GEOL0013 – HYDROGEOLOGY

LIÈGE université
Sciences Appliquées

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Chapter 4 – Copy of the slides for the Hydrogeology course

Reference book: Dassargues A., 2018. Hydrogeology: groundwater science and engineering, 472p. Taylor & Francis CRC press

File: Hydrogeologychap4ENv1

Chapter 4: Groundwater flow in saturated porous media

- *Representative Elementary Volume (REV)*
- *Porosities*
- *Piezometric head*
- *Darcy's law and hydraulic conductivity*
- *Heterogeneity: equivalent values*
- *Applications*
- *Limitations*
- *Transmissivity*
- *Steady state groundwater flow*
- *Storage variation*
- *Transient groundwater flow*
- *References*

Representative Elementary Volume (REV)

2 fundamental properties for groundwater flow

- *porosity* ⟷ *water storage property*
- *permeability* ⟷ *water conduction property*

How to quantify them ?

at which scale ?

- *not too small: no signification*
- *not too large: smoothing all*

(Bachmat et Bear 1986, Bear et Verruijt 1987)

REV concept = considered volume of geological medium for quantifying properties at the appropriate scale (by averaged equivalent values)

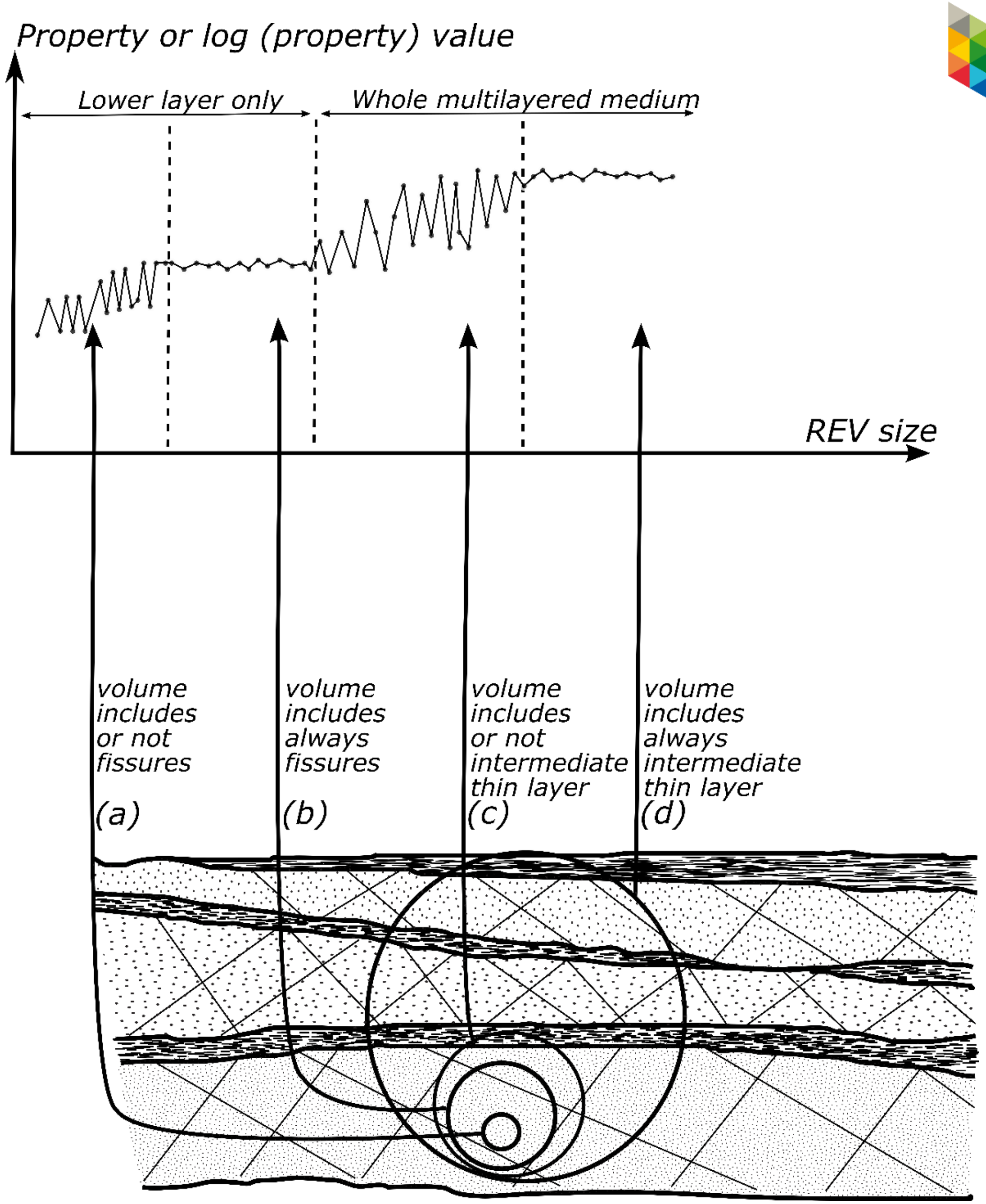
(de Marsily, 1986, Dagan, 1989)

… very useful concept that implicitly assumes a continuum and a porous medium *(Molz 2015)*

Representative Elementary Volume (REV)

- the REV depends on the kind of problem being studied and the study objectives

- the REV is used for groundwater flow and solute transport … but also in all other fields where a quantification is needed for properties of the geological medium



(Dassargues 2018, modified from Bear et Verruijt 1987)

REV concept and problem scale

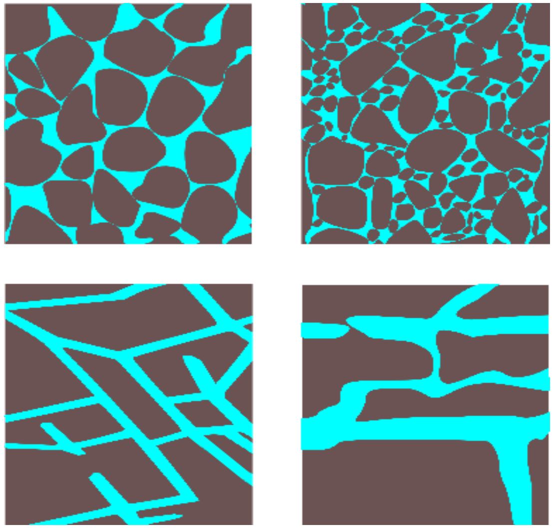
Scale

Scale	Study / model type
Micro	*Fluid dynamic and pore studies*
Homogenization	
Macro	*Lab tests (from dm to a few m)*
Homogenization	
Mega	*Detailed numerical models – Physically consistent simulations of the reservoir*
Homogenization	
Giga	*Balance models, black-box models, transfer functions, etc.*

(Dassargues 2018)

Porosity

… volume of pores divided by total volume (solid grains + pores)

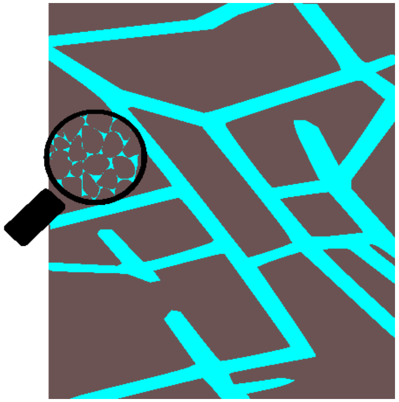


(Dassargues 2018)

total porosity $n = \frac{V_v}{V_t}$

$n = 1 - \frac{V_s}{V_t} = 1 - \frac{\rho_b}{\rho_s}$ with $\rho_b = M_s/V_t$

$\rho_s = M_s/V_s$



… pores volume divided by grains volume:

void ratio : $e = \frac{V_v}{V_s}$

$n = \frac{e}{(1+e)}$ $e = \frac{n}{(1-n)}$

water content: $\theta = V_w/V_t$

Effective porosity

... two components in the total porosity:

- *effective drainage porosity* $n_e = V_m/V_t = S_y$
- *retention capacity or 'specific retention'* $S_r = V_{im}/V_t$

(Castany 1963, de Marsily 1986)

$$n = S_y + S_r = n_e + S_r$$

- *effective porosity ? in practice: drainage*
  (... after which duration ? ... at which pressure ? ...)

- *drainage porosity*
  = effective porosity
  corresponding to the drainable water by gravity
  (mobile water or moving water)
  = "specific yield" S_y

⚠ *for solute transport the 'effective' porosity* n_m *could be different*

(Payne et al. 2008, Hadley & Newell, 2014)

$$n_m < n_e < n < 1$$

Porosities

… effective porosity can be small with regards to the total porosity

… effective porosity can be dependent on the fluid nature: molecules size in relation with the pore size and shape

- *intergranular porosity*
- *fissure porosity*
  in rock traction fissures or joints, stratification planes, etc.
  = secondary porosity

Porosities and scale effect

Example of a Cretaceous chalk aquifer (Hesbaye, Belgium)

Microscopic scale (< cm)

coccolithes micro-skeletons aggregated by diagenesis

$$\boldsymbol{n = 0.40 - 0.42 \textit{ and } n_e = 0.35}$$

Macroscopic scale (< dm)

micro-fractures and stratification

$$\boldsymbol{n = 0.42 - 0.45 \textit{ and } n_e = 0.01 - 0.03}$$

Megascopic scale (e.g., pumping tests, < 100 m)

faults and interconnected discontinuities

$$\boldsymbol{n = 0.42 - 0.45 \textit{ and } n_e = 0.05 - 0.1}$$

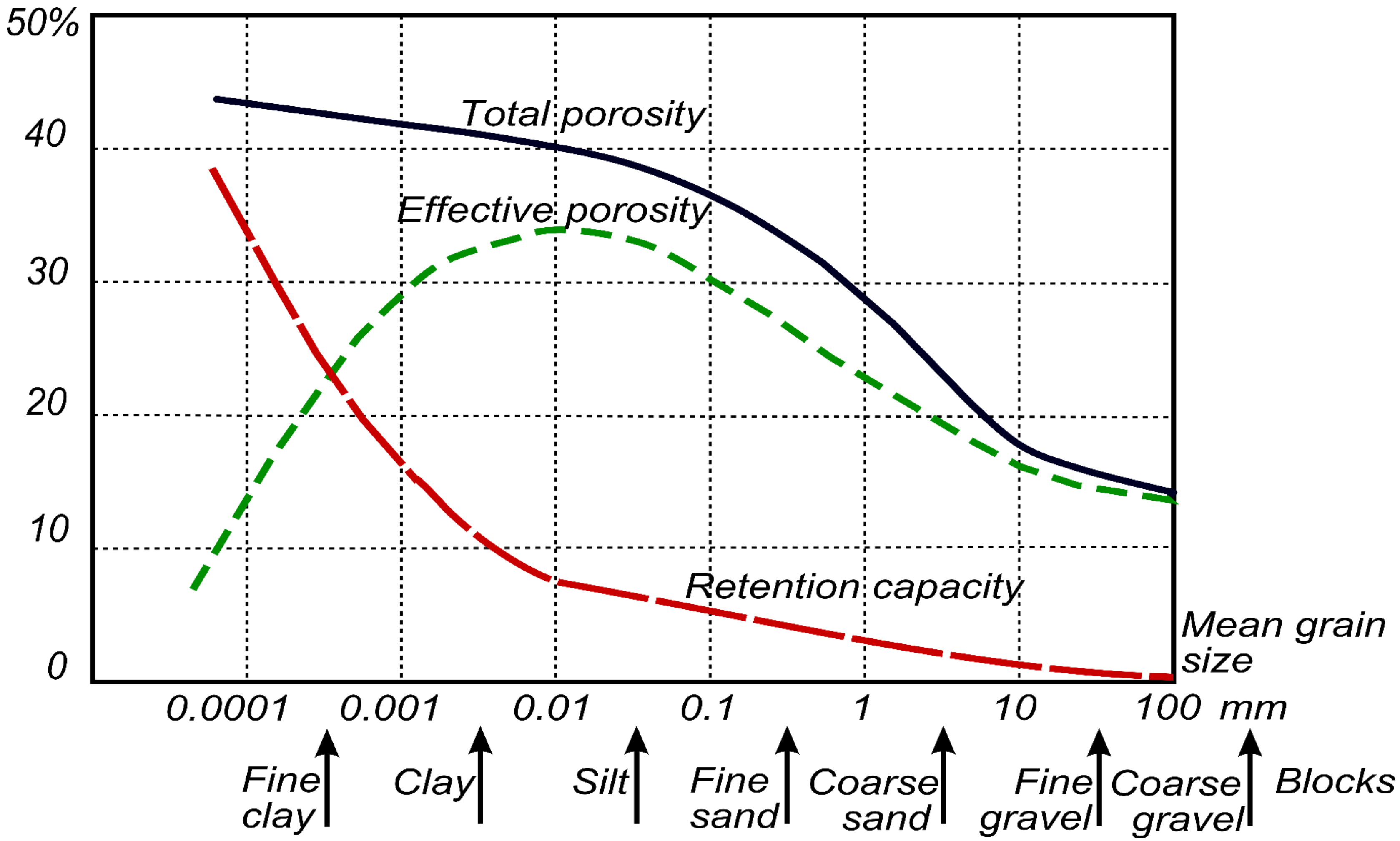
Data from Geer basin in Belgium *(Dassargues & Monjoie, 1993)*

Porosities *(indicative range of values)*

Lithology	n (%)	n_e (%)
granite and gneiss	0.02 - 2	0.1 – 2*
basalt	5 - 30	0.1 – 2*
quartzite	0.5 - 2	0 – 2*
shales	0.1 – 7.5	0.1 – 1*
schists and slates	0.1 – 7.5	0.1 – 2*
limestone and dolomite	0.5 - 15	0.5 – 14*
chalk	0.5 - 45	0.5 – 15*
sandstone, siltstone	3 – 38	3 – 25
volcanic tuff	30 -40	5 – 15
gravels	15 - 25	5 – 25
sands	15 - 35	5 – 25
silts	30 - 45	5 – 15
loams, loess and clays	40 - 70	0.1 – 3
***depends strongly on fractures, fissures**		

(Dassargues 2018, adapted fromFreeze and Cherry 1979, Fetter 2001, among others)

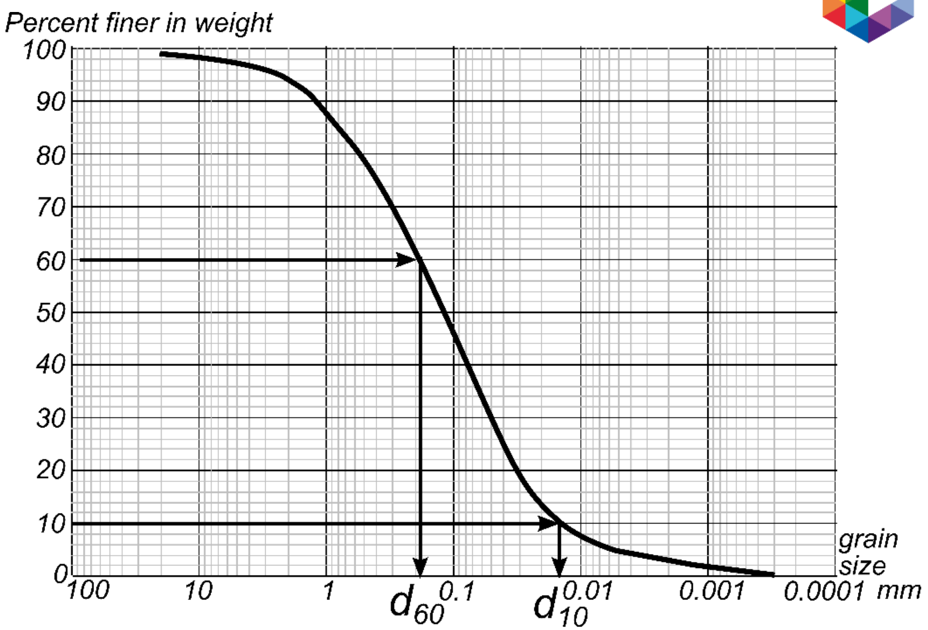
Porosity and mean grain size: Eckis diagram

