The Continuation Power Flow: A Tool for Parametric Voltage Stability and Security Analysis

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Abstract—**This paper considers a toll for assessing voltage-related instability problems due to slow load variations, based on power flow formulation. Most conventional power flow tools break down when used to analyze voltage stability problems. This paper deals with a class of methods for solving nonlinear equations known as path-following (or continuation) methods, i.e. with a continuation power flow as a robust power flow method capable of dealing with sometimes difficult mathematical problems encountered during the analysis of voltage problems in power systems. In particular, the paper presents a simplified continuation power flow with the next four basic elements: first order polynomial predictor (secant predictor), physical parameterization (bus, area or total system load as a continuation parameter), an ordinary Newton-Raphson solver based on the constant impedance load model and well-conditioned Jacobian matrix in that case throughout solution process, and appropriate step length control. It is well known that constant power load model is the most conservative in voltage stability analyses, and four slightly different algorithms which are combination of the constant power and constant impedance load models during a continuation process, are investigated and presented in the paper. In addition, the application of the proposed continuation power flow to compute the saddle node bifurcation points and security margins in the load parameter space of an electric power system, with appropriate eigenvalue and eigenvector calculations, is also presented in the paper. All examples, included in the report are based on the CIGRE 31-bus Test System which originates from the CIGRE Task Force that investigated voltage collapse indices, and real-life Bosnian Electric Power System (configuration in 1996.).**

I. INTRODUCTION

Many of the different tools for assessing voltage-related instability problems due to slow load variations are based on power flow formulation. Mathematically, the power flow problems consist of the solution of a large sparse set of non-linear algebraic equations. In general, non-linearity gives rise to complex and unexpected behaviour in power systems. Existence of solutions, convergence of a solver, multiple solutions, bifurcations, linearity, curvature, approximations and reduction methods are some of the issues that are generally associated with power flow problem solving [1]. A particular difficulty being encountered in such researches is that the Jacobian of Newton-Raphson power flow becomes singular at the steady state voltage stability limit. As a consequence, attempts at power flow solutions near the critical point are prone to divergence and errors. For this reason, double precision and anti-divergence algorithms [2] have been used in an attempt to overcome the numerical instability. A class of methods which overcome the singularity in the Jacobian of the power flow are known as path-following or continuation methods. Continuation methods are extremely robust and powerful, and are suitable not only for solving 'difficult' problems, but can also answer a number non-standard questions about power systems. They are particularly well suited to study severe voltage problems. Power flow formulation which is based on continuation methods was named the Continuation Power Flow. The purpose of the continuation power flow is to find a continuum of power flow solutions for a given load and/or generation variations [3]. That has the ability to find a set of solutions from a base case up to, and after the critical point in a single program run. Continuation power flow can be used in a variety of applications [4] such as:

- to analyze voltage problems due to load and/or generation variations,
- to evaluate maximum interchange capability and maximum transmission capability,
- to simulate power system static behaviour due to load and/or generation variations with/without control devices,
- to conduct co-ordination studies of control devices for steady state security assessment,
- to determinate voltage security margins.

Assessing the load margins before instability, in the paper, is developed in the parameter space, i.e. the space of load demands, rather than in the state space. Determination of margins in the parameter space is still a new concept. An iterative procedure for calculating power margins to the saddle node bifurcation was proposed in [6], and proposed continuation power flow application in the procedure was presented in the paper.

II. CONTINUATION POWER FLOWS

A continuation power flow employs a continuation method to find the solution path of a set of power flow equations. Effective continuation method, and consequently continuation power flow formulation, solves the problem via four basic elements [1]:

- Predictor. Its purpose is to find an approximation for the next solution. ODE (ordinary differential equation) or polynomial extrapolation. Usually first-order ODE or tangent predictor and first-order polynomial or secant predictor.
- Parameterization. Mathematical way of identifying each solution on the solution curve. Three basic types: parameterization by adding an equation, arclength and local parameterization. Each parameterization augments the system of non-linear equations and make them regular at the critical point.
- Corrector. Usually, application of Newton method to the augmented system of equations. Structure of augmented Jacobian depends on the chosen parameterization.
- Step length control. Can be done by optimal fixed step length, which depends on investigated problem ,or by adaptive step length control.

Power flow equations have to be reformulated to include a continuation (varying) parameter. It is desirable that this parameter has physical meaning. In the case of power flow problem, the parameter of interest depends on the study of interest. Typical natural parameters of interest include the following:

- the total system demand,
- the demand at a given bus or within a given area,
- the amount of power transfer between two areas or between two buses,
- some other parameter such as the impedance of a line, etc.

