

## **Updating Fatigue Failure Probability**

Considering Monitoring and Inspection Data

Quang Mai

December 7, 2018

Faculty of Applied Sciences

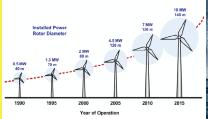


## Introduction

## Wind Energy – the fastest growing energy source

- Less environmental impact
- Fast installation
- Fast development!

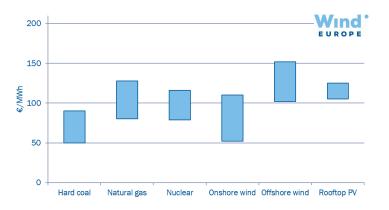




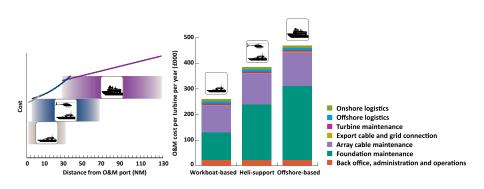


High LCoE

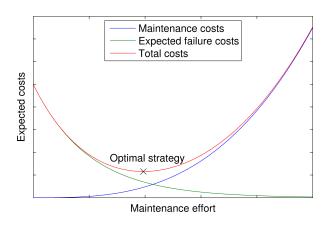
(lifetime costs per unit of electricity - MWh)

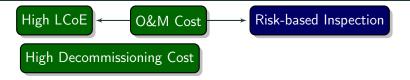




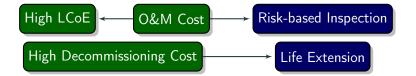




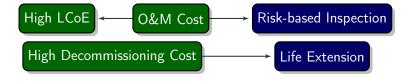












Majority of reported offshore failures are fatigue failures!

⇒ Update Failure Probability in Fatigue Failure Mode!

## Message Objective

Using Failure Assessment Diagram and Occurence of Weather Conditions in the Limit State Function improves the accuracy of the Updated Failure Probability.



## Road Map

Introduction

Background

FAD in Updating Considering Inspection

Occurence of Weather Conditions in Updating Considering Monitoring

Conclusion

## Background

## Road Map

#### Introduction

## Background

Fatigue Assessment

Failure Probability

**Updating Principle** 

FAD in Updating Considering Inspection

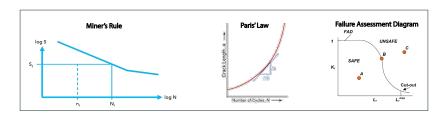
Occurence of Weather Conditions in Updating Considering Monitoring

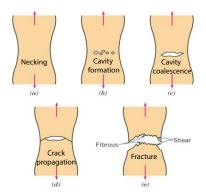
Conclusion

## Background

Fatigue Assessment

## Fatigue Assessment





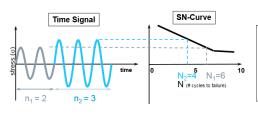
## Fatigue Assessment: Miner's Rule

Fatigue Damage:

$$D = \sum_{i=1}^{k} \frac{n_i}{N_i}$$

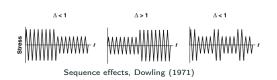
Safety condition:  $D \le \Delta$ , or Limit State Function:

$$g = \Delta - D \ge 0$$



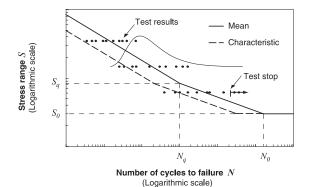


## Fatigue Assessment: Miner's Rule

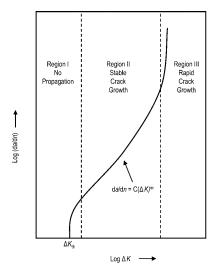




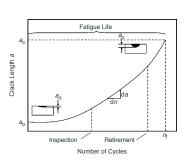
$$g = \Delta - D$$



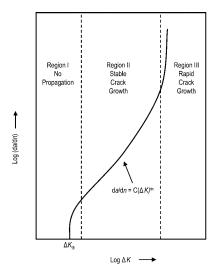
## Fatigue Assessment: Paris' Law



$$\frac{da}{dN} = C \left( \Delta K \right)^m$$
$$\Delta K = Y \Delta \sigma \sqrt{\pi a}$$



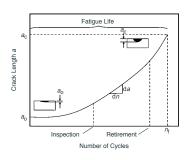
## Fatigue Assessment: Paris' Law



### Limit State Functions:

$$g_1 = a_c - a$$

$$g_2 = K_{mat} - K$$



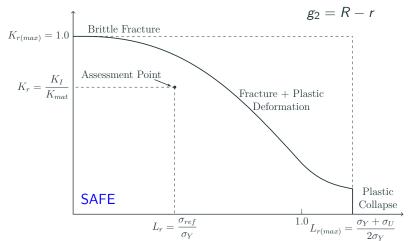
## Fatigue Assessment: Failure Assessment Diagram



## Limit State Function:

$$g_1 = K_{rFAD} - K_r$$

or:



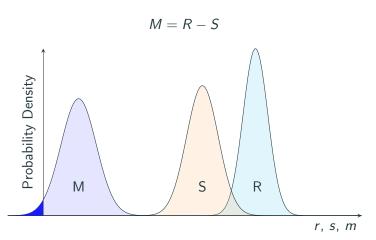
## Background

\_\_\_\_

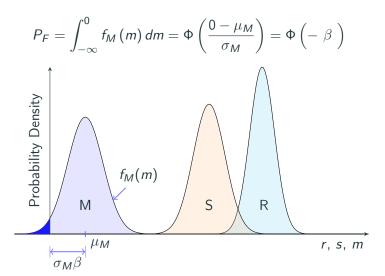
Failure Probability

## Failure Probability

Safety margin (Limit State Function):



