

Profit Luenberger and Malmquist-Luenberger indexes for multi-activity decision-making units: the case of the star-rated hotel industry in China*

May 8, 2018

Abstract

Due to the vigorous economic development of the tourism industry in China, the number of star-rated hotels has rapidly increased. As a result, techniques to evaluate the performances of star-rated hotels have gained in popularity. In this paper, we develop two indexes for dynamic settings: the profit Luenberger and Malmquist-Luenberger indexes. The distinguishing features of our indexes are three-fold. One, we adopt an economic perspective by considering that hotels are profit maximizers. Two, we model hotels as multi-activity decision-making units by considering that they provide multiple services. Three, our indexes are nonparametric, and work when prices are partially observed. We apply our technique to 30 provinces during 2005-2015. We find that star-rated hotels present better performances over time, but not for every activity. Next, we highlight particular patterns for the provinces. These results are useful for managers to better target their investments, and also for policy makers.

Keywords: destination management; profit efficiency; Luenberger index; Malmquist-Luenberger index; multi-activity; hotel; China.

*We thank the Editor Chris Ryan, the Associate Editor Bob McKercher, and the referees for their comments that have improved the paper substantially.

1 Introduction

The tourism sector is playing an increasing important role in China. As proof, China is ranked in fourth position with regard to both intentional tourism arrivals and receipts in 2016 (Travel and Tourism Economic Impact 2017 report), and the growth rate of that sector was higher than the GDP growth rate from 2005 to 2015 (Chinese Tourism Statistic Yearbook 2017). As such, the tourism sector income has increased over past decades, while its contribution to the GDP has importantly increased over time (the contribution of tourism to GDP was 4.11% in 2005 and 5.99% in 2015). Regarded as an influential component of the tourism sector, the hotel industry can be used to foster tourism, and even local economic development, as it contributes to output, job creation and business opportunity. In 2015, the revenue of Chinese star-rated hotels accounted for nearly 60% of the total hotel revenue, and the growth rate of their revenue was important in past decades (Chinese Tourism Statistic Yearbook 2017). This mainly explains why this type of hotels has attracted the major share of international and domestic investments. Therefore, star-rated hotels represent the main segment of the hotel industry in China, implying that the performances of these hotels can accurately reflect the progress of the Chinese hotel industry, as well as the tourism industry.¹

With the booming of the tourism sector, fierce competition is imposed on the hotel industry. As a result, the evaluation of hotel performances has received important attention in the literature. Among the techniques available to conduct a performance analysis, nonparametric efficiency analysis has gained in popularity for the hotel sector, and for the tourism industry in general.² The popularity of this method could be explained by two main reasons. On the one hand, nonparametric efficiency analysis does not require any functional specification of the production process, but rather reconstructs the production possibilities using the observed data. This is attractive as the production process is, in general, unobserved for the hotels; and imposing assumptions about the production process may have huge impacts on the performance results. On the other hand, the performances are evaluated using efficiency scores that are easy to interpret and to compute. From a practical point of view, efficiency scores can be used to increase profit or reduce costs, and also for strategical or tactical purposes.

The distinguishing features of our efficiency analysis are two-fold. Firstly, we adopt an

¹Note that some authors have argued that, in some cases, star-rated hotel performances do not reflect the progress of the hotel industry. See, for example, Nunez-Serrano, Turrion, and Velazquez (2014) for a discussion of star-rated hotels in Spain.

²See, for example, Assaf and Magnini (2012), and Madanoglu and Ozdemir (2016) for the US; Huang et al (2012), Zhang and Cheng (2014), Yang and Cai (2016), and Yang, Xia, and Cheng (2017) for China; Chen (2007), and Shyu and Hung (2012) for Taiwan; Fernandez and Becerra (2015); Oses, Gerrikagoitia and Alzua (2016), and Rodriguez-Algeciras and Talon-Ballesteros (2017) for Spain; and Peypoch (2007), Botti et al (2009), and Zhang, Botti and Petit (2016) for France.

economic (as opposed to engineering) perspective on efficiency. That is, we start from an economic model by assuming that hotels are profit maximizers.³ In general, profit efficiency evaluations are more stringent than, for example, cost efficiency evaluations. Indeed, cost minimization is, by its initial definition, a necessary condition for profit maximization; but the opposite is not true. As a result, profit efficiency evaluations can signal additional potential performance improvements. Next, profit efficiency analysis takes the overall production process (i.e. the input and output sides) into account, while other economic behaviour ignores one of the two sides. This has also been pointed out by Arbelo-Prez, Arbelo and Perez-Gomez (2017) for the hotel industry.

