# Looking for the Tangent Portfolio: Risk Optimization Techniques on Equity Style Buckets

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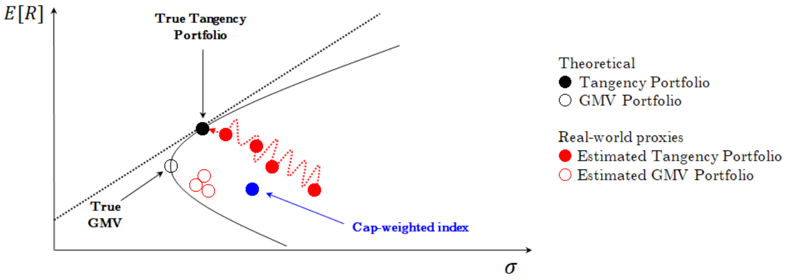
# Agenda

# RESEARCH OBJECTIVE

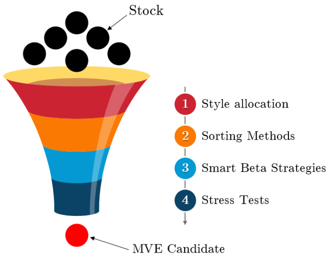
Looking for the tangent portfolio using risk-optimization techniques.

Our objective is to propose a simple intermediary method to proxy for ●



# METHODOLOGY AND RESULTS

- Stratification of the US equity universe (NYSE, AMEX, Nasdaq) into size and book-to-markets equity style buckets
  - Extension to momentum
- Risk-based investment strategies (MV, MD, RP) are shown to provide
  - Better pricing of characteristic-sorted portfolios than existing multifactor models
  - Higher Sharpe ratio than a portfolio made of:
    - market portfolio (Mkt)
    - 30-year US treasury bond (B30)
    - size (SMB) and value (HML) factors



# Motivation

We ground our research into the following papers:

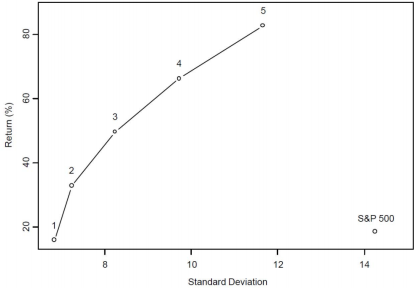
- Daniel et al. (2017, JF)
- Grinblatt and Saxena (2018, JFQA Forthcoming)
- Ao, Li, and Zheng (2018, RFS)

We rely on the following facts and evidence:

1. Caveats over the cap-weighted market benchmarks
2. Sharp rise in multi-factor models and in the number of index-funds and ETFs
3. Inefficiencies of long-short factors
4. Finding MSR is a noisy exercise

# MOTIVATION - CAVEATS OVER THE CAP-WEIGHTED MARKET BENCHMARKS

- "Market indices [...] are if anything inside that [mean-variance] frontier" (Cochrane 2001, Asset Pricing)
- "Cap-weighted stock portfolios are inefficient investments. [...] Even the most comprehensive cap-weighted portfolios occupy positions inside the efficient set" (Haugen and Baker 1991, JPM, p.35)

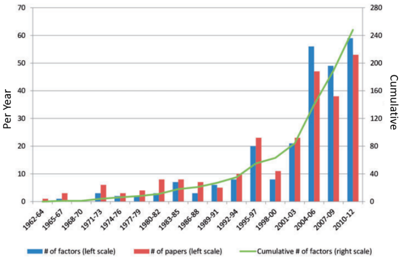



Based on data for the period 1979-1998. The efficient frontier assumes a perfect forecast of the future covariance matrix and of the future mean return. Figure taken from Schwartz(2000, Figure 3, p. 19).

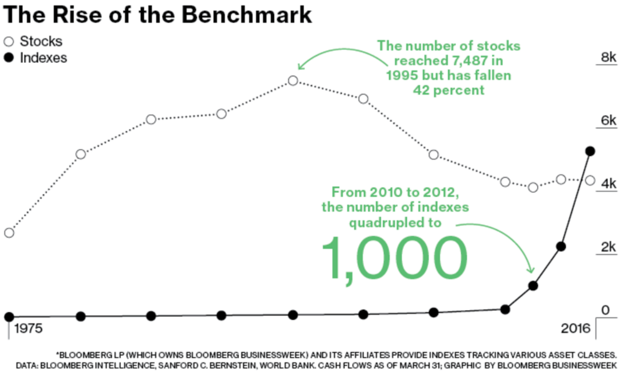
# MOTIVATION - SHARP RISE IN THE NUMBER OF MULTI-FACTOR MODELS

- From 50 significant characteristics (Subrahmanyam, 2010 EFM)
- To **over 300**!
  - 316 anomaly-based firm characteristics, see Harvey and Liu (2016, RFS)
  - 330 characteristics, see Green, Hand, and Zhang (2013, RAS)
  - +430 characteristics, see Hou, Xue, and Zhang (2018, WP)

Source: Harvey and Liu (2016, RFS)



# MOTIVATION - SHARP RISE IN THE NUMBER OF ETFS VERSUS LISTED STOCKS




Source: Bloomberg.com

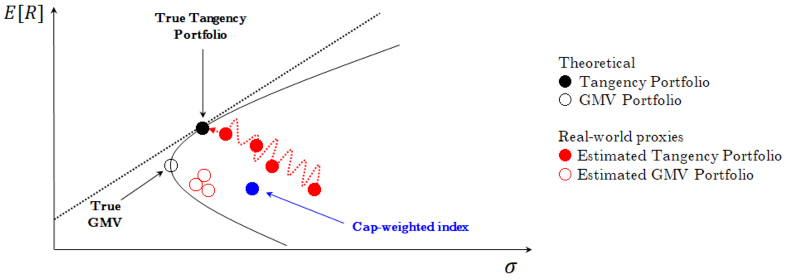
# Motivation – Inefficiencies of long/short factors

Daniel et al. (2017, WP, p. 3):
"This set of portfolios will explain the returns of portfolios sorted on the same characteristics, but are unlikely to span the MVE portfolio of all assets, because they do not take into account the asset covariance structure."

Grinblatt and Saxena (2018, JFQA Forthcoming, p. 5):
"The optimal combination of the factor mimicking portfolios has a significantly lower Sharpe ratio than the optimal combination of the basis portfolios they are created from."

# MOTIVATION – SAMPLE ERRORS WITH MSR ESTIMATE

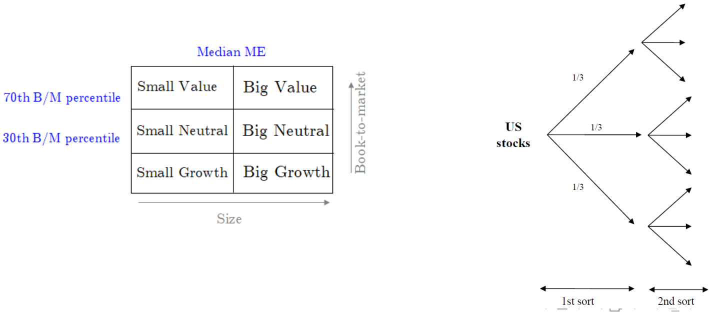
- Sample and specification errors
- Low-risk portfolios : giving up on estimating expected returns
- Robust variance-covariance matrix



# FINDING A CANDIDATE FOR THE MVE PORTFOLIO

1. The opportunity sets: the DNS versus the original Fama-French sorting procedure
   - Independent versus dependent **(D)** sorting
   - NYSE breakpoints vs all names **(N)** breakpoints
   - Double and triple sort (size, value and momentum): 2x3, 3x3 and 3x3x3 (Asymmetric versus **S**ymmetric sort)

