

# Resource Competition and Community Structure



**OCEA0096-1 Modélisation des écosystèmes et des cycles biogéochimiques : Partim Ressources**

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## Resource Competition & Community Structure

Nutrient, light, food, ... Something that is required and consumed for growth.

## Resource **Competition** & **Community** Structure

Different populations depends on the same resource

## Resource Competition & Community Structure

Is cohabitation possible ? Under which conditions ?

## Resource Competition & Community Structure

Abstract mathematical framework: Simple case, assumption, generalities.

# Basics

# What are Resource ?

A factor that

- has a given availability
- leads to higher growth as availability increases
- is consumed by the population(s)

Multiple factors can be considered as resources if they meet the above criteria for some range of other factors availability.

# Consumption for Growth model

For  $n$  species competing for  $k$  resources :

$$\frac{d N_i}{N_i dt} = f_i(R_1, \dots, R_k) - m_i \quad (1)$$

$$\frac{d R_j}{dt} = g_j(R_j) - \sum_{i=1}^n N_i f_i(R_1, \dots, R_k) h_{i,j}(R_1, \dots, R_k) \quad (2)$$

$N_i$  population density of species  $i$

$R_j$  availability of resource  $j$

$f_i$  functional relationship between  $R_j$  and rate of population changes for  $N_i$

$g_j$  resource supply

$m_i$  mortality rate for species  $i$

$h_{i,j}$  amount of resource  $j$  required to create a new individual  $i$

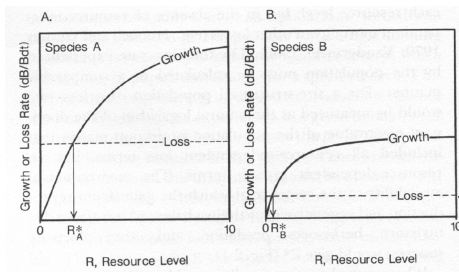


# Assumptions

- pure competition :  $\frac{\partial f_i}{\partial N_j} = 0$
- resources are not interactives :  $\frac{\partial g_i}{\partial R_j} = 0$  for  $i \neq j$

# One resource

# One resource



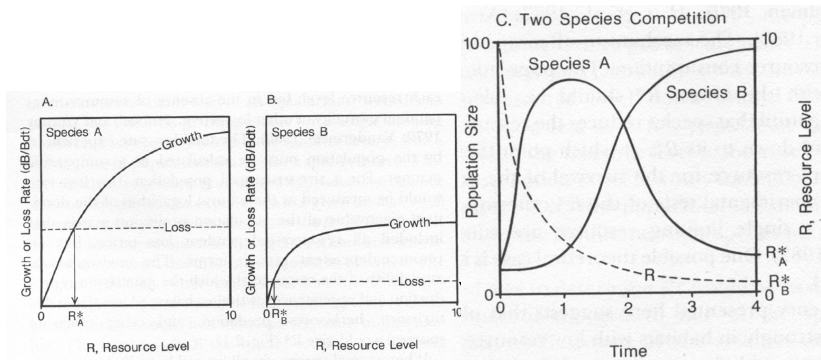
$$\frac{1}{N} \frac{\partial N}{\partial t} = f(R) - m \quad (3)$$

$$f(R^*) = m \Rightarrow \frac{\partial N}{N \partial t} = 0 \quad (4)$$

$R^*$

The resource level for which growth = mortality.

# One resource



$R^*$

The resource level for which growth = mortality.

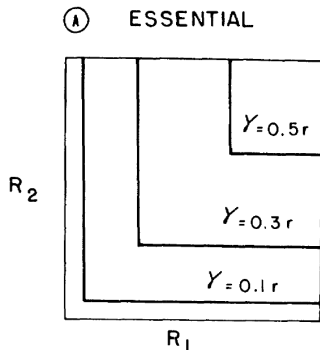
## The $R^*$ Theory

If only one resource, the species with the lower  $R^*$  overcompetes the others.

## Two resource

# Growth Isoclines

In the "resource-space", the  $\gamma$ -growth isocline is the locus  $f_i(R_1, R_2) = \gamma$

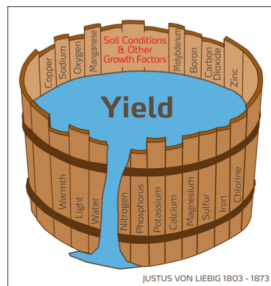


Different shape for different type of resources

Note that  $\gamma$  is the reproduction rate without mortality.