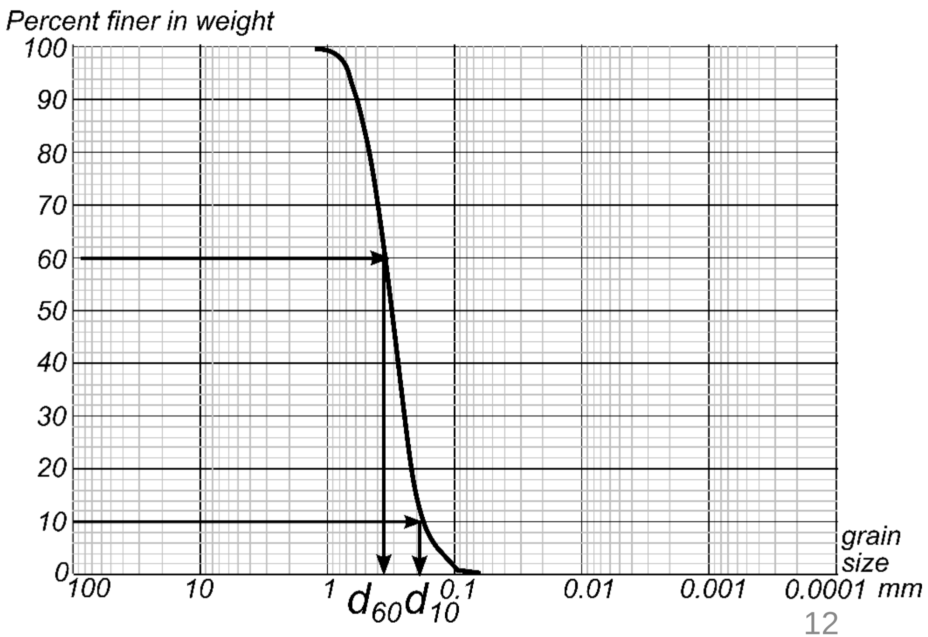


(Eckis, 1934, Castany 1963)

Porosity and grain size distribution

$$C_u = d_{60}/d_{10}$$

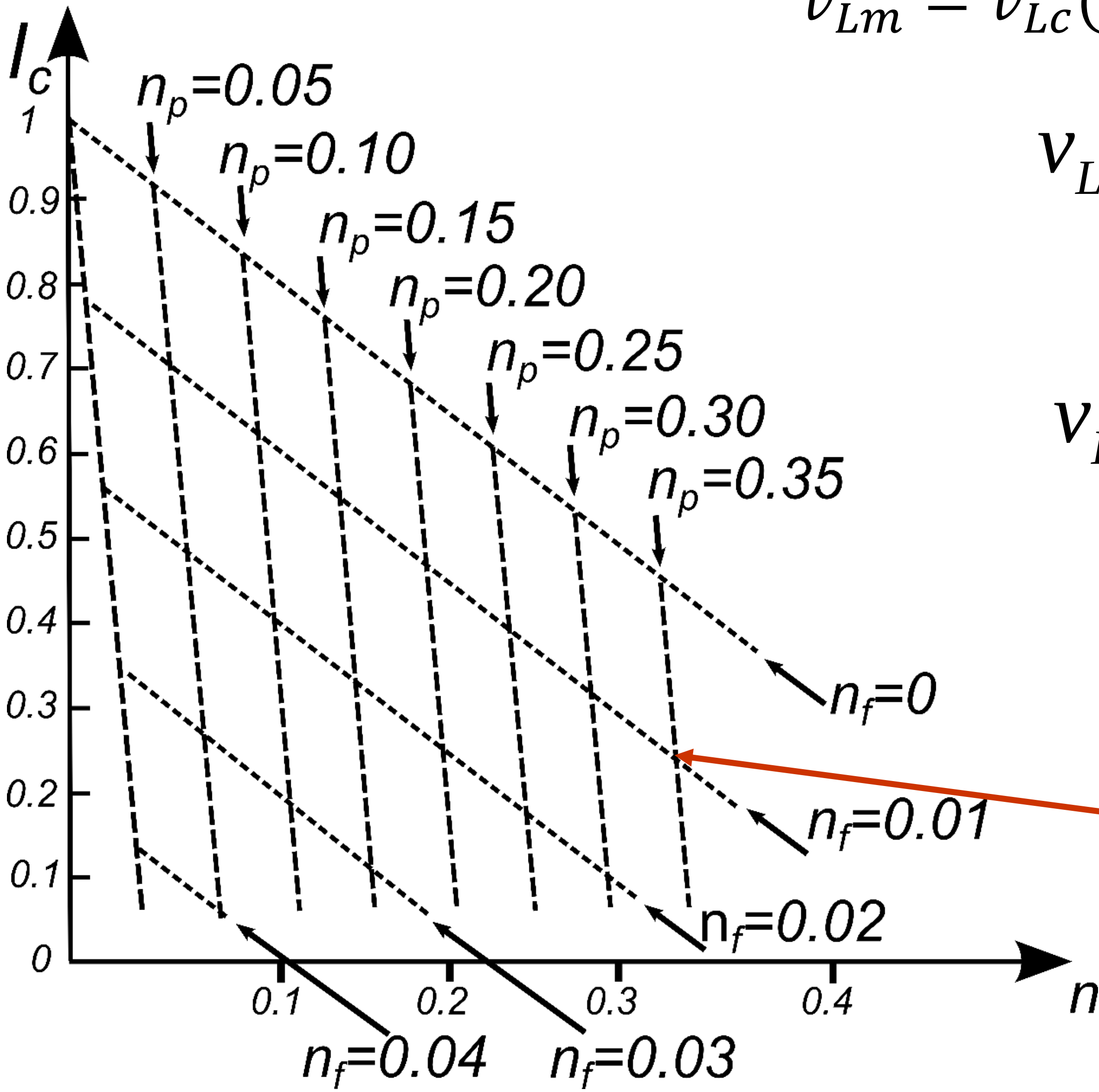




Matrix and fissure porosities

$$I_C = v_{Lm}/v_{Lc}$$

$$v_{Lm} = v_{Lc}(100 - 1.6n_p - 22n_f)/100$$



v_{Lm} ***longitudinal seismic velocity measured on samples***

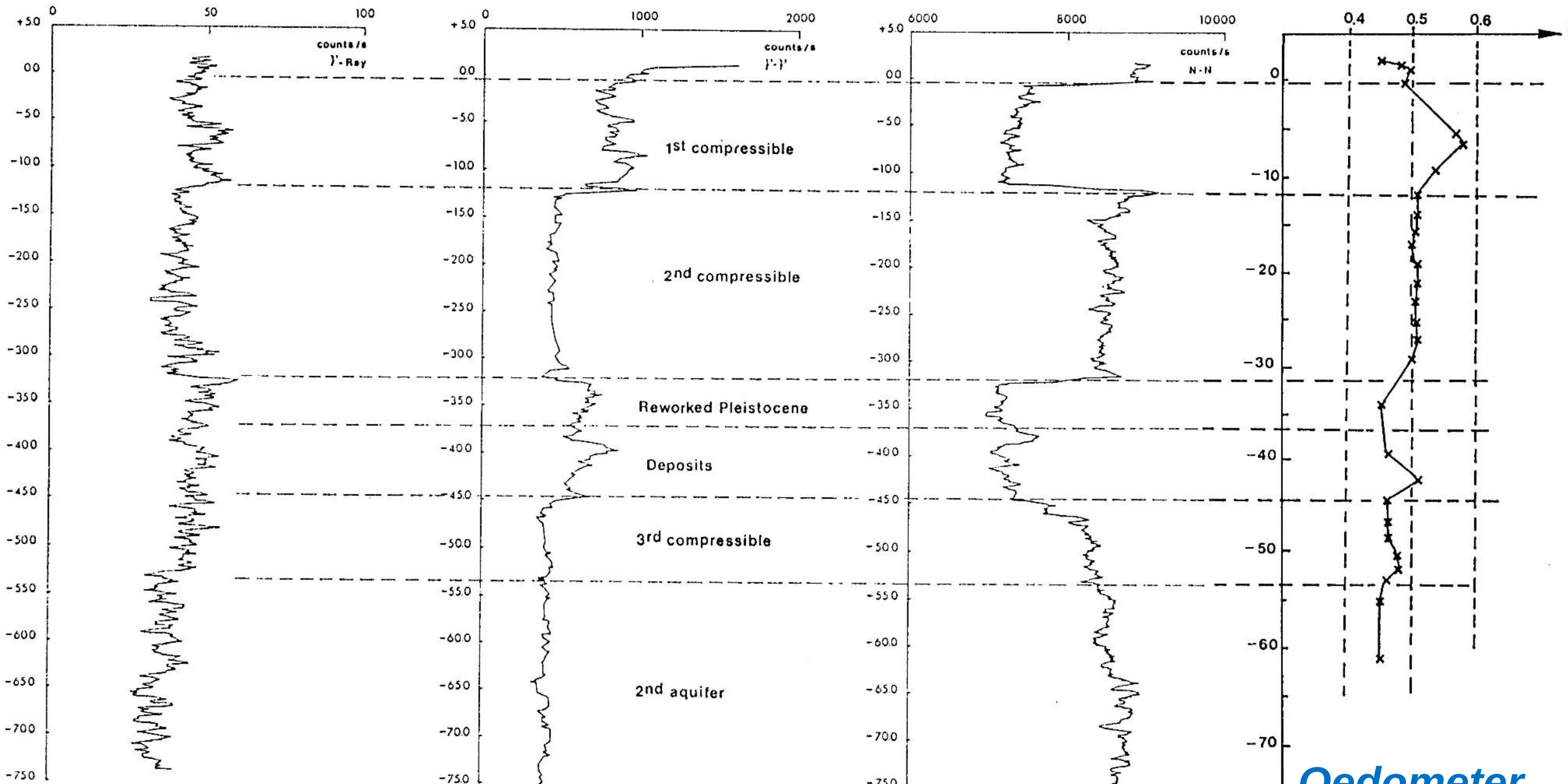
v_{Lc} ***longitudinal seismic theoretical velocity based on each mineralogical composition***

effective porosity in Hesbaye

(Tourenq 1978, Denis et al. 1978, Calembert et al. 1981, Dassargues 2018)

Porosity: measurements as ‘proxies’

Indirect assessment by using combined well-logging results



γ - ray   *γ - γ*   **neutron**   ***Oedometer consolidation tests***

Piezometric head

Introduction

… **pressure :** $p = F/A$ **force (normal to the surface) per surface unit (N/m² or Pa)**

… **density :** $\rho = m/V$ **mass per unit of volume (kg/m³)**

Representative values for density (in kg/m³)

Freshwater (at 4°C)	1×10^{-3}
Seawater (average value at the surface)	1.025×10^{-3}
Petrol	0.660 to 0.760×10^{-3}
Fuel	0.890 to 1.025×10^{-3}
Lamp oil	0.790 to 0.820×10^{-3}
Benzene	0.88×10^{-3}
BTEX	0.86 to 0.88×10^{-3}
Naphthalene (at 15.5 °C)	1.145×10^{-3}
PCE	1.622×10^{-3}
Mercury	13.6×10^{-3}

(Dassargues, 2018)

the relative density of a fluid: ratio between the fluid density and pure water density at 4°C (without dimension)

→ ***example: relative density of seawater: 1.025***

Hydraulic head, piezometric head or level

… groundwater level = hydraulic head h …linked with total energy of the fluid, mainly expressed in meters of water column above a reference datum

Total potential in a given point:

- ***gravity potential:*** $g.z$ ***(m²/s²)***
- ***water pressure potential:*** p/ρ ***(m²/s²)***
- ***groundwater velocity potential:*** $v^2/2$ ***(m²/s²)***

… groundwater is most often considered as a laminar and low velocity flow

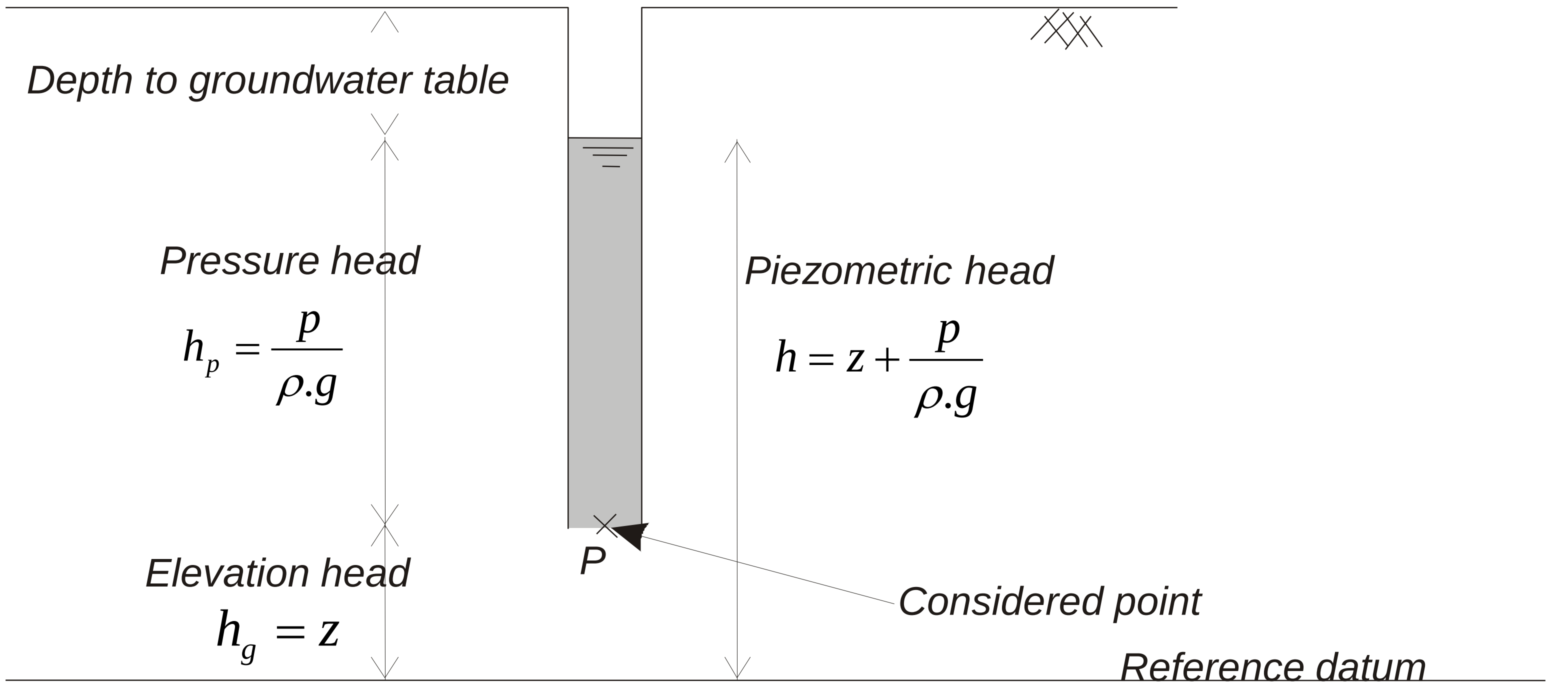
→ ***'Bernoulli' translated in groundwater potential:***

$$\Phi_{tot} = g.z + \frac{p}{\rho} + \cancel{\frac{v^2}{2}}$$

(Bernoulli 1738, Burger et al. 1985, Bear & Cheng 2010)