## Failure Probability



## Background

Updating Principle

Bayes Theorem: 
$$P(A|B) = \frac{P(A,B)}{P(B)}$$

Bayes Theorem: 
$$P(A|B) = \frac{P(A,B)}{P(B)}$$

Event updating

Variable updating

$$M = R - S$$

Event updating

Variable updating

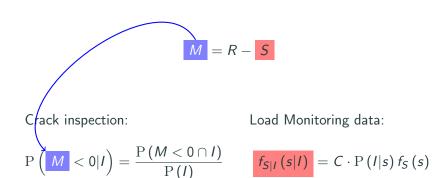
$$M = R - S$$

Crack inspection:

$$P\left(M < 0|I\right) = \frac{P\left(M < 0 \cap I\right)}{P\left(I\right)} \qquad f_{S|I}\left(s|I\right) = C \cdot P\left(I|s\right) f_{S}\left(s\right)$$

Load Monitoring data:

$$f_{S|I}(s|I) = C \cdot P(I|s) f_S(s)$$



$$M = R - S$$

Crack inspection:

$$P\left(M < 0|I\right) = \frac{P\left(M < 0 \cap I\right)}{P\left(I\right)} \qquad f_{S|I}\left(s|I\right) = C \cdot P\left(I|s\right) f_{S}\left(s\right)$$

Load Monitoring data:

$$f_{S|I}(s|I) = C \cdot P(I|s) f_S(s)$$

Inspection

FAD in Updating Considering

## Road Map

Introduction

Background

FAD in Updating Considering Inspection

Motivation & Literature Review

FAD Gives Higher Failure Probability Values

FAD in Updating

Occurrence of Weather Conditions in Updating Considering Monitoring

Conclusion

# FAD in Updating Considering

Inspection

Motivation & Literature Review

## Motivation

LSF 1:  $g = a_c - a$ 

 $a_c$ : critical crack size

a: crack size

#### Motivation

LSF 1:  $g = a_c - a$ 

 $a_c$ : critical crack size

a: crack size

⇒ ambiguous, for ships & pipelines!

#### Motivation

LSF 1:  $g = a_c - a$ 

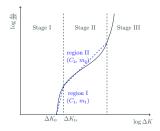
 $a_c$ : critical crack size  $\Rightarrow$  ambiguous, for ships & pipelines!

a: crack size

LSF 2: 
$$\begin{cases} g_1 = a_c - a \\ g_2 = K_{mat} - K_{max} \end{cases}$$

 $K_{mat}$ : fracture toughness

 $K_{max}$ : maximum stress intensity factor



#### Motivation

LSF 1:  $g = a_c - a$ 

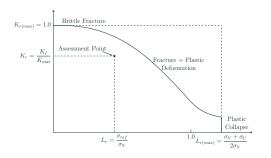
 $a_c$ : critical crack size  $\Rightarrow$  ambiguous, for ships & pipelines!

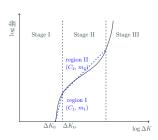
a: crack size

LSF 2: 
$$\begin{cases} g_1 = a_c - a \\ g_2 = K_{mat} - K_{max} \end{cases} \Rightarrow \text{fracture+plastic deformation?}$$

 $K_{mat}$ : fracture toughness

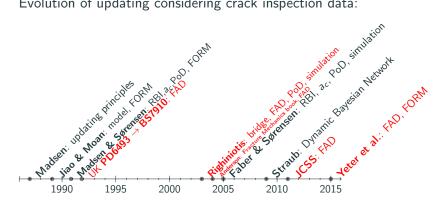
 $K_{max}$ : maximum stress intensity factor





#### Literature Review

Evolution of updating considering crack inspection data:



#### Literature Review

#### Gap:

Advantages and disadvantages of using Failure Assessment Diagram in updating failure probability considering crack inspection data for existing OWT support structures?

# FAD in Updating Considering Inspection

FAD Gives Higher Failure Probability Values

# Road Map

FAD in Updating Considering Inspection

Motivation & Literature Review

FAD Gives Higher Failure Probability Values

Limit State Functions to Compare

Method to Calculate Failure Probabilities

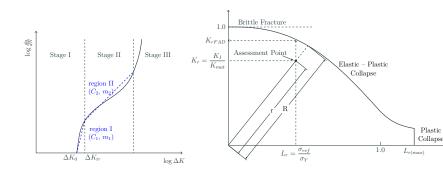
Results & Discussions

FAD in Updating

## Limit State Functions to Compare

LSF 1: LSF 2: LSF 3: 
$$g = a_c - a$$
 
$$\begin{cases} g_1 = a_c - a \\ g_2 = K_{mat} - K_{max} \end{cases}$$
 
$$g = K_{rFAD} - K_r$$
 or: 
$$g = R - r$$

Considered Uncertainties: C,  $a_0$ ,  $a_0/c_0$ , and FAD FAD uncertainty: from Offshore Technology Report (HSE, 2000)



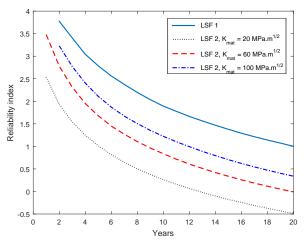
Plastic

#### Method to Calculate Failure Probabilities

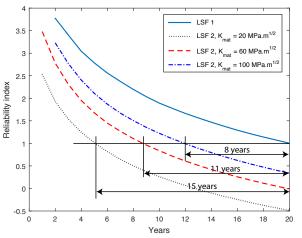
- Monte Carlo Simulation: 10<sup>5</sup> samples of crack propagations
- Crack depth and crack length are coupled
- Constant amplitude stress history
- Failure Probability:

$$P_f = \frac{1}{N} \sum_{j=1}^{N} I[g]$$

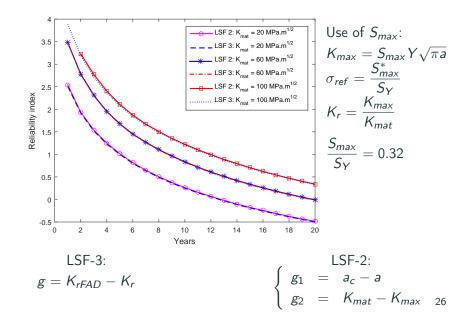
$$I[g] = \begin{cases} 0 & \text{if } g > 0 \\ 1 & \text{if } g \le 0 \end{cases}$$