The results of these studies is often a set of curves illustrating the behaviour of one or more system variables as a function of parameter. Considering of the demand at a single bus as a varying parameter is not realistic, but can be used to detect weak buses and to express system robustness. Using of the total system demand at several buses is more realistic (load increase can be based on the existing load profile or a load forecast). Let, an electric power system be modelled by the power flow equations,

$$f(x, \lambda) = 0 \tag{1}$$

where x is the vector of state variables (voltages and angles at load buses, angles at generator buses) and λ is a vector of parameters. In the continuation power flows the objective is to study how solutions of equations (1) vary as parameters are changed. If one parameter is varied, then a curve results, if two parameters are allowed to change, then a surface is obtained, variation of more parameters results in a higher dimensional hypersurface. To have an applicable continuation power flow one has to take care about three, among many, very important things: physical meaning of parameter λ (usually total system load demand), load modelling and generation rescheduling. Load modelling includes the constant power and non-linear load models. Non-linear load model which includes a voltage dependency, generally has the form,

$$P = P_0\left(\frac{V}{V_0}\right)^{\alpha} \quad ; \quad Q = Q_0\left(\frac{V}{V_0}\right)^{\beta} \tag{2}$$

where, P_0 and Q_0 are initial active and reactive power consumed by the load, α and β are voltage dependency coefficients and V_0 initial voltage at the bus. In order to use this load model in a continuation power flow, the load parameter and load increase multipliers must be somehow added so that various load changes can be simulated. This can be done as follows,

$$P_{Li} = (1 + k_{Li}\lambda)P_{Li0}\left(\frac{V_i}{V_{i0}}\right)^{\alpha i} \tag{3}$$

$$Q_{Li} = (1 + k_{Li}\lambda)Q_{Li0}\left(\frac{V_i}{V_{i0}}\right)^{\beta i} \tag{4}$$

where P_{Li0} and Q_{Li0} are original loads at bus i, active and reactive respectively, k_{Li} multipliers to designate the rate of load change at bus i. Parameter λ in this formulation corresponds to the quantity of connected load (λ is the connection, not the load parameter). The load change can be expressed as,

$$\Delta P_{total} = \sum_{i=1}^{n} P_{Li} - P_{total0} \tag{5}$$

where P_{total} is the total active power load at any given instant and P_{total0} is the total active power load in the base case. If each generator is made to take up a fraction k_{Gi} of the load change, generation at a given bus i is given by,

$$P_{Gi} = P_{Gi0} + k_{Gi}\,\Delta P_{total} \tag{6}$$

When the newly formulated load and generation terms are inserted in the general form of power flow equations the result is,

$$\Delta P_i = P_{Gi0} + k_{Gi}\left(\sum_{i=1}^{n} P_{Li} - P_{total0}\right) - P_{Li} - P_{Ti} \tag{7}$$

$$\Delta Q_i = Q_{Gi0} - Q_{Li} - Q_{Ti} \tag{8}$$

The set of equations that describe the entire system are made up of a combination of these two general equations. A locally parameterized continuation power flow has been proposed in [5] and only constant power load model was considered. The method is based on the tangent predictor. In [3] the method has been extended to include non-linear load models, where load models have included voltage dependent terms. A continuation power flow, named as CPFLOW, has been proposed in [4]. The distinctive feature of the method is combination of the tangent and secant predictor and so-called pseudo arclength parameterization.

III. THE PROPOSED CONTINUATION POWER FLOW

The main idea is to simplify matters at the expense of some, but acceptable accuracy. This simplification aims at using an ordinary Newton solver throughout the solution process (the use of a Newton solver is essential for the 'lower' portion of the PV curve, as the assumptions generally made for fast decoupled solutions no longer hold), and at keeping the λ as the continuation parameter from the beginning to the end of solution process. In the case of λ as the varying parameter, coincide singularity of the augmented and non-augmented (conventional) power flow Jacobian. To avoid the problems with Jacobian singularity, in the vicinity of the 'nose' of PV curve, it is necessary to ensure well-conditioning of the conventional power flow Jacobian. It is well known that in case of constant impedance load models, conventional power flow Jacobian is well-conditioned, even in the vicinity of the PV curve 'nose', so using of the constant impedance load model in the corrector of the continuation power flow is the first simplification that is included in the proposed algorithm. Power loads, at all buses which participate in load increase, are modelled as follows,

$$P_{Li} = (1 + k_{Li}\lambda) P_{Li0} \left(\frac{V_i}{V_{i0}}\right)^2 \tag{9}$$

$$Q_{Li} = (1 + k_{Li}\lambda) Q_{Li0} \left(\frac{V_i}{V_{i0}}\right)^2 \tag{10}$$