## Type of resources

# Essential Resources I



Justus von Liebig's law of the minimum, 1873 : *If one growth factor/nutrient is deficient, plant growth is limited, even if all other vital factor/nutrient are adequate .. plant growth is improved by increasing the supply of the deficient factor/nutrient.*

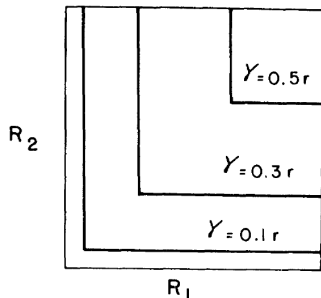
$$f(R_1, \dots, R_k) = \min_{j=1, k} (f_j(R_j)) \quad (3)$$



## Essential Resources II

- No growth possible if one resource is lacking
- $f_i(R_1 = 0, R_2) \leq 0$  for all  $R_2$  and  $f_i(R_1, R_2 = 0) \leq 0$  for all  $R_1$
- iso-growth lines never intersect the axis
- Examples
  - ▶ Nitrate, Phosphate, Light
- Counter-Example :  $[\text{NO}_3^-]$  and  $[\text{NO}_2^{2-}]$

Ⓐ ESSENTIAL



# Essential Resources III

(A) ESSENTIAL

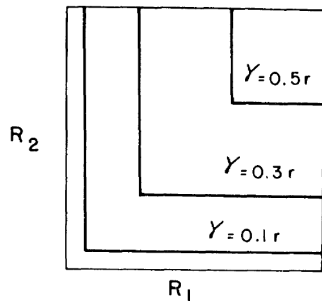


Figure 1: Noninteractive Essential

(B) INTERACTIVE -  
ESSENTIAL

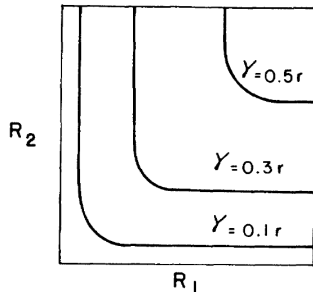
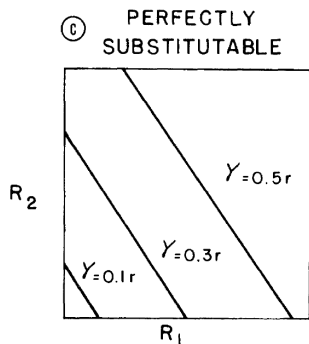


Figure 2: Interactive Essential

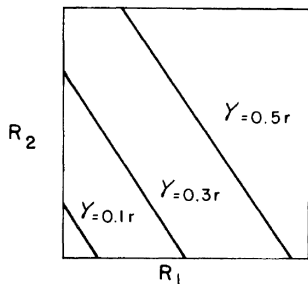
# Substitutable Resources

- Any resources can support growth on its own
- $f_i(R_1 = 0, R_2) > 0$  for some  $R_2$  and  $f_i(R_1, R_2 = 0) > 0$  for some  $R_1$
- iso-growth lines intersect the axis
- Examples
  - ▶ Herbivorous and carnivorous diet

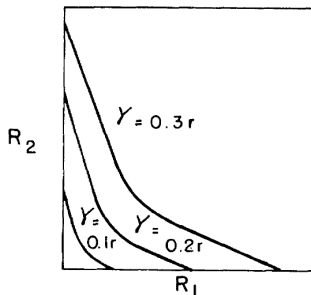


# Substitutable Resources

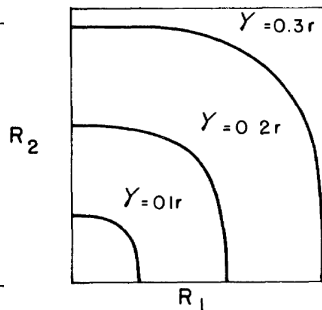
Ⓒ PERFECTLY  
SUBSTITUTABLE



Ⓓ COMPLEMENTARY

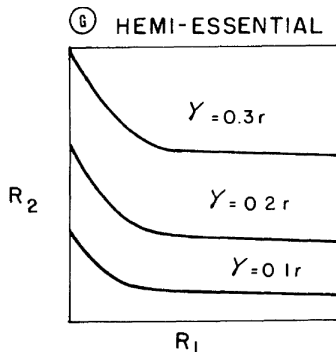


Ⓔ ANTAGONISTIC



# Hemi-essential Resources

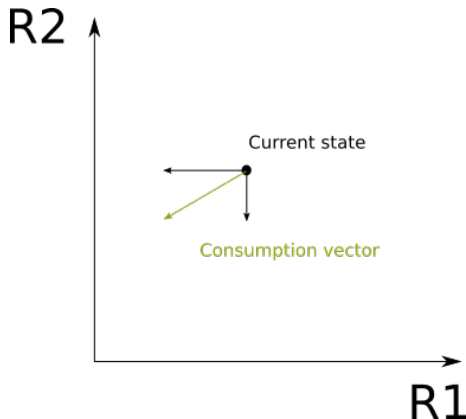
- One is essential and can support growth on its own , the other can only partly substitute
- $f_i(R_1 = 0, R_2) > 0$  for some  $R_2$  and  $f_i(R_1, R_2 = 0) \leq 0$  for all  $R_1$
- iso-growth lines intersect one axis



