TOPOLOGY OPTIMIZATION WITH SELF-WEIGHT LOADING : (UNEXPECTED) PROBLEMS AND SOLUTIONS

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OUTLINE

- INTRODUCTION
- PROBLEM FORMULATION
- FAILURE OF THE STANDARD APPROACH
- NUMERICAL SOLUTION OF THE OPTIMIZATION PROBLEM
- CONVERGENCE PROBLEM OF LOW DENSITY VARIABLES
- NUMERICAL APPLICATION
- CONCLUSION AND ON-GOING WORK

INTRODUCTION

- Compliance minimization with density dependent loads (self-weight, centrifugal forces) is quite usual in practice.
- Literature generally suggests to treat this problem as a simple extension of the dead load problem.
- Only a little number of successful academic and industrial applications are published.
- To our knowledge, only 1 paper devoted specifically to the problem: Turtletaub & Washabaugh, *Int. J. of Solids and Structures*, vol. 36, (1999) , pp 4587-4608.
- Our experience: standard procedure does not work in the self-weight load case !

PROBLEM FORMULATION

- Compliance minimization with self-weight loading

$$\min_{\mu} C = g^T q \quad with \quad \sum_e \mu_e V_e \leq \overline{V}$$

- Power law model (SIMP)

$$< E > = \mu^p \; E^0 \quad and \quad < \rho > = \mu \; \rho^0 \quad p > 1$$

- Sensitivity analysis

$$\frac{\partial C}{\partial \mu_i} = 2q^T \frac{\partial g}{\partial \mu_i} - q^T \frac{\partial K}{\partial \mu_i} q$$

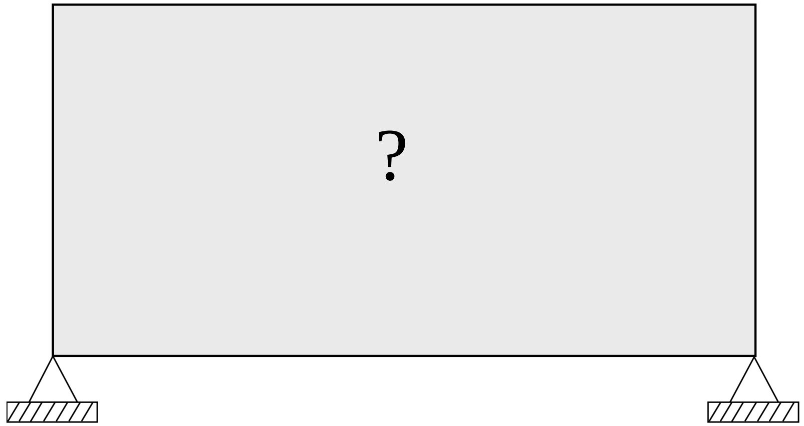
- Sigmund's filter to circumvent the checkerboard and mesh dependency problems (*Mech Struct. & Mach.* , 25 (4), 493-524, 1997)

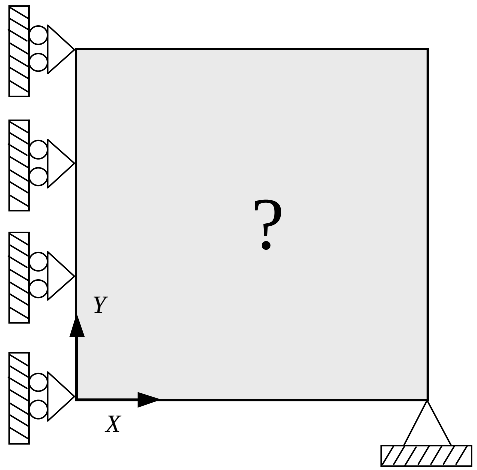
- Mathematical programming approach to solve the optimization problem.

FAILURE OF THE STANDARD APPROACH

- Min Compliance
- Volume fraction < 80%
- p = 2 (SIMP)
- Filter
- Minimum density = 0.2
- CONLIN solver

(Conlin approximation + dual solver)





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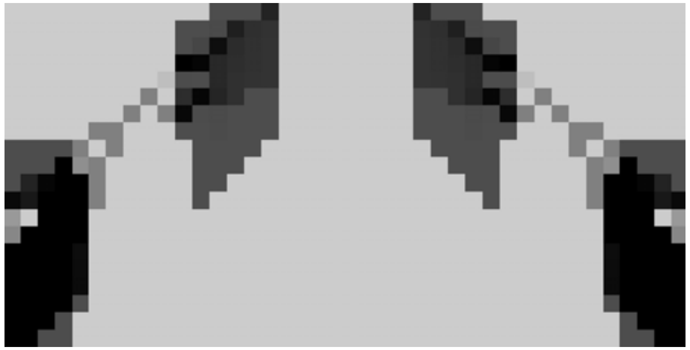
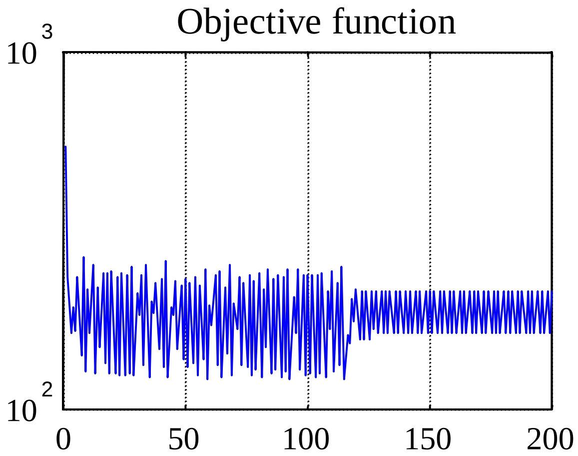
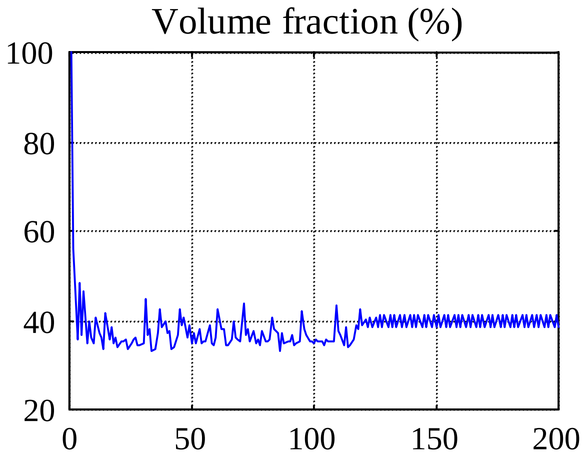
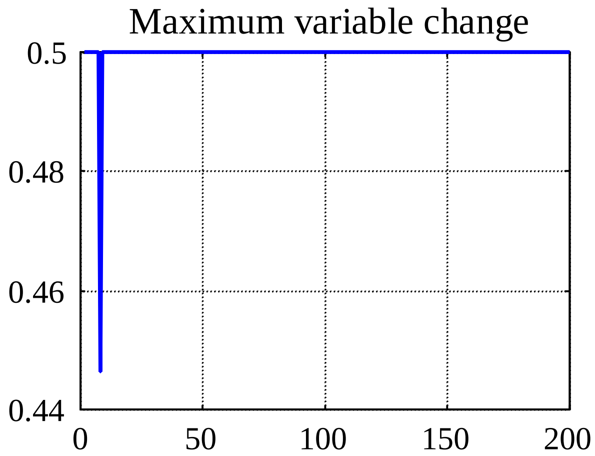