**(a)** Independent Sort **(b)** Dependent Sort



# Finding a candidate for the MVE portfolio

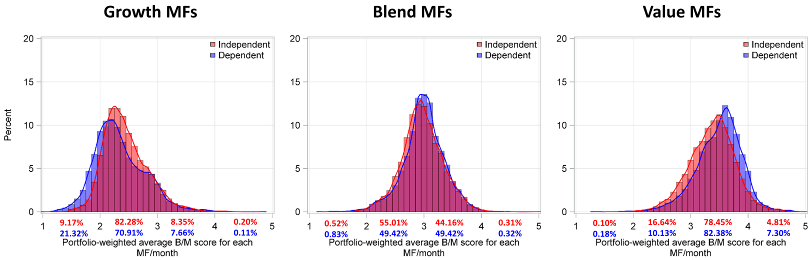
1. The opportunity sets
2. MSR weights replaced by smart beta (risk-based) optimization
   - Minimum Variance (MV) (Clarke, Silva, and Thorley 2013, JPM)
   - Maximum Diversification (MD) (Choueifaty and Coignard 2008, JPM)
   - Risk parity (RP) (Maillard, Roncalli, and Teïletche 2010, JPM)

| Strategy | Objective Function | Constraints |
|---|---|---|
| Minimum Variance (MV) | $\min f(w) = \sum_i^N \sum_j^N w_i \sigma_{ij} w_j$ | |
| Maximum Diversification (MD) | $\max f(w) = \frac{\sum_i^N w_i \sigma_i}{\sqrt{\sum_i^N \sum_j^N w_i \sigma_{ij} w_j}}$ | $w_i \in [0,1]$ and $\sum_{i=1}^N w_i = 1$ |
| Risk parity (RP) | $\min f(w) = \sum_i^N \sum_j^N (w_i \times (\Sigma w)_i - w_j \times (\Sigma w)_j)^2$ | |

# FINDING A CANDIDATE FOR THE MVE PORTFOLIO - THE OPPORTUNITY SETS

The DNS sorting procedure allows for:

- A better stratification of the US equity universe



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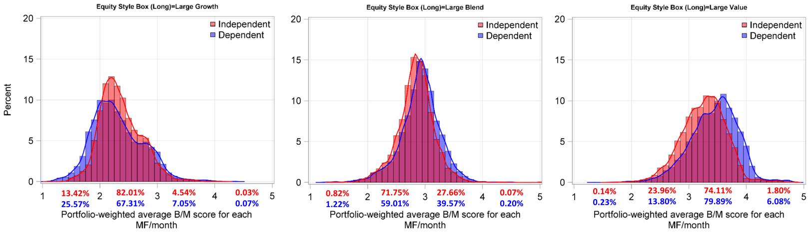

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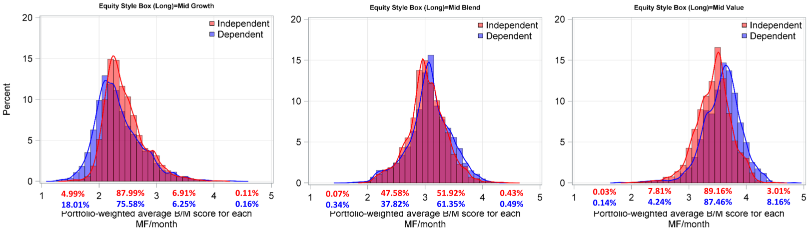

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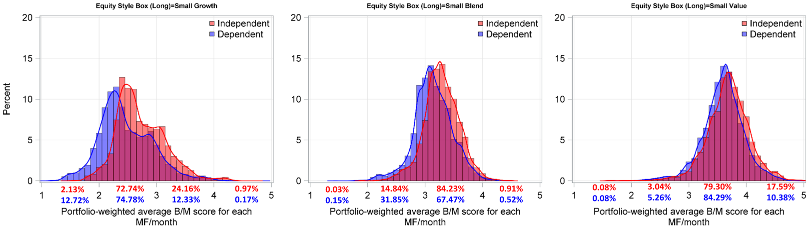


# FINDING A CANDIDATE FOR THE MVE PORTFOLIO - THE OPPORTUNITY SETS

The DNS sorting procedure allows for:

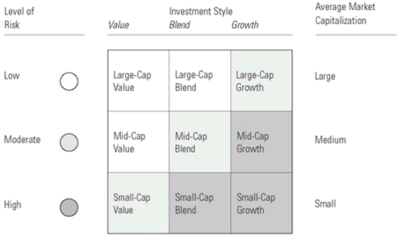
- A better stratification of the US equity universe
- Better diversification

| # Portfolios | Independent Sort (1) | Dependent Sort (2) | Difference (1)-(2) |
|---|---|---|---|
| | Panel A: Cap-weighted Portfolios | | |
| 2x3 | 84.99 | 78.00 | 6.99 |
| 3x3 | 84.99 | 75.81 | 9.18 |
| 3x3x3 | 78.38 | 66.8 | 11.58 |

# FINDING A CANDIDATE FOR THE MVE PORTFOLIO - THE OPPORTUNITY SETS

The DNS sorting procedure allows for:

- A better stratification of the US equity universe
- Better diversification
- Similar to other portfolio sorts, a reduction of the complexity of the universe (consistent with the categorization process of Barberis and Shleifer (2003))



# FINDING A CANDIDATE FOR THE MVE PORTFOLIO - MSR WEIGHTS REPLACED BY SMART BETA (RISK-BASED) OPTIMIZATION

- Long-only investment scheme
- Avoid the empirical challenge of estimating expected returns

# US EQUITIES

**We employ:**

- Dataset from the merge of CRSP and Compustat.
- All stocks listed on NYSE, NASDAQ, and AMEX stocks and share code of 10 or 11.
- Sample period ranges from July 1963 to December 2015.

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- Shares (SHROUT) and price (PRC)
- Stock return (RET) data for month t
- 2 years of listing on COMPUSTAT (survival bias)

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**Characteristics:**

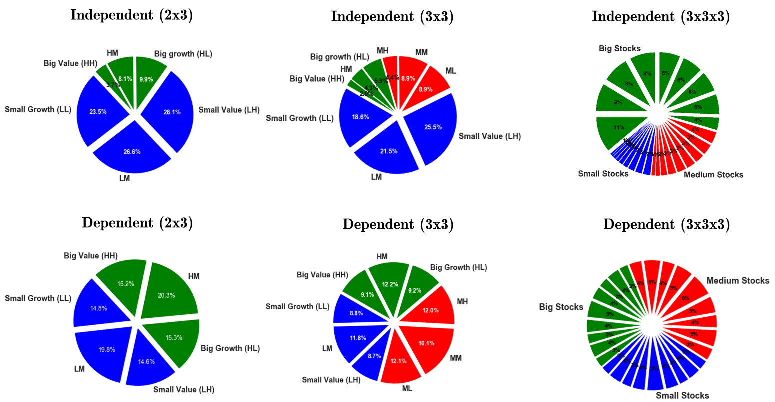
- Market equity (firm size) as $SHARE \times PRICE$
- Book-to-market equity as $BE/ME$
- Momentum is the $t$-2 to $t$-12 cumulative return of stock

# Characteristic-Sorted Portfolios

Each year in June, we sort US stocks on the following traditional characteristics.

- size and value (2x3)
- size and value (3x3)
- size and value and momentum (3x3x3)

**Average distribution of stock in portfolios**



# EMPIRICAL TESTS AND RESULTS IN A NUTSHELL

1. Smart investment strategies on DSN portfolios achieve better diversification return than other smart investment strategies and equally weighted scheme
   - Diversification return framework of Booth and Fama (1992, FAJ) and Willenbrock (2011, FAJ)
2. Strategic beta portfolios constructed on dependent equity style buckets outperform a single-index model (using CW factor), a multi-factor model (FF-3 Factors) and other strategic beta portfolios
   - Mean-variance spanning of Kan and Zhou (2012, AEF)
   - Bootstrap procedure similar to Fama and French (2010, JF) and Harvey and Liu (2016, WP)
   - Factor selection technique from Harvey and Liu (2016, WP)

# DIVERSIFICATION RETURN

Following Booth and Fama (1992, FAJ) and Willenbrock (2011, FAJ), the diversification return is given by,

$$\mathrm{DR}^{FW} = \underbrace{\mu_p - \sum_i^N w_i \mu_i}_{\mathrm{DR}_1^{FW} = 0 \text{ if weights are constant}} + \underbrace{\frac{1}{2}\left(\sum_i^N w_i \sigma_i^2 - \sigma_p^2\right)}_{\mathrm{DR}_2^{FW} = \text{variance reduction benefit}} \quad (1)$$

The relationship assumes that ,

- weights $w_i$ are held constant over the estimation period,
- $i$ stands for the $i^{th}$ security in the portfolio $p$,
- $FW$ denotes Fixed-Weight.

# DIVERSIFICATION RETURN

**Issues:**

- Weights of the low risk strategies are not constant over time. For rebalancing strategies (non fixed weight), Erb and Harvey (2006, FAJ) use of the average of the weights over the sample period ($\overline{w_i} = \frac{1}{T}\sum_1^T w_i^t$).

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- The endogenous fixed weights benchmark used in the FW configuration differ sharply across the strategies.

