TOPOLOGY OPTIMIZATION WITH SELF-WEIGHT LOADING : UNEXPECTED PROBLEMS AND SOLUTIONS

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Abstract:

Topology optimisation of structures has been explored for more than 10 years. Especially, compliance minimisation has been intensively studied. Thus it may seem that everything has been said and done on that topic. Nevertheless, we must admit that we use usually the same benchmarks and test problems. Some practical problems that appear in industrial problems have not been carefully investigated and their detailed solution procedure is not totally available. An example of this is the compliance minimisation of structures subject to their self-weight or to any kind of body forces depending of the volume density. So the problems described in this note also happen in problems with centrifugal forces or overall acceleration forces. For instance, one can encounter these kinds of problems in civil engineering structures, in design of optical mirrors, or in rotating machines as turbo-machines. We were faced to difficulties related to self-weight problem for the first time with academic works given to students. However in this context it is rather difficult to suspect some problems where it is expected to have no difficulties a priori… We became more suspicious when we encountered again the same problems while trying to apply topology optimisation to the reinforcement of optical mirrors. This called for our attention and we started to investigate more carefully the problem. Here are the conclusions of this technical study.

In this study we use a standard procedure (see Bendsøe, 1995), in which the structure is dicretized into finite element and a constant density variable is assigned to each element. For the sake of simplicity, a power law material is considered:

$$< E > = \mu^p \; E^0 \quad and \quad < \rho > = \mu \; \rho^0 \quad p > 1 \tag{1}$$

For the numerical solution of the optimisation problem, the sequential convex programming approach (Duysinx, 1997) is used. Our experience with this method is very good.

From the sensitivity analysis point of view, introducing density dependent loads is rather simple. If **K** is the stiffness matrix, **q** is the generalised displacement vector, and **g** is the load vector, the weak equilibrium equations are:

$$\mathbf{K}\;\mathbf{q} = \mathbf{g} \tag{2}$$

Thus the sensitivity of the generalised displacement with respect to a design variable μ is:

$$K\;\frac{\partial \mathbf{q}}{\partial \mu} = \frac{\partial \mathbf{g}}{\partial \mu} - \frac{\partial \mathbf{K}}{\partial \mu}\;\mathbf{q} \tag{3}$$

Because the body forces depends on the density variables, the sensitivity of the compliance has two terms:

$$\frac{\partial(\mathbf{g}^T \mathbf{q})}{\partial \mu} = 2\frac{\partial \mathbf{g}^T}{\partial \mu}\;\mathbf{q} - \mathbf{q}^T\;\frac{\partial \mathbf{K}}{\partial \mu}\;\mathbf{q} \tag{4}$$

However as the changes in the sensitivity analysis are minor, one expects that the usual solution procedure will give good results in a straight forward manner. Unfortunately, this is not the case as it is illustrated on the following benchmark. One looks for the best structure that is able to withstand its self-weight loading. Two supports are allowed in the corner of the rectangular design domain. The volume constraint is 80 percent of the whole design domain volume. A low penalization (p=2) parameter is taken. CONLIN optimizer (Fleury and Braibant, 1986) is adopted. Convergence curves (see Fig.3) show clearly that convergence is far from being reached!

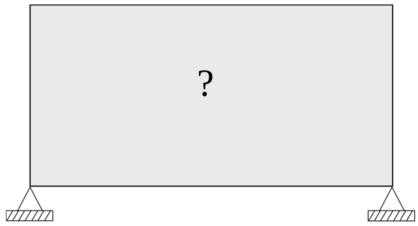

Fig.1 : Design domain and support conditions of test problem.

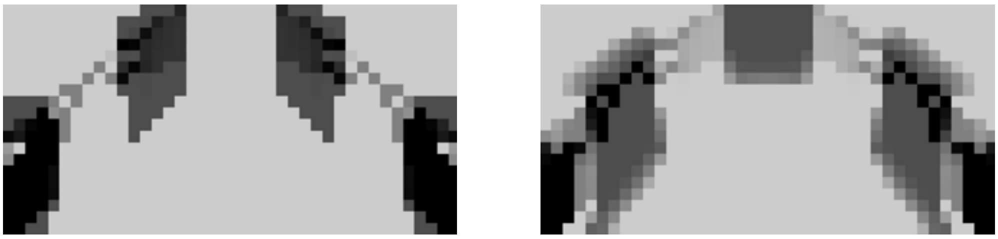

Fig. 2: Distribution of density after 199 iterations (left) and 200 iterations (right)

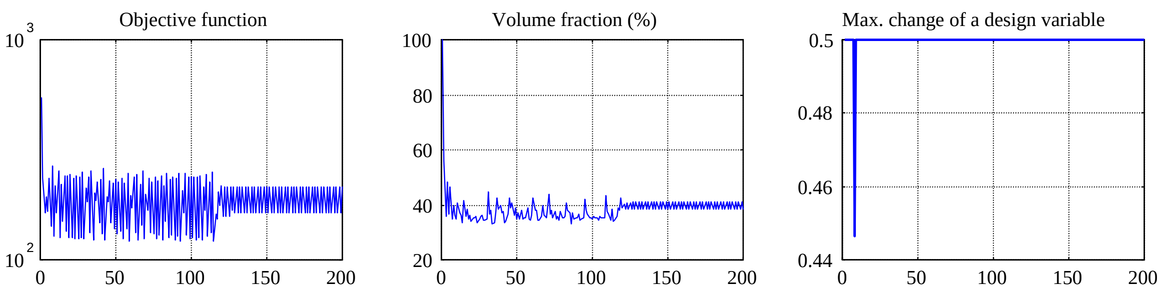

Fig. 3: Convergence history with CONLIN optimizer and power law model.