Iteration 199

Iteration 200

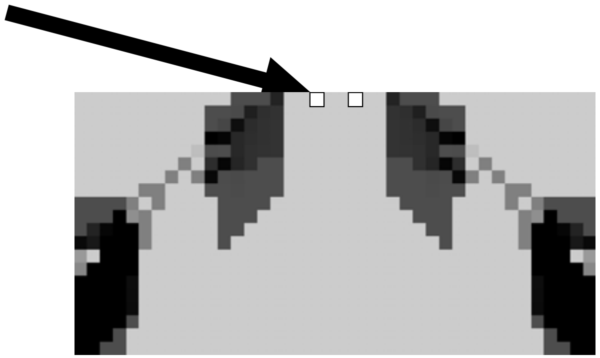






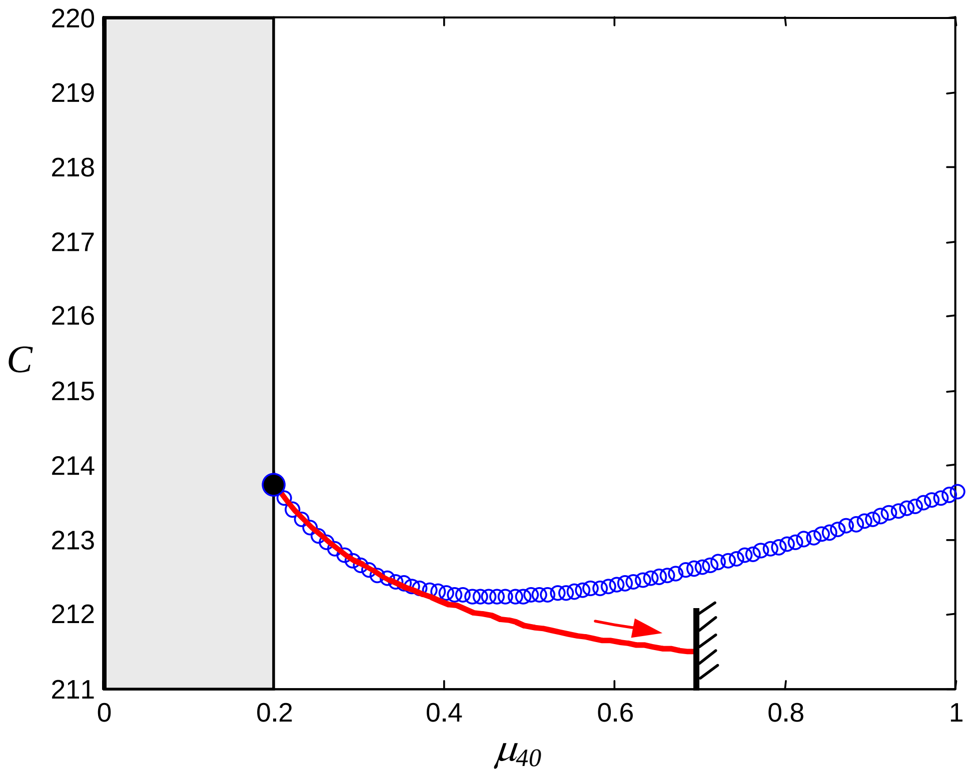
NON MONOTONOUS CHARACTER OF COMPLIANCE

The structural behavior of the compliance is non monotonous in terms of some density variables!