… the energy is expressed usually in ‘water head’ or hydraulic head or piezometric head:

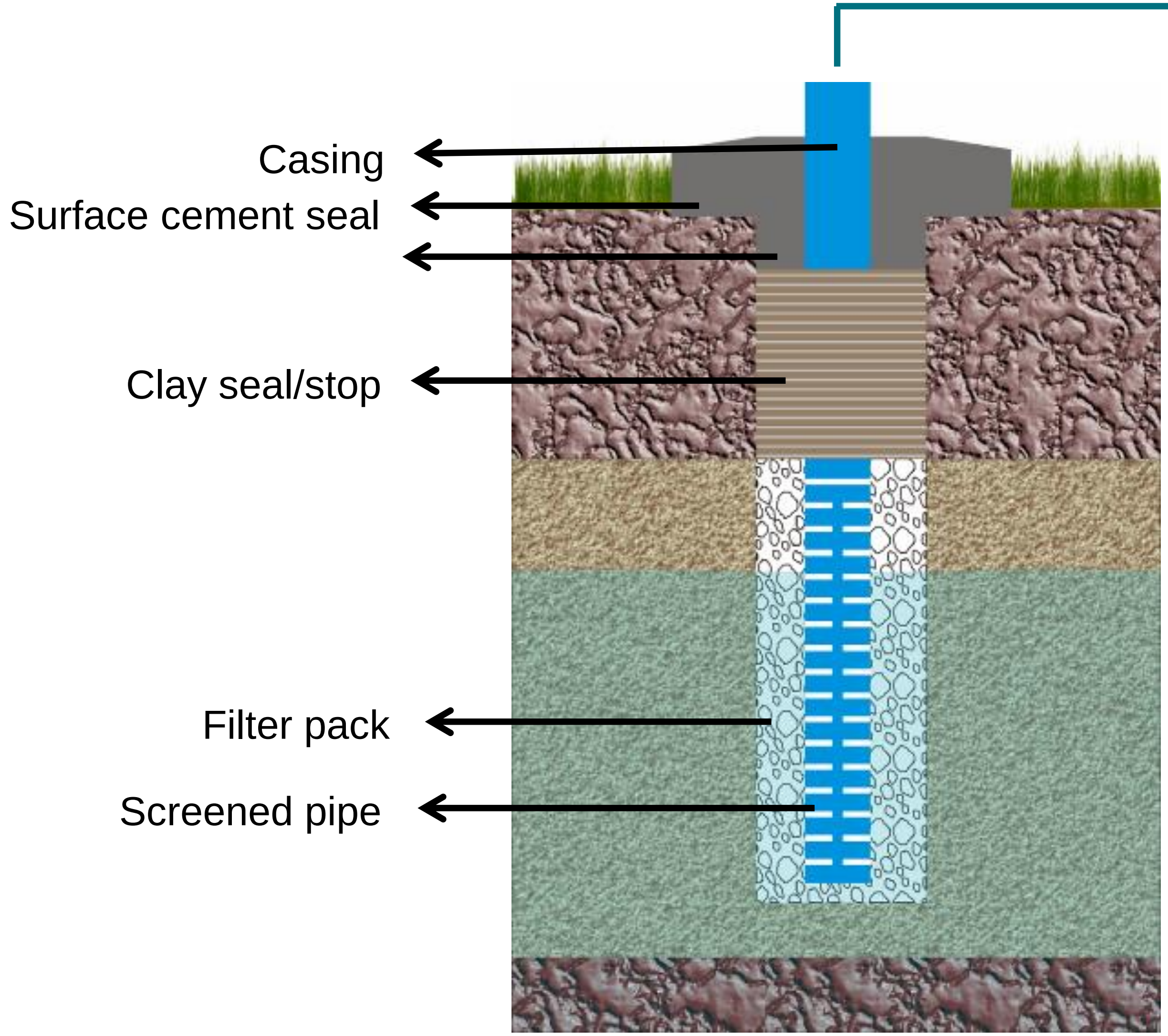
$$h = \frac{\Phi_{tot}}{g} = z + \frac{p}{\rho . g}$$



(Bear & Cheng 2010)

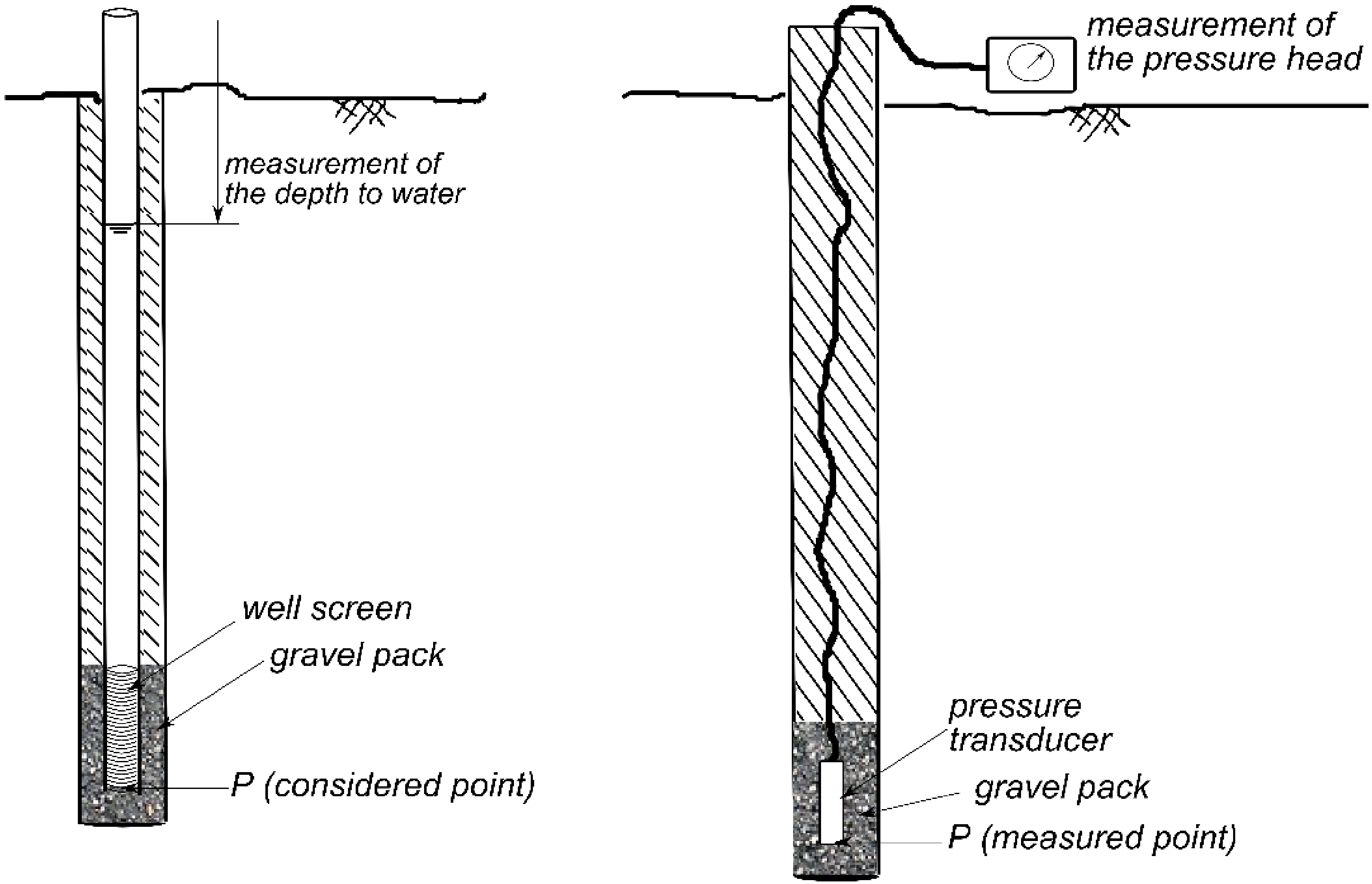
How are measured hydraulic heads ?

Drilling and equipment of boreholes = piezometers



How are measured hydraulic heads ?

… in practice



(Dassargues, 2018, modified from Chapuis 2007)

How are measured hydraulic heads ?

… in practice

Reference elevation

Water pressure vs piezometric head

… a direct link between hydraulic/piezometric head h and water pressure p:

$$p = (h - z)\rho.g$$

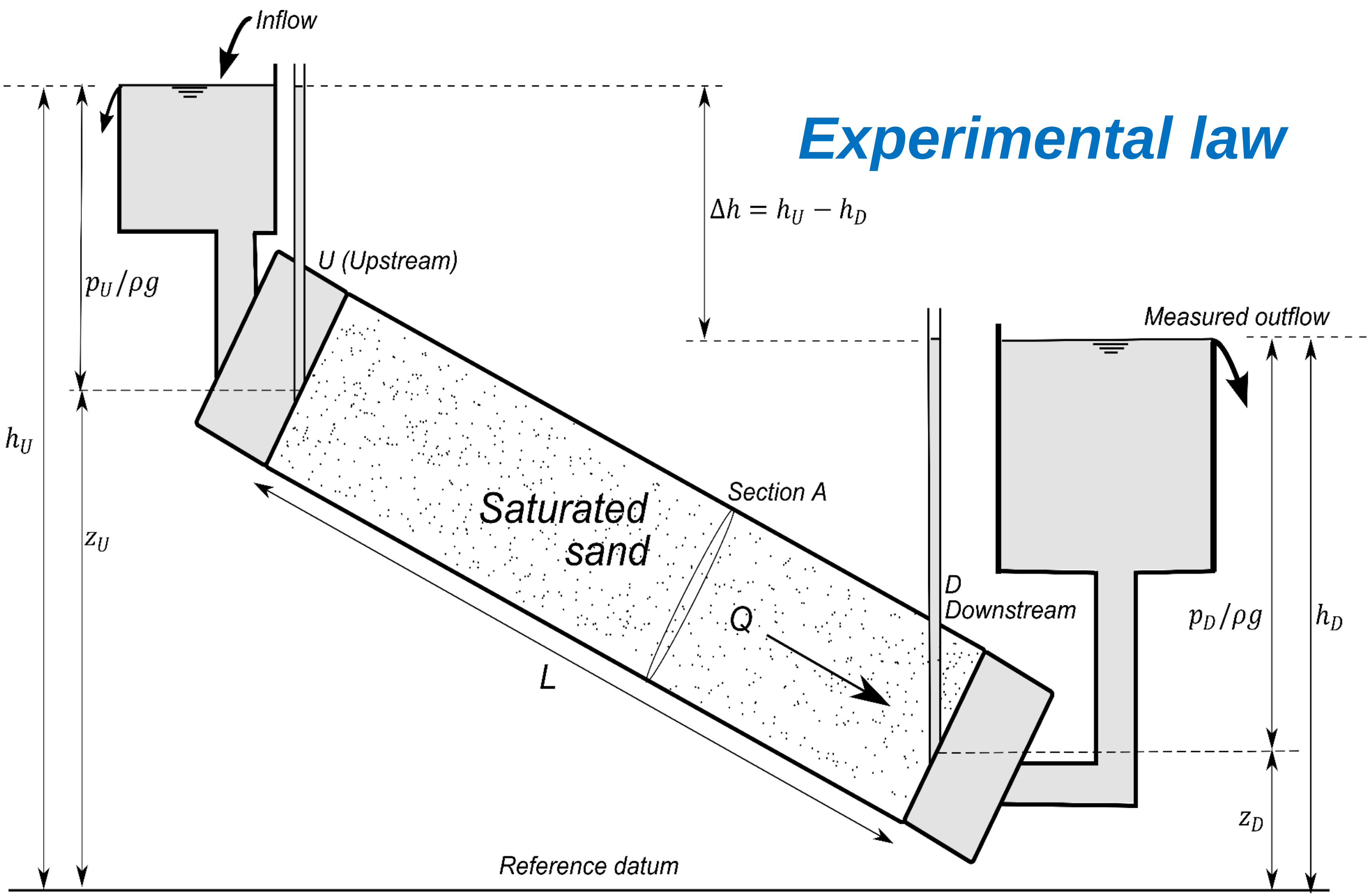
$$\frac{\partial p}{\partial t} = \rho.g\frac{\partial h}{\partial t}$$

- ***for groundwater flow problems, the main variable is the piezometric head or the water pressure***
- ***piezometric heads can be compared only if groundwater has everywhere the same temperature and the same salt content***
- ***if it is not the case, … density will vary … and to a same water pressure correspond different piezometric heads (of groundwater with different salt content)***
- ***work with pressure or with 'equivalent freshwater piezometric head' as the main variable***

(Carabin & Dassargues 1999)

Hydraulic conductivity and Darcy's law

Experimental law



(Darcy 1856, Delleur 1999, Dassargues 2018)

Hydraulic conductivity and Darcy's law

… experimental law

- *quantity of water per time unit through a saturated porous medium:*

$$Q = K.A.\frac{\Delta h}{L}$$

- K *permeability coefficient, hydraulic conductivity, water permeability (by abuse of language: permeability) of the porous medium (m/s)*

- *… specific flux or flow rate (specific discharge):*

$$q = \frac{Q}{A}$$

in $m^3/(m^2.s)$ … so in m/s

Hydraulic conductivity and Darcy's law

This specific discharge is often inapropriately called 'Darcy's velocity' … it is only a flow rate Q divided by a surface A

this surface is not the groundwater flow section

a) a global mass-averaged velocity of water is defined by:

$$v_{avg} = q/n$$ *(Bear & Cheng 2010)*

(b) an effective velocity relative to the mobile fraction of water in drainage and flow problems (i.e balance equations for groundwater flow) is expressed by:

$$v_e = q/n_e$$

(c) a mobile water velocity for solute transport named transport velocity or advection velocity is expressed by:

$$v_a = q/n_m$$ *(Payne et al. 2008)*

$v_a > v_e > v_{avg} > q$ **as** $n_m < n_e < n < 1$

Hydraulic conductivity and intrinsic permeability

K ***depends on:***

- ***fluid properties:***
  - → ***viscosity***
  - → ***density***
- ***porous medium properties:***
  - → ***granular proportions,***
  - → ***grains shapes,***
  - → ***pore distribution and shapes***
  - → ***intergranular porosity***

→

$$K = \frac{k.\rho.g}{\mu}$$

intrinsic permeability or permeability (m^2)

volume mass of the fluid (kg/m^3)

gravity acceleration (m/s^2)

dynamic viscosity (kg/(m.s), N.s/m^2 or Pa.s

Hydraulic conductivity and intrinsic permeability

… in the oil industry ('reservoir engineering'), the 'Darcy' unit is used as intrinsic permeability unit

$$k$$

$$\textbf{1 darcy} = \frac{\dfrac{1cP \times 1cm^3/s}{1cm^2}}{\dfrac{1atm}{1cm}}$$

$$1Cp = 0.01 dyn.s/cm^2$$

$$1atm = 1.01321 0^6 dyn/cm^2$$

➡ ***1 darcy = 9.87 10^{-13} m²***

➡ ***by the fluid viscosity*** ➡ ***K (m/s) is very dependent on the temperature and salt content !***

➡ ***K is not to be used when and where density and/or viscosity can vary***

➡ ***intrinsic permeability k (m²)***

Hydraulic conductivity and intrinsic permeability

Permeability (k) of 1 darcy converted to hydraulic conductivity (K) in m/s depending on T° and TDS influencing viscosity (kg/(m.s)) and density (kg/m³)....