Next, we model hotels as multi-activity decision-making units. The majority of the hotels not only provide accommodation, but also other supplement services such as catering and entertainment. The multi-activity nature of the hotels is closely related to their profit maximization behaviour. Indeed, we consider that the main reason why hotels choose to provide multiple services is because they are looking for more profits. In other words, their multi-activity nature is motivated by their profit maximizer behaviour. Also, considering more than one activity implies the presence of economies of scope for the hotels, which represents a prime economic motivation to propose more than one service. Importantly, the multi-activity nature of the hotels imply that they could use different types of inputs for each activity. Some inputs could be used for all activities, while other inputs could be allocated to specific activity. Overall, considering hotels as multi-activity decision making units increases the realism of the profit efficiency analysis.

When profit efficiency is of interest over several periods of time, a popular method is to rely on indexes. Two main indexes have been used by practitioners for those contexts: the Luenberg index, introduced by Chambers (2002) after Luenberger (1992) that is defined as a difference of profit efficiency measurements; and the Malmquist-Luenberger index, introduced by Chung, Färe and Grosskopf (1997) after Malmquist (1953) and Luenberger (1992) that is defined as a ratio of profit efficiency measurements. While those indexes have demonstrated their usefulness for practical exercises, they do not fit with the requirements of our analysis of the star-rated hotel industry in China.⁴ In particular, those indexes do not take the multi-activity aspect of the hotels into account. As such, to match with our

³For profit efficiency analysis, refer, for example to Sahoo and Tone (2013), Sahoo, Mehdiloozad and Tone (2014), Boussemart, Crainich and Leleu (2015), and Cherchye, De Rock and Walheer (2016). In the context of the hotel industry, refer, for example, to Botti et al (2009), Ratsimbanierana, Sbai and Stenger (2013), and Zhang, Botti and Petit (2016).

⁴For previous works using performance indexes for the tourism industry, see, for example, Barros and Alves (2004) and Barros (2005) for the Malmquist index for Portuguese public-owned hotels, Cracolici, Nijkamp, and Cuffaro (2007) for the Malmquist index for Italian regions, Peypoch (2007) for the Luenberger index to French tourism industry Barros, Peypoch, and Solonandrasana (2009) for the Luenberger index for Portugal hotels, Assaf and Agbola (2011a, b) for a bootstrapped Malmquist index for Australian hotels, Sun et al (2015) for the Malmquist index to Chinese regions.

requirements, we extend those indexes in that direction. The new indexes offer two extra advantages. Firstly, besides providing profit efficiency results for the overall production process, the new indexes also give the option to evaluate profit efficiency for each activity separately. This represents valuable information that can be used to better manage and monitor the hotels. Next, the new indexes give the possibility to allocate the inputs to each activity. All in all, the new indexes offer the advantages of increasing the realism and the discriminatory power of the efficiency analysis. They can therefore be used by managers to better quantify and monitor the performances of the hotels. Also, they represent valuable information for policy makers to better target their policy implementations.

The rest of the paper is structured as follows. In Section 2, we define the profit Luenberg and Malmquist-Luenberger indexes when considering multi-activity decision makers. In Section 3, we analyze the star-rated hotel industry in China using our new indexes. In Section 4, we present a summary and a discussion of our results. Finally, we conclude in Section 5.

2 Methodology

We consider that we observe Decision Making Units (DMUs) during T periods of time. For each period t , every DMU uses N inputs: \mathbf{x}_t , to produce M outputs: \mathbf{y}_t . Also, the input prices of period t is captured by $\mathbf{p}_{x,t}$, and the output prices by $\mathbf{p}_{y,t}$. To facilitate our notation, we regroup the inputs and outputs into a netput vector: $\mathbf{z}_t = \begin{bmatrix} \mathbf{y}_t \\ -\mathbf{x}_t \end{bmatrix}$. Its

corresponding price vector is thus given by $\mathbf{p}_t = \begin{bmatrix} \mathbf{p}_{y,t} \\ \mathbf{p}_{x,t} \end{bmatrix}$. Therefore, the actual profit at time t is given by $\mathbf{p}'_t \mathbf{z}_t = \mathbf{p}'_{y,t} \mathbf{y}_t - \mathbf{p}'_{x,t} \mathbf{x}_t$.

