Abstract This paper shows how an overload followed by generator redispach may be incorporated into the set of power flow equations. A pair of generators is chosen in order to remove the overload according to one of three approaches: most effective, greatest margin and redispach. The main idea of the paper is to illustrate the interaction between various kinds of limits and their respective corrective actions. The methodology proposed here has some important characteristics not addressed before in the literature. The most important is the precise handling of flow limits by explicit inclusion of the limit as an additional equation. The impact of this change on the system Jacobian matrix is illustrated. The results illustrated with the help of the 118 bus-system with all limits considered.

1.0 Introduction – Operating a power system in a stable and secure manner requires adequate information to be available to the operator. Under normal conditions, the system is supposed to be working within its limits. In addition, operators need to take into consideration several other factors, for example:

Tripping of transmission lines and generators – If any kind of contingency happens, the dynamic response of the generators produce an angle swing until the frequency stabilizes [1,2]. Depending on the impact of the contingency, the system may become unstable. Therefore, it is important to determine the contingencies most likely to drive the system to transient instability. The analysis of such phenomenon requires an algebraic-differential set of equations, and will not be studied in this paper.

Voltage Collapse – This problem is relatively new in the literature, dating around 1980. However, because of several occurrences around the world, it became a matter of concern of many researchers. Unlike the transient instability problem, a power flow model may often be employed, provided some assumptions are made. This model enables one to determine the system load margin [3, 4], the critical bus[5] and control actions to avoid the problem.

One of the most powerful tools employed for voltage collapse analysis is the continuation method [6]. Such a method identifies accurately the voltage collapse point and traces the voltage collapse path as well. Another important aspect of continuation methods is with regards to critical bus identification. This information is obtained by tangent vector calculations [3]. Since the bifurcation diagram is traced step by step, considering reactive power limits is not a problem. The computational time associated with its calculation may be a barrier, but the accurate results obtained render this methodology as a benchmark for other methods. The continuation method for voltage collapse analysis is used to determine the bifurcation point associated with a base case. It can also be used to determine the bifurcation points associated with to contingencies, where the critical lines most likely to drive the system to voltage collapse have been identified [7, 8, 9]. Contingency screening is not discussed in this paper. The application described in this paper represents a unique new type of application. In addition to voltage collapse occurrences, the method also handles overloads. When an overload occurs, it is identified and removed, as shown next.

Overloads – Voltage collapse analysis requires that other kinds of limits be taken into account. Checking generator limits is easy to consider, and this is done in most power flow programs. Likewise, reactive generator limits are generally handled by changing the generator bus type to PQ type. This feature is also easily handled by most power flow program. It is also important to check for transmission lines overloads. Controlling and removing overloads is not trivial. This kind of analysis has not been addressed in the literature in a general manner. Control and/or removal of overload conditions can take place many ways, including the possible removal of the overloaded line.

The aim of this paper is to remove overloads by means of generator redispach. The following steps are used:
- With the help of a continuation power flow, trace the system PV curve as a function of a defined load increase or power transfer direction.
- If an overload is identified, choose the generators most likely to respond to relieve the overload. Take control actions to remove the overload with the help of these generators. The loading at the transmission line violated is kept at its upper limit.
- Keep tracing the PV curve up to another overload is identified or the system reaches voltage collapse.

The methodology above will be carefully detailed in the paper, and the results obtained using the IEEE 118-bus system will be analyzed. The work is organized as follows:
Section 2 describes briefly the continuation power flow. Section 3 shows how the inclusion of overload in the set of equations is handled. Also, it introduces the methodology used to pick the generators most likely to act on overload removal. This section also shows the stoppage criterion used in the continuation power flow. Section 4 presents the test results and Section 5 discusses conclusions of this work.

2.0 The Continuation Method

This section describes how continuation methods work and how overloads may be explicitly included in the set of continuation equations.

**Continuation method principle**

Continuation methods may be used to trace the path of a power system from a stable equilibrium point up to a bifurcation point is reached [6]. Such a methodology is based on the following system model:

\[ f(x, \lambda) = 0 \]  

where \( x \) represents the state variables and \( \lambda \) is a system parameter, used to drive a system from one equilibrium point to another. This type of model has been employed for numerous voltage collapse studies, with \( \lambda \) been considered as the system load/generation increase factor or power transfer level. Two steps move the system along the bifurcation path:

1- **Predictor step**, used to indicate a direction to move.