TDS (mg/L) / T (°C)	0	100	500	1000	10000	35000 (seawater)
0	$\rho = 999.868$ $\mu = 1.79\times10^{-3}$ $K = 5.4\times10^{-6}$	$\rho = 999.950$ $\mu = 1.79\text{x}10^{-3}$ $K = 5.4\text{ x}10^{-6}$	$\rho = 1000.278$ $\mu = 1.79\text{ x}10^{-3}$ $K = 5.4\text{ x}10^{-6}$	$\rho = 1000.687$ $\mu = 1.79\text{ x}10^{-3}$ $K = 5.4\text{ x}10^{-6}$	$\rho = 1007.980$ $\mu = 1.83\text{ x}10^{-3}$ $K = 5.3\text{ x}10^{-6}$	$\rho = 1028.131$ $\mu = 1.88\text{ x}10^{-3}$ $K = 5.3\text{ x}10^{-6}$
10	$\rho = 999.728$ $\mu = 1.31\text{ x}10^{-3}$ $K = 7.4\text{ x}10^{-6}$	$\rho = 999.807$ $\mu = 1.31\text{ x}10^{-3}$ $K = 7.4\text{ x}10^{-6}$	$\rho = 1000.122$ $\mu = 1.31\text{ x}10^{-3}$ $K = 7.4\text{ x}10^{-6}$	$\rho = 1000.514$ $\mu = 1.31\text{ x}10^{-3}$ $K = 7.4\text{ x}10^{-6}$	$\rho = 1007.527$ $\mu = 1.34\text{ x}10^{-3}$ $K = 7.3\text{ x}10^{-6}$	$\rho = 1026.979$ $\mu = 1.41\text{ x}10^{-3}$ $K = 7.05\text{ x}10^{-6}$
20	$\rho = 998.234$ $\mu = 1.00\text{ x}10^{-3}$ $K = 9.6\text{ x}10^{-6}$	$\rho = 998.310$ $\mu = 1.00\text{ x}10^{-3}$ $K = 9.6\text{ x}10^{-6}$	$\rho = 998.616$ $\mu = 1.00\text{ x}10^{-3}$ $K = 9.6\text{ x}10^{-6}$	$\rho = 998.997$ $\mu = 1.00\text{ x}10^{-3}$ $K = 9.6\text{ x}10^{-6}$	$\rho = 1005.820$ $\mu = 1.02\text{ x}10^{-3}$ $K = 9.5\text{ x}10^{-6}$	$\rho = 1024.790$ $\mu = 1.08\text{ x}10^{-3}$ $K = 9.2\text{ x}10^{-6}$
30	$\rho = 995.678$ $\mu = 0.80\text{ x}10^{-3}$ $K = 12.2\text{ x}10^{-6}$	$\rho = 995.753$ $\mu = 0.80\text{ x}10^{-3}$ $K = 12.1\text{ x}10^{-6}$	$\rho = 996.053$ $\mu = 0.80\text{ x}10^{-3}$ $K = 12.1\text{ x}10^{-6}$	$\rho = 996.427$ $\mu = 0.80\text{ x}10^{-3}$ $K = 12.1\text{ x}10^{-6}$	$\rho = 1003.122$ $\mu = 0.83\text{ x}10^{-3}$ $K = 11.7\text{ x}10^{-6}$	$\rho = 1021.755$ $\mu = 0.86\text{ x}10^{-3}$ $K = 11.5\text{ x}10^{-6}$
40	$\rho = 992.247$ $\mu = 0.65\text{ x}10^{-3}$ $K = 14.8\text{ x}10^{-6}$	$\rho = 992.322$ $\mu = 0.65\text{ x}10^{-3}$ $K = 14.7\text{ x}10^{-6}$	$\rho = 992.616$ $\mu = 0.65\text{ x}10^{-3}$ $K = 14.7\text{ x}10^{-6}$	$\rho = 992.988$ $\mu = 0.65\text{ x}10^{-3}$ $K = 14.7\text{ x}10^{-6}$	$\rho = 999.602$ $\mu = 0.72\text{ x}10^{-3}$ $K = 13.4\text{ x}10^{-6}$	$\rho = 1017.998$ $\mu = 0.74\text{ x}10^{-3}$ $K = 13.3\text{ x}10^{-6}$

Hydraulic conductivity *(indicative values ranges)*

of a porous medium ...

… of fissured media considered as equivalent porous media

Lithology		K (m/s)
granite and gneiss	with fissures	$1. \times 10^{-7} – 1. \times 10^{-4}$
	without fissures	$1. \times 10^{-14} – 1. \times 10^{-10}$
basalt	with fissures	$1. \times 10^{-7} – 1. \times 10^{-3}$
	without fissures	$1. \times 10^{-12} – 1. \times 10^{-9}$
quartzite	with fissures	$1. \times 10^{-7} – 1. \times 10^{-4}$
	without fissures	$1. \times 10^{-12} – 1. \times 10^{-9}$
shales		$1. \times 10^{-13} – 1. \times 10^{-9}$
schists and slates		$1. \times 10^{-9} – 1. \times 10^{-5}$
limestone and dolomite	karstified	$1. \times 10^{-5} – 1. \times 10^{-1}$
	with fissures	$1. \times 10^{-9} – 1. \times 10^{-3}$
	without fissures	$1. \times 10^{-12} – 1. \times 10^{-9}$
Chalk		$1. \times 10^{-6} – 1. \times 10^{-3}$
sandstone, siltstone	with fissures	$1. \times 10^{-5} – 1. \times 10^{-3}$
	without fissures	$1. \times 10^{-9} – 1. \times 10^{-5}$
volcanic tuff		$1. \times 10^{-7} – 1. \times 10^{-3}$
gravels		$1. \times 10^{-4} – 1. \times 10^{-1}$
sands		$1. \times 10^{-6} – 1. \times 10^{-2}$
silts		$1. \times 10^{-7} – 1. \times 10^{-4}$
loams, loess and clays		$1. \times 10^{-13} – 1. \times 10^{-7}$

(Dassargues 2018)

Hydraulic conductivity and scale effect

Example of a Cretaceous chalk aquifer (Hesbaye, Belgium)

Microscopic scale (< cm)

coccolithes micro-skeletons aggregated by diagenesis

$n = 0.40 - 0.42$ and $n_e = 0.35$ $K \cong 1.10^{-8}$ (m/s)

Macroscopic scale (< dm)

micro-fractures and stratification

$n = 0.42 - 0.45$ and $n_e = 0.01 - 0.03$

$1.10^{-5} \leq K \leq 1.10^{-4}$ (m/s)

Megascopic scale (e.g., pumping tests, < 100 m)

faults and interconnected discontinuities

$n = 0.42 - 0.45$ and $n_e = 0.05 - 0.1$

$1.10^{-4} \leq K \leq 1.10^{-3}$ (m/s)

Data from Geer basin in Belgium (Dassargues & Monjoie, 1993)

3D Darcy's law

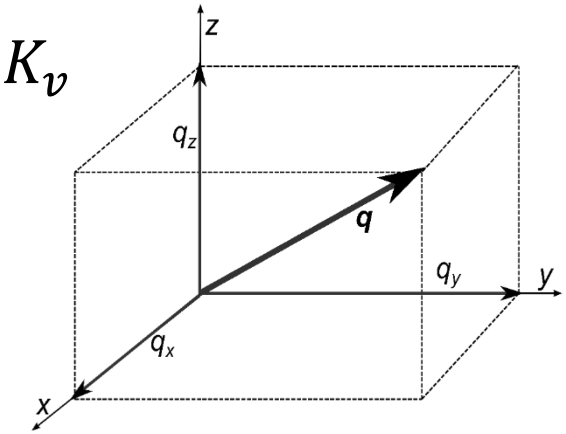
… piezometric gradient: $\boldsymbol{grad}\, h = \nabla h = \left(\frac{\partial h}{\partial x}, \frac{\partial h}{\partial y}, \frac{\partial h}{\partial z}\right)$

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

In a isotropic REV, hydraulic conductivity is a scalar

In an anisotropic medium: $\boldsymbol{K} = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix}$

In most cases: $K_{xx} = K_{yy} = K_h$ ***and*** $K_{zz} = K_v$

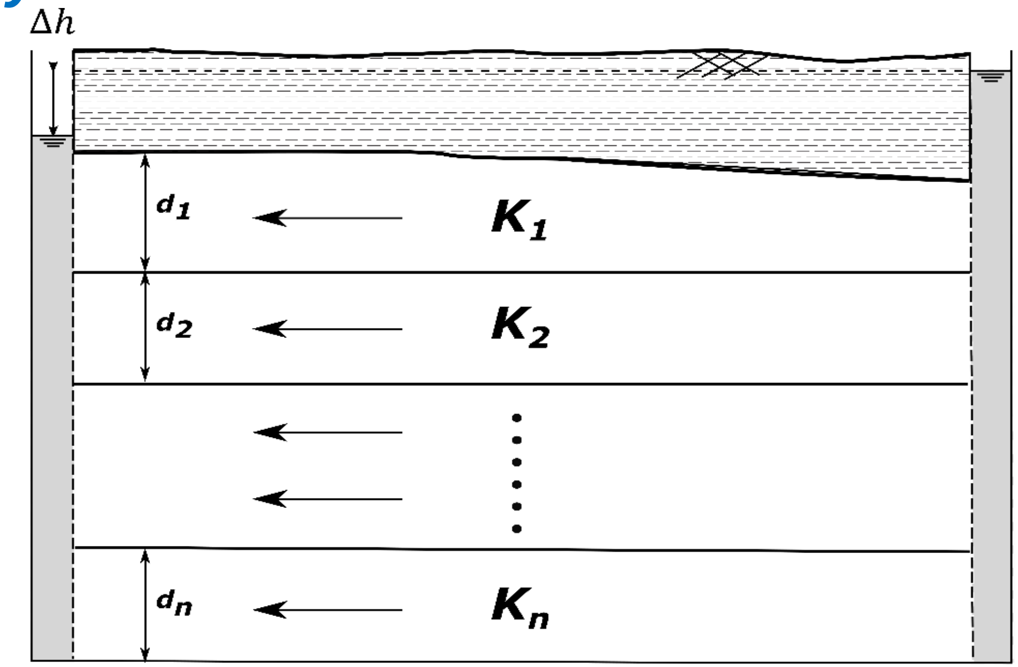


3D Darcy's law:

$$\boldsymbol{q} = -\boldsymbol{K} \cdot \nabla h = -\frac{\boldsymbol{k}\rho g}{\mu} \cdot \nabla h = -\frac{\boldsymbol{k}}{\mu} \cdot (\nabla p + \rho g \nabla z)$$

Hydraulic conductivity: equivalent values

Groundwater flow // to the layers



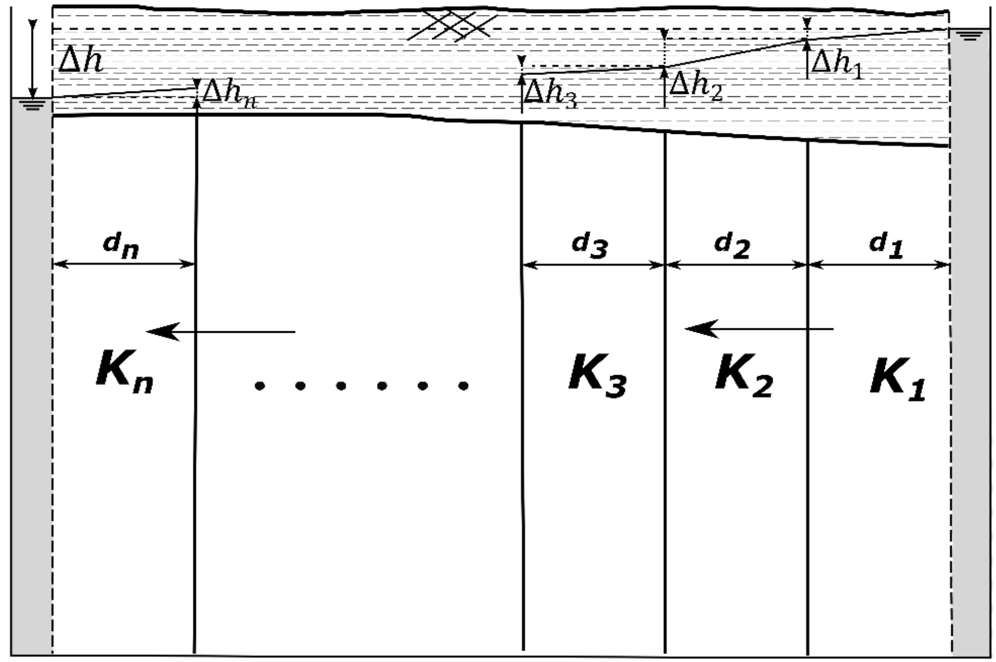
(Dassargues, 2018)

$$K_{eq} = \left(\sum_{i=1}^{n} K_i d_i\right) \Big/ \left(\sum_{i=1}^{n} d_i\right)$$

… equivalence of the total discharge through the medium is expressed

Hydraulic conductivity: equivalent values

Groundwater flow ⊦ to the layers



(Dassargues, 2018)

$$K_{eq} = \left(\sum_{i=1}^{n} d_i\right) \Big/ \left(\sum_{i=1}^{n} \frac{d_i}{K_i}\right)$$

… equivalence of the total discharge through the medium is expressed

Heterogeneity and equivalent/averaged hydraulic conductivity

Remarks

- REV concept/theory: tool for upscaling from microscopic scale to macro and mega scales
- REV concept/theory: tool for homogenization of heterogeneities :
  - ⇒ averaged values calculation
  - ⇒ equivalent values
- the way of calculating equivalent values depends strongly on the final aim of the study
- reminder: heterogeneity scale and problem scale must be considered when choosing the adequate REV size

(Durlofsky 1991, Pickup et al. 1994, Renard and de Marsily 1997, Ringrose and Bentley 2015)