The particularity of our procedure is to consider each activity separately, proxied in practice, by a specific output.⁵ Let us denote the m -th output at time t by y_t^m (i.e. $(\mathbf{y}_t)_m = y_t^m$), the inputs used to produce that particular output by \mathbf{x}_t^m , and their respective price by $p_{y,t}^m$ (i.e. $(\mathbf{p}_{y,t})_m = p_{y,t}^m$) and $\mathbf{p}_{x,t}^m$. As done previously, we can simplify our notation by defining the netput vector associated with output m as $\mathbf{z}_t^m = \begin{bmatrix} y_t^m \\ -\mathbf{x}_t^m \end{bmatrix}$, and its corresponding prices

as $\mathbf{p}_t^m = \begin{bmatrix} p_{y,t}^m \\ \mathbf{p}_{x,t}^m \end{bmatrix}$.

Input allocation. Attractively, the definition of inputs used to produce a specific output m (\mathbf{x}_t^m) can be used to allocate the inputs to the output-specific production processes. In

⁵Note that the indexes could be extended to the cases when several outputs represent an activity. We consider that one output proxies one activity for simplicity.

general, two types of inputs can be used to produce the outputs. Some inputs could be used to produce certain outputs. That is, these inputs are allocated to specific output production processes.⁶ Next, some inputs could be used to produce all the outputs, i.e. these inputs are not allocated to specific output production processes.⁷ These inputs could also be interpreted as public good (they are non-rival and non-exclusive to the output production processes), and, therefore, they give rise to economies of scope in the production process (see Panzar and Willig (1981) and Nehring and Puppe (2004)). As such, these inputs form a prime economic motivation to produce more than one output, i.e. considering more than one activity.⁸

Intuitively, the inputs used to produce every output \mathbf{x}_t^m must be connected to the initial input vector \mathbf{x}_t . Formally, we have the following relationships between the output-specific inputs and the inputs:

$$(\mathbf{x}_t)_n = (\mathbf{x}_t^1)_n + \dots + (\mathbf{x}_t^M)_n = \sum_{m=1}^M (\mathbf{x}_t^m)_n, \text{ if input } n \text{ is allocated,} \quad (1)$$

$$(\mathbf{x}_t)_n = (\mathbf{x}_t^m)_n, \text{ if input } n \text{ is not allocated.} \quad (2)$$

As the inputs used to produce specific outputs can be different, nothing guarantees that, in general, their prices coincide. Moreover, as those inputs are related to the input vector \mathbf{x}_t , their prices must also be related. We obtain the following relationships between the output-specific input price and the input prices:

$$(\mathbf{p}_{x,t})_n = (\mathbf{p}_{x,t}^m)_n, \text{ if input } n \text{ is allocated,} \quad (3)$$

$$(\mathbf{p}_{x,t})_n = (\mathbf{p}_{x,t}^1)_n + \dots + (\mathbf{p}_{x,t}^M)_n = \sum_{m=1}^M (\mathbf{p}_{x,t}^m)_n, \text{ if input } n \text{ is not allocated.} \quad (4)$$

The first condition states that for every allocated input, the output-specific input prices coincide with the input prices. That is, the output production processes face the same input prices for allocated inputs (see Cherchye et al (2013) for more detail). The second condition says that for every non-allocated input, the output-specific input prices must add up to the input prices. As explained previously non-allocated inputs could be interpreted as public good. As such, the output-specific input prices have a similar interpretation as

⁶These types of input have been considered in, for example, Färe and Grosskopf (2000), Färe, Grosskopf and Whittaker (2007), Tone and Tsutsui (2009), Cherchye et al (2013), Walheer (2016a, b, 2018a, f), and Silva (2017).

⁷These types of input have been considered by, for example, Cherchye et al (2013), Cherchye, De Rock, and Walheer (2016), Ding et al (2017), and Walheer (2018 b, c, d, e).

⁸Note that the setting can be fairly easily extended to other types of inputs; as, for example those considered in Salerian and Chan (2005), Despic, Despic, and Paradi (2007), and Cherchye, De Rock, and Walheer (2015).

Lindahl prices that, by definition, sum up to the aggregate prices.

