Proposed design rules for iron columns reinforced by FRP

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ABSTRACT: This paper presents design rules for iron columns under axial compression, strengthened with high modulus carbon fibre reinforced polymer (FRP) sheets. According to experimental and numerical results, it is shown that the resistance and stiffness of iron columns can be significantly increased with the use of longitudinal FRP sheets because of the reduction of the column slenderness, but also that transverse FRP sheets should be used to prevent any local buckling of the longitudinal FRP sheets.

1 INTRODUCTION

The present study is a part of the European project PROHITECH whose main objective is to develop sustainable methodologies for the use of reversible mixed technologies in the seismic protection of existing constructions with particular emphasis to historical and monumental buildings (Mazzolani et al. 2008). In this framework, one of the contributions of the University of Liège (ULg), in collaboration with University of Naples "Federico II", is devoted to derive design rules for the iron columns reinforced by Fibre Reinforced Polymer (FRP) sheets.

In the literature, design rules are available to predict the resistance of steel elements reinforced by FRP sheets subjected to tensile loads or to bending moments (CNR-DT 202/2005), but no rules have yet been addressed to predict the buckling resistance of such elements under bending and/or axial compression, especially when they are submitted to earthquake. Three main buckling problems may occur with such loading: compressive buckling associated to members under axial compression, lateral torsional buckling associated to members under bending and compressive flexural buckling associated to members under bending and axial compression.

For the simplicity's sake, it is possible to solve all these problems through the solutions proposed for the compressive buckling associated to members under axial compression:

a) *Members under bending (Lateral Torsional Buckling – LTB):* no information relative to the resistance of iron members affected by lateral torsional buckling seems available. As an alternative to the study of the actual LTB effects, it is possible to refer, for I-shape elements, to a traditional approach that consists in considering LTB as a transversal buckling of the compression flange.
b) *Members under bending and axial compression*: an iron member in bending and axial compression is affected, at the same time, by compressive buckling and by LTB. Accordingly, it is possible to refer to an elastic interaction criterion to combine these two phenomena.

In addition, lateral force method (i.e. equivalent static loading of earthquake) can be used when a structure satisfies criteria on the regularity and vibration period conditions (Eurocode 8). That is why the priority of this research is first to focus on the investigation of the buckling resistance of iron columns reinforced by FRP under static axial compression.

2 IRON MATERIAL

The mechanical properties of iron material are highly dependent on the origin and production period of the iron. Usually, iron material possesses a relatively ductile behaviour in compression, but a brittle one in tension. The ratio of the two ultimate strengths ($\sigma_{i,u,t}/\sigma_{i,u,c}$), in tension and in compression, may range from 0.1 to 0.2 (Rondal et al. 2003).

Following Rondal et al. 2003 and Ly et al. 2008a, the full behaviour of irons can be expressed by a non linear part in compression, with four parameters E_i, $\sigma_{i,0.2,c}$, n and $\sigma_{i,u,c}$ (Ramberg-Osgood law – formula (1)), and a linear part in tension, with two parameters E_i and $\sigma_{i,u,t}$. Figure 1 shows that the so-defined law permits to represent, with a good accuracy, the behaviour of iron materials if compared to experimental results (curves BT2 to BT5).

$$\varepsilon = \frac{\sigma}{E_i} + 0.002\left(\frac{\sigma}{\sigma_{i,0.2,c}}\right)^n \qquad (1)$$

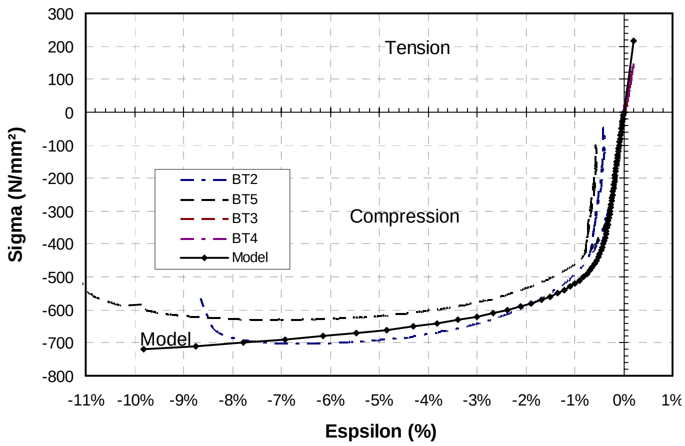

Figure 1 - Comparison of the defined analytical model for the iron behaviour law with experimental test results (Ly et al. 2008a)

Given the mechanical characteristics of iron material described above, it is preferable to assume that this material can only work in the elastic domain, especially when subjected to tensile stresses. Accordingly, elastic analyses should be used to design iron elements reinforced by FRP.