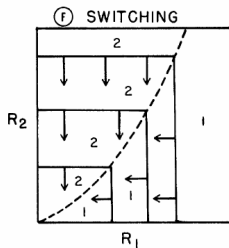
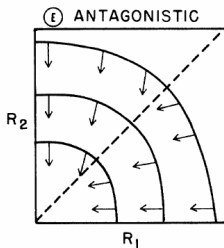
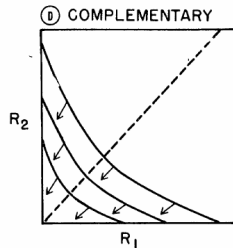
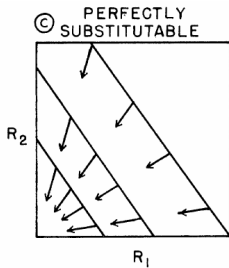
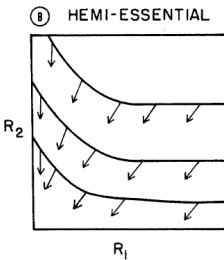
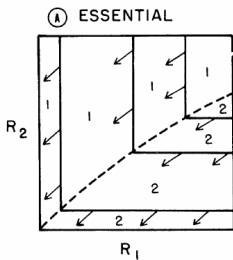
# Dynamics

# Consumption vectors

- Represents changes in resources following consumption for growth
- Depends on
  - ▶ Consumption(s) per individuals ( $\rightarrow$  Characteristic to species)
  - ▶ Number of individuals



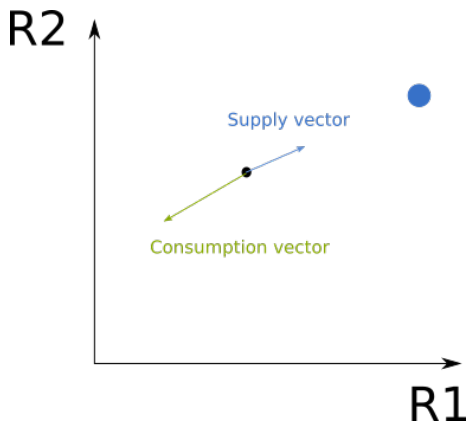
# Consumption vectors



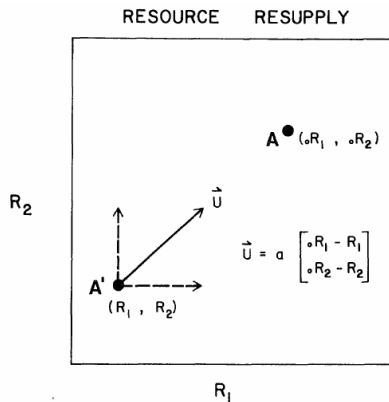


# Supply vectors

- Represents the environmental supply of resources
- Depends on
  - ▶ The environment (Supply point)
  - ▶ The actual state of resources

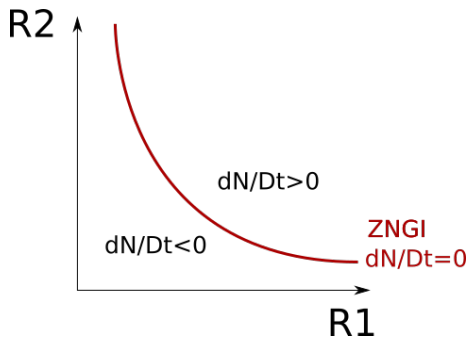


# Supply vectors



- Constant "string" toward supply point
- $\frac{dR_1}{dt}|_{supply} = a(R_{1,0} - R_1)$  &  $\frac{dR_2}{dt}|_{supply} = a(R_{2,0} - R_2)$