**Proposition:** diversification return with regard to an EW benchmark.

$$\mathrm{DR}^{EW} = \underbrace{\mu_p - \frac{1}{N}\sum_i^N \mu_i}_{\mathrm{DR}_1^{EW}} + \underbrace{\frac{1}{2}\left(\frac{1}{N}\sum_i^N \sigma_i^2 - \sigma_p^2\right)}_{\mathrm{DR}_2^{EW}} \tag{2}$$

# DIVERSIFICATION RETURN

We test the difference in the diversification components ($DR_1$, $DR_2$, and $DR$) using the bootstrap method of Ledoit and Wolf (2008, JEF).

Fixed-Weight (FW) Benchmark

| | $DR_1^{FW}$ | | | | $DR_2^{FW}$ | | | | $DR^{FW}$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Ind | Dep | Δ Dep-Ind | p-val[b] | Ind | Dep | Δ Dep-Ind | p-val[b] | Ind | Dep | Δ Dep-Ind | p-val[b] |
| $MD_{2x3}$ | -0.005 | -0.003 | 0.002 | 0.936 | 0.025 | 0.038 | 0.012 | **0.000** | 0.020 | 0.034 | 0.014 | 0.524 |
| $MD_{3x3}$ | 0.004 | -0.032 | -0.035 | 0.257 | 0.031 | 0.050 | 0.019 | **0.000** | 0.034 | 0.018 | -0.016 | 0.594 |
| $MD_{3x3x3}$ | -0.041 | -0.075 | -0.034 | 0.428 | 0.046 | 0.080 | 0.034 | **0.000** | 0.005 | 0.005 | 0.000 | 0.993 |
| $MV_{2x3}$ | 0.013 | -0.025 | -0.038 | 0.554 | 0.024 | 0.031 | 0.007 | 0.165 | 0.037 | 0.006 | -0.031 | 0.635 |
| $MV_{3x3}$ | -0.001 | -0.036 | -0.035 | 0.581 | 0.024 | 0.045 | 0.020 | **0.003** | 0.023 | 0.009 | -0.014 | 0.807 |
| $MV_{3x3x3}$ | -0.018 | -0.115 | -0.097 | **0.093** | 0.047 | 0.070 | 0.023 | **0.001** | 0.029 | -0.045 | -0.074 | 0.191 |
| $RP_{2x3}$ | 0.005 | 0.005 | 0.000 | 0.992 | 0.024 | 0.036 | 0.012 | **0.000** | 0.030 | 0.041 | 0.012 | 0.308 |
| $RP_{3x3}$ | 0.005 | 0.000 | -0.005 | 0.679 | 0.027 | 0.042 | 0.014 | **0.000** | 0.032 | 0.041 | 0.009 | 0.460 |
| $RP_{3x3x3}$ | -0.001 | -0.010 | -0.009 | 0.522 | 0.043 | 0.065 | 0.023 | **0.000** | 0.041 | 0.056 | 0.014 | 0.307 |

*number of bootstraps=4999
** figures are from gross return on a monthly basis (in %)
*** Block size for bootstrap = 10

Bootstrap

# DIVERSIFICATION RETURN

In this framework, results suggests that dependent-sorted portfolios provide significantly

- greater variance reduction benefits
- greater diversification return

Equal-Weight (EW) Benchmark

| | $DR_1^{EW}$ | | | | $DR_2^{EW}$ | | | | $DR^{EW}$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Ind | Dep | Δ Dep-Ind | p-val | Ind | Dep | Δ Dep-Ind | p-val | Ind | Dep | Δ Dep-Ind | p-val |
| $MD_{2x3}$ | 0.005 | 0.088 | 0.083 | **0.012** | 0.027 | 0.038 | 0.011 | **0.003** | 0.032 | 0.126 | 0.094 | **0.007** |
| $MD_{3x3}$ | 0.010 | 0.070 | 0.060 | 0.125 | 0.033 | 0.048 | 0.015 | **0.000** | 0.043 | 0.118 | 0.075 | **0.055** |
| $MD_{3x3x3}$ | -0.034 | 0.063 | 0.097 | 0.107 | 0.046 | 0.061 | 0.014 | **0.010** | 0.012 | 0.124 | 0.112 | **0.067** |
| $MV_{2x3}$ | 0.082 | 0.218 | 0.136 | **0.071** | 0.033 | 0.028 | -0.004 | 0.407 | 0.115 | 0.246 | 0.131 | **0.075** |
| $MV_{3x3}$ | 0.050 | 0.143 | 0.093 | 0.163 | 0.034 | 0.046 | 0.012 | **0.066** | 0.084 | 0.189 | 0.105 | 0.112 |
| $MV_{3x3x3}$ | 0.022 | 0.019 | -0.003 | 0.955 | 0.061 | 0.080 | 0.018 | **0.002** | 0.084 | 0.099 | 0.015 | 0.785 |
| $RP_{2x3}$ | 0.020 | 0.056 | 0.036 | **0.013** | 0.026 | 0.036 | 0.010 | **0.001** | 0.046 | 0.092 | 0.047 | **0.005** |
| $RP_{3x3}$ | 0.019 | 0.048 | 0.029 | **0.063** | 0.029 | 0.042 | 0.013 | **0.000** | 0.048 | 0.090 | 0.041 | **0.015** |
| $RP_{3x3x3}$ | 0.011 | 0.035 | 0.024 | 0.102 | 0.046 | 0.068 | 0.022 | **0.000** | 0.057 | 0.102 | 0.046 | **0.007** |

*number of bootstraps=4999
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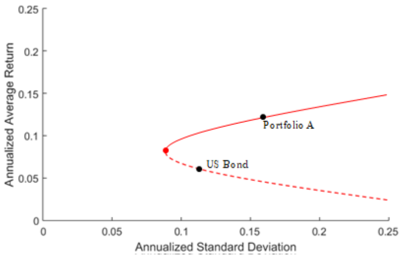
Bootstrap

# TEST OF MEAN-VARIANCE SPANNING

Illustration of Kan and Zhou (2012, AEF) mean-variance spanning test :

$H_0^1$: Tangency Portfolio




Benchmark Assets ($R_1$):

- US Bond
- Portfolio A

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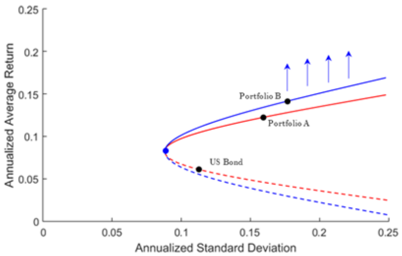



Benchmark Assets ($R_1$):

- US Bond
- Portfolio A

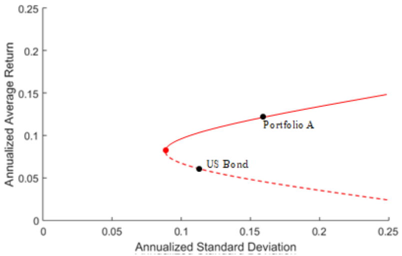
Test Asset ($R_2$):

- Portfolio B

# Test of Mean-Variance Spanning

Illustration of Kan and Zhou (2012, AEF) test of Mean-Variance spanning :

$H_0^2$: GMV Portfolio

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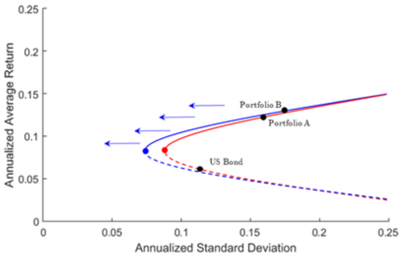


Benchmark Assets ($R_1$):

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Test Asset ($R_2$):

- Portfolio B

# TEST OF MEAN-VARIANCE SPANNING

Huberman and Kandel (1987, JF) define the following regression test:

$$R_2^t = \alpha + \beta R_1^t + e^t \tag{3}$$

The null hypothesis $H_0$ sets $\alpha = 0$ and $\delta = 1 - \beta = 0$.