3 FRP MATERIAL

The applicability and the effectiveness of the strengthening with FRP depend mainly on the material and the nature of the member to be strengthened. When applied as reinforcement, the strengthening material should have a similar or higher stiffness compared to the member to be strengthened. Figure 2 shows stress-strain behaviour laws for different commercial FRPs compared to the steel one.

The strengthening of steel or iron members with FRP may be both mechanically and economically satisfactory in retrofitting due to ease of installation and the potential of eliminating welded and bolted repairs. In particular, for historical buildings, the overall aim is to preserve the appearance of all structural elements to be reinforced, what is possible with the FRP technique.

4 SAFETY APPROACHES

The use of iron as a building material probably dates back to about the year 1800. Cast iron columns were still being produced for limited uses in the early 1930s but were progressively replaced by steel at the beginning of the 20th century. During this period, the design of structural elements was performed according to the "**allowable stresses**" safety approach based on global safety factors applied to the material strengths (values ranging from 4 to 5 as given in the available literature).

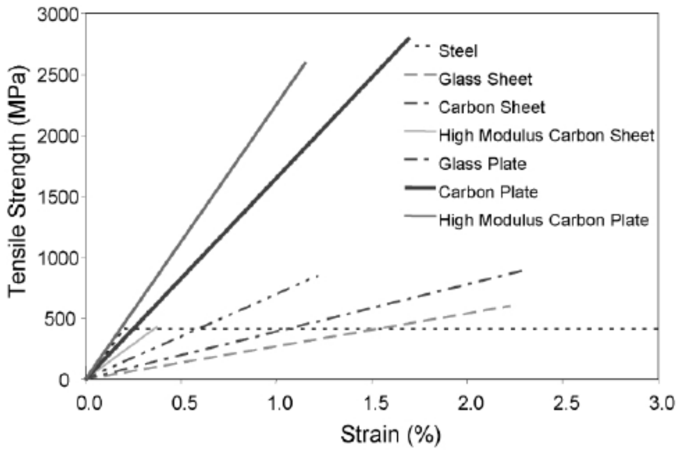

Figure 2: Stress-strain behaviour curves for different FRPs compared to the steel one (Buyukozturk et al. 2004)

Nowadays, another safety approach is proposed and usually used: the **semi-probabilistic approach** based on partial safety factors (safety factors applied to the material strengths and to the actions). For cast iron, values ranging from 2.16 to 2.7 are proposed for the material safety factors (Käpplein et al.) and an average value of about 1.4 for the action safety factors (Eurocode 0).

However, equivalence between the two methods is observed; indeed, if the material safety factors from the semi-probabilistic approach are multiplied by the action safety factors, the obtained values vary from 3.0 to 3.8, what is close to the global safety factors used in the allowable stresses approach. It means that there is no difference between both.

In this report, the proposed analytical procedure is founded on **the semi-probabilistic approach,** used in most recent codes and standards such as the Eurocodes.

5 PROPOSED DESIGN RULES

5.1 *Cross section resistance*

In this paper, the class 4 (according to Eurocode 3) is not considered. Accordingly, a cross section may reach its elastic resistance, under axial forces.

Experimental tests on stocky elements (Ly et al. 2008b) show that within the elastic domain ($\varepsilon \leq 0.2\%$), FRP and iron member behave as different parts of a monolithic cross-section. Then the elastic resistance of a composite cross-section in compression can be calculated with the entire transversal area as follows:

$$N_{e,Rd} = \sigma_{i,0.2,c} A_{eq} \quad (2)$$

A_{eq} is the equivalent cross-sectional area, see formula(6).