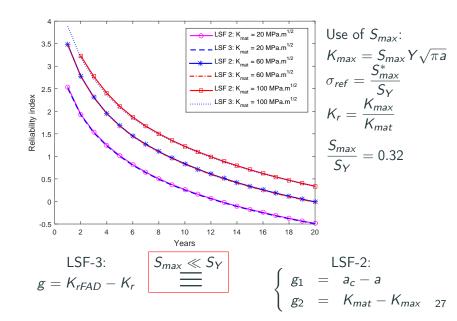


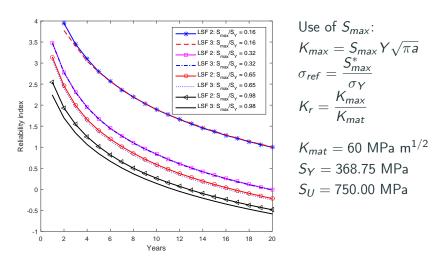
LSF 1: 
$$g = a_c - a$$
 
$$\begin{cases} g_1 = a_c - a \\ g_2 = K_{mat} - K_{mat} \end{cases}$$



LSF-1: 
$$g = a_c - a$$
  $\Rightarrow$  Under-estimate  $P_f$  LSF-2: 
$$\begin{cases} g_1 = a_c - a \\ g_2 = K_{mat} - K_{max} \end{cases}$$



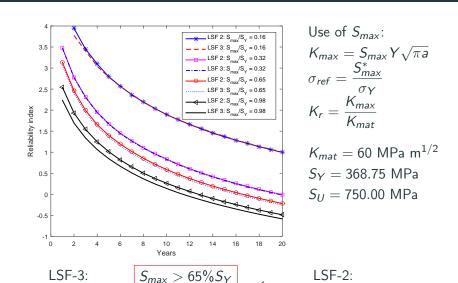




LSF-3:  

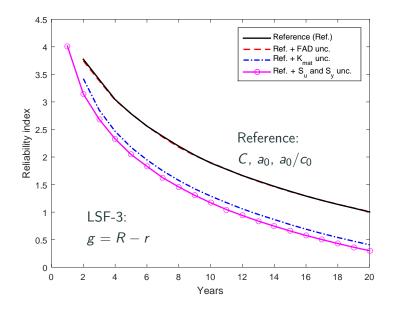
$$g = K_{rFAD} - K_{r}$$

$$\begin{cases}
g_{1} = a_{c} - a \\
g_{2} = K_{mat} - K_{max}
\end{cases}$$

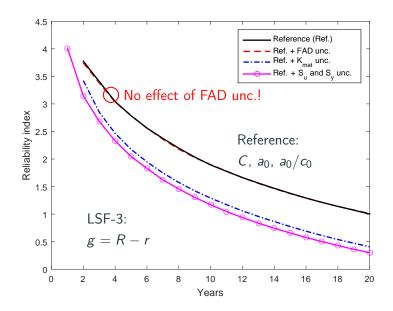


29

## Results & Discussions: FAD Uncertainty



## Results & Discussions: FAD Uncertainty



# FAD in Updating Considering

Inspection

FAD in Updating

## Road Map

FAD in Updating Considering Inspection

Motivation & Literature Review

FAD Gives Higher Failure Probability Values

FAD in Updating

The Updating Problems

Method to Solve

Safety Margin using FAD: 
$$g = K_{rFAD} - K_r$$

Safety Margin using FAD:

Crack Detection Event:

c: crack length,

 $c_d$ : detectable length

$$g = K_{rFAD} - K_r$$

$$I_d = c - c_d$$

Safety Margin using FAD:

Crack Detection Event:

Probability of Detection:

$$g = K_{rFAD} - K_{r}$$

$$I_{d} = c - c_{d}$$

$$P(c_{d}) = 1 - \frac{1}{1 + \left(\frac{c_{d}}{x_{0}}\right)^{b}}$$

Safety Margin using FAD:  $g = K_{rFAD} - K_r$ 

Crack Detection Event:

 $g = \kappa_{rHAL}$   $I_d = c - c_d$   $P(c_d) = 1 - \frac{1}{1 + \left(\frac{c_d}{x_0}\right)^b}$ Probability of Detection:

Update  $P_F$  when No crack is detected:

$$P[g \le 0 | I_d < 0] = ?$$

Safety Margin using FAD:  $g = K_{rFAD} - K_r$ 

Crack Detection Event:

 $g = c_{d}$   $I_{d} = c - c_{d}$   $P(c_{d}) = 1 - \frac{1}{1 + \left(\frac{c_{d}}{x_{0}}\right)^{b}}$ Probability of Detection:

Update  $P_F$  when No crack is detected:

$$P[g \le 0 | I_d < 0] = ?$$

Update  $P_F$  when Crack is detected and repaired imperfectly:

$$P\left[g\leq 0|I_d\geq 0\bigcap R_{im}\right]=?$$

Safety Margin using FAD:

$$g = K_{rFAD} - K_r$$

Crack Detection Event:

$$I_d = c - c_d$$

Probability of Detection:

$$P(c_d) = 1 - \frac{1}{1 + \left(\frac{c_d}{x_0}\right)^b}$$

Update  $P_F$  when No crack is detected:

$$P[g \le 0 | I_d < 0] = ?$$

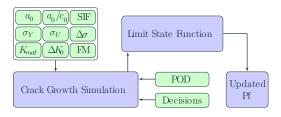
Update  $P_F$  when Crack is detected and repaired imperfectly:

$$P\left[g\leq 0|I_d\geq 0\bigcap R_{im}\right]=?$$

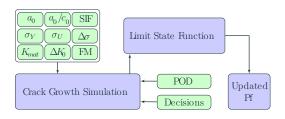
Update  $P_F$  when Crack is detected and repaired perfectly:

$$P\left[g\leq 0|I_d\geq 0\bigcap R_p\right]=?$$

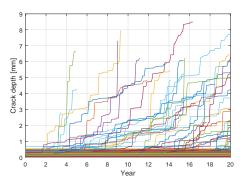
#### Method to Solve: Procedure



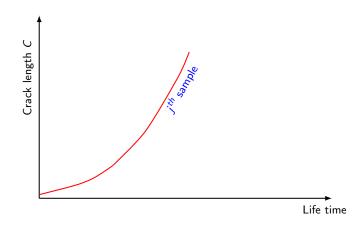
#### Method to Solve: Procedure



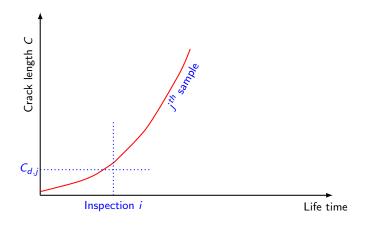
constant amplitude during 1 month!



