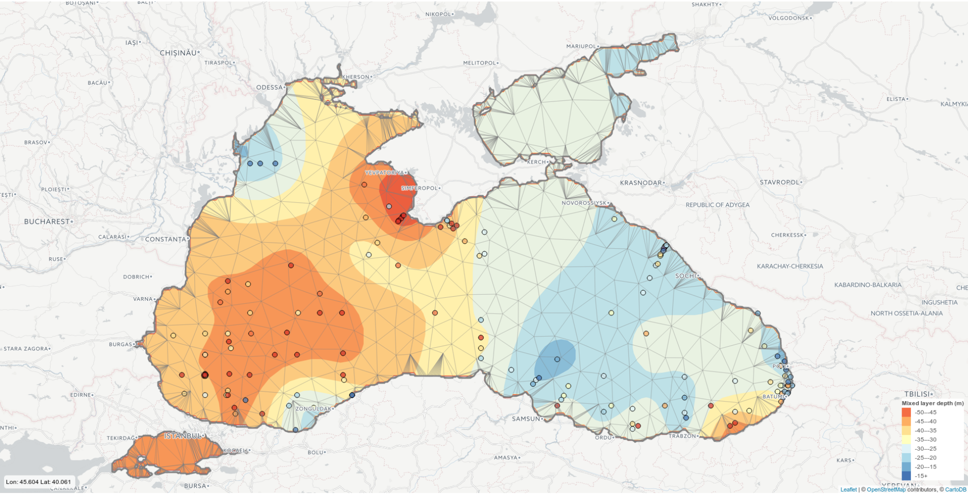
Charles Troupin

Data interpolation using DIVA{nd}

From in situ data to gridded fields



https://github.com/gher-ulg/DIVA

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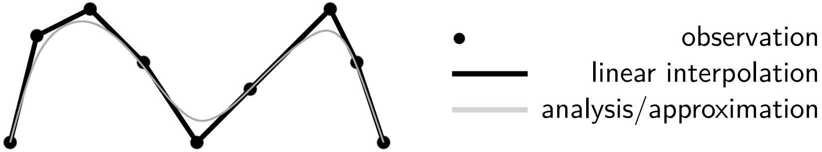
DIVA

Data **I**nterpolating **V**ariational **A**nalysis
(2-dimensional)

Interpolation vs. analysis vs. gridding



Pure interpolation is not always suitable to oceanography

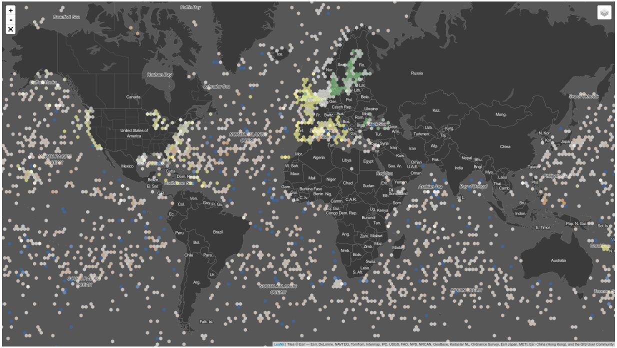
Different types of errors

instrumental: imperfect sensors and instruments

representativeness: what is measured is not what is intended to be analysed

synopticty: measurements assumed to be taken at the same time

Specificities of the ocean



① Big amount of observations

② Not uniformly distributed

③ Different sensors & platforms...

④ Physical boundaries Currents...

Variational method: cost function

Find function φ that minimises over a domain $\mathcal{D}$:

$$J[\varphi] = \sum_{j=1}^{Nd} \mu_j \left[d_j - \varphi(x_j, y_j)\right]^2 + \|\varphi\|^2$$

Variational method: cost function

Find function φ that minimises over a domain $\mathcal{D}$:

$$J[\varphi] = \sum_{\jmath=1}^{Nd} \mu_\jmath \left[d_\jmath - \varphi(x_\jmath, y_\jmath)\right]^2 + \|\varphi\|^2$$

- Proximity to observations

Variational method: cost function

Find function φ that minimises over a domain $\mathcal{D}$:

$$J[\varphi] = \sum_{\jmath=1}^{Nd} \mu_\jmath \left[d_\jmath - \varphi(x_\jmath, y_\jmath)\right]^2 + \|\varphi\|^2$$