Profit efficiency. To define our concept of profit efficiency at the output level, we first characterize the technology at the output level by introducing the notions of output-specific production possibility sets. It is defined for period t and output m as follows:

$$T_t^m = \{\mathbf{z}_t^m \in \mathbb{R}_+^{1+N} \mid \mathbf{z}_t^m \text{ is technically feasible}\}. \quad (5)$$

T_t^m contains all combination of inputs and individual output that are technically feasible at period t . We assume that those sets fulfill some general regularity conditions to perform a profit efficiency evaluation (see Färe and Primont (1995)).⁹ Building on the notion of output-specific production possibility sets, we naturally define the concept of maximal attainable profit at the output level as follows:

$$\pi_t^m(\mathbf{p}_t^m) = \max_{\mathbf{z}_t^m \in T_t^m} \mathbf{p}_t^{m'} \mathbf{z}_t^m. \quad (6)$$

$\pi_t^m(\mathbf{p}_t^m)$ gives the maximum attainable profit given the prices \mathbf{p}_t^m and the technology at time t (as such the subscript t on π_t^m refers to the time period of the technology). By construction, maximal profit can only be greater than actual profit: $\pi_t^m(\mathbf{p}_t^m) \geq \mathbf{p}_t^{m'} \mathbf{z}_t^m$. Profit is at its maximal level for output m when $\pi_t^m(\mathbf{p}_t^m) = \mathbf{p}_t^{m'} \mathbf{z}_t^m$, and profit improvement is possible when $\pi_t^m(\mathbf{p}_t^m) > \mathbf{p}_t^{m'} \mathbf{z}_t^m$.

Using the previous definition of maximum attainable profit of output m , we define the notion of output-specific profit efficiency measurement for period t as follows:

$$PE_t^m(\mathbf{z}_t^m, \mathbf{p}_t^m, \mathbf{g}_{\mathbf{z}_t^m}) = \frac{\pi_t^m(\mathbf{p}_t^m) - \mathbf{p}_t^{m'} \mathbf{z}_t^m}{\mathbf{p}_t^{m'} \mathbf{g}_{\mathbf{z}_t^m}}. \quad (7)$$

$PE_t^m(\mathbf{z}_t^m, \mathbf{p}_t^m, \mathbf{g}_{\mathbf{z}_t^m})$ provides a directional profit efficiency measurement (i.e. profit efficiency in the direction of $\mathbf{g}_{\mathbf{z}_t^m}$). In this definition, $\mathbf{g}_{\mathbf{z}_t^m} = \begin{bmatrix} g_{y_t^m} \\ \mathbf{g}_{\mathbf{x}_t^m} \end{bmatrix}$ stands for the directional vector; $g_{y_t^m}$ is the directional vector for the output, while $\mathbf{g}_{\mathbf{x}_t^m}$ is the directional vector for the inputs. $PE_t^m(\mathbf{z}_t^m, \mathbf{p}_t^m, \mathbf{g}_{\mathbf{z}_t^m})$ indicates profit efficiency of \mathbf{z}_t^m in the direction of $\mathbf{g}_{\mathbf{z}_t^m}$ given the prices \mathbf{p}_t^m and the technology at time t . As pointed out previously maximal profit can only be greater than actual profit (i.e. $\pi_t^m(\mathbf{p}_t^m) \geq \mathbf{p}_t^{m'} \mathbf{z}_t^m$), implying that $PE_t^m(\mathbf{z}_t^m, \mathbf{p}_t^m, \mathbf{g}_{\mathbf{z}_t^m}) \geq 0$.¹⁰ When $PE_t^m(\mathbf{z}_t^m, \mathbf{p}_t^m, \mathbf{g}_{\mathbf{z}_t^m}) = 0$, it means that $\pi_t^m(\mathbf{p}_t^m) = \mathbf{p}_t^{m'} \mathbf{z}_t^m$ reflecting profit efficiency behaviour for output m . On the contrary, profit inefficient behaviour is captured

⁹Note that very weak conditions are needed for profit efficiency evaluation (see, for example, Cherchye, De Rock, and Walheer (2016) for more discussion).

¹⁰Note that, we implicitly assume that $\mathbf{p}_t^{m'} \mathbf{g}_{\mathbf{z}_t^m} > 0$, which in general is fulfilled for standard values of the directional distance vectors. See Section 4 for more discussion.

by $PE_t^m(\mathbf{z}_t, \mathbf{p}_t, \mathbf{g}_{\mathbf{z}_t}) > 0$. As a final remark, we note that $PE_t^m(\mathbf{z}_t^m, \mathbf{p}_t^m, \mathbf{g}_{\mathbf{z}_t^m})$ provides a generalization of the directional profit efficiency measurement introduced by Chambers, Chung, and Färe (1998), and is a very general way of measuring profit efficiency.