Considering an efficient frontier comprising $K + N$ assets, the weights of the $N$ assets into the tangent ($Qw_1$) and GMV ($Qw_2$) portfolios are defined as:

$$\begin{aligned} Qw_1 &= \frac{QV^{-1}\mu}{1'_{N+K}V^{-1}\mu} = \frac{\Sigma^{-1}\alpha}{1'_{N+K}V^{-1}\mu} \\ Qw_2 &= \frac{QV^{-1}1_{N+K}}{1'_{N+K}V^{-1}1_{N+K}} = \frac{\Sigma^{-1}\delta}{1'_{N+K}V^{-1}1_{N+K}} \end{aligned} \tag{4}$$

where $Q = [0_{N\times K}, I_N]$, $I_N$ is an $N \times N$ identity matrix, $\Sigma = V_{22} - V_{21}V_{11}^{-1}V_{12}$, and $V$ is the variance-covariance matrix of the $K$ benchmark assets ($R_1$) plus the $N$ test assets ($R_2$) such that,

$$V = Var[R_1, R_2] = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}$$

# Test of Mean-Variance Spanning

Step-down procedure to test the spanning hypothesis (Kan and Zhou 2012, AEF):

$$H_0^1 = \alpha = 0_N, \text{ such that } Qw_1 = 0.$$

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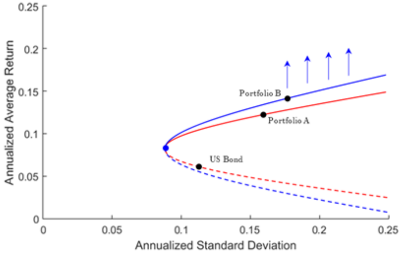
$H_0^1$: Tangency Portfolio




The $F$-test ($H_0^1$):

$$F_1 = \frac{T-K-N}{N}\frac{\hat{a}-\hat{a}_1}{1+\hat{a}_1}$$

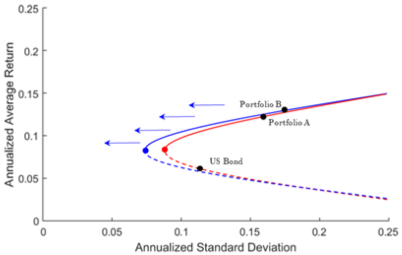
- $T$ is the number of observations
- $K$ is the number of benchmark assets
- $N$ is the number of test assets
- $\hat{a}_1 = \hat{\mu}_1' \hat{V}_{11}^{-1} \hat{\mu}_1$
- $\hat{V}_{11}$: the variance of the benchmark assets
- $\hat{\mu}_1$: the vector of mean return of the benchmark assets
- $\hat{a}$ but refers to the benchmark assets ($R_1$) plus the new test asset ($R_2$)

# TEST OF MEAN-VARIANCE SPANNING

Step-down procedure to test the spanning hypothesis (Kan and Zhou 2012, AEF):

$$H_0^2 : \delta = 1_N - \beta 1_K = 0_N | \alpha = 0_N, \text{ such that } Qw_2 = 0 \text{ conditional on } Qw_1 = 0.$$

$H_0^2$: GMV Portfolio




The $F$-test ($H_0^2$):

$$F_2 = \frac{T-K-N+1}{N}\left[\frac{\hat{c}+\hat{d}}{\hat{c}_1+\hat{d}_1}\frac{1+\hat{a}_1}{1+\hat{a}} - 1\right]$$

- $\hat{a}_1 = \hat{\mu}_1' \hat{V}_{11}^{-1} \hat{\mu}_1$
- $\hat{b}_1 = \hat{\mu}_1' \hat{V}_{11}^{-1} 1_K$
- $\hat{c}_1 = 1_K' \hat{V}_{11}^{-1} 1_K$
- $\hat{d}_1 = \hat{a}_1 \hat{c}_1 - \hat{b}_1^2$
- $\hat{V}_{11}$: the variance of the benchmark assets
- $\hat{\mu}_1$: the vector of mean return of the benchmark assets

$\hat{a}$, $\hat{b}$, $\hat{c}$ and $\hat{d}$ refers to the benchmark assets ($R_1$) plus the new test asset ($R_2$)

# BOOTSTRAP APPROACH ON TEST OF MEAN-VARIANCE SPANNING

The null hypothesis should be true in-sample (Harvey and Liu (2016, WP) and White (2000, ECO)):

$$\rightarrow Qw_1 = 0 \text{ and } Qw_2 = 0.$$

# Bootstrap Approach on test of Mean-Variance Spanning

The null hypothesis should be true in-sample (Harvey and Liu (2016, WP) and White (2000, ECO)):

$$\rightarrow Qw_1 = 0 \text{ and } Qw_2 = 0.$$

**Step 1:** Orthogonalization under the null

$$R_2^t = \alpha + \beta R_1^t + R_2^{t,e} \tag{5}$$

- $R_2^{orth} = R_2^{t,e} + R_{1,MVE}^t$ by construction $\alpha = 0$ and $\beta_{MVE} = 1 \rightarrow Qw_1 = 0$ and $Qw_2 = 0$
- $R_{1,MVE}$ is the proxy for the market portfolio present in $R_1$

# Bootstrap Approach on test of Mean-Variance Spanning

The null hypothesis should be true in-sample (Harvey and Liu (2016, WP) and White (2000, ECO)):

$$\rightarrow Qw_1 = 0 \text{ and } Qw_2 = 0.$$

**Step 1:** Orthogonalization under the null

$$R_2^t = \alpha + \beta R_1^t + R_2^{t,e} \tag{5}$$

- $R_2^{orth} = R_2^{t,e} + R_{1,MVE}^t$ by construction $\alpha = 0$ and $\beta_{MVE} = 1 \rightarrow Qw_1 = 0$ and $Qw_2 = 0$
- $R_{1,MVE}$ is the proxy for the market portfolio present in $R_1$

**Step 2:** Bootstrap (Harvey and Liu 2016, WP)

- preserves the cross-sectional correlations among the benchmark ($R_1$) and test ($R_2^{orth}$) assets
- preserves the uncertainty of the time-series: bootstrap sampling length=original time-series length

# BOOTSTRAP APPROACH ON TEST OF MEAN-VARIANCE SPANNING

**Step 3:** Test of Mean-Variance Spanning

- Apply the test of mean-variance spanning from Kan and Zhou (2012, AEF)
- Outputs:
  - Tangency portoflio: range of $F^b_{1,ind}$ and $F^b_{1,dep}$ with $\{b = 1, 2, \dots, B\}$
  - GMV portoflio: range of $F^b_{2,ind}$ and $F^b_{2,dep}$ with $\{b = 1, 2, \dots, B\}$
  - Where,

$$F_1 = \frac{T-K-N}{N}\frac{\hat{a}-\hat{a}_1}{1+\hat{a}_1}$$

$$F_2 = \frac{T-K-N+1}{N}\left[\frac{\hat{c}+\hat{d}}{\hat{c}_1+\hat{d}_1}\frac{1+\hat{a}_1}{1+\hat{a}} - 1\right]$$

# BOOTSTRAP APPROACH ON TEST OF MEAN-VARIANCE SPANNING

**Step 3:** Mean-Variance Spanning Test (cont'd)

- Harvey and Liu (2016, WP)'s bootstrap approach robust for multiple testing
- Conservative **reference point**:
  - $F_1^b = max(F_{1,ind}^b, F_{1,dep}^b)$ with $\{b = 1, 2, ..., B\}$
  - $F_2^b = max(F_{2,ind}^b, F_{2,dep}^b)$ with $\{b = 1, 2, ..., B\}$
- Bootstrap p-value for the F-tests

$F_1^o$ p-value:

$$\text{p-val}_{ind}^b = \frac{\#\{F_1^b > F_{1,ind}^o\}}{B}$$

$$\text{p-val}_{dep}^b = \frac{\#\{F_1^b > F_{1,dep}^o\}}{B}$$

$F_2^o$ p-value:

$$\text{p-val}_{ind}^b = \frac{\#\{F_2^b > F_{1,ind}^o\}}{B}$$

$$\text{p-val}_{dep}^b = \frac{\#\{F_2^b > F_{1,dep}^o\}}{B}$$

# SPANNING OF SINGLE INDEX AND MULTIFACTOR MODELS

**Sample period:** July 1963 - December 2015

- Benchmark assets ($R_1$)= CW-Market Portfolio (Mkt) + 30-Year US Treasury Bond (B30)
- Test asset ($R_2$)= Smart Beta (SB)