The same exponents in (9,10) preserve constant power factor throughout the continuation process. Fig. 1. illustrates the continuation power flow for the case of constant impedance load model. The constant power load model, in comparison with voltage dependent load model, is more conservative in voltage stability analysis [3], so an additional correction must be somehow added in the continuation process to allow simulation of constant power increase in the simplified algorithm (also, most of the control devices in a power system are trying to restore the voltages to the pre-disturbance level

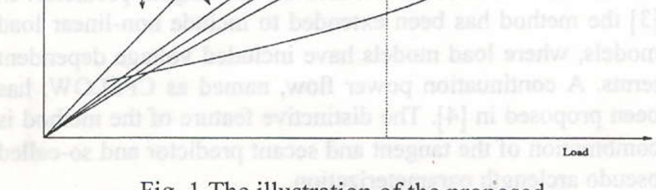

Fig. 1.The illustration of the proposed continuation power flow

and thereby cause voltage dependent loads to act as voltage independent in static conditions). Of course, some 'hysteresis' and errors cannot be avoided. Let (9,10) be rewritten as,

$$\left(\hat{P}_{Li}\right)^{j+1} = (1 + k_{Li}\hat{\lambda}_{j+1}) P'_{Li} \left(\hat{V}_i^{j+1}\right)^2 \tag{11}$$

$$\left(\hat{Q}_{Li}\right)^{j+1} = \left(\hat{P}_{Li}\right)^{j+1} \tan\psi_i \tag{12}$$

where $(\hat{P}_{Li})^{j+1}$, $(\hat{Q}_{Li})^{j+1}$, $\hat{V}_i^{j+1}$, and $\hat{\lambda}_{j+1}$ are predicted values of the active and reactive loads, voltage and parameter λ at the i-th bus in $j+1$ continuation step. The additional correction is made over the term P_{Li}, as

$$P'_{Li} = \frac{P^j_{Li}}{\left(\hat{V}_i^{j+1}\right)^2} \tag{13}$$

where P_{Li}^j is the corrector value from the j-th continuation step and $\hat{V}_i^{j+1}$ is predicted value for bus voltage in $(j+1)$-th continuation step. The main characteristics of the proposed continuation power flow can be summarized as:

- The secant predictor, bacause of its robustness and efficiency,
- Physical parameterization, i.e. parameterization by adding an equation ($\lambda=\lambda_{j+1}$),
- The corrector is ordinary Newton-Raphson solver with/without the additional correction (13) and withconstant impedance load model,
- Appropriate step length control (optimal fixed or adaptive).

Four slightly different algorithms are invetigated in the paper.

<u>Algorithm I</u>

- the secant predictor,
- physical parameterization (λ as the varying parameter),
- Newton-Raphson corrector with constant impedance load model at each bus which participate in a load increase, and without the additional correction (13),
- appropriate step length control.

<u>Algorithm II</u>

- the secant predictor,
- physical parameterization (λ as the varying parameter),
- Newton-Raphson corrector with constant impedance load model at each bus which participate in a load increase with the corrector (13) on the 'upper' portion of PV curve,
- appropriate step length control.

<u>Algorithm III</u>

- the secant predictor,
- physical parameterization (λ as a varying parameter),
- Newton-Raphson corrector with the constant power load model up to 'instant' where number of iterations exceeds the predetermined value (usually 10 iterations), switch to

the constant impedance load model at all buses which participate in a load increase, further load increase is observed without the correction (13), and on the end switch again to the constant power load model on the 'lower' portion of PV curve,
- appropriate step length control.

Algorithm IV

- the secant predictor,
- physical parameterization (λ as the varying parameter),
- Newton-Raphson corrector with the constant power load model up to 'instant' where number of iterations exceeds the predetermined value, switch to the constant impedance load model at all buses which participate in a load increase, further load increase is observed with the correction (13), and on the end switch again to the constant power load model on the 'lower' portion of PV curve,
- appropriate step length control.