Attractively, we can construct a profit efficiency measurement for the overall production process (i.e for \mathbf{z}_t) using only output-specific actual and maximal profits (the proof is given in Appendix A):

$$PE_t(\mathbf{z}_t, \mathbf{p}_t, \mathbf{g}_{\mathbf{z}_t}) = \sum_{m=1}^M \frac{\mathbf{p}_t^{m'} \mathbf{g}_{\mathbf{z}_t^m}}{\mathbf{p}_t' \mathbf{g}_{\mathbf{z}_t}} \times PE_t^m(\mathbf{z}_t^m, \mathbf{p}_t^m, \mathbf{g}_{\mathbf{z}_t^m}). \quad (8)$$

It implies that the profit efficiency measurement is a weighted sum of the output-specific counterparts. The weights $\frac{\mathbf{p}_t^{m'} \mathbf{g}_{\mathbf{z}_t^m}}{\mathbf{p}_t' \mathbf{g}_{\mathbf{z}_t}}$ represent the profit share of output m in the direction of $\mathbf{g}_{\mathbf{z}_t^m}$ and $\mathbf{g}_{\mathbf{z}_t}$. Therefore, those weights allow us to investigate how every output contributes to the overall profit performances. Note that $\mathbf{g}_{\mathbf{z}_t} = \begin{bmatrix} \mathbf{g}_{\mathbf{y}_t} \\ \mathbf{g}_{\mathbf{x}_t} \end{bmatrix}$ stands for the directional vector associated with \mathbf{z}_t ; $\mathbf{g}_{\mathbf{y}_t}$ is the directional vector for the outputs, while $\mathbf{g}_{\mathbf{x}_t}$ is the directional vector for the inputs. Clearly, $PE_t(\mathbf{z}_t, \mathbf{p}_t, \mathbf{g}_{\mathbf{z}_t})$ is bounded from below by 0. When $PE_t(\mathbf{z}_t, \mathbf{p}_t, \mathbf{g}_{\mathbf{z}_t}) = 0$, it shows that every output is produced in a profit efficiency manner, while greater values imply profit inefficient behaviour. As a final remark, we point out that the weights in (8) could be replaced by any other weights reflecting the preferences of the managers or policy makers for each output m . An advantage of the weights $\frac{\mathbf{p}_t^{m'} \mathbf{g}_{\mathbf{z}_t^m}}{\mathbf{p}_t' \mathbf{g}_{\mathbf{z}_t}}$ is that they are based on an economic model.

Luenberger index (LI). The Luenberger index was introduced by Chambers (2002) after Luenberger (1992). That index is defined as the difference of profit efficiency measurements. Attractively, using our notion of profit efficiency measurement for every output, we can naturally define Luenberger indexes at the output level. They allow us to better understand the results found with the aggregate-level indicators. That index is defined as the difference of profit efficiency measurements at \mathbf{z}_t^m and \mathbf{z}_{t+1}^m . In particular, it is defined when time t is chosen as the year for the technology as follows:

$$LI_t^m(\mathbf{z}_t^m, \mathbf{z}_{t+1}^m, \mathbf{p}_t, \mathbf{g}_{\mathbf{z}_t^m}, \mathbf{g}_{\mathbf{z}_{t+1}^m}) = PE_t^m(\mathbf{z}_t^m, \mathbf{p}_t^m, \mathbf{g}_{\mathbf{z}_t^m}) - PE_t^m(\mathbf{z}_{t+1}^m, \mathbf{p}_t^m, \mathbf{g}_{\mathbf{z}_{t+1}^m}). \quad (9)$$

An improvement of profit efficiency between $t+1$ and t implies that $PE_t^m(\mathbf{z}_t^m, \mathbf{p}_t^m, \mathbf{g}_{\mathbf{z}_t^m}) > PE_t^m(\mathbf{z}_{t+1}^m, \mathbf{p}_t^m, \mathbf{g}_{\mathbf{z}_{t+1}^m})$ (recall that a value of zero reflects profit efficient behaviour). As such, $LI_t^m(\mathbf{z}_t^m, \mathbf{z}_{t+1}^m, \mathbf{p}_t^m, \mathbf{g}_{\mathbf{z}_t^m}, \mathbf{g}_{\mathbf{z}_{t+1}^m}) > 0$ captures profit performance improvement between t and $t+1$ for output m . On the contrary, $LI_t^m(\mathbf{z}_t^m, \mathbf{z}_{t+1}^m, \mathbf{p}_t^m, \mathbf{g}_{\mathbf{z}_t^m}, \mathbf{g}_{\mathbf{z}_{t+1}^m}) < 0$ implies profit performance regress. $LI_t^m(\mathbf{z}_t^m, \mathbf{z}_{t+1}^m, \mathbf{p}_t^m, \mathbf{g}_{\mathbf{z}_t^m}, \mathbf{g}_{\mathbf{z}_{t+1}^m}) = 0$ stands for the status quo and thus

represents the benchmark value of the index.