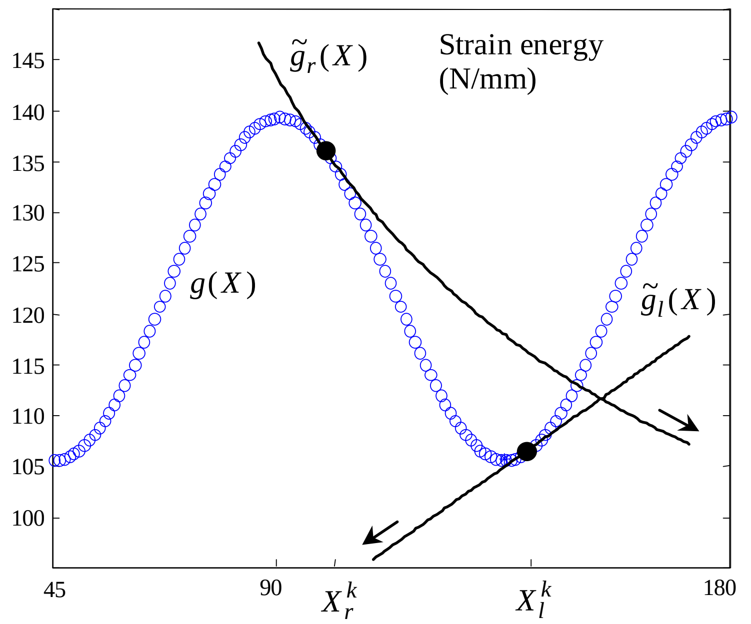


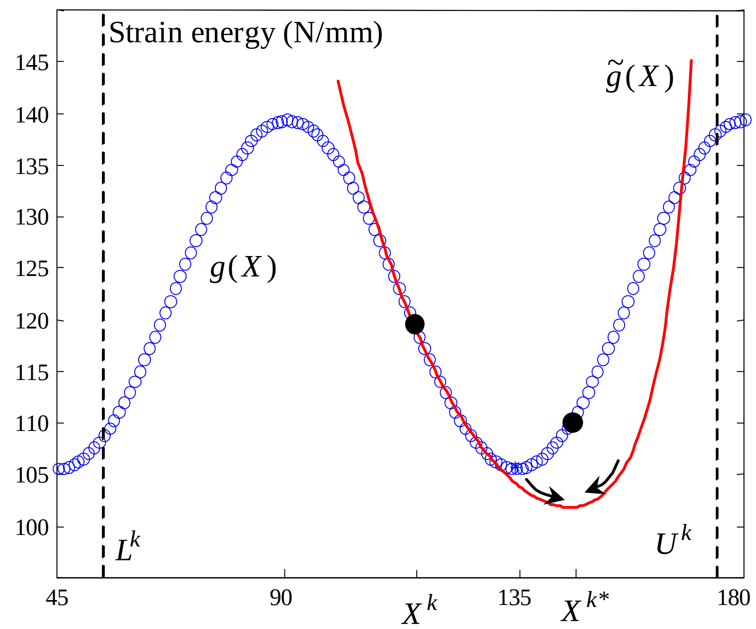
Itération 199

Itération 200



STRUCTURAL APPROXIMATIONS



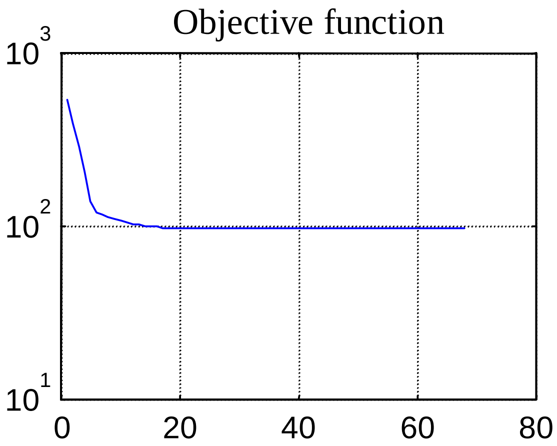


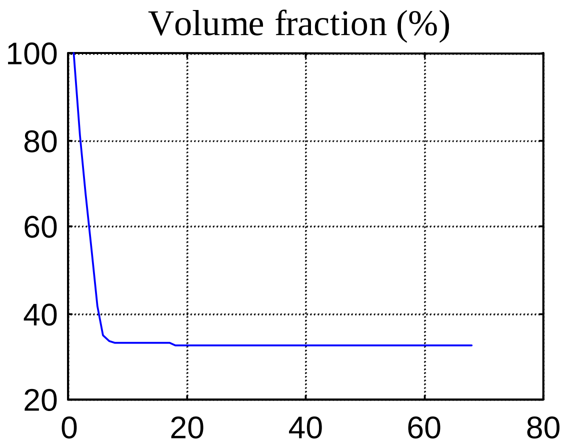
CONLIN (Fleury & Braibant, 1986)

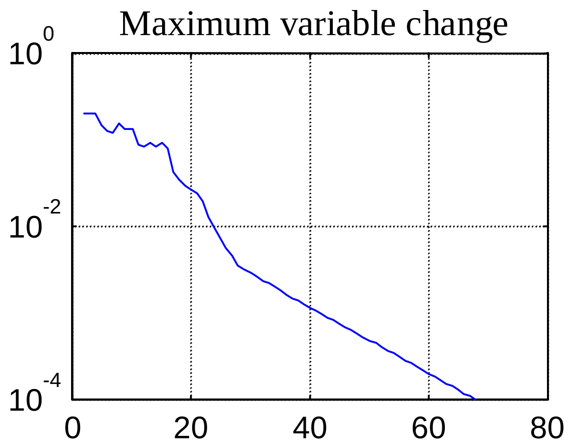
GCMMA (Svanberg, 1995)