- Proximity to observations
- Regularity of the field

Variational method: cost function

Find function φ that minimises over a domain $\mathcal{D}$:

$$J[\varphi] = \sum_{\jmath=1}^{Nd} \mu_\jmath \left[d_\jmath - \varphi(x_\jmath, y_\jmath)\right]^2 + \|\varphi\|^2$$

- Proximity to observations
- Regularity of the field
- Weight on data points

Variational method: cost function

$$\|\varphi\|^2 = \int_D (\alpha_2 \boldsymbol{\nabla}\boldsymbol{\nabla}\varphi : \boldsymbol{\nabla}\boldsymbol{\nabla}\varphi + \alpha_1 \boldsymbol{\nabla}\varphi \cdot \boldsymbol{\nabla}\varphi + \alpha_0 \varphi^2) \, dD$$

Variational method: cost function

$$\|\varphi\|^2 = \int_D \left(\alpha_2 \nabla\nabla\varphi : \nabla\nabla\varphi + \alpha_1 \nabla\varphi \cdot \nabla\varphi + \alpha_0 \varphi^2 \right) dD$$

- penalizes variability

Variational method: cost function

$$\|\varphi\|^2 = \int_D (\alpha_2 \nabla\nabla\varphi : \nabla\nabla\varphi + \alpha_1 \nabla\varphi \cdot \nabla\varphi + \alpha_0 \varphi^2) \, dD$$

- penalizes variability
- penalizes gradients

Variational method: cost function

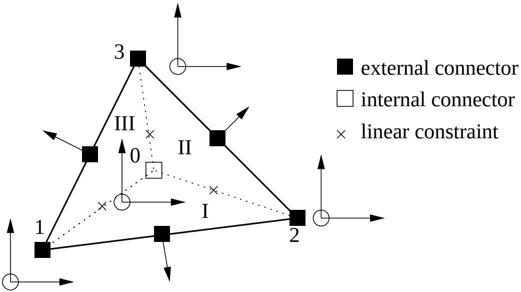
$$\|\varphi\|^2 = \int_D (\alpha_2 \boldsymbol{\nabla\nabla}\varphi : \boldsymbol{\nabla\nabla}\varphi + \alpha_1 \boldsymbol{\nabla}\varphi \cdot \boldsymbol{\nabla}\varphi + \alpha_0 \varphi^2) \, dD$$

- penalizes variability
- penalizes gradients
- penalizes the field itself (anomalies)

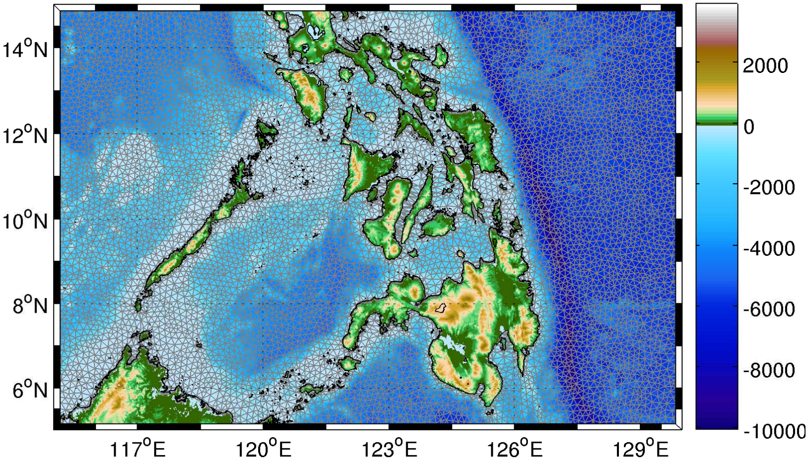
Solver: finite-element mesh

Problem solved in triangular elements:

$$J[\varphi] = \sum_{e=1}^{N_e} J_e(\varphi_e)$$

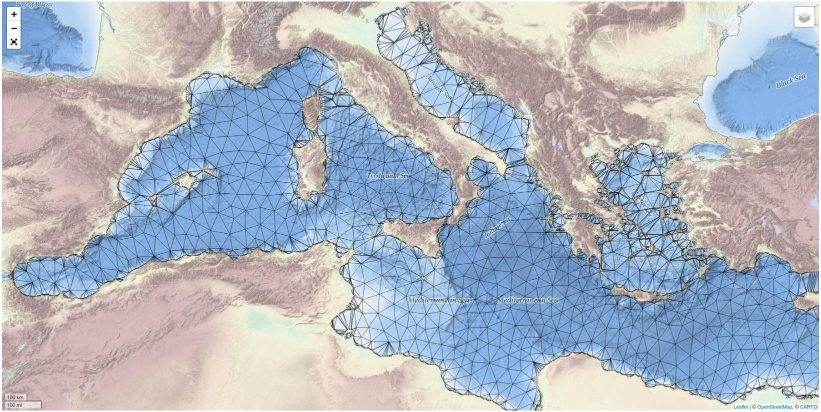


Solver: finite-element mesh



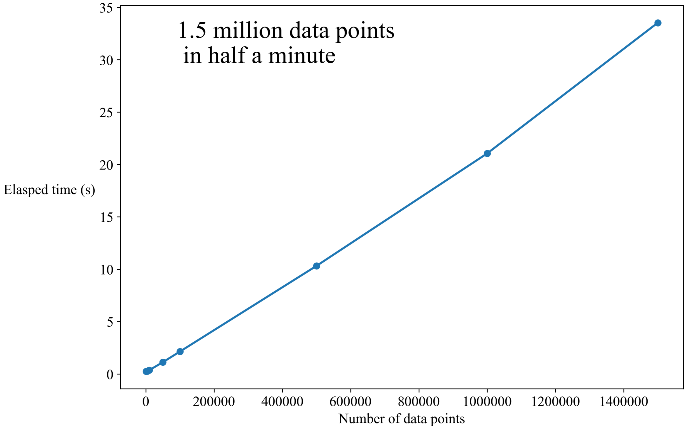
Why use finite-elements?

1. Information cannot cross physical boundaries



Why use finite-elements?

1. Information cannot cross physical boundaries
2. Numerical cost almost independent on the number of data



Analysis parameters: related to the observations

1. Correlation length L: distance over which an observation influences its neighborhood
2. Signal-to-noise ratio λ: relative confidence one can have in the observations

Bonus: can add other constraints to the cost function

Example: advection along currents

$$\tilde{J} = J(\varphi) + \frac{\theta}{U^2 L^2} \int_{\tilde{D}} \left[\mathbf{u} \cdot \tilde{\nabla}\varphi - \frac{\mathcal{A}}{L} \tilde{\nabla} \cdot \tilde{\nabla}\varphi \right]^2 d\tilde{D} \qquad (1)$$

Weaknesses

1. Large number of input files
2. Code to be compiled (Fortran)
3. 3D analysis (lon, lat, depth) done by stacking 2D layers

2 DIVAnd

$n-$dimensional interpolation

DIVAnd: generalised, n-dimensional interpolation

https://www.geosci-model-dev.net/7/225/2014/gmd-7-225-2014.pdf
https://github.com/gher-ulg/divand.jl

divand-1.0: *n*-dimensional variational data analysis for ocean observations

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Languages

DIVA (1991–...) : Fortran + bash

DIVAnd (2013–2016) : GNU Octave or MATLAB

DIVAnd (2016–...): Julia faster, better, stronger

Notebooks: interactive computational environments

Notebooks combine:

1. code fragments that can be executed,
2. text for the description of the application and
3. figures illustrating the data or the results.

In [2]:
```
import numpy as np
import matplotlib.pyplot as plt
```

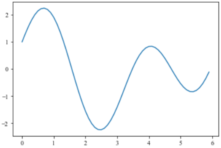
Data

Let's create a simple function.

In [6]:
```
x = np.arange(0, 6, .1)
y = np.cos(x) + 1.5 * np.sin(2 * x)
```

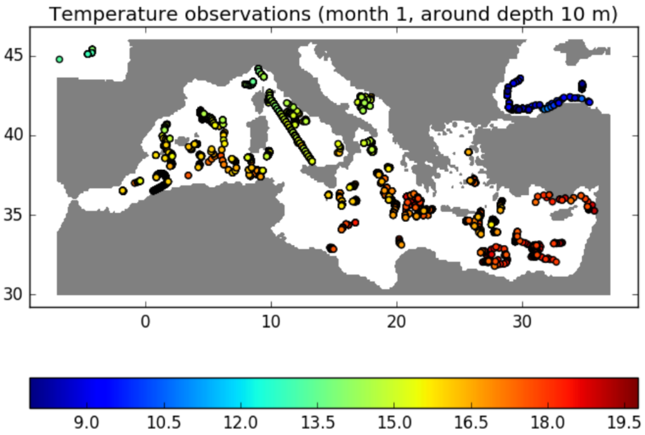
Make a simple plot

In [7]:
```
plt.plot(x, y)
plt.show()
```



DIVAnd in a notebook

In [36]:
```
# sets the correct aspect ratio
gca()[:set_aspect](1/cos(mean(latr) * pi/180))
```



Setup the domain using the bathymetry from the file `bathname`.