Similarly, we can define the Luenberger index to compare \mathbf{z}_t^m and \mathbf{z}_{t+1}^m taking time $t+1$ for the year of the technology:

$$LI_{t+1}^m(\mathbf{z}_t^m, \mathbf{z}_{t+1}^m, \mathbf{p}_{t+1}^m, \mathbf{g}_{z_t^m}, \mathbf{g}_{z_{t+1}^m}) = PE_{t+1}^m(\mathbf{z}_t^m, \mathbf{p}_{t+1}^m, \mathbf{g}_{z_t^m}) - PE_{t+1}^m(\mathbf{z}_{t+1}^m, \mathbf{p}_{t+1}^m, \mathbf{g}_{z_{t+1}^m}). \quad (10)$$

Clearly, $LI_{t+1}^m(\mathbf{z}_t^m, \mathbf{z}_{t+1}^m, \mathbf{p}_{t+1}^m, \mathbf{g}_{z_t^m}, \mathbf{g}_{z_{t+1}^m})$ has to be interpreted in an analogous manner to $LI_t^m(\mathbf{z}_t^m, \mathbf{z}_{t+1}^m, \mathbf{p}_t^m, \mathbf{g}_{z_t^m}, \mathbf{g}_{z_{t+1}^m})$, but with respect to the technology of time $t+1$.

To avoid an arbitrary choice of the reference period for technology, it is commonly agreed to take the arithmetic average of the Luenberger indexes when t and $t+1$ are chosen for the reference year of the technology:

$$LI^m(\mathbf{z}_t^m, \mathbf{z}_{t+1}^m, \mathbf{p}_t^m, \mathbf{p}_{t+1}^m, \mathbf{g}_{z_t^m}, \mathbf{g}_{z_{t+1}^m}) = \frac{1}{2} \left[LI_t^m(\mathbf{z}_t^m, \mathbf{z}_{t+1}^m, \mathbf{p}_t^m, \mathbf{g}_{z_t^m}) + LI_{t+1}^m(\mathbf{z}_t^m, \mathbf{z}_{t+1}^m, \mathbf{p}_{t+1}^m, \mathbf{g}_{z_{t+1}^m}) \right]. \quad (11)$$

$LI^m(\mathbf{z}_t^m, \mathbf{z}_{t+1}^m, \mathbf{p}_t^m, \mathbf{p}_{t+1}^m, \mathbf{g}_{z_t^m}, \mathbf{g}_{z_{t+1}^m})$ measures profit performance change between \mathbf{z}_t^m and \mathbf{z}_{t+1}^m irrespective of the reference year for the technology. A value greater (smaller) than 0 implies an improvement (decline) of the profit performances. A value of 0 implies no performance change, and thus captures the benchmark situation.

In a similar vein, using the relationship between the profit efficiency measurement and the output-specific profit efficiency measurements (see (8)), we can define a Luenberger index for the aggregate level such that it only depends on output-specific profit efficiency measurements as follows:

$$LI(\mathbf{z}_t, \mathbf{z}_{t+1}, \mathbf{p}_t, \mathbf{p}_{t+1}, \mathbf{g}_{z_t}, \mathbf{g}_{z_{t+1}}) = \frac{1}{2} \left(LI_t(\mathbf{z}_t, \mathbf{z}_{t+1}, \mathbf{p}_t, \mathbf{g}_{z_t}, \mathbf{g}_{z_{t+1}}) + LI_{t+1}(\mathbf{z}_t, \mathbf{z}_{t+1}, \mathbf{p}_{t+1}, \mathbf{g}_{z_t}, \mathbf{g}_{z_{t+1}}) \right). \quad (12)$$