Geostatistically derived equivalent averaged hydraulic conductivity values

- *in porous media 'uniformly heterogeneous'*
- *property log K* → *normal (Gaussian) distribution*

mean (averaged) value (equivalent on the REV) =
geometric mean of measured K

$$K_{eq} = \sqrt[n]{\prod_{i=1}^{n} K_i}$$

also valid in anisotropic conditions

⚠ *applications: many measurements are needed*
do not forget geological structures
→ *example: horizontally stratified media*
arithmetic mean

(de Marsily 1986, Ringrose and Bentley 2015)

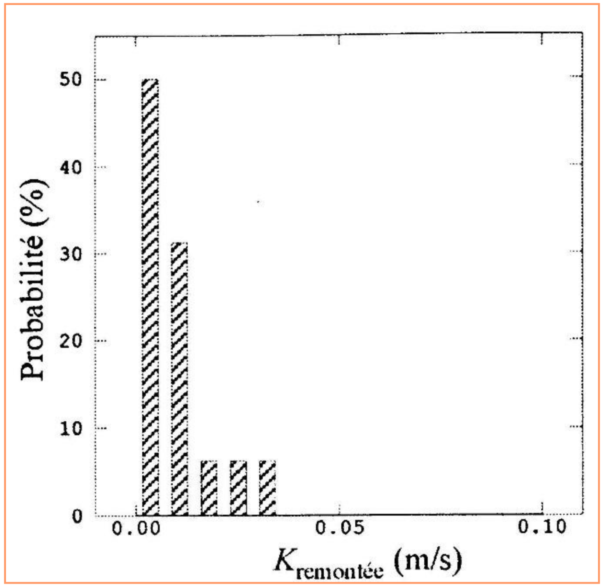
Hydraulic conductivity

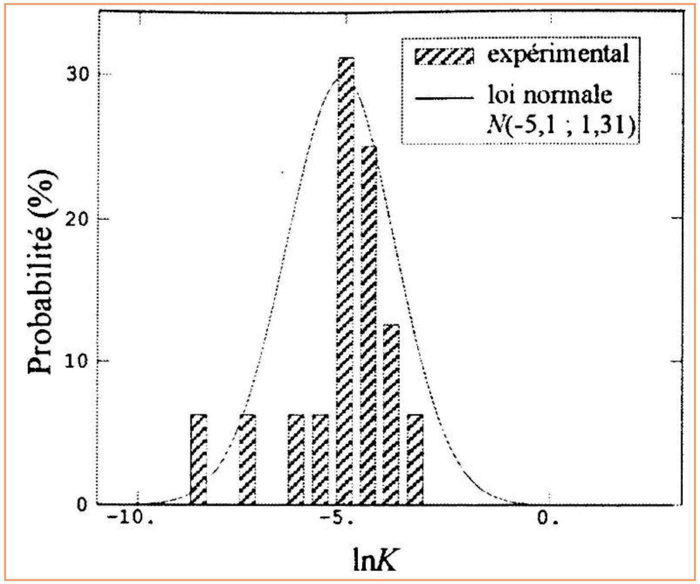
equivalent values, mean (averaged) values

Example:

Vivegnis

(Belgium)





(after Rentier 2012)

More frequent values, …

- more frequent value = harmonic mean of measured K

$$K_{mh} = \frac{n}{\sum_{i=1}^{n} (1/K_i)}$$

Darcy's law application

Different piezometers in different locations

↓

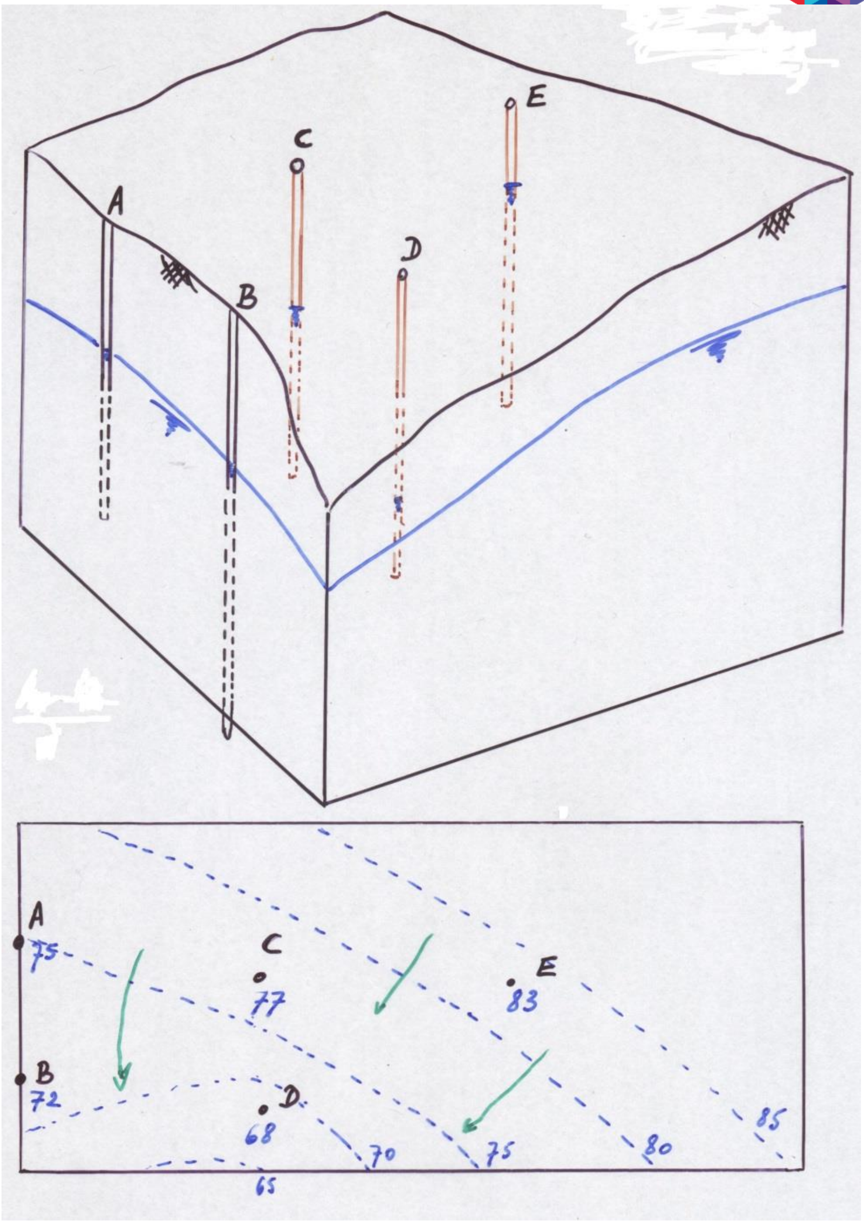
Piezometric heads vary in space and time

↓

Piezometric map

↓

Main directions of groundwater flow



Darcy's law application

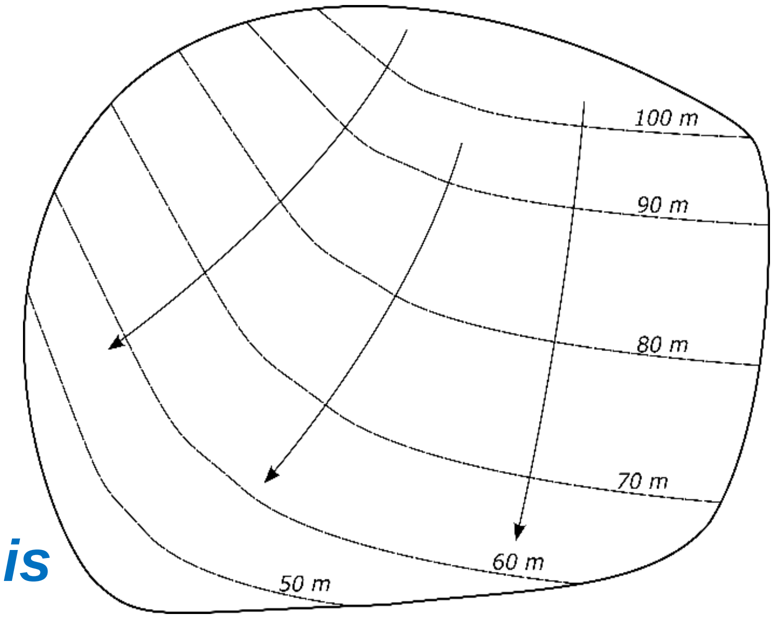
Most often: no mention of the depth at which this horizontal section is drawn

➡ ***implicit assumption that the vertical component is negligible***

= the Dupuit assumption:

(Dupuit 1863)

$$\frac{\partial h}{\partial z} = 0 \qquad \frac{\partial p}{\partial z} = -\rho g \qquad p = -\rho g z$$



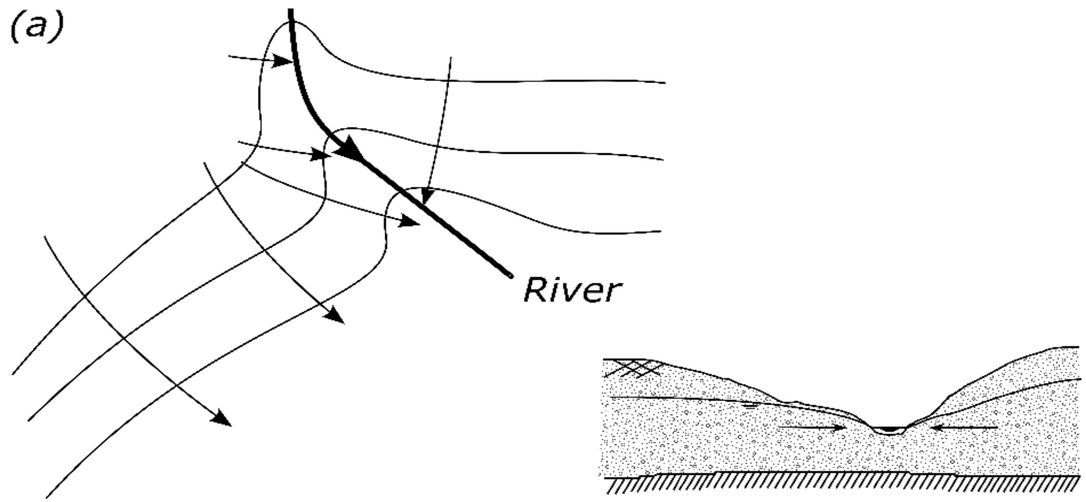
If isotropic medium: groundwater flow is perpendicular to the head isolines

➡ ***if anisotropic medium: not the case***

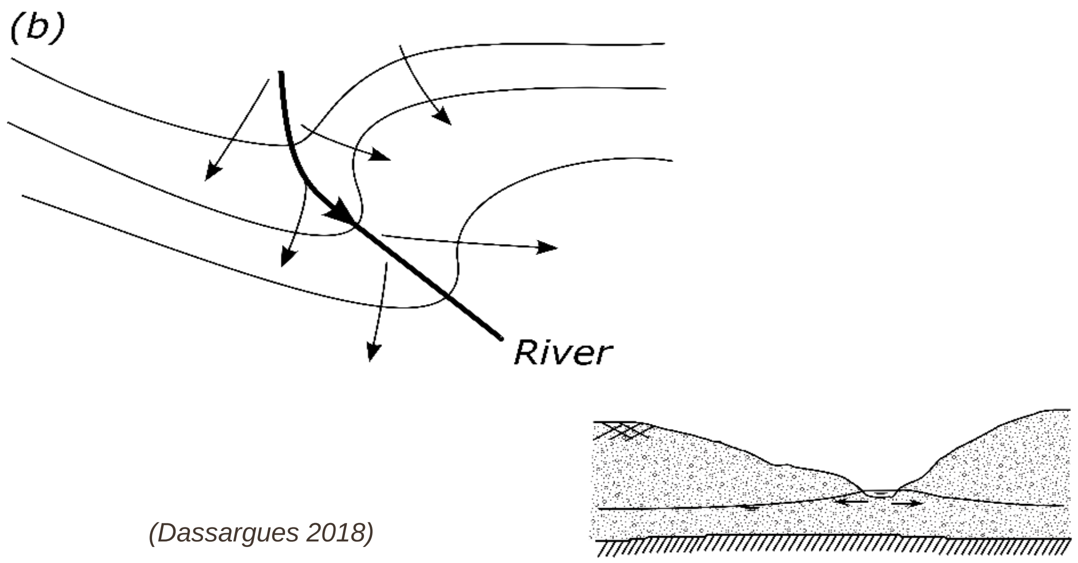
Darcy's law application

River – groundwater interactions detected by interpretation of a piezometric map:

(a) converging streamlines towards a 'gaining' or 'draining' river



(b) diverging streamlines from a 'loosing' or 'feeding' river



(Dassargues 2018)

Darcy's law application

piezometric gradient can be influenced by :

- *heterogeneous nature of the geological layer corresponding to the aquifer*

$$Q = K.A.\frac{\Delta h}{L}$$

- ↗ - ↘

- *groundwater flow section*

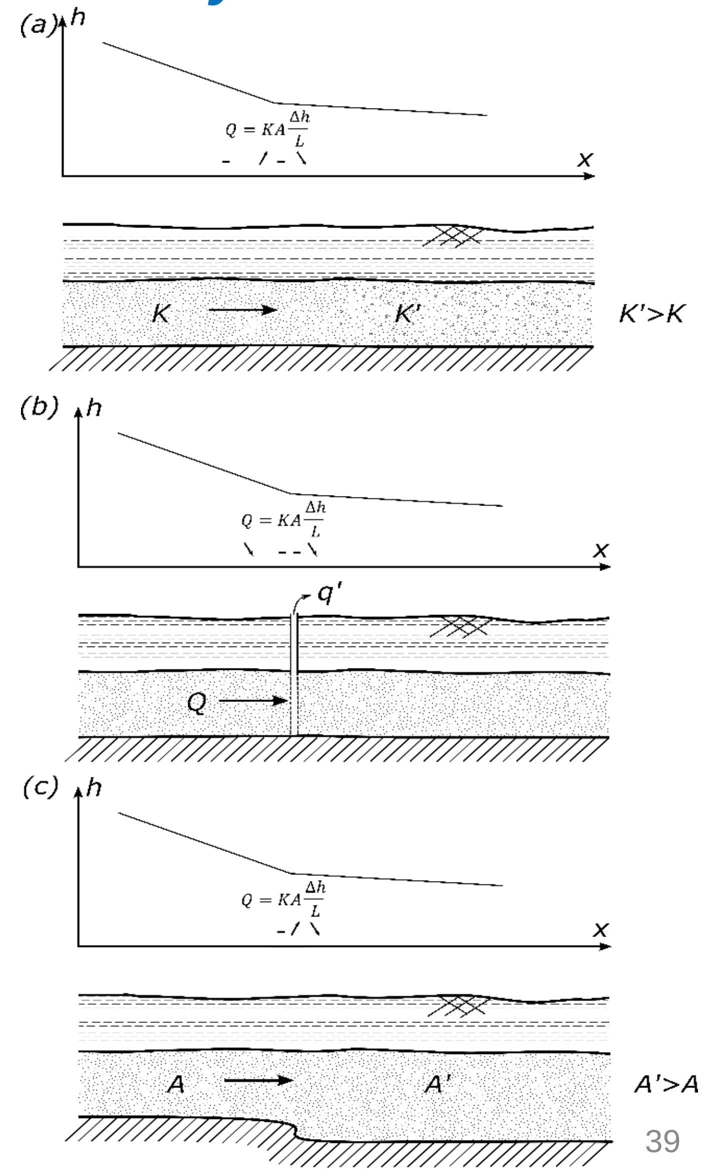
$$Q = K.A.\frac{\Delta h}{L}$$

- - ↗ ↘

- *stress factors*

$$Q = K.A.\frac{\Delta h}{L}$$

↘ - - ↘



(Dassargues 2018)