Full example on the Adriatic

https://github.com/gher-ulg/Diva-Workshops

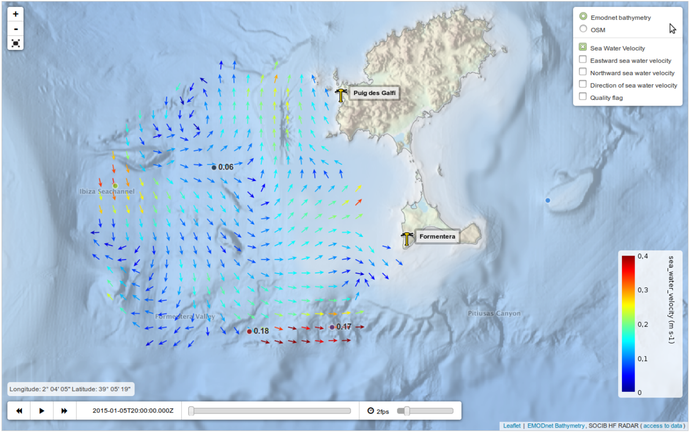
Weaknesses

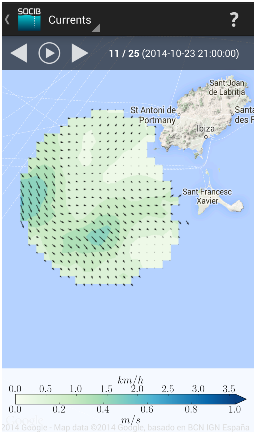
1. Installing and learning a new language

3

HF radar currents

Working on HF radar radial data





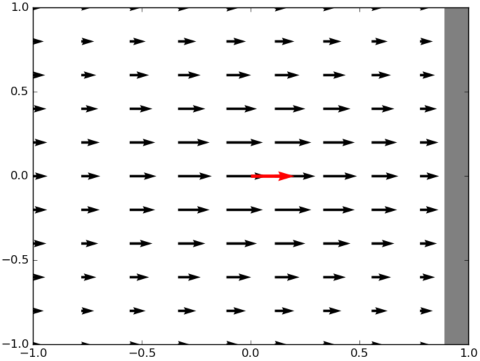
Data: SOCIB HF radar in the Ibiza Channel

http://www.socib.es/

SOCIB Balearic Islands Coastal Observing and Forecasting System

New product: currents

→ hypothetical measurement
→ analyzed field



- Analysis of radial currents to derive total currents
- Observation operator links the radial currents of the different

Formulation: couple velocity components

$$\text{Norm}: \quad |\varphi|^2 = \int_\Omega (\alpha_2 \nabla\nabla\varphi : \nabla\nabla\varphi + \alpha_1 \nabla\varphi \cdot \nabla\varphi + \alpha_0 \varphi^2)\, d\Omega$$

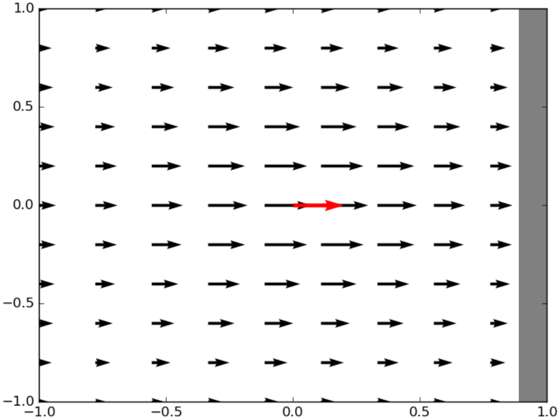
$$\text{Cost function:} \quad J(\vec{u}) = |u|^2 + |v|^2 + \sum_{\imath=1}^{N} \frac{(\vec{u}_\imath \cdot \vec{p}_\imath - u_{r\imath})^2}{\epsilon_\imath^2}$$