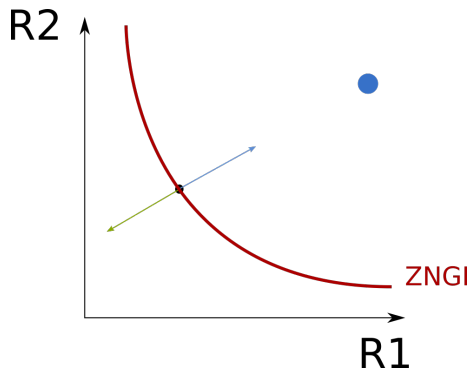
# One species, two resources



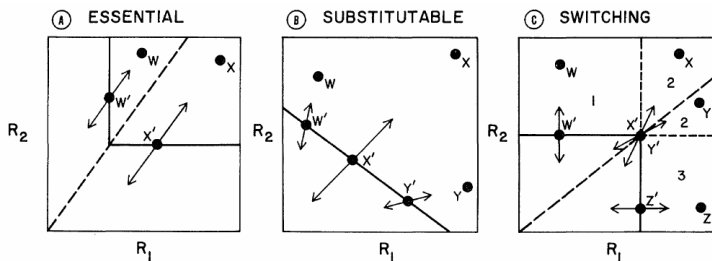
## ZNGI

Zero Net Growth Isoline: where  $\gamma =$  mortality

# One species, two resources



# Equilibrium point: One species, two resources



Equilibrium points set by

- Consumption vectors
- Supply point
- ZNGI

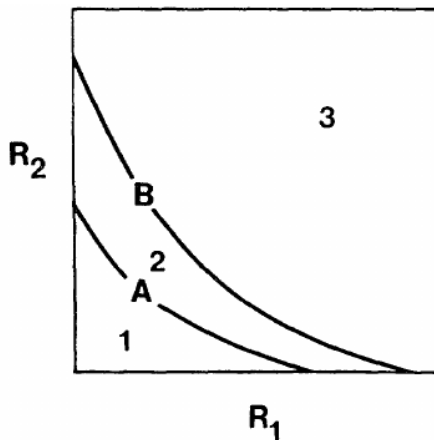
# Equilibrium Points

Examples in R with TILMAN1 “Tilman\_1species.R”

- Present the program
- iso-growth
- Consumption vectors
- Supply Vectors
- Equilibrium points (show that supply below ZNGI prohibits survival)

# Competition

## 2 Species : competition

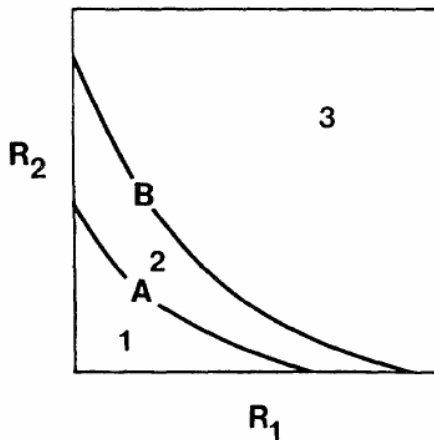


### Question

What's the outcome of supply point in 1, 2, 3 ?

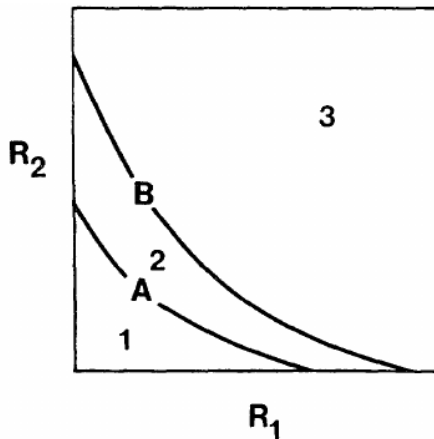


## 2 Species : competition



- 1 Neither species able to survive
- 2 Only A can survive
- 3 A removes B by competition

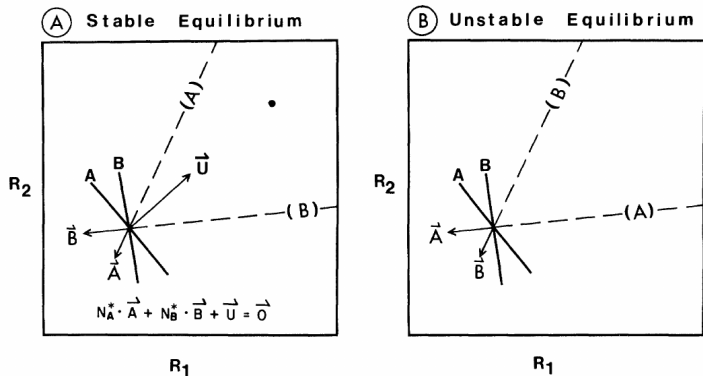
## 2 Species : competition



### Question

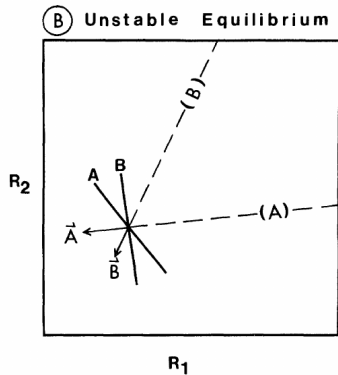
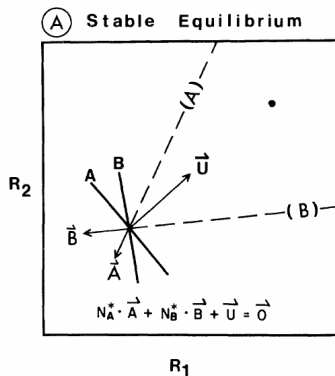
Which condition for coexistence ?