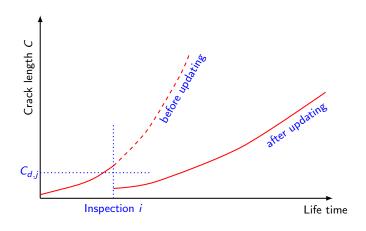
# Method to Solve: Updating

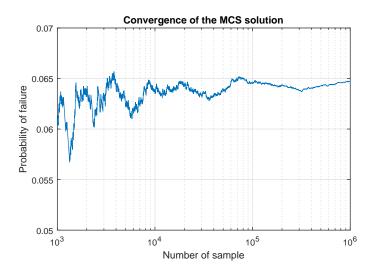


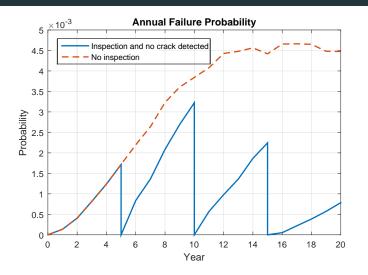
# Method to Solve: Updating



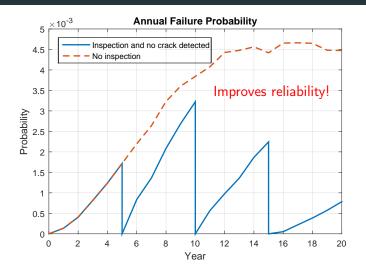
# Method to Solve: Updating



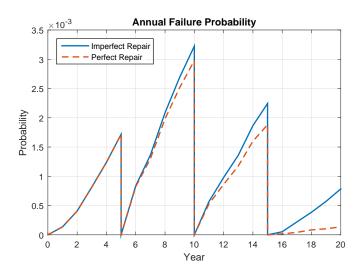


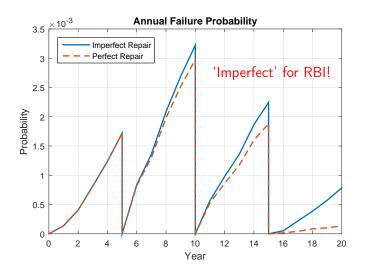


36



36





## Road Map

FAD in Updating Considering Inspection

Motivation & Literature Review

FAD Gives Higher Failure Probability Values

Limit State Functions to Compare

Method to Calculate Failure Probabilities

Results & Discussions

FAD in Updating

The Updating Problems

Method to Solve

#### Conclusion on FAD

#### Gap:

Advantages and disadvantages of using Failure Assessment Diagram in updating failure probability considering crack inspection data for existing OWT support structures?

#### Conclusion on FAD

#### Disadvantages

- Time consuming.
- Fails to find very small failure probability such as 'detected & not repaired'

#### Advantages

- Releases the assumption about  $a_c$
- more conservative P<sub>F</sub> results ⇒ better for inspection planning!

# Monitoring

in Updating Considering

Occurence of Weather Conditions

## Road Map

Introduction

Background

FAD in Updating Considering Inspection

Occurence of Weather Conditions in Updating Considering Monitoring

Motivation & Literature Review

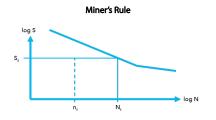
Methodology

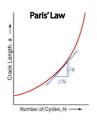
Application

Conclusion

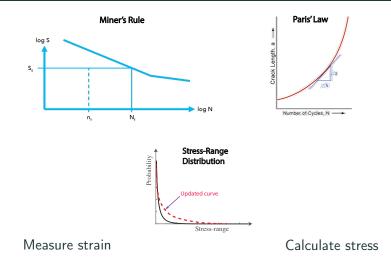
Occurence of Weather Conditions in Updating Considering Monitoring

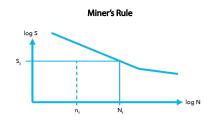
Motivation & Literature Review

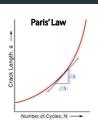




What to do with Load monitoring data?



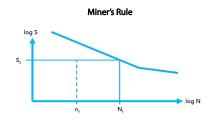


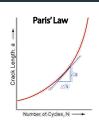




Measure strain: not everywhere! long-term!

Calculate stress

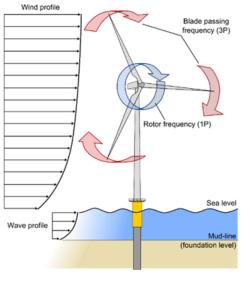


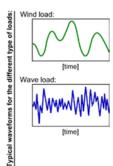




Measure strain: not everywhere! long-term! Calculate stress: time consuming! too much uncertainties!