OPTIMIZATION SOLUTION USING GCMMA

- **USING A NON MONOTONOUS APPROXIMATION: GCMMA**





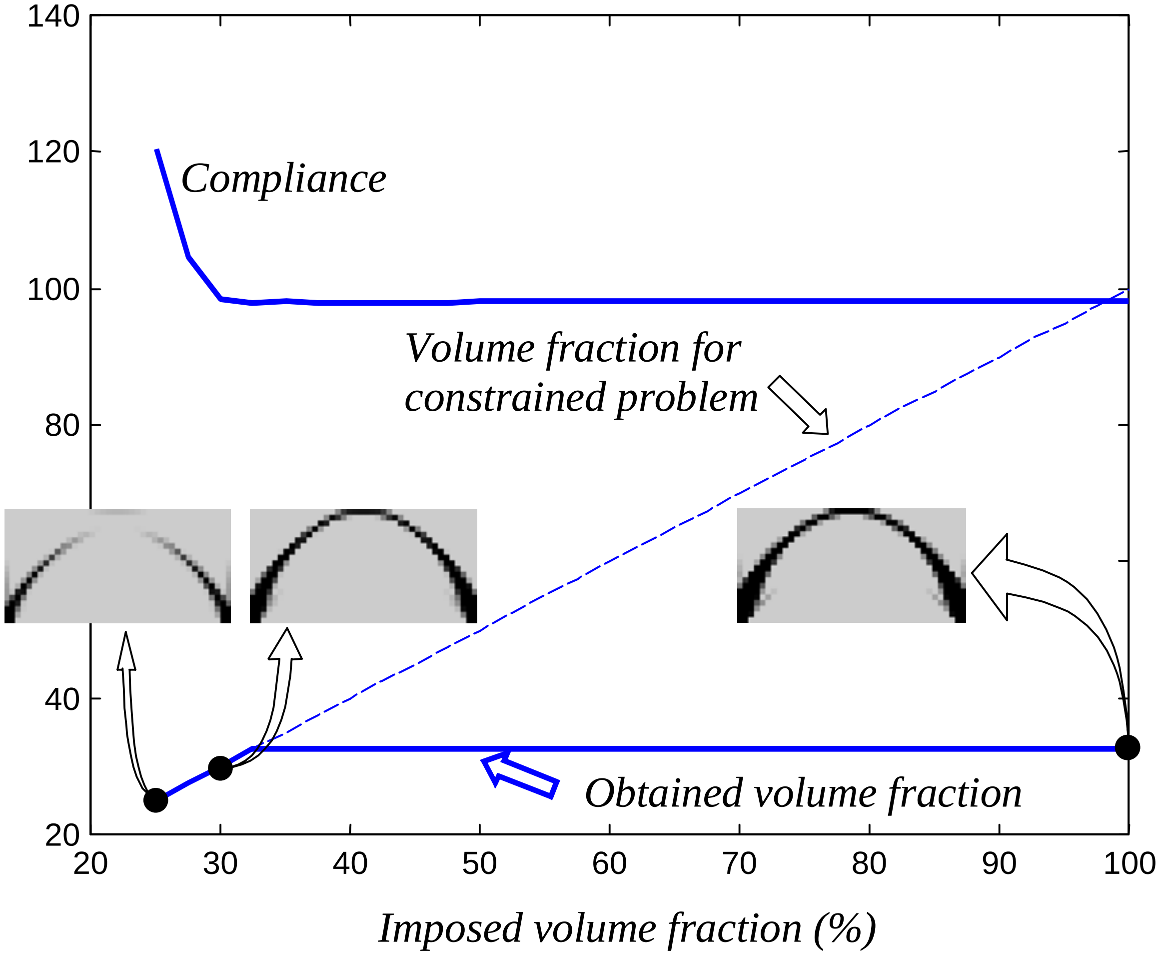


But this is still with a solution with minimum density of 0.2 !

THE OPTIMIZATION PROBLEM SOLUTION

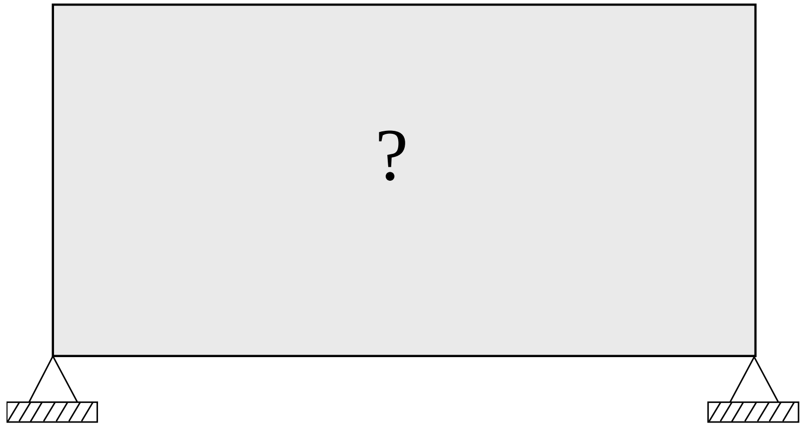
For volume fraction > 32.5% optimum is unconstrained

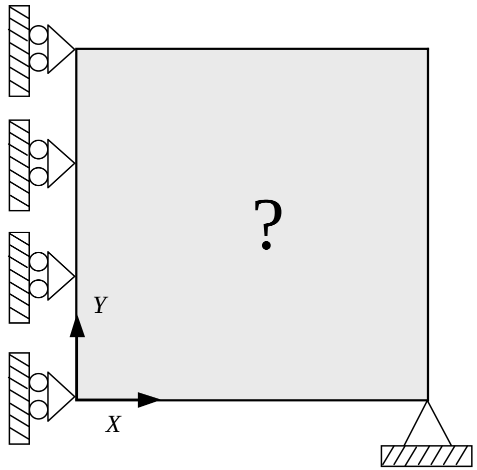
In agreement with conclusions of Turtletaub & Washabaugh, *Int. J. of Solids and Structures*, vol. 36, (1999) , pp 4587-4608



CONVERGENCE PROBLEM FOR LOW DENSITY VARIABLES

- Min Compliance
- Volume fraction < 80%
- p = 2 (SIMP)
- Filter
- Minimum density = 0.01
- GCMMA solver