5.2 *Members under axial compression*

5.2.1 *Member imperfections*

Like other columns, cast iron columns also suffer of geometrical imperfections. An initial crookedness (δ_0) taken as the maximum deviation of the column axis from a straight line connecting the ends can be assumed as given in Rondal et al. 2003:

$$\delta_{0,\max} = \frac{L}{750} \quad (3)$$

5.2.2 *Cross-section imperfections*

In circular hollow cast iron sections, the internal and external diameters are usually eccentric, as shown in figure 3. Irregular wall thickness is the result of lifting forces, dislocations and/or deflections of the casting core used for producing the hole of the member during casting in the horizontal position. This geometrical eccentricity of the hole leads to an eccentricity (g_i) of the load with reference to the centroid of the cross-section. The eccentricity g_i can be obtained by the following formula:

$$g_i = j\frac{d_i^2}{d_e^2 - d_i^2} \quad (4)$$

with d_e, the external diameter, d_i, the internal diameter and j calculated as follows:

$$j = \frac{d_e - d_i}{2} - t_{\min} \quad (5)$$

t_{min} is the minimum thickness, value which is difficult to estimate as the latter is dependant on how the casting core can move during the iron member casting.

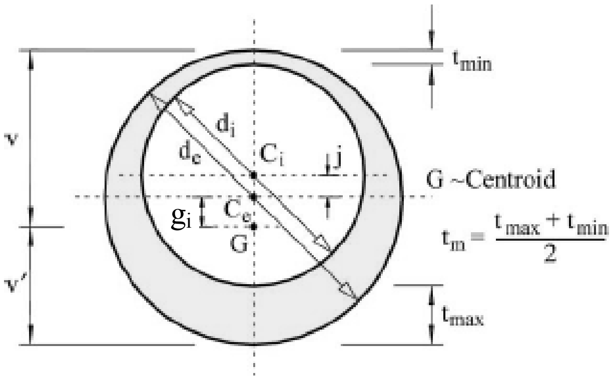

Figure 3: Cross-section imperfection in hollow cast iron column

5.2.3 *Analytical formulation*

An analytical formulation is proposed by Rondal et al. 2003 to predict the buckling resistance of iron columns subjected to axial compression. Its extension to FRP reinforced iron is here contemplated. As iron is quite resistant in compression, but relatively weak in tension, two possible failure modes have to be successively considered (figure 4):

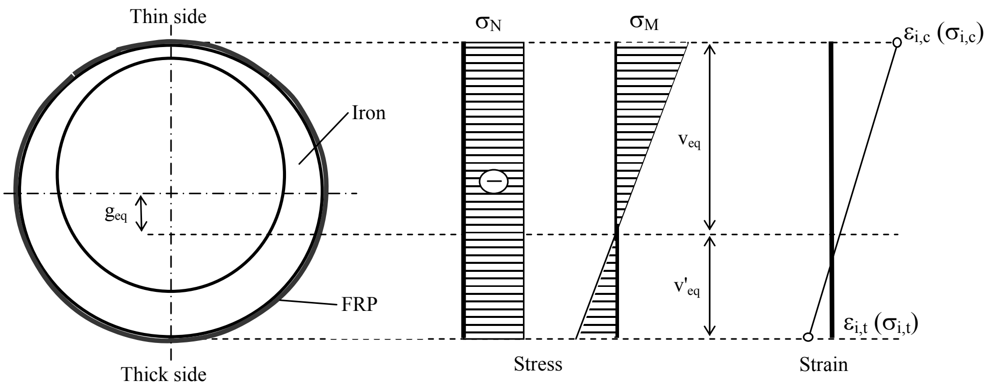

Figure 4: Strain and stress distribution for FRP-iron composite section

- failure by excess of compression on the thin side;
- failure by excess of tension on the thick side.