Tangent vector may be used for this purpose, and is given by:

\[ TV = \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix} \frac{1}{\Delta \lambda} = J^{-1} \begin{bmatrix} P_o \\ Q_o \end{bmatrix} \]  

where \( J \) is the load flow Jacobian, \( \theta \) and \( V \) are the state variables (angle phase and voltage magnitude, respectively), and \( P_o \) and \( Q_o \) are the net active and reactive powers connected to each bus. \( TV \) is the shortcut for tangent vector. The predictor step is then given by:

\[ \Delta \lambda = 1/\|TV\| \]

where \( \|\| \) stands for tangent vector norm. From the expression above, the steeper the curve, the smaller the predictor step, and vice versa. The method takes bigger steps when the system is far way from the bifurcation point, and smaller steps as the bifurcation is approached. The actual operating point is obtained with the help of the corrector stage, described next:

2- **Corrector step**, in general obtained with the inclusion of an extra equation. Such an equation comes from the fact that the predictor and corrector vectors are perpendicular to each other. However, as shown in [5], if the predictor step is given as an initial guess for a power flow program, it converges rapidly for a feasible operating point.

3.0- Special Continuation Process Features

The methodology described in Section 2 enables one to trace PV curves associated with any bus of interest. In this work, some special features have been incorporated to the method. These features are described below.

3.1- The problem of including overload

As the aim of this paper is to remove overload, it will be considered explicitly in the set of equations. Such a consideration is reflected in the inclusion of a new row \( I \) and a new column \( (Gen) \) in the system Jacobian, as shown in equation (3). Sub-matrices \( H \), \( N \), \( M \), \( L \) and \( G_m \) represent the conventional power flow Jacobian

\[ \begin{bmatrix} \Delta P \\ \Delta Q \\ \Delta \lambda \end{bmatrix} = \begin{bmatrix} H & N & G_m \\ M & L & 0 \\ I & 0 \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \\ \Delta R \end{bmatrix} \]  

The row vector \( I \) contains the partial derivatives of the active power equation associated with the transmission line overloaded with respect to the system state variables. It is a row with no more than four nonzero elements. As soon as an overload is identified, the transmission line loading is kept at its upper limit by the explicit inclusion of this equation. This new equation becomes a permanent addition to the set of equations for the system, resulting on the new Jacobian row \( I \).

In order to be able to regulate this flow, redispacth is performed. Redispatch is modeled by the inclusion of a new column \( (Gen) \) associated with the generators chosen to remove the overload. The values associated with these generators are set to 1 (generator assigned to increase generation) and -1 (generator assigned to decrease generation). The rest of vector \( Gen \) equals zero. As one generator is assigned to increase generation by an (unspecified) amount \( \Delta R \), another generation is assigned to reduce its generation by the same \( \Delta R \) amount. In this paper, only two generators are redispached. However, extending the results to a large number of generators is straightforward. Likewise, the method may handle more than one overload at the same time. For each overload, a new row and a new column should be added. As is the case for row \( I \), column \( G_m \) becomes a permanent addition to the set of equations.

This new augmented set of equations is solved by Newton-Raphson method. Its output consists of the regular state variables (phase angles at PV and PQ buses and voltage levels at PQ buses) and the active power deviation at the machines chosen to remove the overload \( \Delta P \). Notice, from equation (3), that the overload is not an explicit function of the redispatch. At every iteration, the loading in the
transmission line violated must be calculated, and the process converges for a value of redistancing associated with an upper loading limit at the transmission line overloaded. Hence, as an overload is identified, its upper loading is kept constant during the system load increase, and the set of equations (3) is always considered.