## 2 Species : competition



- Crossings of the ZNGI defines cohabitation equilibrium point
- Might be stable or unstable

## 2 Species : competition

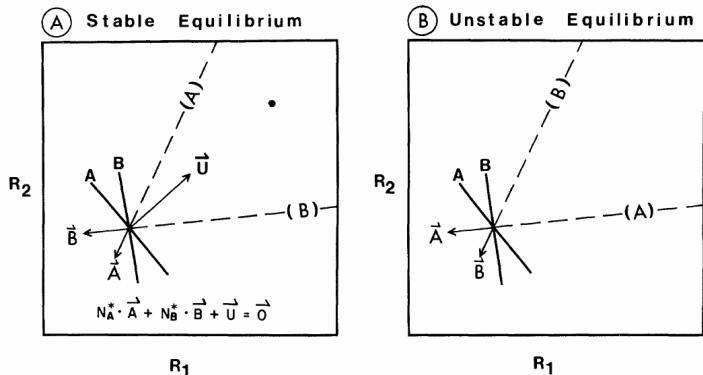


### Trade-offs

In resource poor environments, organisms allocate resource for efficiency on one resource → less efficiency regarding other resources.

Example : Leaves or roots ?

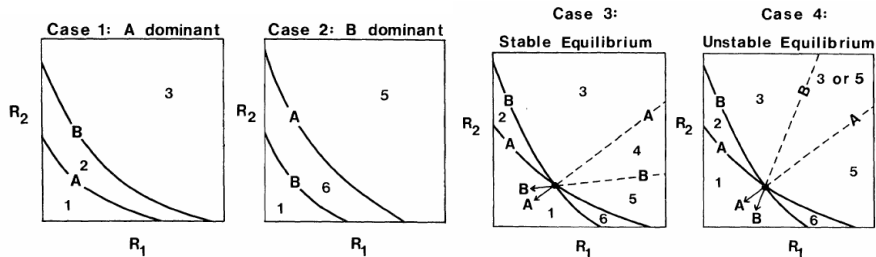
## 2 Species : competition



Stable if

- 1 Each species consumes proportionately more of the resources that limits its own growth
- 2 The amounts of resource consumed by individual changes only slightly in response to small changes in  $R_j$

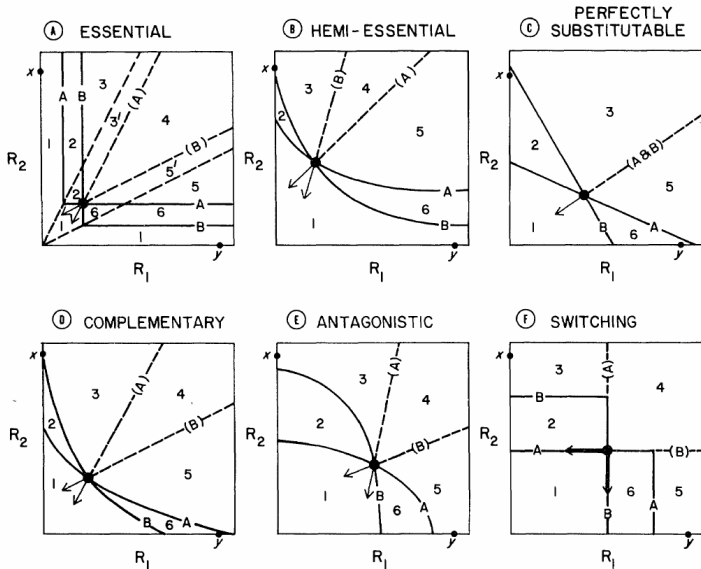
## 2 Species : competition



- 1 Neither species able to survive
- 2 Only A can survive
- 3 A removes B by competition

- 4 Coexistence
- 5 B removes A by competition
- 6 Only A can survive

## 2 Species : Competition



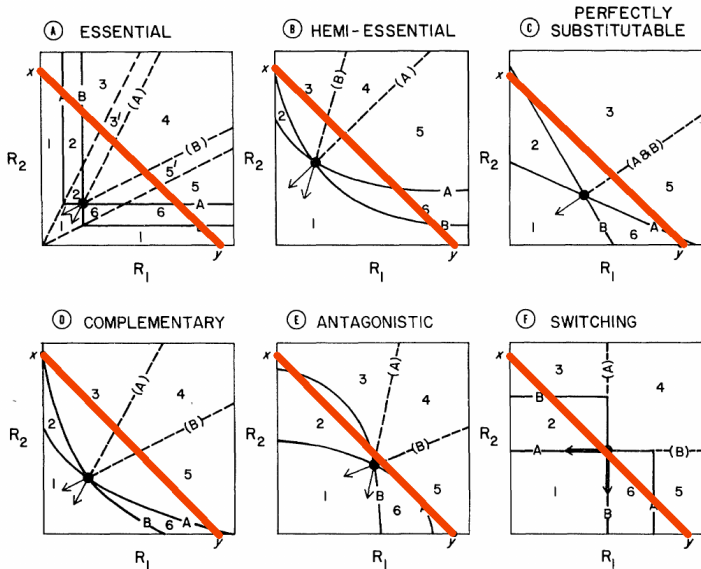
## 2 Species : Competition

Examples in R with TILMAN1 “Tilman\_2species.R”