where

$$\begin{aligned} LI_t(\mathbf{z}_t, \mathbf{z}_{t+1}, \mathbf{p}_t, \mathbf{g}_{z_t}, \mathbf{g}_{z_{t+1}}) &= \sum_{m=1}^M \frac{\mathbf{p}_t^{m'} \mathbf{g}_{z_t^m}}{\mathbf{p}_t' \mathbf{g}_{z_t}} \times PE_t^m(\mathbf{z}_t^m, \mathbf{p}_t^m, \mathbf{g}_{z_t^m}) \\ &\quad - \sum_{m=1}^M \frac{\mathbf{p}_t^{m'} \mathbf{g}_{z_{t+1}^m}}{\mathbf{p}_t' \mathbf{g}_{z_{t+1}}} \times PE_t^m(\mathbf{z}_{t+1}^m, \mathbf{p}_t^m, \mathbf{g}_{z_{t+1}^m}), \\ LI_{t+1}(\mathbf{z}_t, \mathbf{z}_{t+1}, \mathbf{p}_t, \mathbf{g}_{z_t}, \mathbf{g}_{z_{t+1}}) &= \sum_{m=1}^M \frac{\mathbf{p}_{t+1}^{m'} \mathbf{g}_{z_t^m}}{\mathbf{p}_{t+1}' \mathbf{g}_{z_t}} \times PE_{t+1}^m(\mathbf{z}_t^m, \mathbf{p}_{t+1}^m, \mathbf{g}_{z_t^m}) \\ &\quad - \sum_{m=1}^M \frac{\mathbf{p}_{t+1}^{m'} \mathbf{g}_{z_{t+1}^m}}{\mathbf{p}_{t+1}' \mathbf{g}_{z_{t+1}}} \times PE_{t+1}^m(\mathbf{z}_{t+1}^m, \mathbf{p}_{t+1}^m, \mathbf{g}_{z_{t+1}^m}). \end{aligned} \quad (13)$$

This definition provides a disaggregation of the Luenberger index in terms of the output-specific profit efficiency measurements. That is, it only depends on output-specific profit maximization conditions. $LI(\mathbf{z}_t, \mathbf{z}_{t+1}, \mathbf{p}_t, \mathbf{p}_{t+1}, \mathbf{g}_{\mathbf{z}_t}, \mathbf{g}_{\mathbf{z}_{t+1}})$ measures profit performance change between \mathbf{z}_t and \mathbf{z}_{t+1} irrespective of the reference year for the technology. A value greater (smaller) than 0 implies an improvement (regress) of the profit performances. A value of 0 implies no performance change, and thus captures the benchmark situation.

Malmquist-Luenberger index (MLI). The Malmquist-Luenberger index, introduced by Chung, Färe, and Grosskopf (1997) after Malmquist (1953) and Luenberger (1992), is defined as the ratio of profit efficiency measurements. In particular, to compare \mathbf{z}_t^m and \mathbf{z}_{t+1}^m when taking period t or $t + 1$ for the technology, the index is defined as:

$$\begin{aligned} MLI_t^m(\mathbf{z}_t^m, \mathbf{z}_{t+1}^m, \mathbf{p}_t^m, \mathbf{g}_{\mathbf{z}_t^m}, \mathbf{g}_{\mathbf{z}_{t+1}^m}) &= \left(\frac{1 + PE_t^m(\mathbf{z}_{t+1}^m, \mathbf{p}_t^m, \mathbf{g}_{\mathbf{z}_{t+1}^m})}{1 + PE_t^m(\mathbf{z}_t^m, \mathbf{p}_t^m, \mathbf{g}_{\mathbf{z}_t^m})} \right)^{-1}, \\ MLI_{t+1}^m(\mathbf{z}_t^m, \mathbf{z}_{t+1}^m, \mathbf{p}_{t+1}^m, \mathbf{g}_{\mathbf{z}_t^m}, \mathbf{g}_{\mathbf{z}_{t+1}^m}) &= \left(\frac{1 + PE_{t+1}^m(\mathbf{z}_{t+1}^m, \mathbf{p}_{t+1}^m, \mathbf{g}_{\mathbf{z}_{t+1}^m})}{1 + PE_{t+1}^m(\mathbf{z}_t^m, \mathbf{p}_{t+1}^m, \mathbf{g}_{\mathbf{z}_t^m})} \right)^{-1}. \end{aligned} \quad (14)$$