Darcy's law application and heterogeneity

in 2D horizontal, flow from medium 1 towards medium 2

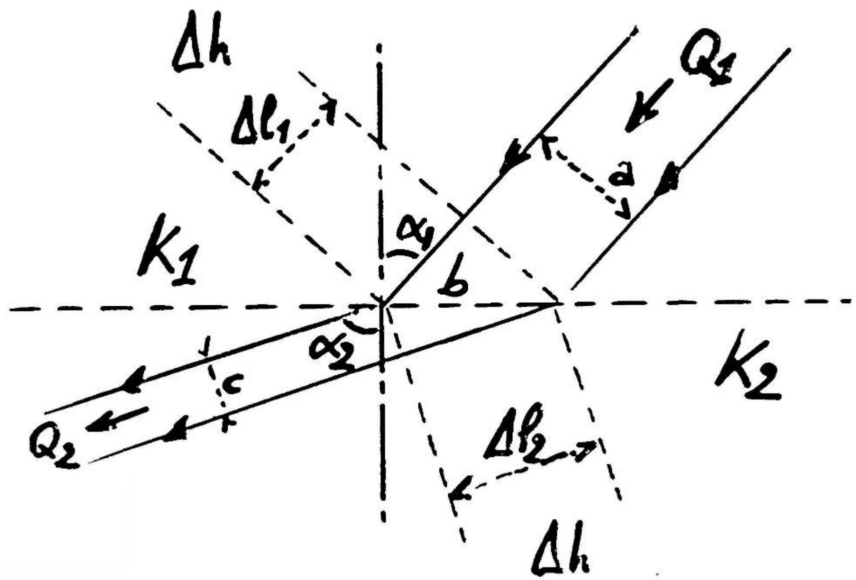
- in medium 1: $Q = -K_1 \, . \, a \, . \, \dfrac{\Delta h}{\Delta l_1}$

with $\dfrac{\Delta h}{\Delta l_1}$ piezometric gradient in this medium

- in medium 2: $Q = -K_2 \, . \, c \, . \, \dfrac{\Delta h}{\Delta l_2}$

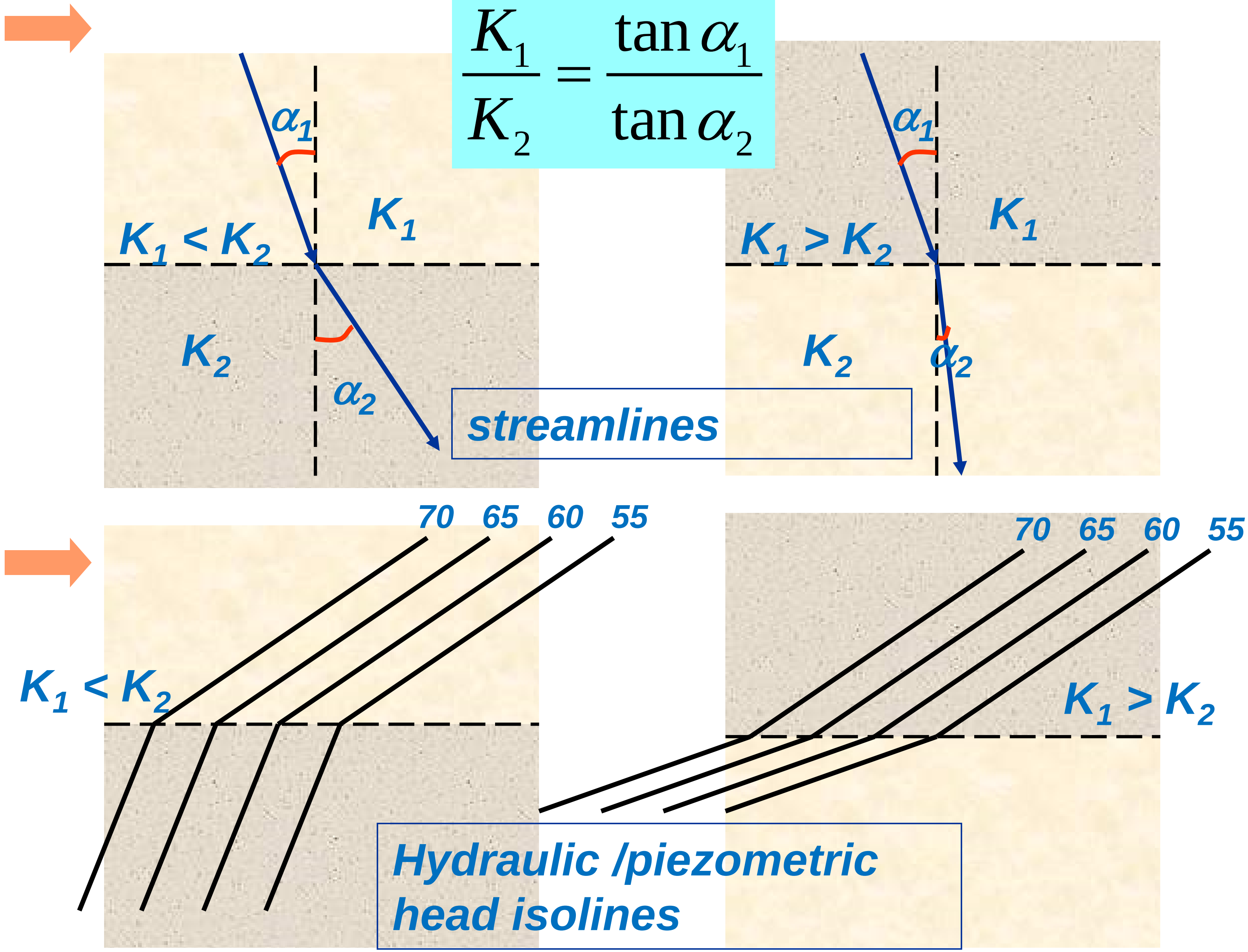
with $\dfrac{\Delta h}{\Delta l_2}$ in medium 2

$$\boxed{\frac{K_1}{K_2} = \frac{\tan \alpha_1}{\tan \alpha_2}}$$

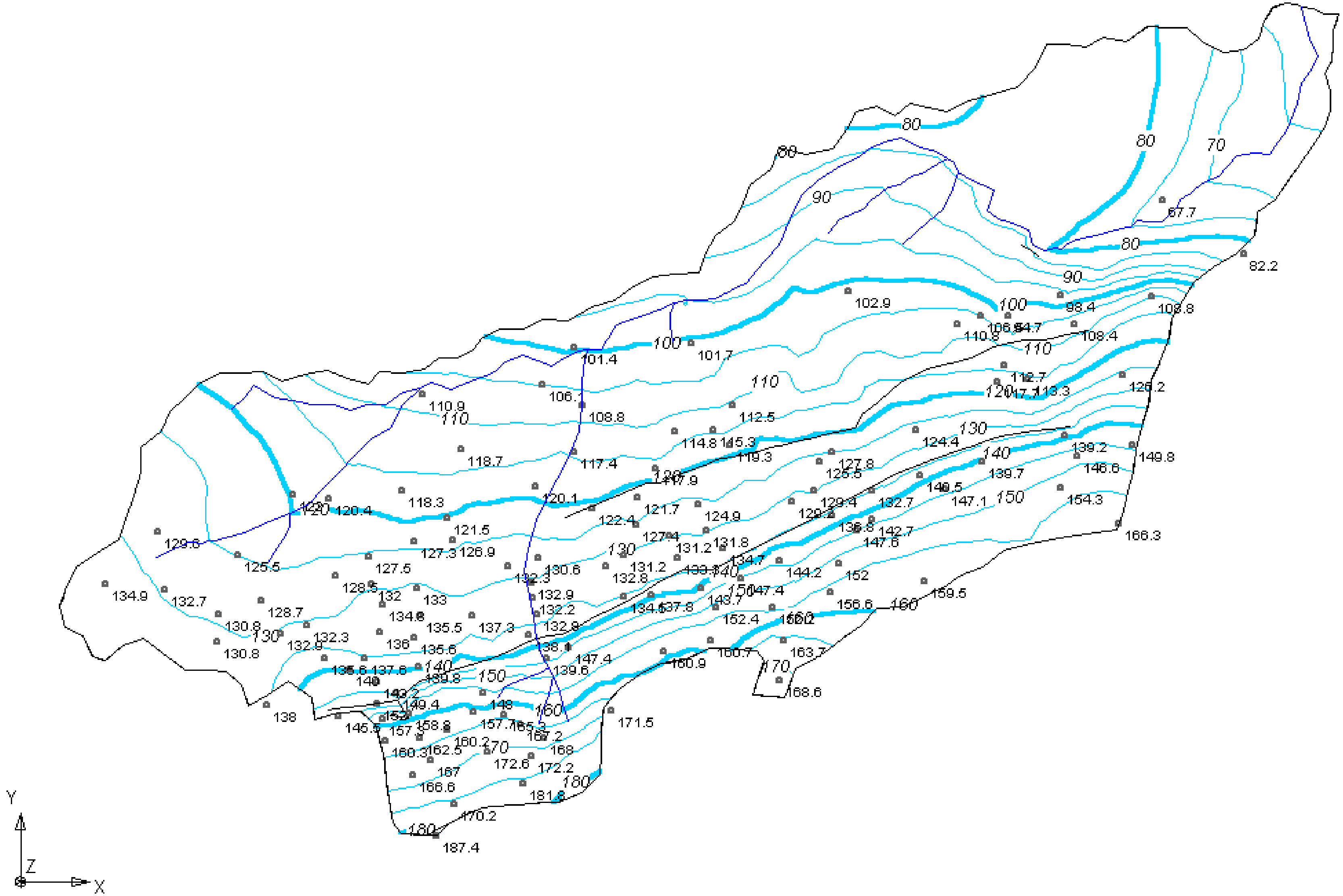


Darcy's law application and heterogeneity

$$\frac{K_1}{K_2} = \frac{\tan \alpha_1}{\tan \alpha_2}$$



Piezometric maps and Darcy's law application



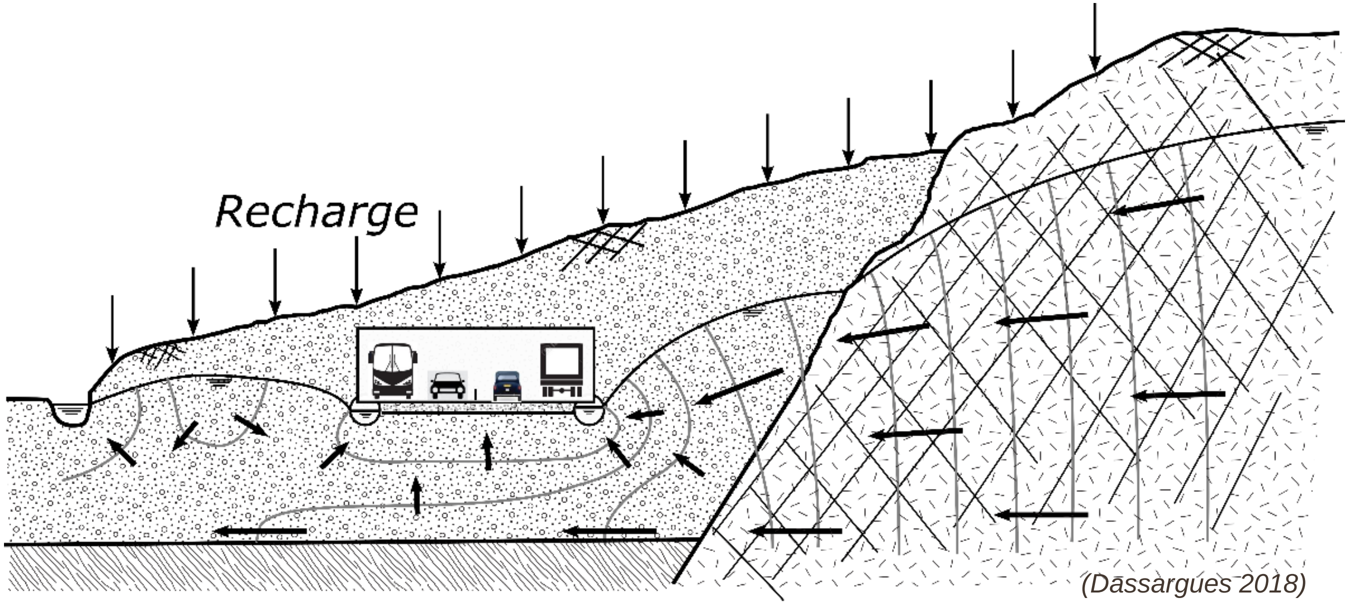
Darcy's law application:

2D vertical flownets to show how the water table controls regional groundwater flow and locations of discharge and recharge areas

(among others: Tóth 1962, 1963, Freeze and Witherspoon 1967)

but be careful to conditions involving temperature differences may induce buoyancy effects at low water table gradients (< 0.0005)

2D cross-sections: very useful to understand hydrogeological conditions



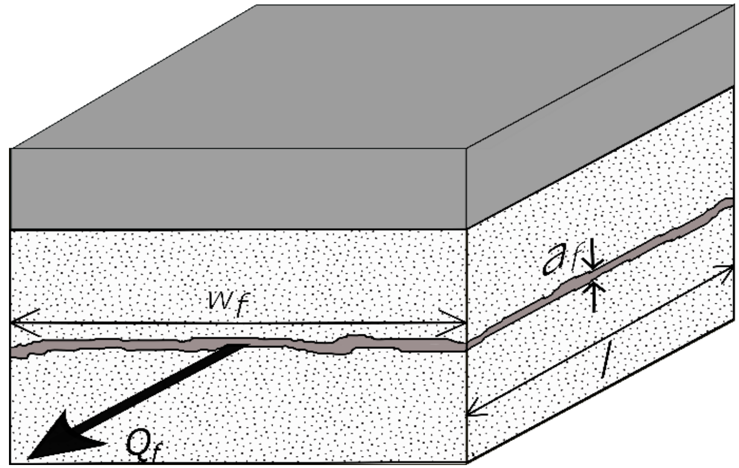
(Dassargues 2018)

Hydraulic conductivity and groundwater flow in fractured rocks

➡ Q_f the flow along axis l of a fracture presenting a flow section A_f is written

$$Q_f = \left(\frac{\rho g}{12\mu} a_f^2\right) A_f \frac{\partial h}{\partial l}$$

$$A_f = a_f w_f$$



(Rausch et al. 2002, Ringrose and Bentley 2015)

➡ a ‘cubic law’ is found:

$$Q_f = \left(\frac{\rho g}{12\mu} a_f^3\right) w_f \frac{\partial h}{\partial l}$$