3.2 - The choice of the generators

The tangent vector calculated in equation (2) is used here as a tool for choosing the generators most likely to reduce the overload. Such an approach is an extension of the idea proposed in [10], where the aim is to carry out a loss sensitivity analysis. The outcome of the method in [10] is the list of buses most likely to experiment shunt capacitance compensation in order to reduce the system loss. The idea is easily adapted in this work to identify the generators most effective to remove overloads. The approach is based on the information provided by tangent (or sensitivity) vector, i.e., how state variables vary as a function of system parameter. The active power at a certain transmission line \( k \) is given by:

\[
P = V_i V_j (B_{ij} \sin(\delta_{ij}) + G_i \cos(\delta_{ij}))
\]

where

\( V_i \) and \( V_j \) are the voltage level at ends \( (i) \) and \( (j) \) of transmission line \( k \).

\( G_i \) is the transmission line \( k \) conductance.

\( B_{ij} \) is the transmission line \( k \) suscetance.

\( \delta_{ij} \) represents the phase angle between buses \( i \) and \( j \).

If equation (4) is derived in relation to system parameter \( \lambda \), it is obtained:

\[
\frac{dP}{d\lambda} = \frac{dV_i}{d\lambda} \frac{P}{V_i} + \frac{dV_j}{d\lambda} \frac{P}{V_j} + V_i V_j (B_{ij} \cos(\delta_{ij}))
\]

\[
(-\frac{d\theta_i}{d\lambda} - \frac{d\theta_j}{d\lambda}) + G_i \cos(\delta_{ij}) (-\frac{d\theta_j}{d\lambda} - \frac{d\theta_j}{d\lambda})
\]

Equation (5) shows how active power varies as a function of system parameter. All the partial derivatives of equation (5) consist of tangent vector components, already calculated from equation (2). Therefore, computing equation (3) is not time consuming.

Assume in equation (2) that the right-hand side is slightly perturbed through a generation increase at a generic generation “\( g \)”. The new tangent vector may be obtained with no need of calculating the new operating point. If equation (5) is calculated, the active power variation at transmission line \( k \) as a function of parameter \( \lambda \) (generation increase at bus “\( g \)” ) is known. Taking “\( g \)” as all system generators, one by one, computing equations (2) and (5) indicate the generators whose redistancing reduce the overload the most. In this process, a power flow program is executed only for the base case.

The criterion above is technical. Two other criteria are also employed in the paper:

- Two generators are simply chosen by the user. This option enables one to reproduce a real situation, when the operator is assigned to pick to generators.

- After the sensitivity approach described above is executed, the “cheapest” generator pair is chosen. This option will be tested with the other options previously mentioned.

3.3 - The stopping criteria

One definition of voltage collapse and maximum loadability is when the Jacobian matrix becomes singular. The method by [11] directly computed this point by appending an explicit Jacobian singularity condition to the continuation problem. Such an approach has excellent convergence characteristics, but only in the immediate vicinity of the nose point. An alternative that is often used in continuation methods is to swap of variables. As the Jacobian becomes near-singular, the loadability parameter \( \lambda \) ceases to be an independent variable. The continuation parameter becomes something else, usually a voltage. Furthermore, there is never a need to solve a set of equations larger than the optimal set.

Reaching the nose point is often impractical unless voltage collapse occurs at a high voltage. In most systems, there are many practical and operational reasons why a simple constraint on the voltage magnitude is a more significant and limiting constraint. From the perspective of this paper, a constraint on voltage limits can be handled the same way as a flow limit: by the inclusion of an extra equation for \( |V| \) related to the redispatch of generator pairs.

4.0 - Test Results

This section presents the tests carried out when the methodologies previously discussed are used. These methods have been implemented in MATLAB. The IEEE 118-bus system is used for testing. Active and reactive power limits are considered. The loading upper limit at each transmission line is randomly chosen. In order to show the multi-purpose capability of the program used here, several cases are considered. Five tests are done:

Voltage collapse problems

This test provides the load margin when the loading upper limit in all transmission lines is neglected. The
reactive power generation limits, in this case, are multiplied by 0.6. Figure 1 shows the PV curves associated with several load buses. The bifurcation point occurs at a total load amount of 8618 MW. In this case, the stopping criterion flagged to stall the process is the smallest eigenvalue calculation.