After investigation, two problems have been identified. The first one is related to the numerical solution of the optimisation problem and more precisely to the quality of the approximations used for the compliance. The second difficulty comes from the physics of the problem and from the effective stiffness and volume laws in terms of the micro-structural parameter. It is related to the ratio between the stiffness and the weight properties for low densities and the remedy to the problem calls for a modification of the power model in the zero density neighbourhood.

The first problem is related to the non-monotonous character of compliance when self-weight loading is included. As observed in equation (4) giving the sensitivity, the second term of the right hand side is negative while the first term, which is introduced by the self-weight, is positive. So depending on the displacement and energy values, the derivatives can be either positive or negative and the compliance can be monotonous or non-monotonous with respect to the design variables. Due to non-monotonous nature of the compliance, monotonous approximations like CONLIN (Fleury and Braibant, 1986) or MMA (Svanberg, 1987) are not well adapted to the problem and these schemes can lead to oscillating optimisation processes as observed in Fig 3. Even after a large number of iterations (e.g. 200 iterations), one can observe large modifications of the design variables. According to our experience, the rather simple solution of using a move limits strategy (even small ones) does not provide a satisfactory answer to the oscillation problem. The real solution is to resort to non-monotonous approximations like the globally convergent method of moving asymptotes (GCMMA) proposed by Svanberg (1995). However as the GCMMA scheme is generally very conservative, the convergence speed is sometimes slow. So we are adapting a family of new approximation schemes built on the MMA family presented in Bruyneel et al., (2001) and originally tested for composite problems. On top of all, this new kind of approximation scheme allows mixing monotonous and non-monotonous expansions. For each design variable, the best suited scheme is selected automatically on the basis of an algorithm using the available information at the current and previous design points. In these conditions, one recovers the good convergence conditions that are usual in compliance minimisation with dead loads.

The second problem with self-weight loading comes from the ratio between the stiffness and the body loads when going to zero density. For power law material, this ratio is:

$$\frac{\|g\|}{\|K\|} \div \frac{\mu}{\mu^p} \frac{\rho^0}{E^0} \qquad (5)$$

When going to zero-density, the weight dominates the stiffness and the displacements become very large. The phenomenon is similar to the one that arises in natural frequency problems (Duysinx, 1996, N. Pedersen, 2000). In self-weight problems, it leads to brutal jumps of the design variables which would like to converge to a low density value. To circumvent the problem, we implemented a modified power law model as suggested by N. Pedersen (2001).

$$< E > = \begin{cases} \mu^p \, E^0 & if \;\; \mu \geq \mu_c = 0.2 \\ (\mu_c)^{p-1} \, \mu \, E^0 & if \;\; \mu < \mu_c = 0.2 \end{cases} \qquad (6)$$

With the two kinds of modifications that have been presented, we can assure the stability and the smoothness of the optimisation procedure when solving self-weight compliance minimisation problems. The successful procedure is illustrated in the following figures (Fig. 4 and 5).

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Fig. 4: Optimal material distribution with the GCMMA and the modified power law.

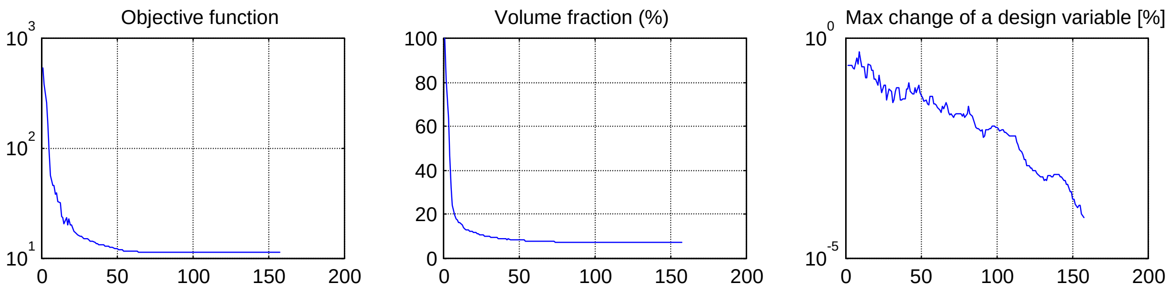

Fig. 5: Convergence history with the GCMMA and the modified power law.