Selection of algorithm depends on the application area. The Algorithm I is quite applicable in the case of load increase at a single bus, so can be used to detect weak buses and to express system robustness. In this algorithm λ is the connection parameter. The Algorithms II, III and IV are generally applicable in any case of load increase scenario, but slight differences in the algorithms result in slight differences in the results. Parameter λ, in these algorithms is the load parameter. Steady state voltage stability is closely related to fold or saddle node type of bifurcation [3], and calculation of the exact bifurcation point or the points which are very close to that point gives the useful information in analysis of voltage instability related problems [6]. How one can recognize, from the data obtained from a continuation power flow, whether a bifurcation is close? The required information is provided by a test function [1], which has to be evaluated during the continuation process. A bifurcation is indicated by a zero of the test function, that is

$$\tau(x_*, \lambda_*) = 0 \tag{14}$$

It is usually unlikely to hit the exact bifurcation point, so bifurcation point passing is checked, by the criterion

$$\tau(x_i - 1, \lambda_{i-1})\tau(x_i, \lambda_i)\langle 0 \tag{15}$$

A natural choice for τ is the maximum of all real parts of the eigenvalues of the power flow Jacobian,

$$\tau = max\{\alpha_1, \ldots, \alpha_n\} \tag{16}$$

In the proposed algorithms the test function is (16) which corresponds to Jacobian matrix in different continuation steps, calculated (in 'background') with all voltage dependency exponents equal to zero. Also, the proposed test function provides the criterion for switching from the constant impedance to constant power load model on the 'lower' portion of PV curve, in the algorithms III and IV. The criterion is a few (usually from 3 to 5) iterations after the test function (16) has passed the zero value. In the proposed continuation power flow, the inverse and simultaneous iterations algorithms are combined to compute the dominant real eigenvalue and the left eigenvector relative to that eigenvalue [7].

IV. APPLICATION TO THE TEST CASES

In order to demonstrate the proposed continuation power flow, different tests were performed on the CIGRE-31 bus test system and power system of Bosnian Electric Power Utility. CIGRE test system originates from the CIGRE Task Force that investigated voltage collapse indices. Fig. 2. presents one-line diagram of the CIGRE test system and Fig. 3. of the Bosnian Power System. All tests are based on simultaneous load increase at several system buses (N201, N202, N203, N204, N205, N206 and N207 for CIGRE test system and Centar, Djurdjevik, HAK, Lukavac, Dubrave, Banovici and Kladanj for the Bosnian Power System) and with assumption that all generators serve the load increase by equal participation factors of each generator. Fig. 4., 5. and 6. present the results of algorithm II application (fixed step size equal to 0.02) on the CIGRE test system. Fig. 4. presents voltages (in pu) at buses N203, N204, and N207 as a function of the total load increase, Fig. 5. load demands at the same buses as a function of the total load increase (the 'hysteresis' is particularly stressed at the bus N207 which is critical one for the simulated pattern of load increase (all buses participate in the load increase with equal participation coefficients.

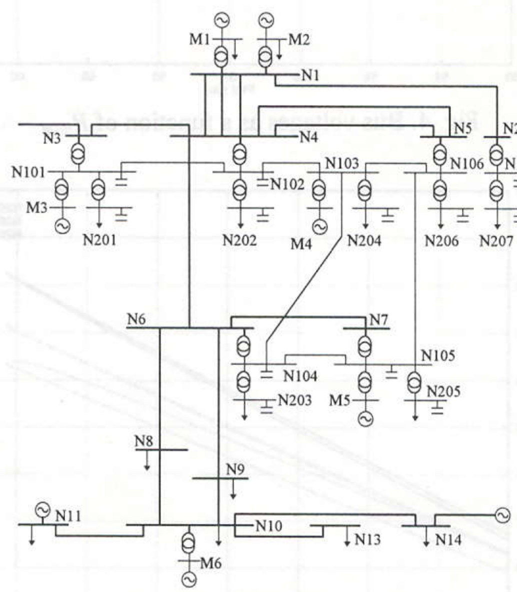

Fig. 2. CIGRE test system

Fig. 6. presents the influence of the chosen step size to the continuation process. With bigger step size number of steps is decerased, number of corrector iterations is increased and, because of more stressed load power voltage dependency, maximum power load is increased (in this case by 0.246 pu). This fact points out that one should be careful in the fixed

