses

t_{wx}, t_{wy} = web thickness of longitudinals or transverses

β = $\dfrac{b}{t}\sqrt{\dfrac{\sigma_o}{E}}$

ν = Poisson's ratio

σ_o = yield stress

σ_{rcx}, σ_{rcy} = compressive (negative) residual stress in the x or y direction

σ_{xav}, σ_{yav} = average longitudinal or transverse axial stress (compression: negative, tension: positive)

σ_{xE}, σ_{yE} = elastic longitudinal or transverse compressive buckling stress

σ_{xB}, σ_{yB} = buckling based capacity for σ_{xav} or σ_{yav}

σ_{xu}, σ_{yu} = ultimate strength based capacity for