#### Motivation: Uncertainties in FEM





Sources of Uncertainties: Type 1: load calculation

Type 2: calibrated FEM

#### Motivation: The Idea

Use measured data:  $\Rightarrow$  No load calculation

Use FEM to extrapolate stress:  $\Rightarrow$  No need to measure everywhere

Use Occurence of Weather Conditions in LSF:

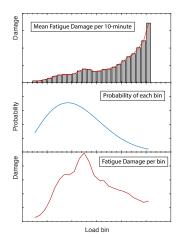
⇒ Wind & Wave instead of strain

#### Motivation: The Idea

Use measured data:  $\Rightarrow$  No load calculation

Use FEM to extrapolate stress: ⇒ No need to measure everywhere Use Occurence of Weather Conditions in LSF:

⇒ Wind & Wave instead of strain



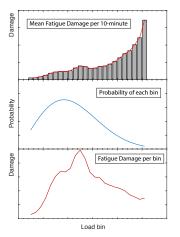


#### Motivation: The Idea

Use measured data:  $\Rightarrow$  No load calculation

Use FEM to extrapolate stress: ⇒ No need to measure everywhere

Use Occurence of Weather Conditions in LSF:



⇒ Wind & Wave instead of strain

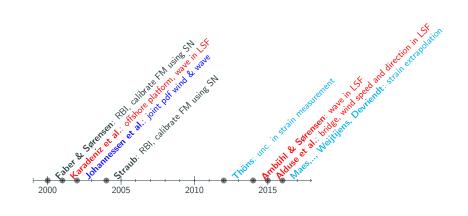
$$g = \Delta - D_{total}$$

$$= \Delta - \sum_{e=1}^{n} D_{bin_e}$$

$$= \Delta - \sum_{i=e}^{n} P_{bin_e} \cdot D_{10m,bin_e} \cdot n_{10m,yr}$$



#### Literature Review



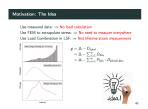
#### Literature Review

#### Gap:

How to perform reliability assessment of existing offshore wind turbine support structures using directly the Occurence of Weather Conditions (wind and wave)? Occurence of Weather Conditions in Updating Considering Monitoring

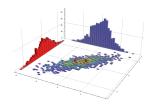
ivioring

$$g = \Delta - \sum_{e=1}^{n} P_{bin_e} \cdot D_{mod,bin_e}$$



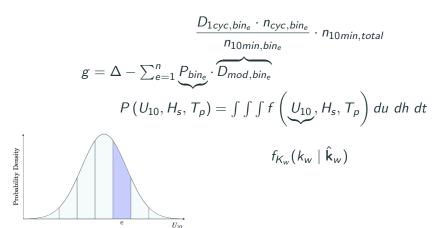
$$g = \Delta - \sum_{e=1}^{n} \underbrace{P_{bin_e}} \cdot D_{mod,bin_e}$$

$$P\left(U_{10},H_{s},T_{p}
ight)=\int\int\int f\left(U_{10},H_{s},T_{p}
ight)du~dh~dt$$

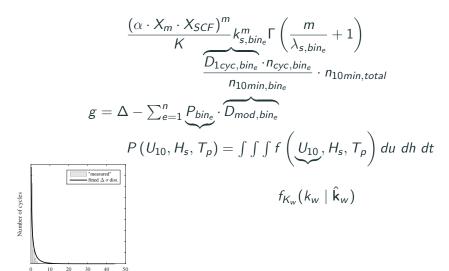


Posterior Distribution

$$g = \Delta - \sum_{e=1}^{n} \underbrace{P_{bin_e} \cdot D_{mod,bin_e}}_{P(U_{10}, H_s, T_p)} = \int \int \int f\left(\underbrace{U_{10}, H_s, T_p}\right) du \ dh \ dt$$
Likelihood
$$f_{K_w}(k_w \mid \hat{\mathbf{k}}_w)$$
Bayes' Theorem



Stress range [MPa]



## Methodology: Limit State Function

$$g = \Delta - \sum_{i=1}^{T} \sum_{j=1}^{n_{U_{10}}} \sum_{k=1}^{n_{H_{s}}} \sum_{l=1}^{n_{T_{p}}} \frac{\left(\alpha_{f} X_{m} X_{SCF}\right)^{m}}{K} k_{s,jkl}^{m} \Gamma\left(\frac{m}{\lambda_{s,jkl}} + 1\right) \times \cdots$$

$$P\left(U_{10,j}, H_{s,k}, T_{p,l} | k_{w,i}\right) \frac{n_{c,jkl}}{n_{m,jkl}} n_{m}^{*}$$

## **Update Wind Speed Distribution**

Given three years (or more) of measured wind speed, how to update the design wind speed distribution?

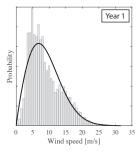
$$f_{K_{w}}(k_{w}|\mu,\sigma) = f_{N}(k_{w}|\mu,\sigma)$$

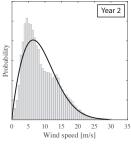
$$= \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{k_{w}-\mu}{\sigma}\right)^{2}\right)$$

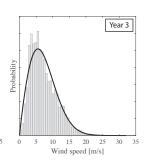
The predictive density function of  $k_w$  given measured data becomes a Student's t-distribution.

## Methodology: Update Wind Speed Distribution

## Keep the 'design' shape parameter:

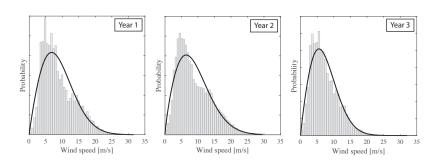






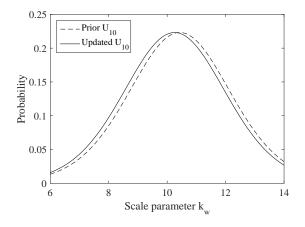
## Methodology: Update Wind Speed Distribution

## Keep the 'design' shape parameter:



$$\hat{\mathbf{k}}_w = [10.005 \quad 9.993 \quad 8.176] \text{ m/s}$$

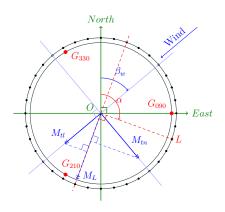
# Methodology: Update Wind Speed Distribution



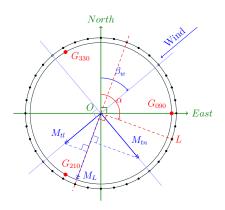
Occurence of Weather Conditions in Updating Considering