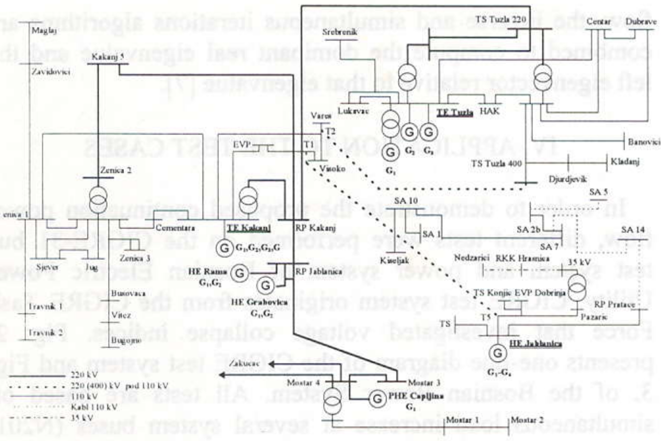

Fig. 3. Bosnian Power System

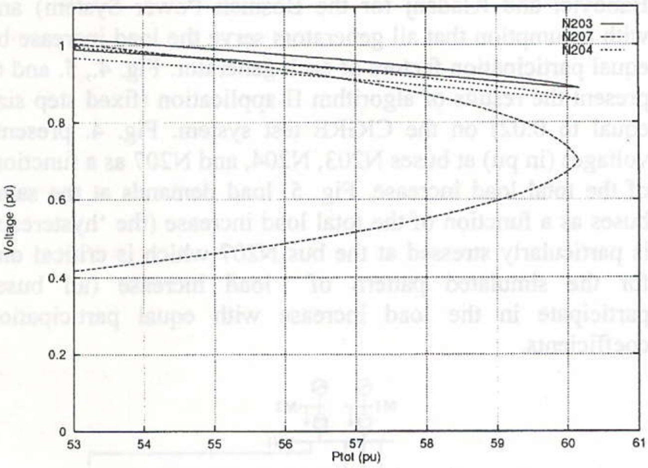

Fig. 4. Bus voltages as a function of P_{total}

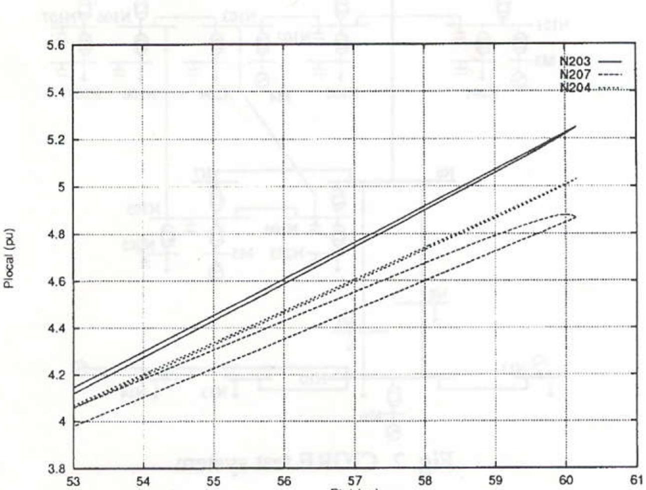

Fig.5. Bus load demands as a function of P_{total}

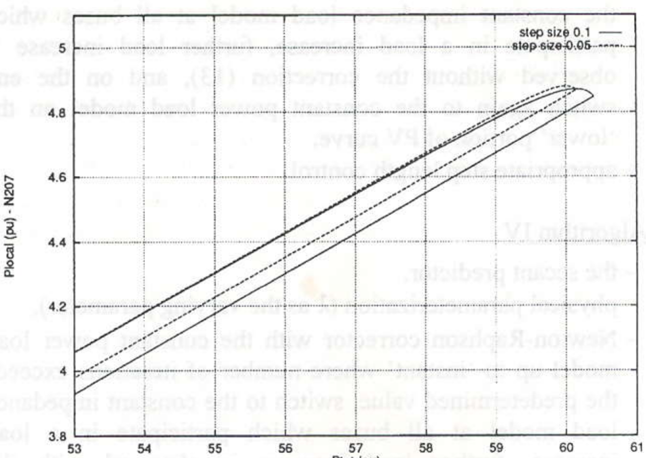

Fig. 6. Influence of different step sizes

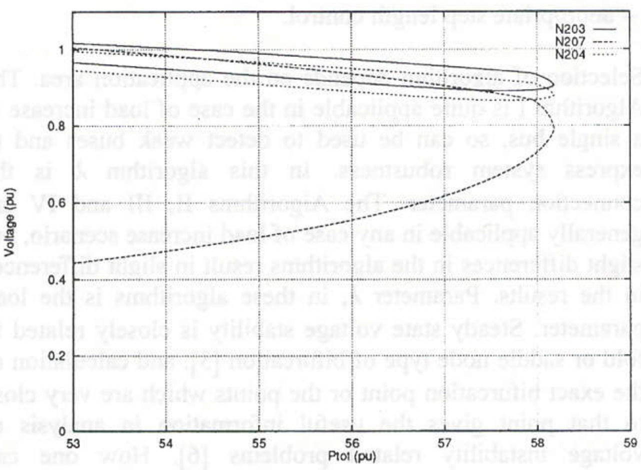

Fig. 7. Bus voltages as a function of P_{total} (generator reactive power limitation)