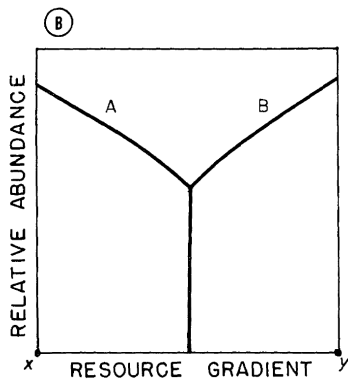
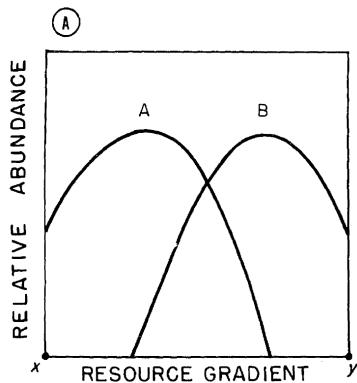
- Trajectories
- Equilibrium points (show that cohabitation conditions depends on resource supply point)



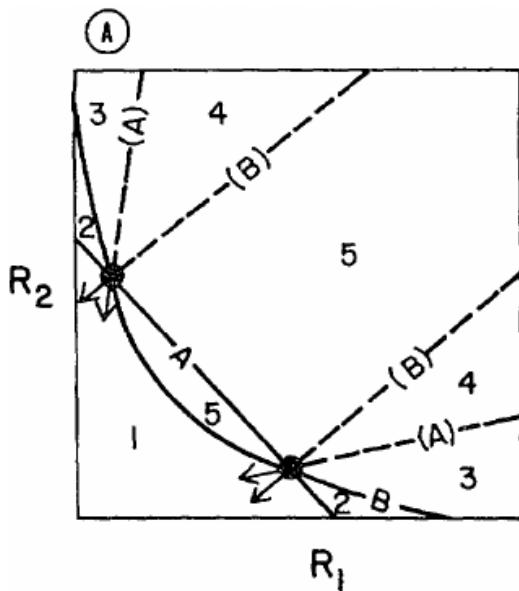
# Resource spatial gradients



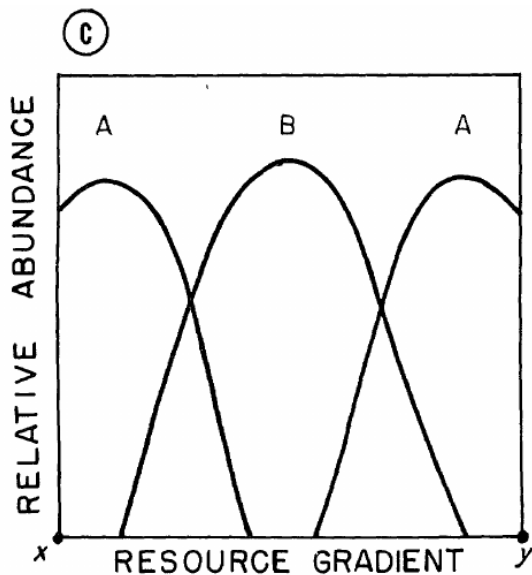
# Resource spatial gradients



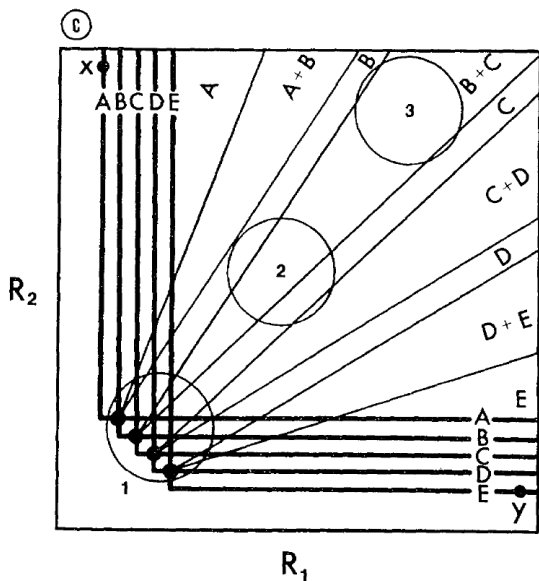
## Mixed Resource types



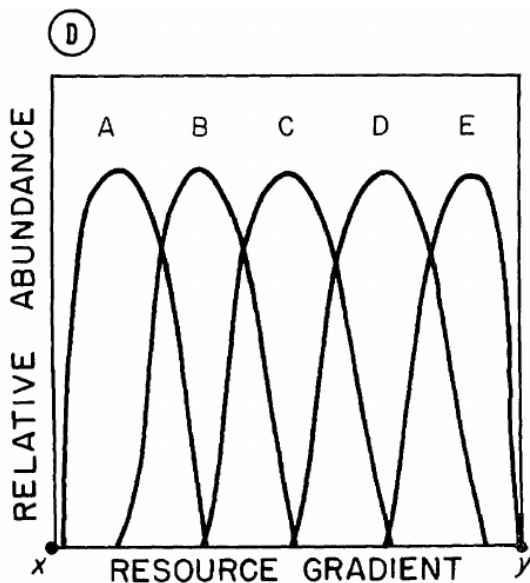
## Mixed Resource types



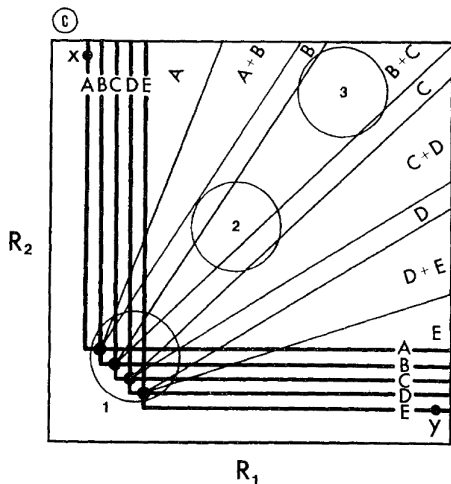
# Multi Species



# Multi Species



# Multi Species



## Heterogeneity

Variations in resource RATIOS supports species variety

# Experiments



## A Critical Review of Twenty Years' Use of the Resource-Ratio Theory

Thomas E. Miller,<sup>1,\*</sup> Jean H. Burns,<sup>1,†</sup> Pablo Munguia,<sup>1,‡</sup> Eric L. Walters,<sup>1,§</sup> Jamie M. Kneitel,<sup>1,||</sup>  
Paul M. Richards,<sup>1,\*</sup> Nicolas Mouquet,<sup>2,\*\*</sup> and Hannah L. Buckley<sup>1,††</sup>

**Table 1:** Predictions from the resource-ratio theory, based on studies that cite Tilman (1980, 1982)

Prediction

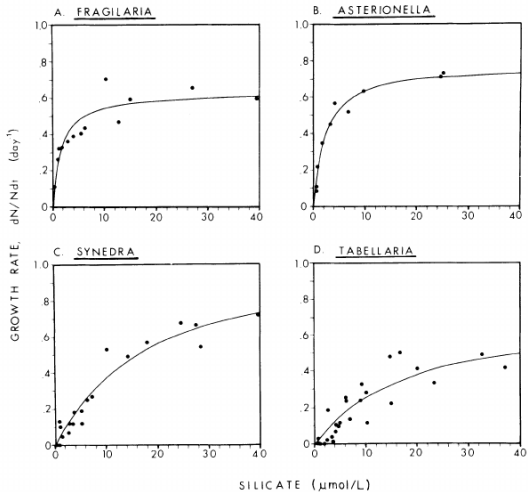
1. The species that can survive at the lowest levels of a limiting resource will be the best competitor for that resource
2. Species dominance varies with the ratio of the availabilities of two resources
3. The number of coexisting species is less than or equal to the number of limiting resources