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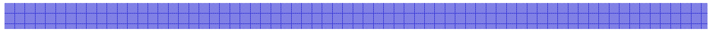
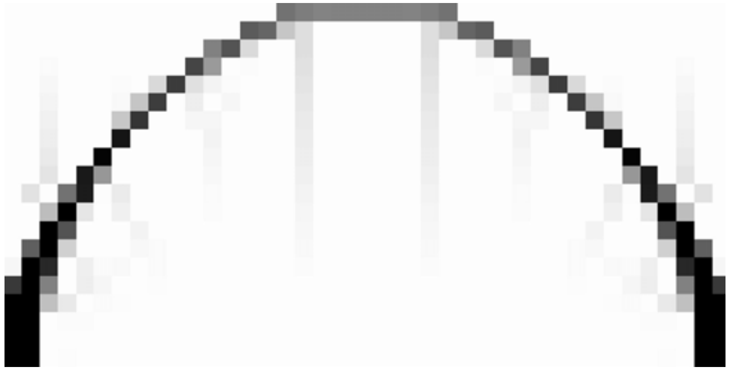
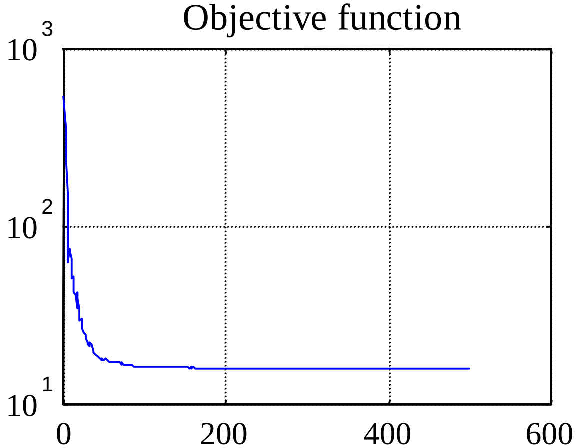
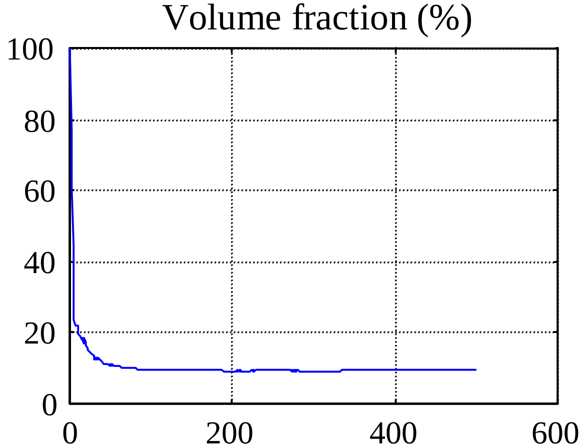
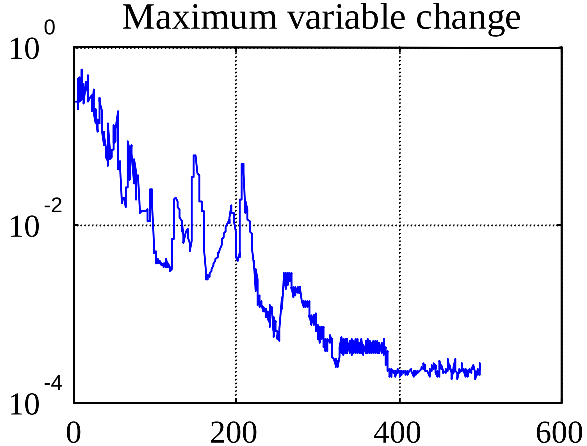