![Figure 1 - PV curves for voltage collapse (eigenvalue case)](image)

**Overload removed**

This time, at each equilibrium point, the loading at every transmission line is checked, whereas the reactive power limits are kept as in the previous case. At the load amount of 8504.78 MW an overload at the transmission line 43-44 is identified. The procedures described in Section 3.2 are then employed.

By inspection of the system topology, generator 54 is chosen to increase its generation, and generator 40 is assigned to reduce its generation. (These generators are the closest ones to ending nodes of the overloaded line).

Next, sensitivity analysis is carried out. As expected, because of the criterion choice used above, the same generators are chosen.

Finally, after executing the sensitivity analysis, the generation incremental redispacth costs are considered. In this example, this did not change the choice of generators.

After the identification of these generators, the following approach is used:

- Redispacth takes place, and a new operating point is evaluated.
- From this operating point, the system parameter is increased.
- The loading at transmission line 43-44 is kept in its upper limit as the load is increased. It is done through the set of equations (3).

Figure 2 shows this result. Because of the redispacth, the system may experiment further load increase.

![Figure 2 - PV curve for overload removed](image)

**Multiple overloads**

In this case, an overload is identified and removed, as in the previous case. However, during the system load increase, another overload is identified, and the program stops tracing the PV curve. At a level of 6223.8 MW, the transmission line 49-51 overloads. Redispacth successfully takes place, and load is further increased. However, at a load level of 6500MW transmission line 43-44 also overloads. At this point we stop. Figure 3 shows the PV curve of Buses 95 (dotted one) and 53, represented by the solid line.

![Figure 3 - PV curve for the case of multiple overloads](image)
Overload not removable

One problem when dealing with overload removal is the lack of generators available to act. This can happen for two reasons:

- There is no generator available, since all the machines have already reached their P limit (active power generation limit).
- The sensitivity analysis indicates that the generators still available are electrically far away from the transmission line overloaded, and the redispacth is not effective.

To illustrate this case, consider that the transmission line 35-36 is overloaded at the load amount of 7843.51MW. The sensitivity analysis provides generators 40 and 31 to increase and decrease their active power generation, respectively. The effect, however, is trivial. This is because the overload has not been removed (equation (3) does not converge), and the problem has no solution. The PV curve depicted in Figure 4 refers to Bus 76.

![Figure 4 - PV curve when overload is not removed](image)

Voltage limits

This section presents some tests when the V limit stoppage criterion takes place. As already pointed out, the process may be stalled for two stoppage criteria: 1) A vanishing eigenvalue is identified (Figure 1) and a V limit is encountered. Both features are simultaneously enabled in the program. The case considered in this subsection is obtained when no transmission line loading is monitored and the reactive power limits are multiplied by 5.

Figure 5 shows the results. In contrast to the results in Figure 1, a larger load margin is obtained. This is expected, since a larger amount of reactive power is available to maintain the system voltage profile. The solid line in Figure 5 represents Bus 95, whereas the dotted line shows the PV curve of Bus 53.

![Figure 5 - PV curves for voltage collapse (V limit case)](image)

5.0 Conclusions

This paper has dealt with some special features associated with continuation power flow. Some original contributions have been proposed, rendering the tool as a powerful and accurate helper for operating a power system within its security constraints. The program employed handles a range of different problems, like voltage collapse and overload in transmission lines. In order to trace the stable operating points of a PV curve, the continuation power flow developed for this work incorporates two stopping criteria, based on the vanishing eigenvalue and low voltage limits. As for the overload problem, the operator is allowed to identify the most effective pair of generators according to three different options, based on topology analysis, sensitivity studies or costs consideration. The results carried out with the help of the IEEE 188-bus system indicate that the methodology is effective.

The features presented in the methodology provide important new insights in the area focused in the paper, and including other features, like area interchange control is straightforward.

Acknowledgements

A. C. Zambroni de Souza thanks FINEP/RECOPE (project 0626/96 – SAGE), CNPq, CAPES and FAPEMIG for the financial support.

Mevludin Glavic is the Fulbright Postdoctoral Scholar at University of Wisconsin-Madison, and he acknowledges support from Council of International Exchange of Scholars (CIES).

Fernando Alvarado has been supported in part under NSF grant EEC-9815325 directed by Ian Dobson.
6.0 References