4. The vector describing the resource supply rate to an environment will affect whether competing species coexist and, if not, which species will competitively exclude the other
5. The vectors describing the consumption rates of resources for two species will determine whether competing species coexist or, if not, which species will dominate competitively
6. Trade-offs in resource use must occur for species to coexist along a gradient of ratios of the availabilities of two resources
7. The highest diversity of competing species will occur at an intermediate ratio of the availabilities of two resources

**Table 2:** Number of individual tests of the seven predictions of the resource-ratio theory listed in table 1

Prediction number	Test adequate?			Total
	Yes; prediction supported?		No	
	Yes	No		
1	8	5	9	22
2	13	3	31	47
3	1	1	1	3
4	5	1	5	11
5	2	0	1	3
6	2	1	2	5
7	0	0	10	10
Total	31	11	59	101

Note: The 101 overall tests were published in 68 different articles. Tests were classified as adequate if a clear and sufficient experimental design and appropriate replication were used. Studies were classified as supporting the prediction on the basis of our interpretation of the results presented in the corresponding article.

# Experiments



# Experiments

TABLE 1. Silicate and phosphate kinetic parameters for four Lake Michigan diatoms.  $K$  is the half saturation constant;  $r$  the maximal growth rate;  $Q$  the quotient; and  $R^*$  the calculated amount of nutrient required to grow at  $D = 0.25 \text{ d}^{-1}$ . The 95% confidence intervals are in parentheses.