Monitoring

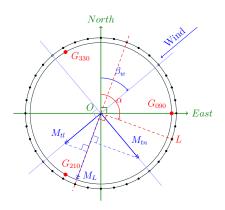
• 3 MW offshore wind turbine



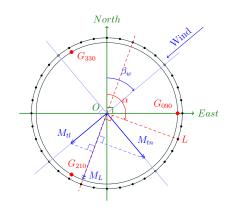
- 3 MW offshore wind turbine
- Monopile, diameter of 5.2 m



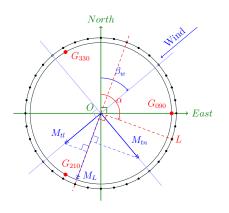
- 3 MW offshore wind turbine
- Monopile, diameter of 5.2 m
- Optical strain sensors



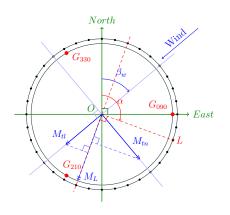
- 3 MW offshore wind turbine
- Monopile, diameter of 5.2 m
- Optical strain sensors
- Before construction: 15 years of wind data



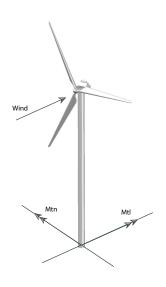
- 3 MW offshore wind turbine
- Monopile, diameter of 5.2 m
- Optical strain sensors
- Before construction: 15 years of wind data
- After construction: 3 years of wind and wave data + 1 year strain data (concurrently measured with the wind)



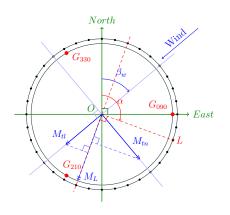
- 3 MW offshore wind turbine
- Monopile, diameter of 5.2 m
- Optical strain sensors
- Before construction: 15 years of wind data
- After construction: 3 years of wind and wave data + 1 year strain data (concurrently measured with the wind)
- The design wind speed distribution



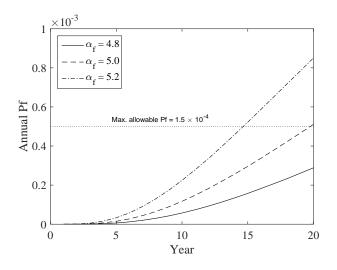
- 3 MW offshore wind turbine
- Monopile, diameter of 5.2 m
- Optical strain sensors
- Before construction: 15 years of wind data
- After construction: 3 years of wind and wave data + 1 year strain data (concurrently measured with the wind)
- The design wind speed distribution



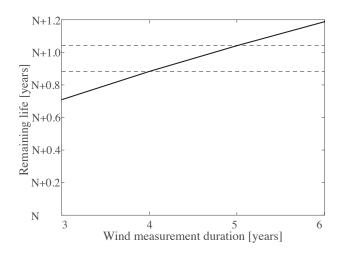
- 3 MW offshore wind turbine
- Monopile, diameter of 5.2 m
- Optical strain sensors
- Before construction: 15 years of wind data
- After construction: 3 years of wind and wave data + 1 year strain data (concurrently measured with the wind)
- The design wind speed distribution

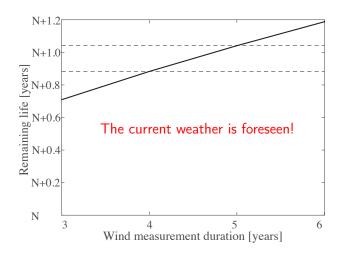


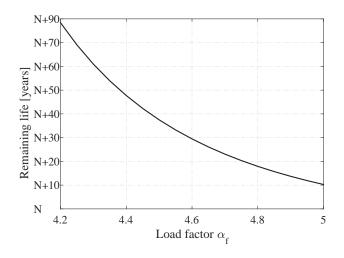
# Application: Estimating Remaining Fatigue Life

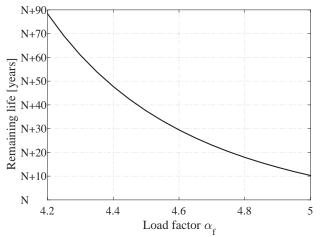


# Application: Results & Discussion



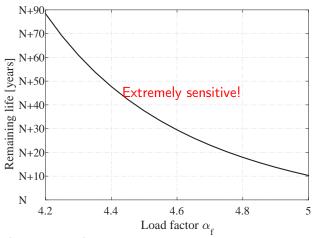






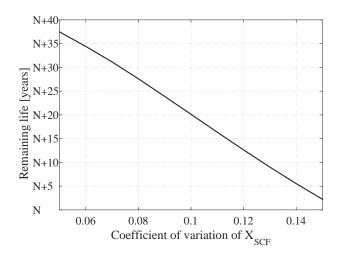
 $\alpha_f$  is deterministic!

represents: Stress Concentration Factor (SCF) at hot-spots, or Stress Extrapolating Factor for unmeasured locations

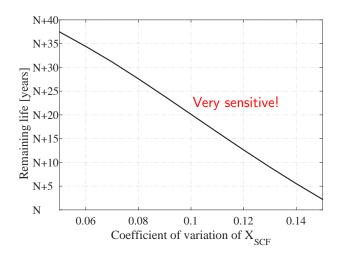


 $\alpha_f$  is deterministic!