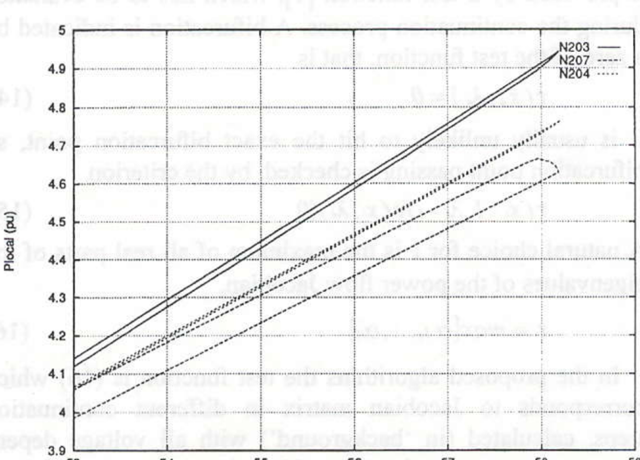

Fig. 8. Bus load demand as a function of P_{total} (generator reactive power limitation)

step size choosing, in this algorithm. Fig. 7. and 8. illustrate application of the algorithm IV on the same system in the case when upper reactive generation limits are imposed to system generators (699 MVAr at buses M1 and M2, 799 MVAr at buses M3 and M4, 899 MVAr at buses M5 and M6). Each sharp corner on the PV curve correspond to reaching of upper reactive generation of one generator (in this case generators at buses M1, M2 and M6 reach its limit). Algorithm IV, generally, has less stressed voltage dependency, in addititon in the case of reactive generation limitation voltage magnitudes are generally higher in vicinity of turning point, so in this case algorithm IV has excelent characteristics. The results of the proposed continuation power flow (algorithm IV) application in the iterative procedure to compute a closest saddle node bifurcation point (adopted from [6]) on the Bosnian Power System, are given in table Table I.

TABLE I
THE RESULTS OF ALGORITHM IV APPLICATION IN THE ITERATIVE PROCEDURE FROM [6]

Pmax	n	Dju	HAK	Dub	Ban	Luk	Cen	Kla
585.6	n0	0.137	0.128	0.127	0.118	0.067	0.268	0.128
547.5	n1	0.043	0.045	0.042	0.035	0.026	0.798	0.034
537.8	n2	0.031	0.037	0.036	0.027	0.022	0.820	0.028
537.5	n3	0.026	0.031	0.029	0.024	0.020	0.848	0.028

The initial ray direction corresponds to the algebraically largest eigenvalue of Jacobian matrix at base case (P_{total}=400 MW). Only three iterations were enough to reach the closest saddle node bifurcation point. The load power margin in the case is 137 MW. The inspection of $n_3=n_*$ shows that Lukavac is the bus most influental on the load power margin and that is load which can most effectively be shed.

V. CONCLUSIONS

The continuation power flow based on secant predictor and using an ordinary Newton-Raphson solver with constant impedance or combination of it with constant power load model, has presented in this paper. Four different algorithms have been investigated, and application of two of them on CIGR test system and Bosnian Power System illustrated. Generally, the algorithm IV has the best characterisitcs, because load dependency is not too stressed and acceptable results can be obtained. Depending on application area, three other algorithms can be also applied. Application of the algorithm IV in the iterative procedure to compute the closest saddle node bifurcation point, on the Bosnian Power System, resulted in the satisfactory results. All tests were performed with fixed step size during the continuation process, so further developments must include appropriate step size control. The future developments, also must include investigations of a good strategy for sharing increase in load among generators. This investigations can be directed to inclusion of predicted (extrapolated) power loses and a real generators participation patern in generation rescheduling. The proper implementation of the proposed methodology with further developments, mentioned above, can result in a continuation power flow that is not substantially slower than conventional power flows for well behaved problems.

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