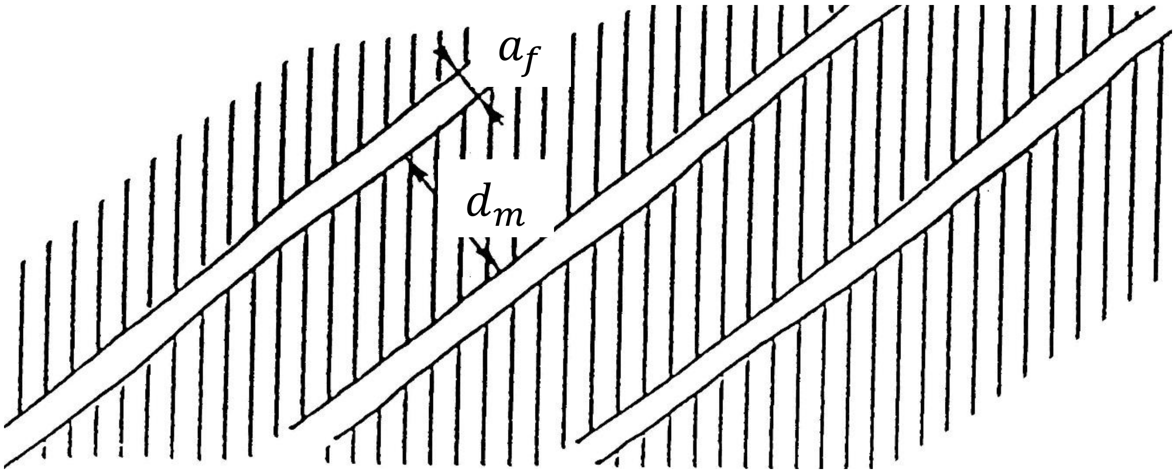
➡ ‘transmissivity’ of the fracture:

$$T_f = \left(\frac{\rho g}{12\mu} a_f^3\right)$$

Hydraulic conductivity fractured rocks

➡ ***equivalent values in fissured media taking into account K_{matrix} ,$K_{fissure}$, opening of the fissures a_f, and rock bank thickness d_m:***

$$K_{eq} = \frac{a_f}{a_f + d_m} K_f + \frac{d_m}{a_f + d_m} K_m$$



➡ ***if $a_f << d_m$*** $\quad K_{eq} \cong (a_f/d_m)K_f + K_m$

(de Marsily 1986, Chen et al. 2015, Rausch et al. 2002, Maini and Hocking 1977, Singhal and Gupta 2010)

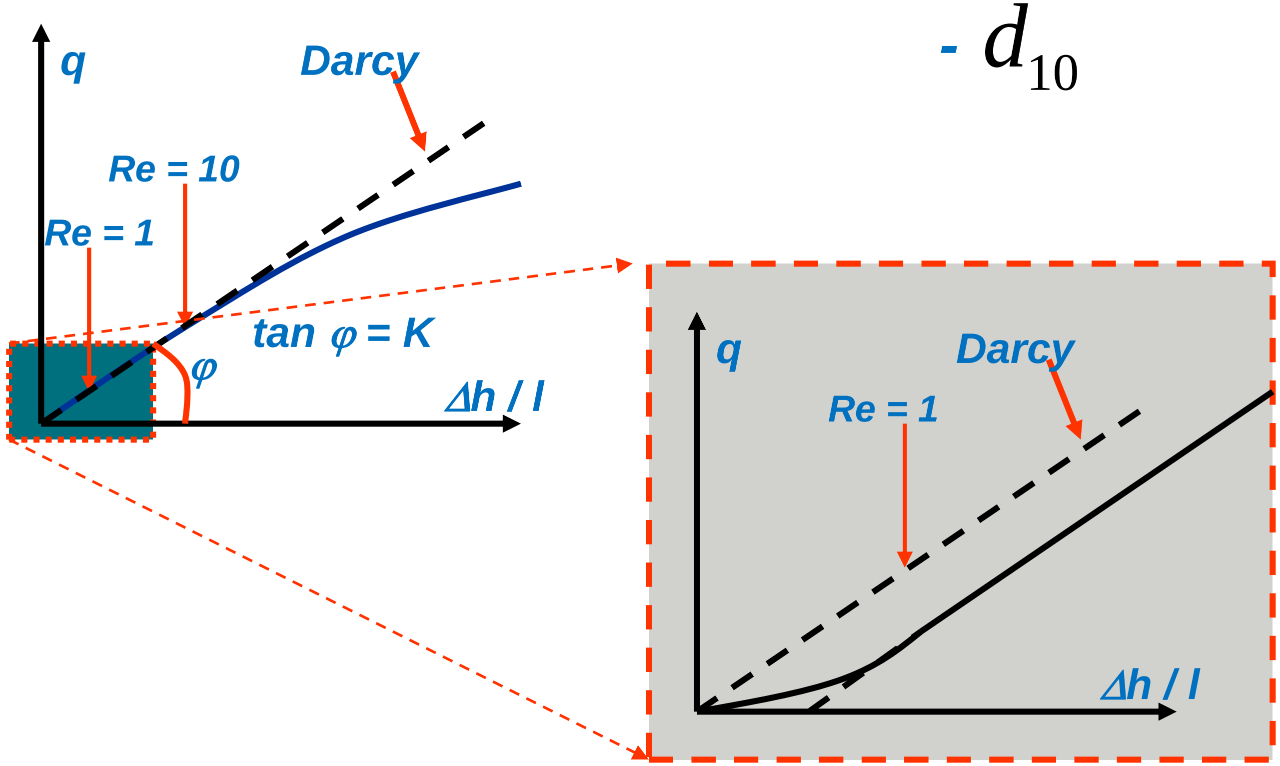
Darcy's law limitations

Flow characterisation by a Reynolds number

$$Re = \frac{q\,.D_h\,.\rho}{\mu}$$

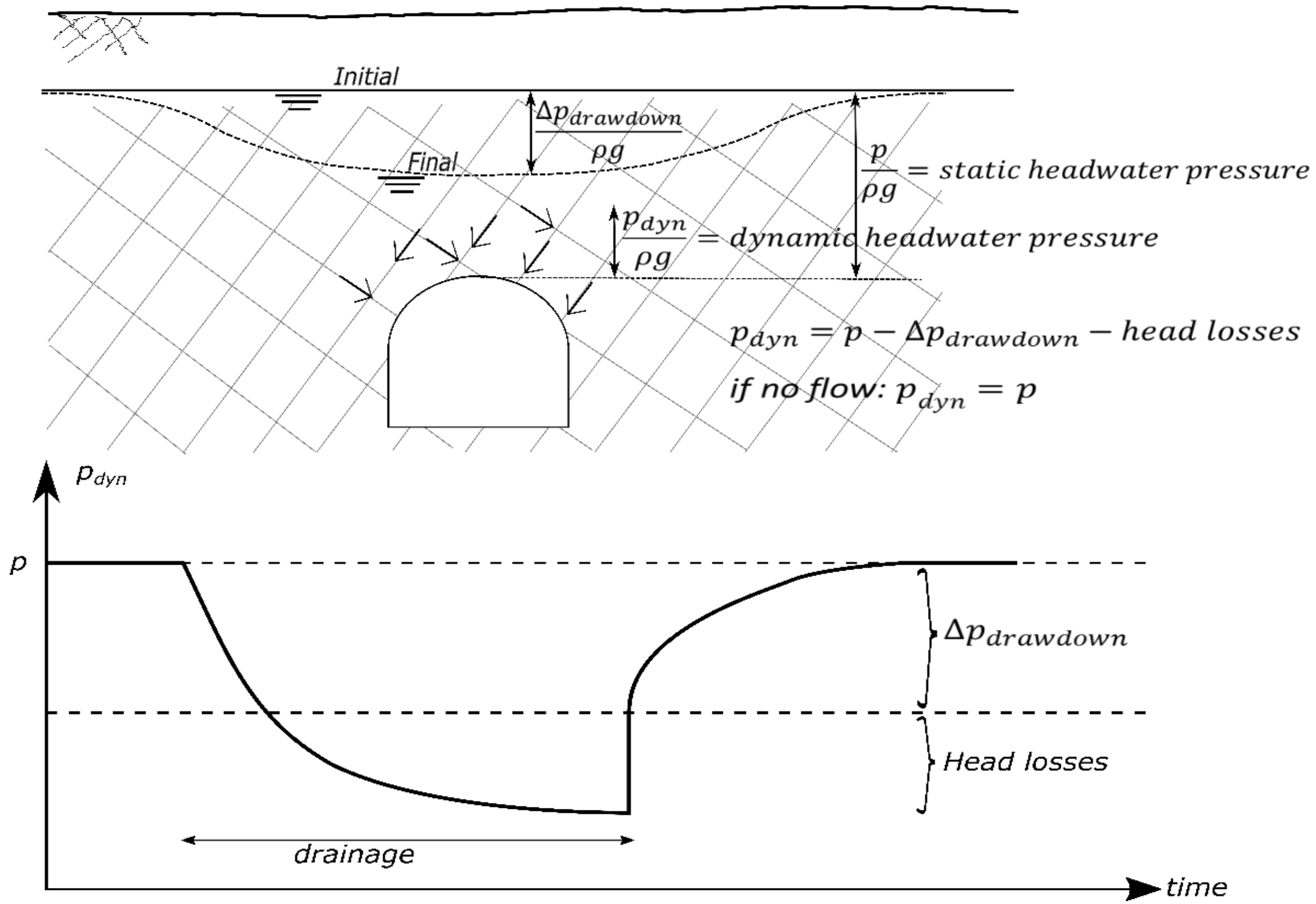
with D_h **hydraulic diameter**

- $D_h = \sqrt{k/n}$
- d_{10}



(Jacquin 1965, de Marsily 1986, Wagner and Egloffstein 1990, Fitts 2002, Bear 2007, Bear and Cheng 2010, Liu 2014)

Flow in fractured rocks and head losses



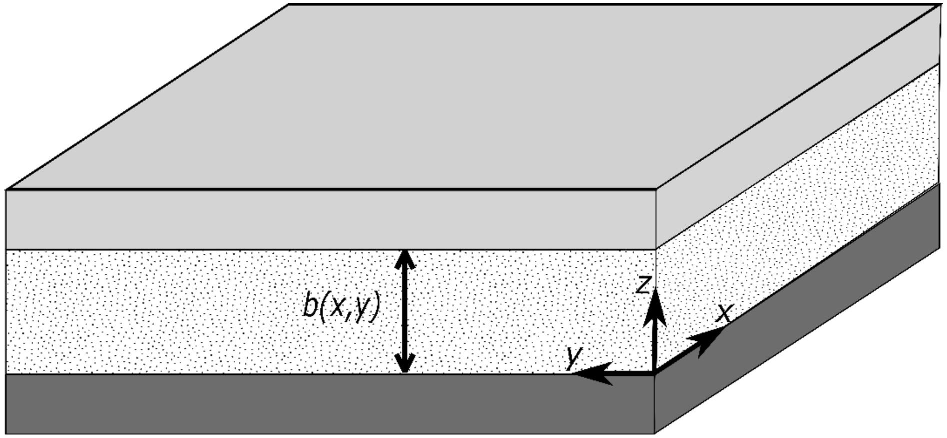
(Dassargues 2018)

➡ ***head losses induced by active drainage in a tunnel help to decrease the (dynamic) head water pressure when drainage is stopped, head losses cease instantaneously and the dynamic water pressure increases until recovering to the static water pressure***

(Maréchal and Perrochet 2003)

Transmissivity

... for a confined aquifer



transmissivity
(m^2/s)

$$T(x,y) = \int_0^{b(x,y)} K(x,y,z)dz = K_{avg}(x,y)b(x,y)$$

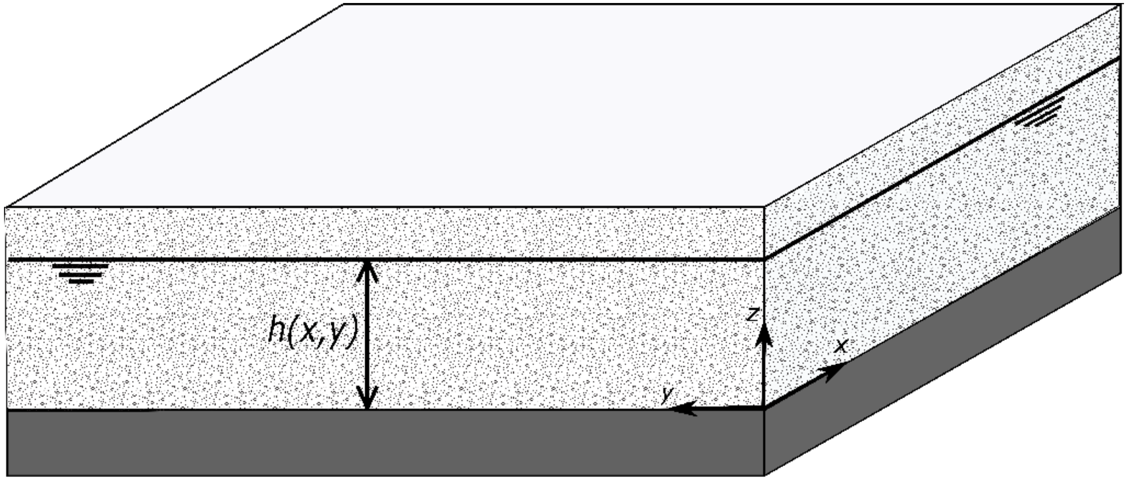
thickness of the confined aquifer at the considered point

mean value of the hydraulic conductivity on the vertical of the considered point

'depth-averaged' conditions
Dupuit assumption

Transmissivity

… for an unconfined aquifer



the saturated thickness of the unconfined aquifer at the point (of horizontal coordinates x and y)

depends on piezometric head

$$T(x,y) = \int_0^{h(x,y)} K(x,y,z)dz = K_{avg}(x,y)h(x,y)$$

Transmissivity

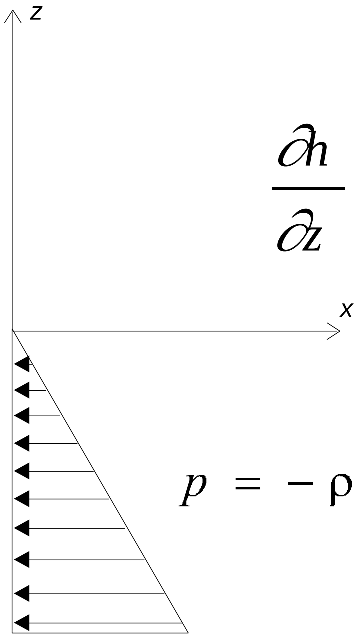
unconfined aquifer

… the following Dupuit assumption is needed :

$$h(x,y) = h(x,y,z_1) = h(x,y,z_2) = h(x,y,z_3)$$

only the horizontal component of the groundwater flow is considered

only vertical head isolines
'hydrostatic' pressure distribution



$$p = -\rho . g . z$$

$$\frac{\partial h}{\partial z} = 0 = 1 + \frac{1}{\rho . g} . \frac{\partial p}{\partial z} \qquad \frac{\partial p}{\partial z} = -\rho . g$$ ***and*** $p = -\rho . g . z + Cst$

acceptable where and when
piezometric gradient lower than $1/1000$

(Delleur 1999)