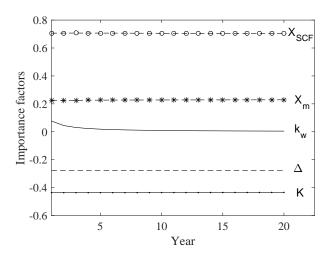
represents: Stress Concentration Factor (SCF) at hot-spots, or Stress Extrapolating Factor for unmeasured locations

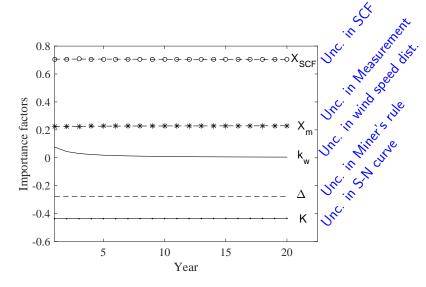


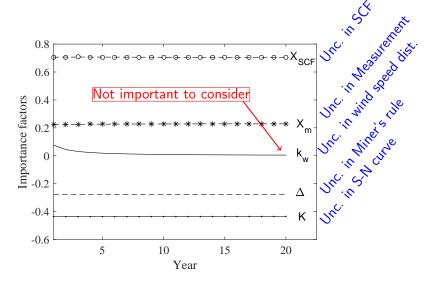
 $X_{SCF}$  is stochastic part of SCF! Coefficient of variation = std./mean



 $X_{SCF}$  is stochastic part of SCF! Coefficient of variation = std./mean







# Road Map

Occurence of Weather Conditions in Updating Considering Monitoring

Motivation & Literature Review

Methodology

Application

## Conclusion on Using Occurence of Weather Conditions in LSF

#### Gap:

How to perform reliability assessment of existing offshore wind turbine support structures using directly the Occurence of Weather Conditions (wind and wave)?

#### Conclusion on Using Occurence of Weather Conditions in LSF

#### Disadvantages

- Assumed that fatigue damage caused by each Weather Condition is constant.
- Depends on the stress extrapolation method to derive stress for locations that is not measured.

#### Advantages

- Fast
- Less uncertainty than a time domain analysis.

Conclusion

## Road Map

FAD in Updating Considering Inspection

Motivation & Literature Review

FAD Gives Higher Failure Probability Values

FAD in Updating

Occurence of Weather Conditions in Updating Considering Monitoring

Motivation & Literature Review

Methodology

Application

• Combining two types of new information in RBI,

- Combining two types of new information in RBI,
- Corrosion and crack inspection in updating failure probability,

- Combining two types of new information in RBI,
- Corrosion and crack inspection in updating failure probability,
- Load extrapolation for other types of OWT support structures,

- Combining two types of new information in RBI,
- Corrosion and crack inspection in updating failure probability,
- Load extrapolation for other types of OWT support structures,
- Quantifying uncertainty of load extrapolation methods,

- Combining two types of new information in RBI,
- Corrosion and crack inspection in updating failure probability,
- Load extrapolation for other types of OWT support structures,
- Quantifying uncertainty of load extrapolation methods,
- Considering the random process of the peak tensile stress in calculating failure probability

## Message Objective

Using Failure Assessment Diagram and Occurence of Weather Conditions in the Limit State Function improves the accuracy of the Updated Failure Probability.



 $P_f$  results are higher when  $K_{mat}$  is included in the LSF of  $a_c \Rightarrow$  it needs to consider to be conservative.

The peak tensile stress affects the safety state of any crack size  $\Rightarrow$  the time when a high peak tensile stress occurs is important. This is a **first passage time** problem where the random process of the peak tensile stress first encounters a threshold.

This is a challenge of considering the fracture toughness criterion.

FAD approach predicts higher  $P_f$  values when the applied peak tensile stress is larger than 65% the yield strength, in comparison to the LSF using  $(a_c, K_{mat}) \Rightarrow$  the use of FAD should be recommended for reliability assessment of existing offshore structures with high stress (designed to the limit, corroded, damage tolerant design)

When FAD approach is utilized, the uncertainties in yield and ultimate strengths are important because they define the region of plastic collapse  $\Rightarrow$  they should be investigated to improve the reliability of the structure.

The information about cracks and intervention actions helps to improve our belief in the structural safety (reducing the probabilty of failure). It is the basic to optimizing inspection plans to reduce the O&M costs of offshore wind turbines.

An imperfect repair leads to a higher failure probability than a perfect repair.  $\Rightarrow$  an imperfect repair should be considered in the decision tree for a conservative inspection plan.

Updating using Monitoring data: the impact of the year-to-year variation of the annual mean wind speed becomes negligible after 4 years. ⇒ it can be ignored in the LSF to reduce significantly calculation time and give a chance to consider a finer descretized Occurence of Weather Conditions.

The value of the predicted remaining fatigue life obtained from the present methodology can be useful for decision making to down-rate, curtail, or extend the lifetime of the wind turbine support structures.

To apply the proposed method for locations where strain gauges cannot be installed, a load extrapolation method is needed, which inturn requires a good calibrated finite element model. A model uncertainty is also needed in the LSF.

vbox Histogram of measured strain is distorted by high frequencies of small strain cycles, by considering the corresponding accumulated fatigue damage during fitting process, the weighting factor of each bin can be modified to preserve total fatigue damage.

#### The use of Miner's Rule

Fatigue damage accumulated by one load cycle is calculated as:

$$D_i = \frac{1}{N_i} = \frac{1}{K_c} S_i^m$$

For a large number of stress cycle, the expected fatigue damage can be estimated as:

$$E[D_i] = \frac{1}{K_c} \sum_{i=0}^{\infty} S_i^m P(S_i)$$
$$= \frac{1}{K_c} \int_{0}^{\infty} S^m f(s) ds$$

If the stress-range is Weibull distributed  $(k, \lambda)$ , the expected fatigue damage per cycle becomes:

$$\mathsf{E}\left[D_{i}\right] = \frac{1}{K_{c}} k^{m} \Gamma\left(\frac{m}{\lambda} + 1\right)$$

People may ask:

• Why not to use measured strain directly?

#### People may ask:

- Why not to use measured strain directly?
- Why not to use wind and wave to get strain from a Finite Element Model and then quantify the model uncertainty using the measured strain?

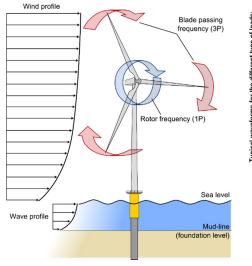
Why not to use measured strain directly?

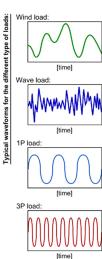
• You may need to measure strain for the whole lifetime.

Why not to use measured strain directly?

- You may need to measure strain for the whole lifetime.
- In offshore structures, there are locations where you cannot install strain gauges.