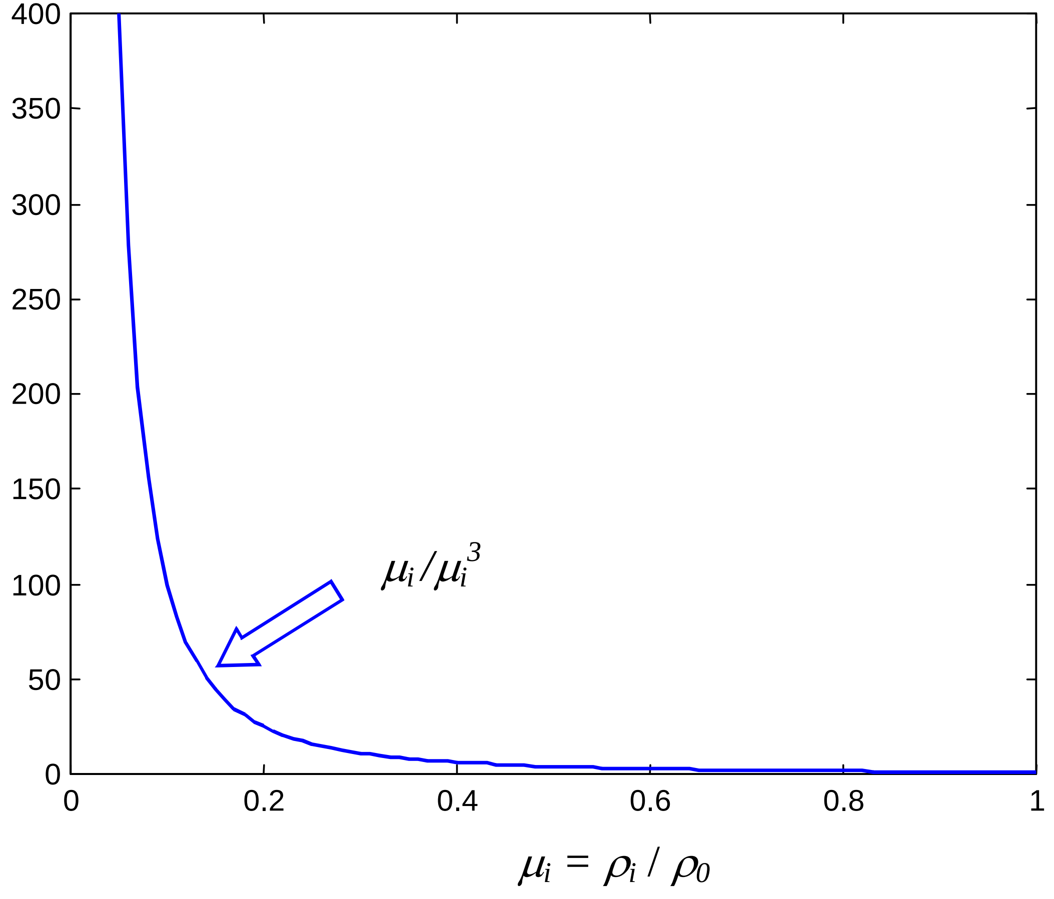


CONVERGENCE PROBLEM FOR LOW DENSITY VARIABLES

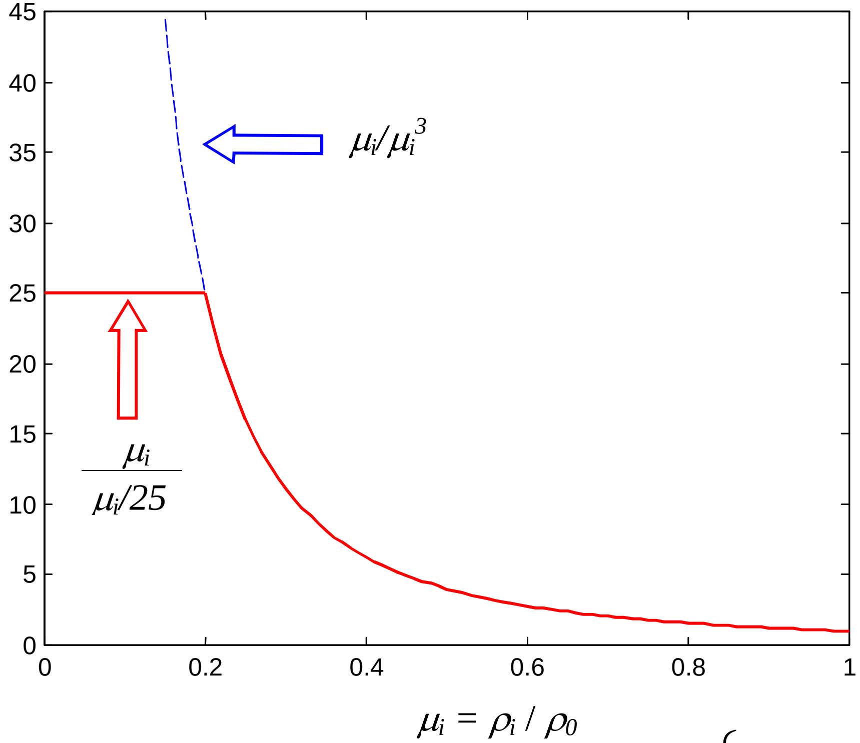
- **EXPLANATION**

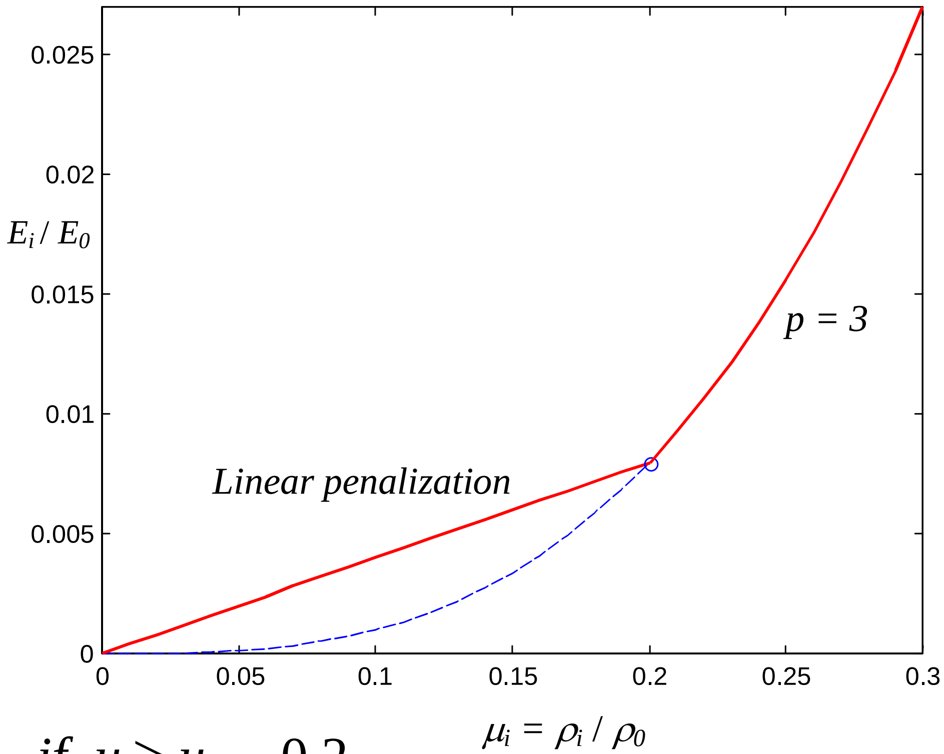
$$\frac{\|g\|}{\|K\|} \div \frac{\mu_i}{\mu_i^p} \frac{\rho^0}{E^0}$$

Problem similar to natural frequency problems !