$\vec{u} = (u, v)$
$\vec{p}_\imath$ = normalized vector pointing toward the correspond HF radar site of the $\imath$-th radial observation $u_{r\imath}$

Coastline as a boundary condition ($\vec{u} \cdot \vec{n} = 0$)

Cost function (OFF)

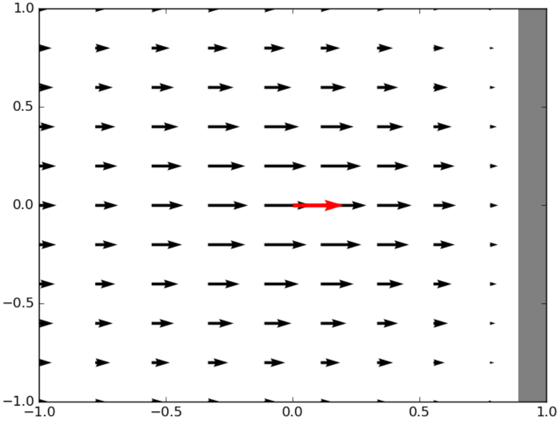
$$J_{bc}(\vec{u}) = \frac{1}{\epsilon_{bc}^2} \int_{\partial\Omega} (\vec{u} \cdot \vec{n})^2 ds$$



Coastline as a boundary condition ($\vec{u} \cdot \vec{n} = 0$)

Cost function (ON)

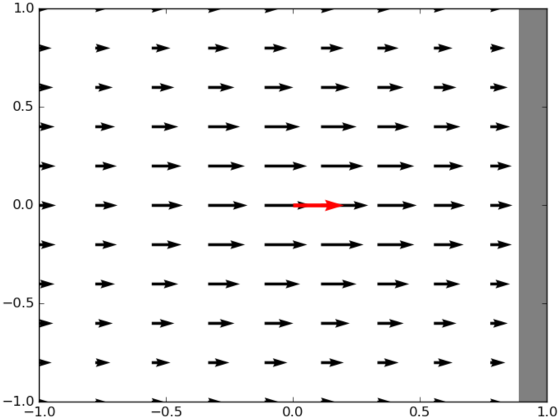
$$J_{\mathrm{bc}}(\vec{u}) = \frac{1}{\epsilon_{\mathrm{bc}}^2} \int_{\partial\Omega} (\vec{u} \cdot \vec{n})^2 ds$$



Low horizontal divergence of currents ($\nabla \cdot \vec{n} = 0$)

Cost function (OFF)

$$J_{\mathrm{d}iv}(\vec{u}) = \frac{1}{\epsilon_{\mathrm{d}iv}^2} \int_\Omega (\vec{\nabla} \cdot \vec{u})^2 dx$$



Low horizontal divergence of currents ($\nabla \cdot \vec{n} = 0$)

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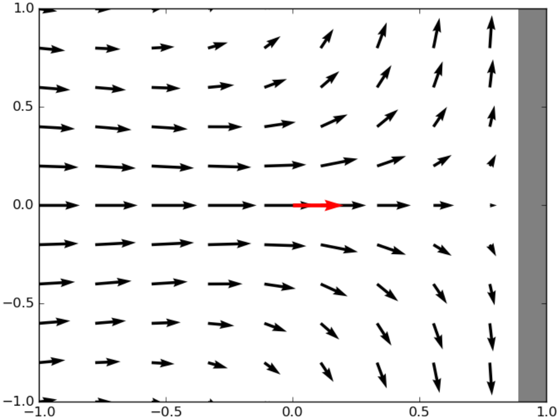

3D analysis: longitude, latitude and time

- Include the data the hour before and after
- Temporal correlation length
- Coriolis force

Coriolis force and geostrophically balanced mean flow

$$\begin{aligned}\frac{\partial u}{\partial t} &= fv - g\frac{\partial \eta}{\partial x} \\ \frac{\partial v}{\partial t} &= -fu - g\frac{\partial \eta}{\partial y}\end{aligned}$$

f $=$ Coriolis frequency
$\eta =$ sea surface elevation

Conclusions

1. Two efficient, open-source tools
2. Interpolation using physical constraints
3. Developments for other types of data

sediments, phytoplankton, ...

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