$$R_2^t = \alpha + \beta_1 B30^t + \beta_2 Mkt^t + R_2^{t,e}$$

- $R_2^{orth} = R_2^{t,e} + Mkt^t$

| MVE candidates | $\alpha_{ind}$ | $F_{1,ind}$ | p-val[b] | $F_{2,ind}$ | p-val[b] | $\alpha_{dep}$ | $F_{1,dep}$ | p-val[b] | $F_{2,dep}$ | p-val[b] |
|---|---|---|---|---|---|---|---|---|---|---|
| | Panel A: $R_1 = MKT + B30$, $R_2 = SB$, $R_2^{orth} = SB^e + Mkt$ | | | | | | | | | |
| $MD_{2x3}$ | 0.0020 | 11.69 | **0.00** | 2.37 | 0.31 | 0.0031 | 13.00 | **0.00** | 13.52 | 0.01 |
| $MD_{3x3}$ | 0.0022 | 11.89 | **0.00** | 2.96 | 0.25 | 0.0034 | 10.66 | **0.00** | 21.30 | 0.00 |
| $MD_{3x3x3}$ | 0.0019 | 7.21 | **0.01** | 1.42 | 0.58 | 0.0038 | 9.45 | **0.00** | 19.39 | 0.00 |
| $MV_{2x3}$ | 0.0032 | 13.63 | **0.00** | 11.83 | 0.01 | 0.0049 | 15.55 | **0.00** | 23.12 | 0.00 |
| $MV_{3x3}$ | 0.0029 | 10.60 | **0.00** | 6.68 | 0.07 | 0.0044 | 13.39 | **0.00** | 28.21 | 0.00 |
| $MV_{3x3x3}$ | 0.0029 | 13.60 | **0.00** | 10.24 | 0.02 | 0.0034 | 10.02 | **0.00** | 19.87 | 0.00 |
| $RP_{2x3}$ | 0.0021 | 13.03 | **0.00** | 1.62 | 0.42 | 0.0027 | 10.60 | **0.00** | 7.48 | 0.04 |
| $RP_{3x3}$ | 0.0022 | 11.57 | **0.00** | 0.49 | 0.72 | 0.0029 | 8.70 | **0.00** | 8.49 | 0.02 |
| $RP_{3x3x3}$ | 0.0023 | 12.09 | **0.00** | 0.25 | 0.84 | 0.0031 | 10.28 | **0.00** | 7.34 | 0.04 |

* Mkt, B30 and SB are taken in excess of the risk-free rate (one-month T-bill from Ibbotson)

## Spanning of single index and multifactor models

**Sample period:** July 1963 - December 2015

- Benchmark assets ($R_1$)= CW-Market Portfolio (Mkt) + 30-Year US Treasury Bond (B30) + SMB + HML
- Test asset ($R_2$)= Smart Beta (SB)

$$R_2^t = \alpha + \beta_1 B30^t + \beta_2 Mkt^t + \beta_3 SMB^t + \beta_4 HML^t + R_2^{t,e}$$

- $R_2^{orth} = R_2^{t,e} + Mkt^t$

| MVE candidates | $\alpha_{ind}$ | $F_{1,ind}$ | p-val[b] | $F_{2,ind}$ | p-val[b] | $\alpha_{dep}$ | $F_{1,dep}$ | p-val[b] | $F_{2,dep}$ | p-val[b] |
|---|---|---|---|---|---|---|---|---|---|---|
| | Panel B: $R_1 = MKT + B30 + SMB + HML, R_2 = SB, R_2^{orth} = SB^e + Mkt$ | | | | | | | | | |
| $MD_{2x3}$ | 0.0001 | 0.21 | 0.85 | 1419.96 | 0.00 | 0.0011 | 4.89 | **0.05** | 578.95 | 0.00 |
| $MD_{3x3}$ | 0.0002 | 0.66 | 0.63 | 1392.77 | 0.00 | 0.0011 | 3.13 | 0.14 | 365.03 | 0.00 |
| $MD_{3x3x3}$ | -0.0003 | 0.65 | 0.63 | 824.06 | 0.00 | 0.0012 | 2.22 | 0.22 | 285.69 | 0.00 |
| $MV_{2x3}$ | 0.0009 | 2.22 | 0.22 | 317.78 | 0.00 | 0.0022 | 6.21 | **0.02** | 249.22 | 0.00 |
| $MV_{3x3}$ | 0.0006 | 1.02 | 0.47 | 438.93 | 0.00 | 0.0018 | 4.59 | **0.06** | 254.99 | 0.00 |
| $MV_{3x3x3}$ | 0.0007 | 2.22 | 0.23 | 564.35 | 0.00 | 0.0010 | 1.92 | 0.28 | 310.91 | 0.00 |
| $RP_{2x3}$ | 0.0002 | 0.92 | 0.47 | 1786.24 | 0.00 | 0.0007 | 3.21 | 0.11 | 882.21 | 0.00 |
| $RP_{3x3}$ | 0.0002 | 0.88 | 0.52 | 2015.93 | 0.00 | 0.0007 | 2.36 | 0.20 | 801.67 | 0.00 |
| $RP_{3x3x3}$ | 0.0003 | 1.16 | 0.44 | 1895.22 | 0.00 | 0.0010 | 4.27 | **0.06** | 811.01 | 0.00 |

* Mkt, B30 and SB are taken in excess of the risk-free rate (one-month T-bill from Ibbotson)

# SPANNING OF SINGLE INDEX AND MULTIFACTOR MODELS

**Sample period:** July 1963 - December 2015

- Benchmark assets ($R_1$)= Smart Beta (SB) + 30-Year US Treasury Bond (B30)
- Test asset ($R_2$)= CW-Market Portfolio (Mkt)

$$R_2^t = \alpha + \beta_1 B30^t + \beta_2 SB^t + R_2^{t,e}$$

- $R_2^{orth} = R_2^{t,e} + SB^t$

| MVE candidates | $\alpha_{ind}$ | $F_{1,ind}$ | p-val[b] | $F_{2,ind}$ | p-val[b] | $\alpha_{dep}$ | $F_{1,dep}$ | p-val[b] | $F_{2,dep}$ | p-val[b] |
|---|---|---|---|---|---|---|---|---|---|---|
| | Panel C: $R_1 = SB + B30$, $R_2 = Mkt$, $R_2^{orth} = Mkt^e + SB$ | | | | | | | | | |
| $MD_{2x3}$ | -0.0013 | 5.92 | **0.01** | 7.93 | 0.03 | -0.0016 | 4.20 | **0.05** | 8.11 | 0.03 |
| $MD_{3x3}$ | -0.0014 | 5.51 | **0.02** | 9.40 | 0.02 | -0.0013 | 1.99 | 0.22 | 10.40 | 0.01 |
| $MD_{3x3x3}$ | -0.0010 | 2.22 | 0.18 | 17.25 | 0.01 | -0.0010 | 0.92 | 0.49 | 19.97 | 0.01 |
| $MV_{2x3}$ | -0.0017 | 3.97 | **0.05** | 12.51 | 0.01 | -0.0016 | 2.31 | 0.17 | 21.01 | 0.00 |
| $MV_{3x3}$ | -0.0014 | 2.63 | 0.13 | 18.31 | 0.00 | -0.0014 | 1.87 | 0.22 | 15.30 | 0.00 |
| $MV_{3x3x3}$ | -0.0017 | 4.60 | **0.05** | 10.31 | 0.01 | -0.0012 | 1.45 | 0.30 | 14.54 | 0.00 |
| $RP_{2x3}$ | -0.0014 | 6.82 | **0.01** | 9.52 | 0.02 | -0.0014 | 3.44 | **0.07** | 10.80 | 0.01 |
| $RP_{3x3}$ | -0.0014 | 5.35 | **0.03** | 16.33 | 0.00 | -0.0011 | 1.81 | 0.21 | 16.07 | 0.00 |
| $RP_{3x3x3}$ | -0.0014 | 5.50 | **0.02** | 19.14 | 0.00 | -0.0013 | 2.46 | 0.14 | 17.64 | 0.00 |

* Mkt, B30 and SB are taken in excess of the risk-free rate (one-month T-bill from Ibbotson)