The location where failure occurs in the section (thin or thick side) results from the eccentricity g_{eq} between the centroid and the load introduction axis.

a) Mechanical characteristics of a composite cross-section

The strain and stress distribution within a composite section is described in figure 4. The equivalent area of the composite cross-section can be calculated with the following formula:

$$A_{eq} = A_i + n_{eq} A_f \quad (6)$$

where

- the equivalent coefficient n_{eq} is given by

$$n_{eq} = \frac{E_f}{E_i} \quad (7)$$

- the areas of FRP A_f (assuming that the thickness of FRP sheets t_f is much smaller than the outer diameter of iron member d_e) and iron section A_i are given by

$$A_f = t_f 2\pi\left(r_e + \frac{t_f}{2}\right) \tag{8}$$

and

$$A_i = \pi\left(r_e^2 - r_i^2\right) \tag{9}$$

The equivalent second moment of inertia for the composite cross-section can be estimated by the following formula:

$$I_{eq} = I_i + n_{eq} I_f \tag{10}$$

where

- the second moment of inertia for the iron member section I_i is given by

$$I_i = \frac{\pi}{4} r_e^4 + \pi r_e^2 g_{eq} - \left[\frac{\pi}{4} r_i^4 + \pi r_i^2 \left(g_{eq} + j\right)^2\right] \tag{11}$$

with the position of the gravity centre g_{eq}, according to the centre of the outer perimeter of the iron member, estimated through formula (12)

$$g_{eq} = \frac{\pi j r_i^2}{n_{eq} A_f + A_i} \tag{12}$$

- the second moment of inertia for the FRP area If is estimated by

$$I_f = \pi t_f \left(r_e + \frac{t_f}{2}\right)^3 + A_f g_{eq}^2 \tag{13}$$

The distances v_{eq} and v_{eq}' between the gravity centre g_{eq} and the extreme fibres of the iron member, see figure 4, are equal to

$$v_{eq} = r_e + g_{eq} \tag{14}$$

$$v_{eq}' = r_e - g_{eq} \tag{15}$$

b) Compression failure

Working with the equivalent iron cross-section, the nominal buckling compressive stress $\sigma_{b,c}$ (N_u/A_{eq}), when the column reaches the buckling resistance (N_u), can be derived through the following formula:

$$\sigma_{b,c} = \chi_c \sigma_{i,0.2,c} \tag{16}$$

where $\sigma_{i,0.2,c}$ is the 0.2% proof stress of iron in compression and χ_c, the slenderness reduction factor calculated when the most stressed iron or FRP fibre (the farthest fibre) reaches its elastic strength ($\sigma_{i,0.2,c}$ or $\sigma_{f,u,c}$). In other words, the farthest fibre of the equivalent cross-section reaches a stress $\sigma_{i,c}$ corresponding to a strain $\varepsilon_{i,c}$, the latter being defined as the minimum of the two values $\varepsilon_{i,0.2,c}$ and $\varepsilon_{f,u,c}$ corresponding to the ultimate strain for the iron material and the FRP respectively (figure 4). If f_c designates the ratio $\sigma_{i,c}/\sigma_{i,0.2,c}$, χ_c can be calculated as follows:

$$\chi_c = \frac{f_c}{\varphi_c + \sqrt{\varphi_c^2 - f_c \bar{\lambda}^2}} \tag{17}$$

with

$$\varphi_c = \frac{1}{2}(1 + \eta_c + f_c \bar{\lambda}^2) \tag{18}$$

where

$$\bar{\lambda} = \frac{\lambda}{\lambda_e} \tag{19}$$

$$\lambda = \frac{L}{r_{eq}} \tag{20}$$

$$r_{eq} = \sqrt{\frac{I_{eq}}{A_{eq}}} \tag{21}$$

and

$$\lambda_e = \pi\sqrt{\frac{E_i}{\sigma_{i,0.2,c}}} \tag{22}$$

The imperfection parameter η_c is given by

$$\eta_c = \alpha\left[\left(\lambda_c - \lambda_1\right)^\beta - \lambda_0\right] + g_{eq} A_{eq} \frac{v_{eq}}{I_{eq}} \tag{23}$$

(α, β, λ_0, λ_1), accounting for the column imperfection, depend on the material parameters n and $e = \sigma_{i,0.2,c}/E_i$ (see Rasmussen et al. 2000). The term $g_{eq} A_{eq} v_{eq} / I_{eq}$ in formula (23) accounts for the cross-section imperfections.