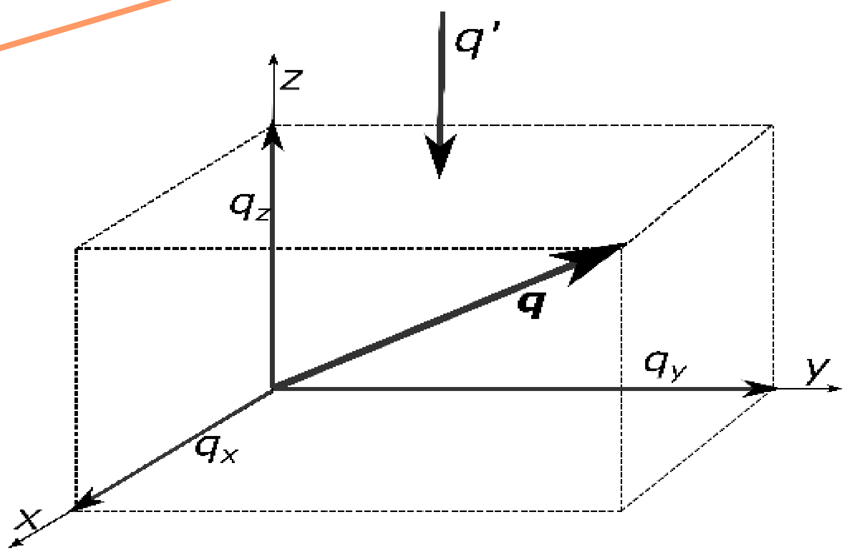
Equations of the steady-state groundwater flow (saturated conditions)

… water mass conservation : input = output

$$-div(\rho \boldsymbol{q}) = -\nabla \cdot (\rho \boldsymbol{q}) = -\rho q'$$

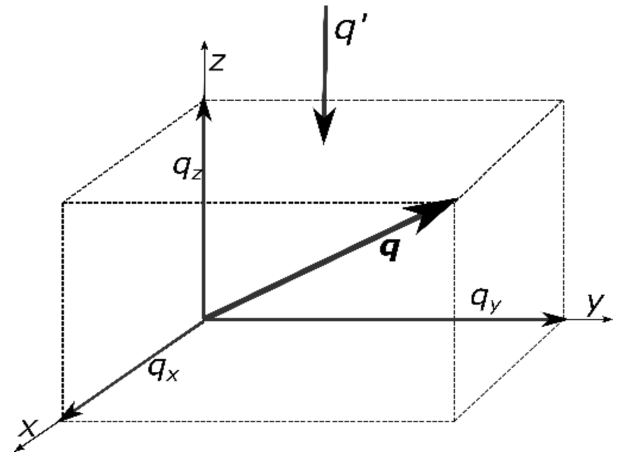
specific flux / discharge (m/s)

sink/source flow rate' per volume unit (s^{-1}), negative for pumping, etc. and positive for infiltration, injection, etc.



$$div(\boldsymbol{q}) = \nabla \cdot \boldsymbol{q} = \left(\frac{\partial q_x}{\partial x}, \frac{\partial q_y}{\partial y}, \frac{\partial q_z}{\partial z}\right)$$

Equation in steady state



$$\nabla \cdot (\rho\, \boldsymbol{K} \cdot \nabla h) + \rho q' = 0$$

terms are kg/(m³s)

$$\frac{\partial}{\partial x_i}\left(\rho K_{ij} \frac{\partial h}{\partial x_j}\right) + \rho q'_i = 0$$

in indicial notation

$$\frac{\partial}{\partial x}\left(K_{xx} \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy} \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{zz} \frac{\partial h}{\partial z}\right) + q' = 0$$

if density is assumed constant and the principal anisotropy directions of the K tensor are known and aligned with the selected coordinate system – terms are in s^{-1}

$$\frac{\partial}{\partial x}\left(K_{xx} \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial z}\left(K_{zz} \frac{\partial h}{\partial z}\right) + q' = 0$$

if 2D vertical flow, terms are in s^{-1}

$$\frac{\partial}{\partial x}\left(T_{xx} \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_{yy} \frac{\partial h}{\partial y}\right) + q'' = 0$$

if 2D horizontal flow, terms are in m/s

Storage variation under saturated conditions

… transient groundwater flow,
variation of the storage in function of time : $\dfrac{\partial(n\,\rho)}{\partial t}$

specific storage coefficient (m^{-1})

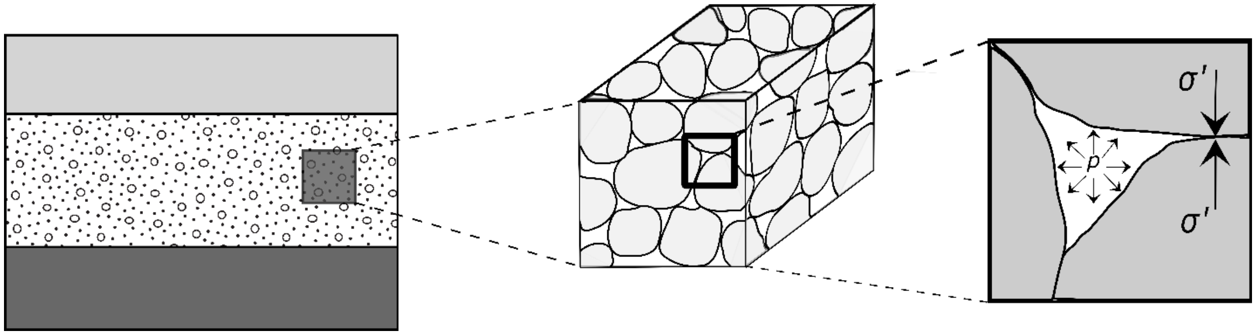
$$\Rightarrow \quad \frac{\partial(n\,\rho)}{\partial t} = \rho^2 g\left(\alpha + n\beta_s + n\beta_w\right)\frac{\partial h}{\partial t} = \rho\, S_s\,\frac{\partial h}{\partial t}$$

volume compressibility of the porous medium (Pa^{-1})

compressibility of the solid grains (matrix) (Pa^{-1})

water compressibility (Pa^{-1})

Terzaghi principle and volume compressibility



$$\sigma = \sigma' + p$$

(Terzaghi 1943, Biot, 1941, Verruijt 1982, Dassargues 2018)

volume compressibility (Pa^{-1}):

$$-\frac{1}{V}\frac{\partial V}{\partial t} = \alpha \frac{\partial \sigma'}{\partial t}$$

Lithology	Volume compressibility α (Pa^{-1})
Highly organic alluvial clays and peats, underconsolidated clays	1.5×10^{-6} – $1.\ \mathrm{x}10^{-6}$
Normally consolidated alluvial clays	$1.\ \mathrm{x}10^{-6}$ – $3.\ \mathrm{x}10^{-7}$
Clays of lake deposits/outwash, normally consolidated clays at depth, weathered marls	$3.\ \mathrm{x}10^{-7}$ – $1.\ \mathrm{x}10^{-7}$
Tills and marls	$1.\ \mathrm{x}10^{-7}$ – $5.\ \mathrm{x}10^{-8}$
Over-consolidated clays	$5.\ \mathrm{x}10^{-8}$ – $1.\ \mathrm{x}10^{-8}$
Sand	$5.\ \mathrm{x}10^{-7}$ – $1.\ \mathrm{x}10^{-9}$
Gravel	$5.\ \mathrm{x}10^{-8}$ – $1.\ \mathrm{x}10^{-10}$
Fractured rock	$5.\ \mathrm{x}10^{-8}$ – $1.\ \mathrm{x}10^{-10}$
Hard rock	$5.\ \mathrm{x}10^{-9}$ – $1.\ \mathrm{x}10^{-11}$

(modified from Freeze and Cherry 1979, Carter and Bentley 1991)

Specific storage coefficient

The specific storage coefficient corresponds to the water volume (m^3) liberated or stored per volume unit of porous medium (m^3) for a unit change of piezometric head (m)

➡ ***with the following assumptions :***

- ***isothermal conditions***
- ***homogeneous fluid***
- ***geomechanical behaviour of the porous medium is described by the volume compressibility***
- ***total stress is considered as constant***
- ***Terzaghi's principle is applied***
- ***the REV concept is used***
- ***the specific discharge (Darcy) is a relative flow rate through the porous medium***

Specific storage coefficient

… often, the influence of the water compressibility and the solid grain compressibility can be neglected with regards to the volume compressibility of the porous medium (as a whole)

$$\Rightarrow \quad S_s = \rho\, g\, \alpha$$

… this link between the volume compressibility and the specific storage coefficient is showing clearly the direct coupling between saturated transient groundwater flow and geomechanical behaviour in compressible porous media

the volume compressibility is dependent on

- ***effective stress variation***
- ***the effective preconsolidation stress of the porous medium***

Equation of transient groundwater flow

input = output + storage variation
input – output = storage variation

in saturated conditions

$$-\nabla \cdot \rho(\boldsymbol{q}_r) + \rho q' = \rho S_s \frac{\partial h}{\partial t}$$

$$\nabla \cdot \rho(\boldsymbol{K} \cdot \nabla h) + \rho q' = \rho S_s \frac{\partial h}{\partial t}$$

$$\frac{\partial}{\partial x_i}\left(\rho K_{ij} \frac{\partial h}{\partial x_j}\right) + \rho q'_i = \rho S_s \frac{\partial h}{\partial t}$$

in indicial notation

$$\frac{\partial}{\partial x}\left(K_{xx} \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy} \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{zz} \frac{\partial h}{\partial z}\right) + q' = S_s \frac{\partial h}{\partial t}$$

if density is assumed constant and the principal anisotropy directions of the K tensor are known and aligned with the selected coordinate system – terms are in s^{-1}

Storage coefficient

Storage coefficient = water volume (m^3) stored or drained per aquifer surface unit (m^2) for a unit variation of piezometric head (m)

… vertical integration

confined aquifer

$$S\,(x,y) = \int_0^{b(x,y))} S_s(x,y,z)dz = S_{s\,avg}(x,y)b(x,y)$$

$$S = S_s b = \rho g \alpha b$$

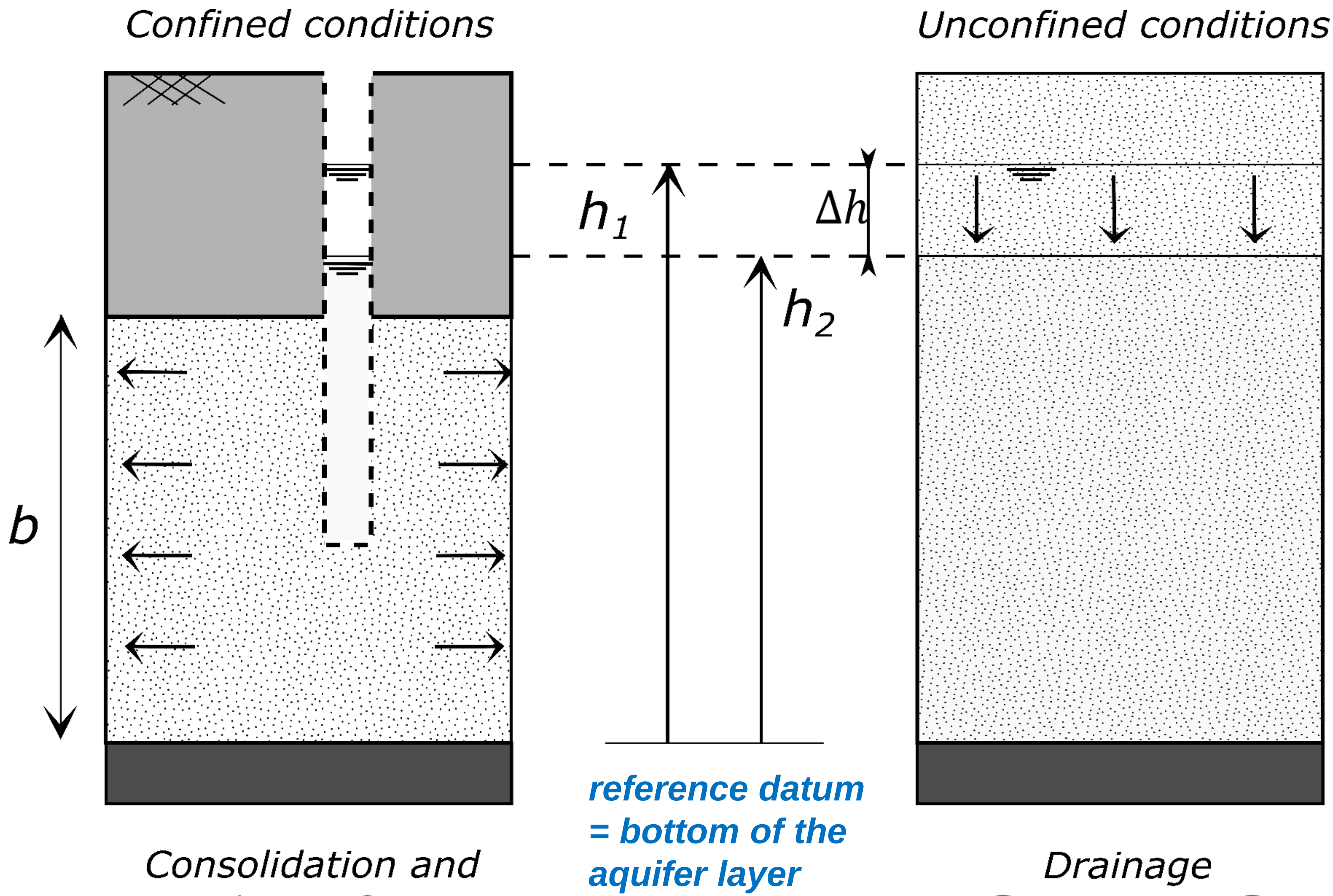
unconfined aquifer

$$S = n_e + \int_0^h S_s dz = n_e + S_s h$$

$$S \cong n_e = S_y$$

the most important part of the storage is due to saturation/drainage of the porous medium

Storage coefficient



Consolidation and expulsion of water

$$S = S_s b = \rho g \alpha b$$

S < 0.005

Drainage

$$S \cong n_e = S_y$$

S = 0.02 – 0.30

2D groundwater flow equations in transient conditions (horizontal flow)

confined aquifer

$$\nabla \cdot (\boldsymbol{T} \cdot \nabla h) + q'' = S\frac{\partial h}{\partial t}$$

terms are in m/s

$$\frac{\partial}{\partial x_i}\left(T_{ij}\frac{\partial h}{\partial x_j}\right) + q''_i = S\frac{\partial h}{\partial t}$$

in indicial notation

$$\frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right) + q'' = S\frac{\partial h}{\partial t}$$

principal anisotropy directions aligned with the selected coordinate system

unconfined aquifer

$$\nabla \cdot (\boldsymbol{T}(h) \cdot \nabla h) + q'' = n_e\frac{\partial h}{\partial t} = S_y\frac{\partial h}{\partial t}$$

terms are in m/s

$$\frac{\partial}{\partial x_i}\left(T_{ij}\frac{\partial h}{\partial x_j}\right) + q''_i = n_e\frac{\partial h}{\partial t} = S_y\frac{\partial h}{\partial t}$$

in indicial notation

$$\frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right) + q'' = S\frac{\partial h}{\partial t}$$

principal anisotropy directions aligned with the selected coordinate system

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