# SPANNING OF SINGLE INDEX AND MULTIFACTOR MODELS

**Sample period:** July 1993 - December 2015

- Benchmark assets ($R_1$)= CW-Market Portfolio (Mkt) + 30-Year US Treasury Bond (B30)
- Test asset ($R_2$)= Smart Beta (SB)

$$R_2^t = \alpha + \beta_1 B30^t + \beta_2 Mkt^t + R_2^{t,e}$$

- $R_2^{orth} = R_2^{t,e} + Mkt^t$

| MVE candidates | $\alpha_{ind}$ | $F_{1,ind}$ | p-val[b] | $F_{2,ind}$ | p-val[b] | $\alpha_{dep}$ | $F_{1,dep}$ | p-val[b] | $F_{2,dep}$ | p-val[b] |
|---|---|---|---|---|---|---|---|---|---|---|
| Panel A: $R_1 = MKT + B30, R_2 = SB, R_2^{orth} = SB^e + Mkt$ | | | | | | | | | | |
| $MD_{2x3}$ | 0.0019 | 3.77 | 0.11 | 6.48 | 0.09 | 0.0048 | 12.70 | **0.00** | 16.12 | 0.00 |
| $MD_{3x3}$ | 0.0024 | 5.08 | **0.06** | 7.26 | 0.05 | 0.0049 | 9.14 | **0.01** | 16.55 | 0.00 |
| $MD_{3x3x3}$ | 0.0016 | 1.86 | 0.33 | 5.87 | 0.17 | 0.0053 | 9.38 | **0.01** | 16.66 | 0.02 |
| $MV_{2x3}$ | 0.0039 | 9.03 | **0.00** | 19.93 | 0.00 | 0.0071 | 14.13 | **0.00** | 17.80 | 0.00 |
| $MV_{3x3}$ | 0.0038 | 7.55 | **0.02** | 11.39 | 0.01 | 0.0066 | 13.10 | **0.00** | 21.37 | 0.00 |
| $MV_{3x3x3}$ | 0.0042 | 11.36 | **0.00** | 18.15 | 0.00 | 0.0050 | 9.92 | **0.00** | 19.77 | 0.00 |
| $RP_{2x3}$ | 0.0021 | 4.73 | **0.07** | 4.94 | 0.12 | 0.0039 | 9.39 | **0.01** | 9.07 | 0.03 |
| $RP_{3x3}$ | 0.0024 | 5.02 | **0.05** | 3.44 | 0.21 | 0.0039 | 6.64 | **0.02** | 7.98 | 0.05 |
| $RP_{3x3x3}$ | 0.0027 | 6.19 | **0.03** | 4.37 | 0.13 | 0.0043 | 8.69 | **0.01** | 9.42 | 0.02 |

* Mkt, B30 and SB are taken in excess of the risk-free rate (one-month T-bill from Ibbotson)

# SPANNING OF SINGLE INDEX AND MULTIFACTOR MODELS

**Sample period:** July 1993 - December 2015

- Benchmark assets ($R_1$)= CW-Market Portfolio (Mkt) + 30-Year US Treasury Bond (B30) + SMB + HML
- MVE market portfolio proxy in is Mkt ($R_{1,MVE}$)
- Test asset ($R_2$)= Smart Beta (SB)

$$R_2^t = \alpha + \beta_1 B30^t + \beta_2 Mkt^t + \beta_3 SMB^t + \beta_4 HML^t + R_2^{t,e}$$

- $R_2^{orth} = R_2^{t,e} + Mkt^t$

| MVE candidates | $\alpha_{ind}$ | $F_{1,ind}$ | p-val$^b$ | $F_{2,ind}$ | p-val$^b$ | $\alpha_{dep}$ | $F_{1,dep}$ | p-val$^b$ | $F_{2,dep}$ | p-val$^b$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | Panel B: $R_1 = MKT + B30 + SMB + HML$, $R_2 = SB$, $R_2^{orth} = SB^e + Mkt$ | | | | | | | | | |
| $MD_{2x3}$ | 0.0006 | 1.52 | 0.40 | 384.37 | 0.00 | 0.0036 | 16.78 | **0.00** | 118.87 | 0.00 |
| $MD_{3x3}$ | 0.0010 | 3.71 | 0.11 | 382.82 | 0.00 | 0.0036 | 9.31 | **0.00** | 63.08 | 0.00 |
| $MD_{3x3x3}$ | 0.0001 | 0.02 | 0.99 | 249.23 | 0.00 | 0.0041 | 8.88 | **0.01** | 39.76 | 0.00 |
| $MV_{2x3}$ | 0.0027 | 6.52 | **0.03** | 42.73 | 0.00 | 0.0056 | 14.39 | **0.00** | 44.38 | 0.00 |
| $MV_{3x3}$ | 0.0025 | 5.89 | **0.02** | 92.31 | 0.00 | 0.0052 | 13.19 | **0.00** | 43.67 | 0.00 |
| $MV_{3x3x3}$ | 0.0028 | 11.43 | **0.00** | 124.93 | 0.00 | 0.0038 | 9.86 | **0.00** | 51.57 | 0.00 |
| $RP_{2x3}$ | 0.0008 | 3.50 | 0.11 | 563.63 | 0.00 | 0.0026 | 14.80 | **0.00** | 217.80 | 0.00 |
| $RP_{3x3}$ | 0.0010 | 5.07 | **0.05** | 652.92 | 0.00 | 0.0026 | 9.23 | **0.00** | 174.22 | 0.00 |
| $RP_{3x3x3}$ | 0.0013 | 7.72 | **0.02** | 618.11 | 0.00 | 0.0031 | 13.91 | **0.00** | 158.94 | 0.00 |

* Mkt, B30 and SB are taken in excess of the risk-free rate (one-month T-bill from Ibbotson)

# HORSE RACE BETWEEN SMART INVESTMENT PORTFOLIOS

- Panel A: MVE market portfolio proxy is $SB^{\textbf{net}}_{dep}$ ($R_{1,MVE}$)
- Panel B: MVE market portfolio proxy is $SB^{\textbf{net}}_{ind}$ ($R_{1,MVE}$)

| | Panel A: $R_1 = B30 + SB^{\textbf{net}}_{dep}$, $R_2 = SB^{\textbf{net}}_{ind}$ | | | | Panel B: $R_1 = B30 + SB^{\textbf{net}}_{ind}$, $R_2 = SB^{\textbf{net}}_{dep}$ | | | | MVE | GMV |
|---|---|---|---|---|---|---|---|---|---|---|
| | $F^o_{1,ind}$ | p-val$^b$ | $F^o_{2,ind}$ | p-val$^b$ | $F^o_{1,dep}$ | p-val$^b$ | $F^o_{2,dep}$ | p-val$^b$ | Candidate | Candidate |
| $MD_{2x3}$ | 0.801 | 0.448 | 0.064 | 0.944 | 5.823 | 0.012 | 11.175 | 0.013 | Dependent | Dependent |
| $MD_{3x3}$ | 0.016 | 0.973 | 0.121 | 0.914 | 4.160 | 0.049 | 17.576 | 0.000 | Dependent | Dependent |
| $MD_{3x3x3}$ | 0.001 | 1.000 | 2.470 | 0.361 | 5.959 | 0.026 | 18.900 | 0.005 | Dependent | Dependent |
| $MV_{2x3}$ | 0.118 | 0.871 | 6.515 | 0.054 | 5.254 | 0.015 | 10.170 | 0.015 | Dependent | Dependent |
| $MV_{3x3}$ | 0.035 | 0.956 | 0.090 | 0.954 | 6.095 | 0.011 | 24.206 | 0.000 | Dependent | Dependent |
| $MV_{3x3x3}$ | 1.490 | 0.285 | 3.210 | 0.178 | 0.788 | 0.471 | 9.723 | 0.010 | Dep ≈ Ind | Dependent |
| $RP_{2x3}$ | 0.006 | 0.984 | 0.299 | 0.773 | 1.743 | 0.173 | 5.814 | 0.061 | Dep ≈ Ind | Dependent |
| $RP_{3x3}$ | 0.035 | 0.931 | 0.116 | 0.885 | 1.497 | 0.204 | 9.125 | 0.019 | Dep ≈ Ind | Dependent |
| $RP_{3x3x3}$ | 0.009 | 0.984 | 0.087 | 0.922 | 2.315 | 0.109 | 9.240 | 0.021 | Dep ≈ Ind | Dependent |

* SB strategies are **net of transactions costs** estimated according to Hasbrouck (2009, JF)'s model as in Novy-Marx and Velikov (2016, RFS) Transaction Costs

# Cross-sectional asset pricing test

Harvey and Liu (2016, WP) define a scaled intercept (SI) to

- measure the incremental contribution of an augmented model *w.r.t.* a baseline model to explain the cross-sectional variations of the $J$ test assets returns
- overcome the over-rejection issues of the GRS test

$$SI_{ew}^{med} = \frac{median(\{|a_i^g|/s_i^b\}_{i=1}^J) - median(\{|a_i^b|/s_i^b\}_{i=1}^J)}{median(\{|a_i^b|/s_i^b\}_{i=1}^J)} \tag{6}$$