c) Tension failure

Cast iron is relatively weak and brittle in tension; a column failure by excess of tension may be observed, because of the development of significant second-order bending moment in slender columns. The verification of the tension failure mode can be achieved through the following resistance formula:

$$\sigma_{b,t} = \chi_t \sigma_{i,0.2,c} \tag{24}$$

As in the previous paragraphs, χ_t should be calculated when the farthest iron or FRP fibre reaches its elastic strength in tension ($\sigma_{i,u,t}$ or $\sigma_{f,u,t}$). But in practice the FRP strength $\sigma_{f,u,t}$ is much higher than the iron one; so the tension failure takes place in the iron part. If f_t designates $\sigma_{i,u,t}/\sigma_{i,0.2,c}$, the slenderness reduction factor χ_t can be calculated through the following formula:

$$\chi_t = \frac{f_t}{\varphi_t + \sqrt{\varphi_t^2 + f_t \overline{\lambda}^2}} \quad (25)$$

where

$$\varphi_t = \frac{1}{2}(-1 + \eta_t + f_t \overline{\lambda}^2) \quad (26)$$

$$\eta_t = \alpha\left[(\lambda_c - \lambda_1)^\beta - \lambda_0 \right] + g_{eq} A_{eq} \frac{v'_{eq}}{I_{eq}} \quad (27)$$

The term $g_{eq} A_{eq} \upsilon'_{eq} / I_{eq}$ in formula (27) accounts for the cross-section imperfections in case of tension failure mode.

5.3 *Validation of the proposed rules with numerical simulations*

To validate the proposed design rules, they are compared with numerical simulations performed through the homemade finite element software FINELG. In fact, in another paper (Ly et al. 2008c) comparing the numerical predictions to experimental tests obtained at the University of Liège, it is illustrated that the proposed numerical model is able to provide a safe prediction of the buckling resistance of iron members with or without FRP, what validated the used numerical tool.

In order to compare easily results obtained for iron columns respectively with and without FRP, all the buckling curves will be presented in a "N_B - Lambda B_i" format, "N_B" (= $\overline{N}$) being the non-dimensional resistance defined by formula (28)

$$\overline{N} = \frac{N_u}{\sigma_{i,0.2,c} A_i} \quad (28)$$

and "Lambda B_i" (= $\overline{\lambda}_i$), the non-dimensional slenderness of the corresponding columns without FRP defined by formula (29)

$$\overline{\lambda}_i = \frac{L}{\sqrt{\frac{I_i}{A_i}}} \frac{1}{\lambda_e} \quad (29)$$

N_u (= $\min(\sigma_{b,t}; \sigma_{b,c}).A_{eq}$) is predicted either numerically or analytically through the procedure described previously.

In order to make easier the comparison between numerical and analytical results, some assumptions have been done, in particular concerning the imperfections; accordingly, the properties that have been used within this study are the following:

- Materials:
  - Iron material behaviour when subjected to compression is approximated through a Ramberg-Osgood law with the following parameters: E_i = 88000 N/mm², n = 6, $\sigma_{i,0.2,c}$ = 375 N/mm² in compression; and when subjected to tension, through a linear elastic law with the following parameters: $\sigma_{i,u,t}$ = 75 N/mm² and E_i = 88000 N/mm².
  - The FRP sheets used to reinforced the iron columns are CFRP 530 ones, with an elastic modulus (E_f) equal to 640 GPa, a tensile strength ($\sigma_{f,u,t}$) equal to 2650 MPa and a Poisson's coefficient equal to 0.28. The compressive strength ($\sigma_{f,u,c}$) is assumed to be equal to the tensile strength, as no information is available concerning this property.
- For the iron columns reinforced by FRP, it is assumed that three longitudinal FRP sheets (3x0.19 mm) are set up along the outer perimeter. In addition, one transversal FRP is placed to prevent the out-of-plan buckling of the longitudinal FRP sheets.

- Member imperfection: an initial crookedness equal to 1/1000 of the column length is assumed.

- Geometry of the cross section: the latter has been defined with the help of a segment extracted from a tested column: d_e = 126.5 mm, d_i = 94 mm, t_{min} = 14.5 mm, t_{max} = 18 mm, j = 1.75 mm and g_i = 2.16 mm (see figure 3). These values have been assumed, for the simplicity's sake, to be constant along the length of the columns.

a) Iron columns without FRP

Axial compression buckling curves for iron columns without FRP obtained through numerical and analytical models are reported in figure 5. The dashed curve represents analytical buckling resistances in the tension failure mode on the thick side; the continuous curve, the compression failure on the thin side; whereas the dots represent numerical results. It has to be noted that the risk of having a tension failure mode increases with the increase of the column slenderness.