MODIFICATION OF POWER LAW

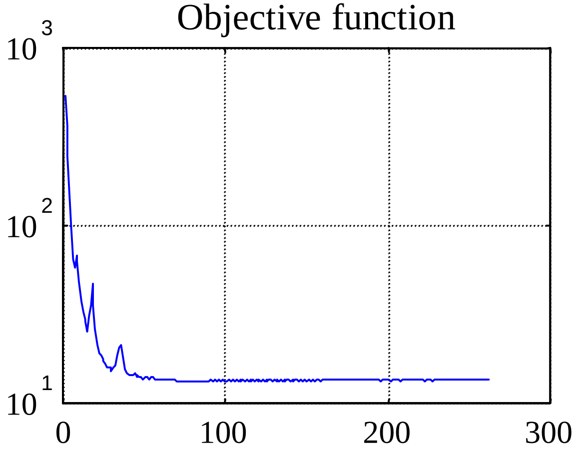


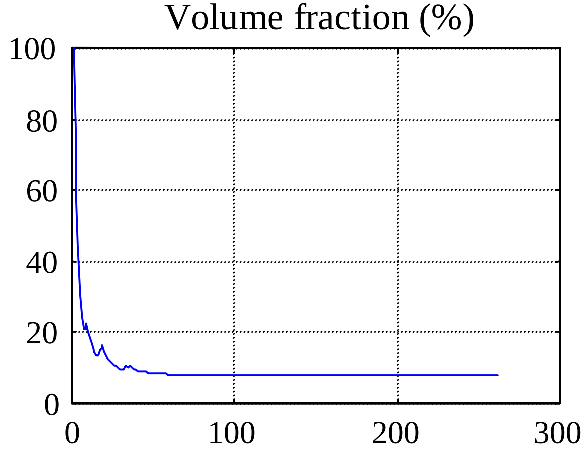


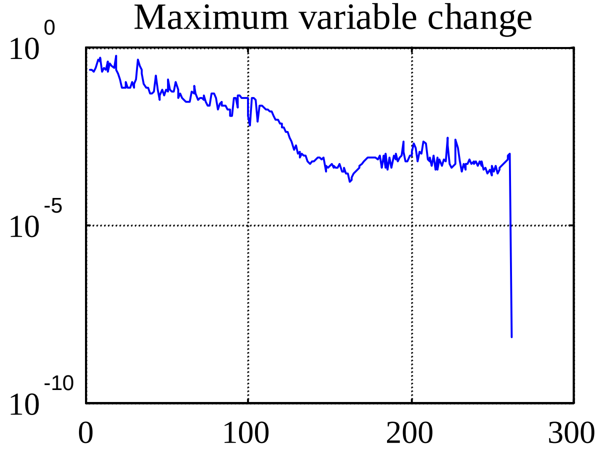
$$< E > = \begin{cases} \mu^p \; E^0 & if \;\; \mu \geq \mu_c = 0.2 \\ (\mu_c)^{p-1} \; \mu \; E^0 & if \;\; \mu < \mu_c = 0.2 \end{cases}$$

see N. Pedersen (2000) “Maximization of eigenvalues using topology optimization” Struc. Multidisc. Optim. 20, 2-11

GCMMA + MODIFIED OF POWER LAW



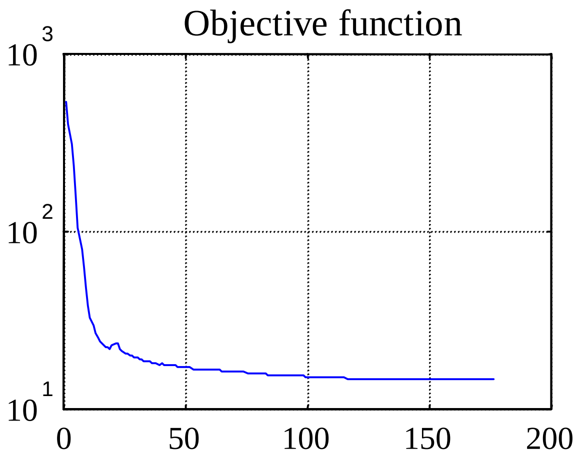


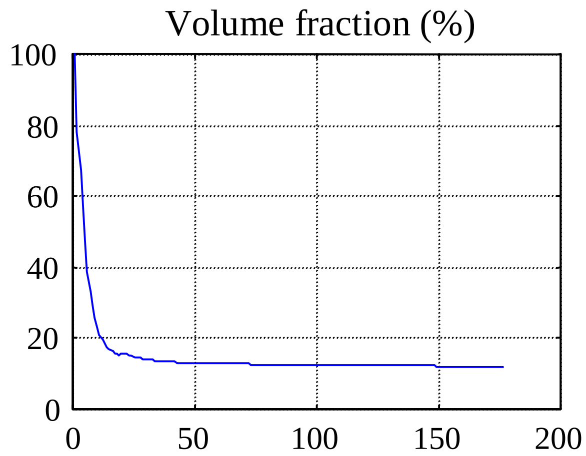


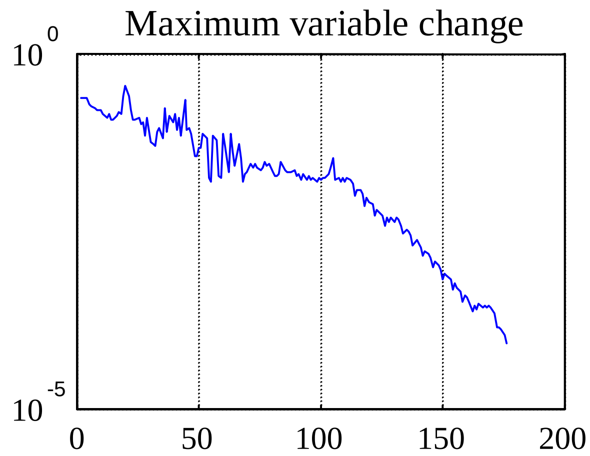
- Power law modified accordingly to N. Pedersen (2000) “Maximization of eigenvalues using topology optimization” Struc. Multidisc. Optim. 20, 2-11