Species	$r \text{ (d}^{-1}\text{)}$	$K \text{ (}\mu\text{mol/L)}$	$Q \text{ (}\mu\text{mol/cell)}$	$R^* \text{ (}\mu\text{mol/L)}$
Silicate-limited experiments				
<i>Fragilaria</i>	0.62 (.54-.70)	1.5 (.7-2.5)	$9.7 \times 10^{-7}$	1.0 (.7-1.5)
<i>Asterionella</i>	0.78 (.72-.84)	2.2 (1.6-2.9)	$1.5 \times 10^{-6}$	1.0 (.8-1.3)
<i>Synedra</i>	1.11 (.87-1.36)	19.7 (12.7-30.3)	$5.8 \times 10^{-5}$	5.7 (4.0-8.3)
<i>Tabellaria</i>	0.74 (.44-1.04)	19.0 (9.0-41.7)	$6.3 \times 10^{-6}$	9.7 (5.3-23.0)
Phosphate-limited experiments				
<i>Fragilaria</i>	0.80 (.72-.88)	0.011 (0-.024)	$4.7 \times 10^{-8}$	.005 (.002-.008)
<i>Asterionella</i>	0.59 (.53-.64)	0.006 (.002-.011)	$2.6 \times 10^{-8}$	.004 (.003-.007)
<i>Synedra</i>	0.65 (.61-.69)	0.003 (0-.015)	$1.1 \times 10^{-7}$	.002 (.001-.006)
<i>Tabellaria</i>	0.36 (.30-.43)	0.008 (0-.04)	$1.9 \times 10^{-7}$	.02 (.006-.07)

# Experiments

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Ecology, Vol. 62, No. 3

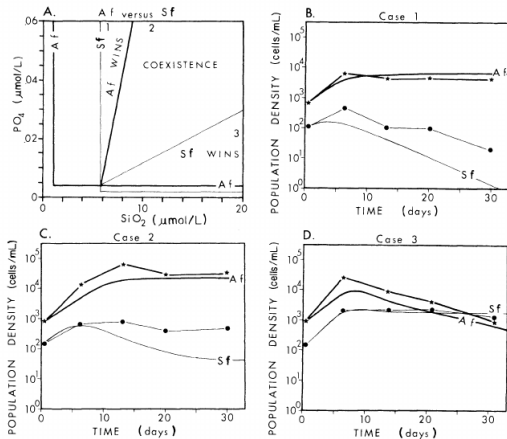


FIG. 4. (A) The predicted outcomes of resource competition between *Asterionella formosa* (Af) and *Synedra filiformis* (Sf). (B) Observed results of competition experiments for Case 1 (low silicate) are shown with stars for Af and dots for Sf. A broken line joins the observed points. The continuous thick line, labeled Af, and the continuous thin line, labeled Sf, show the predicted population dynamics. (C) The same notation as above is used for Case 2 (intermediate concentrations of  $SiO_2$  and  $PO_4$ ). (D) The same notation as above is used for Case 3 (low  $PO_4$  experiments).

# Experiments

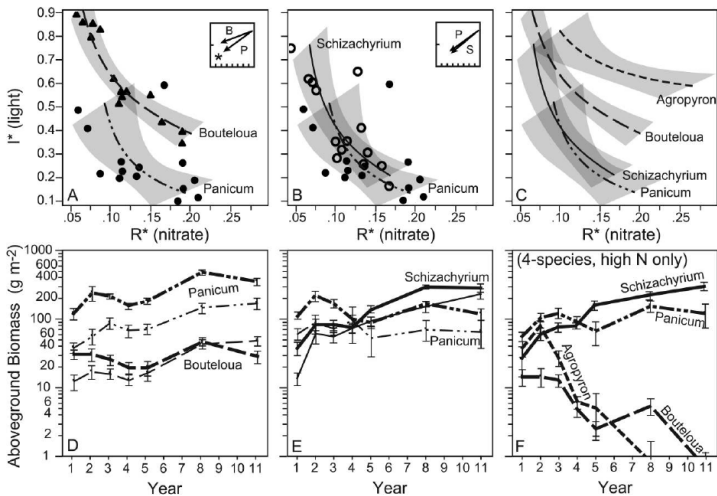


Figure 3: Ray Dybzinski and David Tilman, 2007, *The American Naturalist*

# Propositions of investigations

- Extend R framework for two species, illustrate cases, Monte Carlo to visualize coexistence.
- Mixotrophy/Allelopathy
- Heterogeneity
- Investigate Plankton dominance in BS model outputs
- Fit and identify growth parameters for various species.



RESOURCES: A GRAPHICAL-MECHANISTIC APPROACH TO  
COMPETITION AND PREDATION

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*Submitted August 7, 1978; Accepted April 27, 1979*

In such circumstances the question as to the causes of the victory of certain forms over others presents itself in the following aspect: By aid of what morphological and physiological advantages of the process of the individual does one plant suppress another? [G.F. Gause (1934)].

Recent application : <http://www.biogeosciences.net/14/2877/2017/>