Why not to use wind and wave to get strain from a Finite Element Model and then quantify the model uncertainty using the measured strain?





Why not to use wind and wave to get strain from a Finite Element Model and then quantify the model uncertainty using the measured strain?

- You consider one irrelevant uncertainty more than the method proposed in this thesis.
- You take a lot of time to perform time domain analyses.

#### Joint Distribution of Wind and Wave

The probability of occurrence of  $jkl^{th}$  bin which is used to link to fatigue damage is:

$$P(U_{10,j}, H_{s,k}, T_{z,l}) = \int \int \int f(U_{10}, H_s, T_z) dw dh dt$$

this integration need to be calculated numerically.

If only  $U_{10}$  is considered in the bin, the probability of  $j^{th}$  bin becomes:

$$P(U_{10,j}) = F_W(a_j \le U_{10} < b_j; k_w, \lambda_w)$$

$$= \exp\left(-\left(\frac{a_j}{k_w}\right)^{\lambda_w}\right) - \exp\left(-\left(\frac{b_j}{k_w}\right)^{\lambda_w}\right)$$

#### Joint Distribution of Wind and Wave

$$f(U_{10}, H_s, T_z) = f(U_{10}) \times f(H|U_{10}) \times f(T_z|H_s|U_{10})$$

where:

- $f\left(U_{10}\right)$  marginal distribution of the 10-minute mean wind speed, Weibull  $(k_w, \lambda_w)$ ,
- $f\left(H_{s}|U_{10}\right)$  conditional distribution of significant wave height given  $U_{10}$ , Weibull (scale =  $func\left(U_{10}\right)$ ), shape =  $func\left(U_{10}\right)$ ),
- $f\left(T_z|H_s\;U_{10}\right)$  conditional distribution of mean wave period given  $H_s$  and  $U_{10}$ , Lognormal (mean =  $func\left(H_s,\,U_{10}\right)$ , std =  $func\left(H_s,\,U_{10}\right)$ ).

#### The use of Miner's Rule

The assumption that stress-ranges follow a Weibull distribution is not perfect!

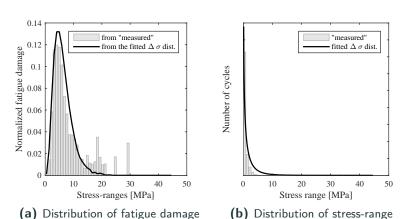


Figure 1: Fitting stress-range in wind class [0 to 5 m/s]

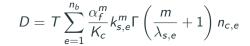
## **Total Fatigue Damage**

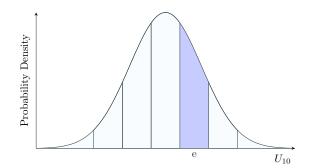
$$D = \sum_{i=1}^{T} \sum_{j=1}^{n_{U_{10}}} \sum_{k=1}^{n_{H_{s}}} \sum_{l=1}^{n_{T_{p}}} \frac{\alpha_{f}^{m}}{K_{c}} k_{s,jkl}^{m} \Gamma\left(\frac{m}{\lambda_{s,jkl}} + 1\right) \times \cdots$$

$$P\left(U_{10,j}, H_{s,k}, T_{p,l} | k_{w,i}\right) \frac{n_{c,jkl}}{n_{m,jkl}} n_{m}^{*}$$

- $n_{cj} = n_{U_{10}} \times n_{H_s} \times n_{T_z}$  is total number of bins;
- $n_{c,jkl}$  is number of stress cycles in the bin number jkl;
- n<sub>m,jkl</sub> is number of oceanographic records in the bin number jkl;
- $n_m^* = \sum_{j=1}^{n_{cl}} n_{m,j}$  is total of observed oceanographic data per year;
- P  $(U_{10,j}, H_{s,k}, T_{p,l}|k_{w,i})$  is the probability of the bin jkl given the scale parameter of the wind speed distribution  $k_{w,i}$  in the  $i^{th}$  year.

## **Total Fatigue Damage**





# **Equality vs. Inequality Events**

Equality: when crack is measured a certain value. Not considered here because it is a very small failure probability problem, MCS is not suitable.

## Importance Factors

The 'importance factor' of a random variable is a measure of the sensitivity of the reliability index to randomness of that random variable at the design point.

The 'importance factors' offer a way to rank the importance of the input variables with respect to the failure event of the welded joint.

The vector of 'importance factors' is denoted as  $\alpha$ ,

$$\alpha = -\frac{\triangledown g(\mathbf{x})}{|\triangledown g(\mathbf{x})|} \tag{1}$$

where  $\nabla g(\mathbf{x})$  is the gradient vector of the limit state function at the design point  $\mathbf{x}$ , which is assumed to exist, as shown in Eq.(2):

$$\nabla g(\mathbf{x}) = \begin{pmatrix} \frac{\partial g}{\partial x_1}(\mathbf{x}), & \cdots, & \frac{\partial g}{\partial x_n}(\mathbf{x}) \end{pmatrix}$$
 (2)

# Minimum Number of Stress Cycles

- Weibull (scale = k, shape =  $\lambda$ ) of stress-range distributions in:
  - Case 1: wind speeds in bin 1 (5-10 m/s): k=1.922,  $\lambda=0.6172$
  - Case 2: wind speeds in bin 2 (10-15 m/s): k = 4.2385,  $\lambda = 0.7793$
  - Case 3: wind speeds in bin 3 (20-30 m/s): k = 9.408,  $\lambda = 1.0774$
- SN curve:  $\log a_2 = 15.606$ ;  $\log a_1 = 11.764$ ;  $m_1 = 3$ ;  $m_2 = 5$

No. of cycles (n)	Case 1	Case 2	Case 3
10 <sup>7</sup>	5.5%	3.4%	1.4%
$5 \times 10^6$	7.3%	4.5%	1.7%
10 <sup>6</sup>	19.5%	9.3%	4.3%

**Table 1:** Error in fatigue damage

# Minimum wind measurement for design

15 years is not a long data set for design because, to estimate the 50-year return period wind speed, a minimum 20 years of data is required (Coles et al. 2001)