## Cross-sectional asset pricing test

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$$SI^{med}_{ew} = \frac{median(\{|a^g_i|/s^b_i\}^J_{i=1}) - median(\{|a^b_i|/s^b_i\}^J_{i=1})}{median(\{|a^b_i|/s^b_i\}^J_{i=1})} \quad (6)$$

where,

- $median(.)$ is the median value of the ratio $|a^g_i|/s^b_i$ or $|a^b_i|/s^b_i$
- $s$ denotes the standard errors for the regression intercept $a$
- superscript $b$ is for the baseline model
- superscript $g$ is for the augmented model
- subscript $i$ refers to the i-th portfolio among the $J$ test assets

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**Outputs:**

- if SI<0 then au<u>g</u>mented > <u>b</u>aseline model

# CROSS-SECTIONAL ASSET PRICING TEST

To test the significance of the model improvement, Harvey and Liu (2016, WP) define the following boostrap procedure,

**Step 1:** Orthogonalization of the list of $K$ candidates

- Baseline assets ($R_1$)= 30-Year US Treasury Bond (B30) + SMB + HML
- Test asset ($R_2^i$)= i-th candidate among the list of $K$ candidates

## Cross-sectional asset pricing test

To test the significance of the model improvement, Harvey and Liu (2016, WP) define the following boostrap procedure,

**Step 1:** Orthogonalization of the list of $K$ candidates

- Baseline assets ($R_1$)= 30-Year US Treasury Bond (B30) + SMB + HML
- Test asset ($R_2^i$)= i-th candidate among the list of $K$ candidates

$$\begin{aligned} R_2^i &= \alpha^i + \beta^i R_1 + e^i \\ R_2^{\alpha,i} &= R_2^i - \alpha^i = \beta^i R_1 + e^i \end{aligned} \tag{7}$$

Such that, $R_2^{\alpha,i}$ does not bring any additional information to the baseline model.

## CROSS-SECTIONAL ASSET PRICING TEST

**Step 2:** Bootstrap (Similar to the method presented earlier)

In each sample of the $B$ bootstrap:

- a score for the scaled intercept $SI_{ew}^{med}$ can be obtained for the $K$ number of orthogonalized candidates (i.e, $R_2^{\alpha,i}$ with the $i = \{1, 2, ..., K\}$ candidate)
- take the minimum value among the b-th bootstrap to control for multiple testing

$$SI^{b,*} = \underbrace{min}_{i \in \{1,2,\ldots,K\}} \{SI^{b,i}\} \quad (8)$$

## CROSS-SECTIONAL ASSET PRICING TEST

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- take the minimum value among the b-th bootstrap to control for multiple testing

$$SI^{b,*} = \underbrace{min}_{i \in \{1,2,\ldots,K\}} \{SI^{b,i}\} \tag{8}$$

**Step 3:** Single test p-value

Select the candidate with the lowest $SI^o$ value and significant p-val

$$\text{p-val} = \frac{\#\{SI^o > SI^b\}}{B} \tag{9}$$

# Cross-sectional asset pricing test

**Step 2:** Bootstrap (Similar to the method presented earlier)

In each sample of the $B$ bootstrap:

- a score for the scaled intercept $SI_{ew}^{med}$ can be obtained for the $K$ number of orthogonalized candidates (i.e, $R_2^{\alpha,i}$ with the $i = \{1, 2, ..., K\}$ candidate)
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$$SI^{b,*} = \underbrace{min}_{i \in \{1,2,\ldots,K\}} \{SI^{b,i}\} \tag{8}$$

**Step 3:** Single test p-value

Select the candidate with the lowest $SI^o$ value and significant p-val

$$\text{p-val} = \frac{\#\{SI^o > SI^b\}}{B} \tag{9}$$

**Step 4:** Multiple test p-value

$$\text{p-val} = \frac{\#\{SI^o > SI^{b,*}\}}{B} \tag{10}$$

# Cross-sectional asset pricing test

| MVE Candidates → | Mkt | $MV_{dep}$ | $MD_{dep}$ | $RP_{dep}$ | $MV_{ind}$ | $MD_{ind}$ | $RP_{ind}$ |
|---|---|---|---|---|---|---|---|
| | | | | Baseline = US30 + SMB + HML | | | |
| | | | | Panel A: 2x3 cap-weighted independent portfolios as test assets | | | |
| GRS | 4.836 | 4.189 | 4.391 | 4.538 | 4.155 | 4.445 | 4.341 |
| p-value | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Scaled intercept (SI) | 0.042 | 0.070 | 0.041 | 0.036 | 0.066 | -0.012 | -0.842 |
| Single test p-value | 0.701 | 0.984 | 0.929 | 0.894 | 0.893 | 0.325 | 0.000 |
| SI sequence | 2 | 2 | 2 | 2 | 2 | 2 | 1 |
| Selected candidate(s) | | | | $\mathbf{RP}_{ind}$ | | | |
| Multiple test p-value | | | | [0.000] | | | |
| | | | | Panel B: 2x3 cap-weighted dependent portfolios as test assets | | | |
| GRS | 12.058 | 10.947 | 11.344 | 11.589 | 11.552 | 11.888 | 11.962 |
| p-value | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Scaled intercept (SI) | 0.049 | 0.061 | -0.838 | 0.023 | 0.009 | -0.001 | -0.008 |
| Single test p-value | 0.880 | 0.865 | 0.000 | 0.782 | 0.569 | 0.547 | 0.468 |
| SI sequence | 2 | 2 | 1 | 2 | 2 | 2 | 2 |
| Selected candidate(s) | | | | $\mathbf{MD}_{dep}$ | | | |
| Multiple test p-value | | | | [0.000] | | | |

# Cross-sectional asset pricing test

| MVE Candidates → | Mkt | $MV_{dep}$ | $MD_{dep}$ | $RP_{dep}$ | $MV_{ind}$ | $MD_{ind}$ | $RP_{ind}$ |
|---|---|---|---|---|---|---|---|
| | Baseline = US30 + SMB + HML | | | | | | |
| | Panel C: 3x3 cap-weighted independent portfolios as test assets | | | | | | |
| GRS | 3.403 | 2.792 | 2.892 | 2.975 | 2.909 | 2.918 | 2.929 |
| p-value | 0.000 | 0.003 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| Scaled intercept (SI) | -0.006 | 0.041 | 0.255 | 0.373 | -0.876 | 0.100 | 0.169 |
| Single test p-value | 0.542 | 0.755 | 0.883 | 0.895 | 0.000 | 0.697 | 0.755 |
| SI sequence | 2 | 2 | 2 | 2 | 1 | 2 | 2 |
| Selected candidate(s) | $\mathbf{MV}_{ind}$ | | | | | | |
| Multiple test p-value | [0.000] | | | | | | |
| | Panel D: 3x3 cap-weighted dependent portfolios as test assets | | | | | | |
| GRS | 8.228 | 7.757 | 7.959 | 7.950 | 8.116 | 8.108 | 8.108 |
| p-value | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Scaled intercept (SI) | 0.210 | 0.249 | 0.221 | -0.806 | 0.001 | 0.150 | 0.107 |
| Single test p-value | 0.937 | 0.979 | 0.916 | 0.000 | 0.528 | 0.861 | 0.849 |
| SI sequence | 2 | 2 | 2 | 1 | 2 | 2 | 2 |
| Selected candidate(s) | $\mathbf{RP}_{dep}$ | | | | | | |
| Multiple test p-value | [0.000] | | | | | | |

# Cross-sectional asset pricing test

| MVE Candidates → | Mkt | $MV_{dep}$ | $MD_{dep}$ | $RP_{dep}$ | $MV_{ind}$ | $MD_{ind}$ | $RP_{ind}$ |
|---|---|---|---|---|---|---|---|
| | Baseline = US30 + SMB + HML | | | | | | |
| | Panel E: 3x3x3 cap-weighted independent portfolios as test assets | | | | | | |
| GRS | 2.262 | 2.135 | 2.164 | 2.213 | 2.085 | 2.332 | 2.110 |
| p-value | 0.000 | 0.001 | 0.001 | 0.000 | 0.001 | 0.000 | 0.001 |
| Scaled intercept (SI) | 0.245 | 0.030 | -0.790 | 0.301 | 0.283 | 0.111 | 0.225 |
| Single test p-value | 0.943 | 0.661 | 0.000 | 0.970 | 0.962 | 0.816 | 0.952 |
| SI sequence | 2 | 2 | 1 | 2 | 2 | 2 | 2 |
| Selected candidate(s) | | | | **$MD_{dep}$** | | | |
| Multiple test p-value | | | | [0.000] | | | |
| | Panel F: 3x3x3 cap-weighted dependent portfolios as test assets | | | | | | |
| GRS | 4.591 | 4.366 | 4.335 | 4.283 | 4.335 | 4.608 | 4.379 |
| p-value | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Scaled intercept (SI) | -0.015 | 0.007 | -0.668 | -0.039 | 0.006 | 0.013 | 0.020 |
| Single test p-value | 0.566 | 0.571 | 0.000 | 0.431 | 0.610 | 0.686 | 0.739 |
| SI sequence | 2 | 2 | 1 | 2 | 2 | 2 | 2 |
| Selected candidate(s) | | | | **$MD_{dep}$** | | | |
| Multiple test p-value | | | | [0.000] | | | |