GCMMA + MODIFIED OF POWER LAW

- **SIMP with p=3**

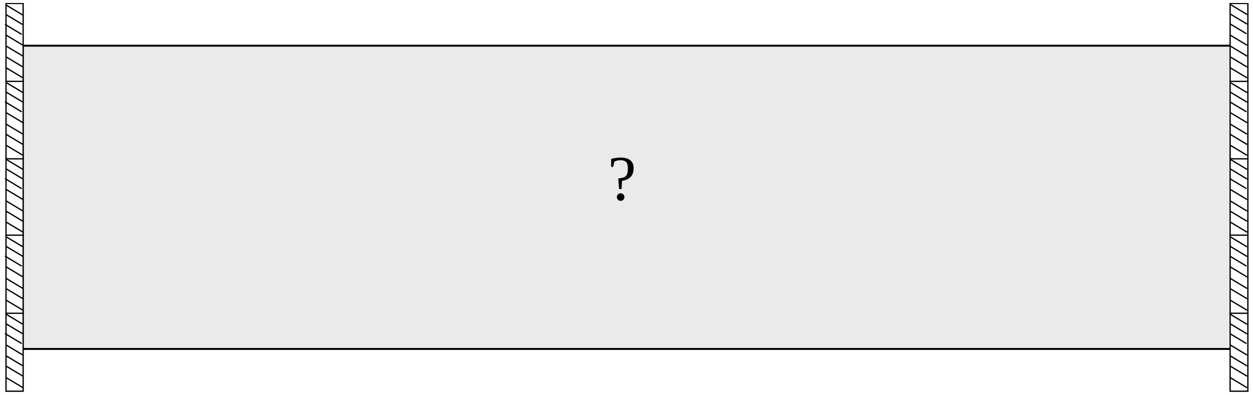


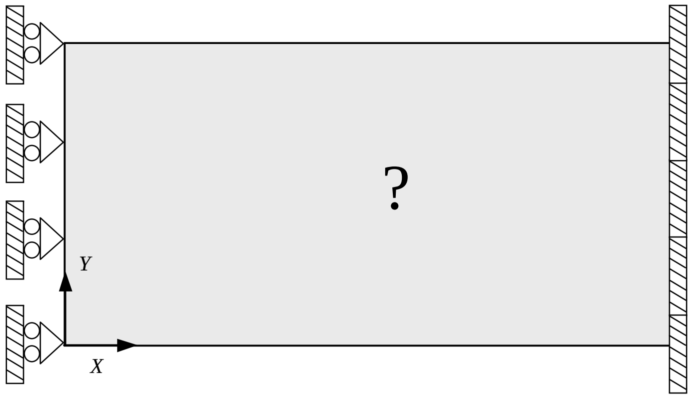




- **Power law modified accordingly to N. Pedersen (2000) “Maximization of eigenvalues using topology optimization” Struc. Multidisc. Optim. 20, 2-11**

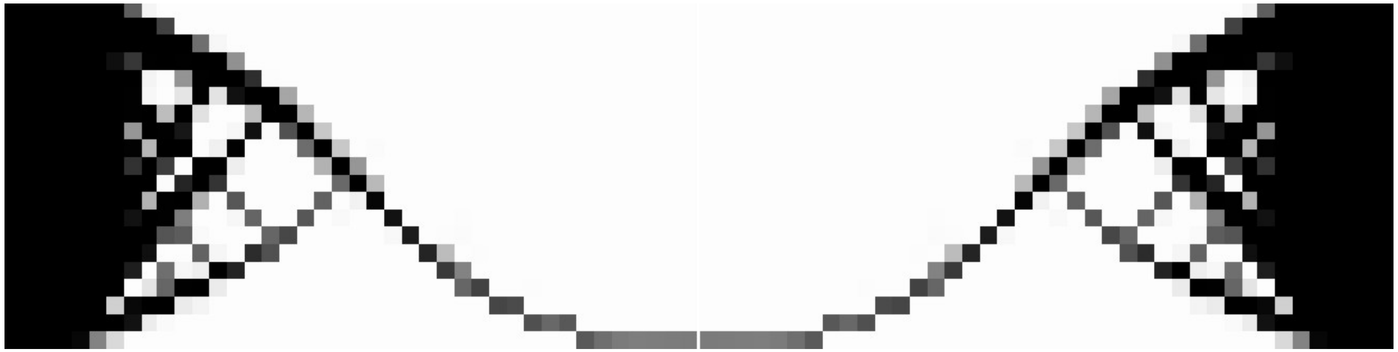
NUMERICAL APPLICATION 2

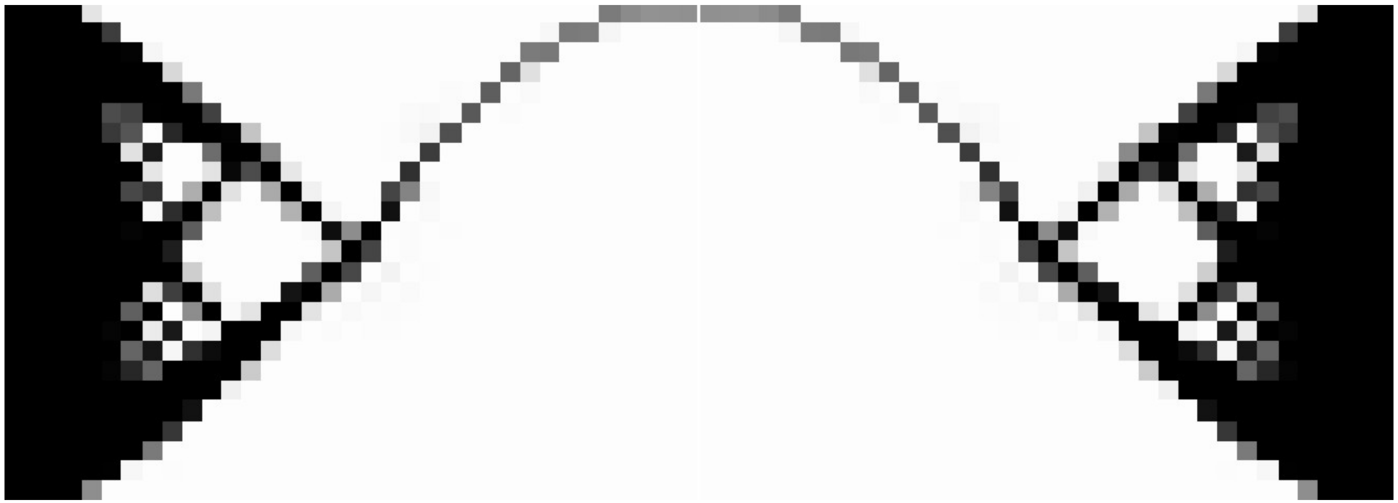




- Volume fraction < 100%
- p = 3 (SIMP)
- Min Compliance
- Filter
- minimum density = 0.01

NUMERICAL APPLICATION 2





CONCLUSION AND ON-GOING WORK

- **NUMERICAL SOLUTION PROBLEM**
  - Compliance can be non-monotonous with density dependent loads
  - Use non-monotonous approximations e.g. GCMMA
  - Increase solution performance with **GBMMA** approximations (Bruyneel, Duysinx, and Fleury, 2001)
    - high quality approximations using the information at previous design points (value + gradient)
    - mixed scheme with monotonous and non-monotonous expansions + an automatic selection strategy

- **LOW DENSITY PROBLEM**
  - Modifcation of power low model
  - Further investigation of the problem in the light of ε-relaxation
  - Convergence properties

- **INDUSTRIAL APPLICATIONS**
  - Mirror stiffening, design of rotating machine parts...