The good agreement between the numerical and the analytical results, whatever the failure modes, means that the analytical model permits a good prediction of the buckling resistance of iron columns.

b) Iron columns reinforced by FRP sheets

With the presence of the transversal FRP sheets, the longitudinal FRP sheets can reach the maximum strain $\varepsilon_{f,u,c}$ (= $E_f/\sigma_{f,u,c}$) equal to 0.414%. By introducing this value in the numerical and analytical models, results shown in figure 6 are obtained: the two dashed curves represent analytical buckling resistances of iron columns without FRP, while the two continuous curves correspond to analytical buckling resistances of FRP reinforced columns; again, the dots indicate numerical buckling resistances of iron columns with FRP.

The obtained numerical and analytical results are in good agreement for stocky or very slender columns, but not very optimal for the range of medium slenderness in which buckling resistance significantly depends on every imperfection factors. As the imperfection parameters (α, β, λ_0, λ_1) have been proposed for iron material, but not for the composite one composed of iron and FRP, new imperfection levels should be found so as to improve the proposed analytical formulation. Anyway, the actual analytical formulation gives safe results.

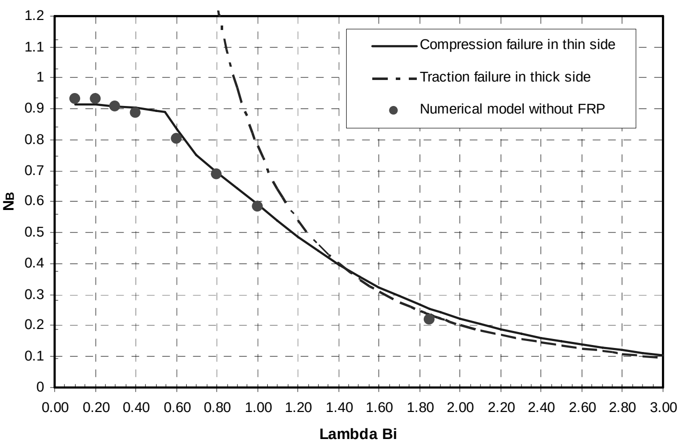

Figure 5: Axial compression buckling curves for iron columns without FRP

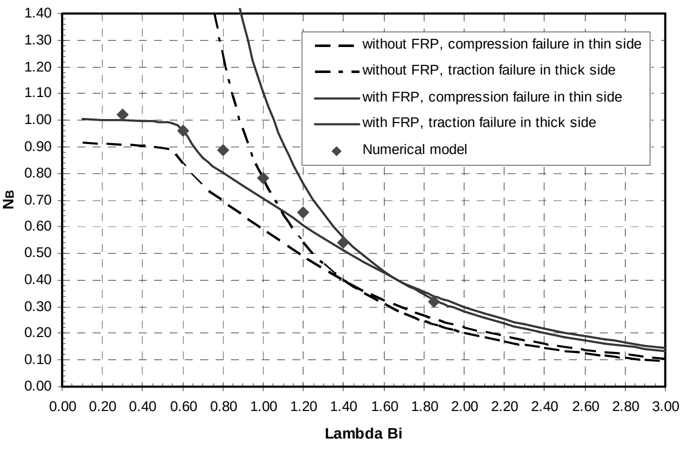

Figure 6: Axial compression buckling curves for iron columns with FRP not experiencing local buckling

6 CONCLUSIONS

Design rules able to predict the buckling resistance of iron columns reinforced or not with FRP sheets has been developed and validated. The presented design method is based on the modern buckling curve approach, as for steel columns, and is able to predict the resistance of iron columns, taking into account of the asymmetric behaviour of the iron material when subjected to tension or compression.

It was demonstrated herein that the proposed design rules produces safe results if compared to the numerical ones; it can be concluded that the proposed design rules permit then to predict a safe value of the buckling resistance of iron columns with or without FRP if compared to its actual resistance. It was also demonstrated that the accuracy of the model could be improved by defining appropriate imperfection parameters (α, β, λ_0, λ_1) for FRP-iron composite columns. To achieve that, further developments are requested, what constitutes a perspective to the presented paper.

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