# CONCLUSION AND TAKEWAYS

**Testing the MVE of Smart Beta strategies on characteristic-sorted portfolios**

- Context
  - Multidimensional market risks, especially after 1993
  - Inefficiencies of long-short factors
  - Sample errors for estimating MVE
  - Need for long-only solutions
- Contribution
  - Risk-based optimization on DNS opportunity sets span a single-index model, other MVE candidates (market-cap and other risk-based strategies) and improves a 3-factor model
  - Risk-based strategies on DNS opportunity sets have incremental significance for pricing characteristics-sorted portfolios
  - Dependent-sorted portfolios provide a better investment opportunity set to investors compared to independent-sorted portfolio
- Robustness
  - Out-of-sample, multiple testing

END...

Thank you for your attention!

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## DIVERSIFICATION RETURN: BOOTSTRAP

**Method:**

- block-bootstrap method from Politis and Romano (1992)
- studentized test statistic following Ledoit and Wolf (2008)

**Bootstrap:**

1. block length = 10 observations (robust to other length $\{2, 4, 6, 8, 10\}$)
2. match the length of the original time-series (630 observations)
3. randomly resample with replacement the original time-series
4. keep the same sequence for all assets in each sample (cross-dependence)
5. 4999 simulations similar to Ledoit and Wolf (2008)

# Testing the incremental diversification return

Hypothesis testing the spread in Sharpe ratio between the strategy $i$ and $j$,

$$\hat{\Delta} = \frac{\hat{\mu}_i}{\hat{\sigma}_i} - \frac{\hat{\mu}_j}{\hat{\sigma}_j} \tag{11}$$

**Assumption:**
Difference between the first and second moments of the distributions between the two series converge towards zero

$$\sqrt{T}(\hat{u} - u) \xrightarrow{d} N(0, \Omega) \tag{12}$$

- $\hat{u} = (\hat{\mu}_i, \hat{\mu}_j, \hat{\sigma}_i{}^2, \hat{\sigma}_j{}^2)$ are the sample estimates of $u = (\mu_i, \mu_j, \sigma_i^2, \sigma_j^2)$
- $\xrightarrow{d}$ refers to the convergence in distribution of the parameters
- $\Omega$ not valid when returns exhibit non-normal distribution or serial autocorrelation

**Solution:**

$$\sqrt{T}(\hat{v} - v) \xrightarrow{d} N(0, \Psi) \tag{13}$$

where $\hat{v} = (\hat{\mu}_i, \hat{\mu}_j, \hat{\gamma}_i, \hat{\gamma}_j)$ is the sample estimates of $v = (\mu_i, \mu_j, \gamma_i, \gamma_j)$, $\hat{\gamma}_i = E(r_i^2)$ and $\hat{\gamma}_i = E(r_j^2)$ and a HAC kernel estimate of $\Psi$.

**Spread in Sharpe ratio:**

$$f(a,b,c,d) = \frac{a}{\sqrt{c-a^2}} - \frac{b}{\sqrt{d-b^2}} \tag{14}$$

With $a = \hat{\mu}_i$, $b = \hat{\mu}_j$, $c = \hat{\gamma}_i$, and $d = \hat{\gamma}_j$.

Gradient of this function (delta-method) is

$$\nabla' f(\hat{v}) = (\frac{c}{(c-a^2)^{1.5}}, -\frac{d}{(d-b^2)^{1.5}}, -\frac{1}{2}\frac{a}{(c-a^2)^{1.5}}, \frac{1}{2}\frac{b}{(d-b^2)^{1.5}})$$

The standard error is of delta estimate is,

$$s(\hat{\Delta}) = \sqrt{\frac{\nabla' f(\hat{v})\hat{\Psi}\nabla f(\hat{v})}{T}} \tag{15}$$

The kernel estimator $\hat{\Psi}$ ensures that the estimation of the standard error is robust to heteroskedasticity and autocorrelation (HAC).

## Diversification Return: Hypothesis Testing

**Studentized test statistic:**

On the original time-series,

$$d = \frac{|\hat{\Delta}|}{s(\hat{\Delta})} \tag{16}$$

On the b-th bootstrap sample,

$$d^b = \frac{|\hat{\Delta}^b - \hat{\Delta}|}{s(\hat{\Delta}^b)} \tag{17}$$

The boostrap 1-$\alpha$ confidence interval is defined as:

$$\left[\hat{\Delta} - z^b_{|.|,1-\alpha/2} s(\hat{\Delta}), \hat{\Delta} + z^b_{|.|,1-\alpha/2} s(\hat{\Delta})\right] \tag{18}$$

with $z^b_{|.|,1-\alpha}$ the quantile of the distribution of $d^b$ denoted $\mathcal{L}(d^b)$.

**p-value:**

$$\text{p-val} = \frac{\#\{d^b \geq d\} + 1}{B + 1} \tag{19}$$

# Transaction Costs: Gibbs Estimates

Hasbrouck (2009) extend Roll (1984)'s price dynamics model with a market factor

$$\Delta p_t = c\Delta q_t + \beta_{rm} rm_t + u_t \tag{20}$$

with

$$\begin{aligned} \Delta p_t &= p_t - p_{t-1} \\ &= m_t + cq_t - m_{t-1} - cq_{t-1} \\ &= c\Delta q_t + u_t \end{aligned} \tag{21}$$

- $m_t$ is the log midpoint of the prior bid-ask price
- $p_t$ is the log trade price
- $q_t$ is the sign of the last trade of the day ($+1$ for a buy and $-1$ for a sale)
- $u_t$ is assumed to be unrelated to the sign of the trade ($q_t$)
- $rm_t$ is the market return on day $t$
- $\beta_{rm}$ is the slope on the marker return
- $c$ is the effective cost

## TRANSACTION COSTS: GIBBS ESTIMATES

$$\Delta p_t = c\Delta q_t + \beta_{rm} rm_t + u_t \tag{22}$$

Iterative Bayesian methodology to estimate the effective costs ($c$):

1. Initialize $q_1$ to $+1$ and $\sigma_u^2$ to 0.001.
   - if no trade $q_t$=0 (in CRSP, PRC$<$0) else $q_t = sign(\Delta p_t)$
   - minimum of 60 to a max 250 daily observations

2. Initialize the distribution from where the values $c$, $\beta_{rm}$, and $\sigma_u^2$ will be drawn:
   - $c \sim N^+ (\mu = 0.01, \sigma^2 = 0.01^2)$
   - $\beta_{rm} \sim N (\mu = 1, \sigma^2 = 1)$
   - $\sigma_u^2 \sim IG (\alpha = 10^{-12}, \beta = 10^{-12})$

# Transaction Costs: Gibbs Estimates

**for 1 to 1000 sweeps**

1. Perform a Bayesian OLS regression on a 250-day of lagged observations to estimate the new values of $c$ and $\beta_{rm}$, update the posterior distribution of the parameters and make a new draw of the coefficients.
2. Back out $u_t$ according to $c$, $\beta_{rm}$, $\Delta p_t$, $rm_t$, $\Delta q_t$

$$u_t = \Delta p_t - \beta_{rm} rm_t - c\Delta q_t \tag{23}$$

   - update $\sigma_u^2$

3. Draw new series of $q_t$ according to the posterior $\sigma_u^2$

$$u_t = \Delta p_t - \beta_{rm} rm_t - cq_t + cq_{t-1} \tag{24}$$

   - estimate $u_t(q_t = +1)$ and $u_t(q_t = -1)$ given $u_t \sim N(0, \sigma_u^2)$

$$\text{Odds} = \frac{f(u_t(q_t = +1))}{f(u_t(q_t = -1))} \begin{cases} q_t = +1 & \text{if Odds} > 1 \\ q_t = -1 & \text{if Odds} < 1 \end{cases} \tag{25}$$

**end**

→ $c$ is the average of the last 800 estimations ("burn in" the 200 first obs.)