As noticed previously, we have $PE_t^m(\mathbf{z}_t^m, \mathbf{p}_t^m, \mathbf{g}_{\mathbf{z}_t^m}) > PE_t^m(\mathbf{z}_{t+1}^m, \mathbf{p}_t^m, \mathbf{g}_{\mathbf{z}_{t+1}^m})$ when profit efficiency increases between years t and $t + 1$. As a result, $MLI_t^m(\mathbf{z}_t^m, \mathbf{z}_{t+1}^m, \mathbf{p}_t^m, \mathbf{g}_{\mathbf{z}_t^m}, \mathbf{g}_{\mathbf{z}_{t+1}^m}) > 1$ means that profit performance improvement occurs. A value smaller than 1 reflects the opposite, and 1 is thus the benchmark value for this index. A similar reasoning can be made for $MLI_{t+1}^m(\mathbf{z}_t^m, \mathbf{z}_{t+1}^m, \mathbf{p}_{t+1}^m, \mathbf{g}_{\mathbf{z}_t^m}, \mathbf{g}_{\mathbf{z}_{t+1}^m})$. We obtain $MLI_{t+1}^m(\mathbf{z}_t^m, \mathbf{z}_{t+1}^m, \mathbf{p}_{t+1}^m, \mathbf{g}_{\mathbf{z}_t^m}, \mathbf{g}_{\mathbf{z}_{t+1}^m}) > (<)$ 1 reflects profit performance improvement (regress) for output m , and a value of 1 indicates the status quo, with respect to the technology of year $t + 1$.

To avoid an arbitrary choice of the reference year for technology, we rely, this time, on the geometric average:

$$MLI^m(\mathbf{z}_t^m, \mathbf{z}_{t+1}^m, \mathbf{p}_t^m, \mathbf{p}_{t+1}^m, \mathbf{g}_{\mathbf{z}_t^m}, \mathbf{g}_{\mathbf{z}_{t+1}^m}) = \left[MLI_t^m(\mathbf{z}_t^m, \mathbf{z}_{t+1}^m, \mathbf{p}_t^m, \mathbf{g}_{\mathbf{z}_t^m}) \times MLI_{t+1}^m(\mathbf{z}_t^m, \mathbf{z}_{t+1}^m, \mathbf{p}_{t+1}^m, \mathbf{g}_{\mathbf{z}_{t+1}^m}) \right]^{1/2}. \quad (15)$$

$MLI^m(\mathbf{z}_t^m, \mathbf{z}_{t+1}^m, \mathbf{p}_t^m, \mathbf{p}_{t+1}^m, \mathbf{g}_{\mathbf{z}_t^m}, \mathbf{g}_{\mathbf{z}_{t+1}^m})$ captures the ratio change in profit efficiency between periods t and $t + 1$ for output m , and is not dependent on a specific period for the technology. A value greater than 1 implies profit improvement, a value smaller than 1 profit regress, and a value of 1 similar profit performances between years t and $t + 1$.

Next, we can provide a disaggregation of the Malmquist-Luenberger indexes for the

aggregate level in terms of the output-specific profit efficiency measurements as follows:

$$MLI(\mathbf{z}_t, \mathbf{z}_{t+1}, \mathbf{p}_t, \mathbf{p}_{t+1}, \mathbf{g}_{\mathbf{z}_t}, \mathbf{g}_{\mathbf{z}_{t+1}}) = [MLI_t(\mathbf{z}_t, \mathbf{z}_{t+1}, \mathbf{p}_t, \mathbf{g}_{\mathbf{z}_t}, \mathbf{g}_{\mathbf{z}_{t+1}}) \times MLI_{t+1}(\mathbf{z}_t, \mathbf{z}_{t+1}, \mathbf{p}_{t+1}, \mathbf{g}_{\mathbf{z}_t}, \mathbf{g}_{\mathbf{z}_{t+1}})]^{1/2}. \quad (16)$$

where

$$MLI_t(\mathbf{z}_t, \mathbf{z}_{t+1}, \mathbf{p}_t, \mathbf{g}_{\mathbf{z}_t}, \mathbf{g}_{\mathbf{z}_{t+1}}) = \left(\frac{1 + \sum_{m=1}^M \frac{\mathbf{p}_t^{m'} \mathbf{g}_{\mathbf{z}_{t+1}}^m}{\mathbf{p}_t^m \mathbf{g}_{\mathbf{z}_{t+1}}^m} \times PE_t^m(\mathbf{z}_{t+1}^m, \mathbf{p}_t^m, \mathbf{g}_{\mathbf{z}_{t+1}}^m)}{1 + \sum_{m=1}^M \frac{\mathbf{p}_t^{m'} \mathbf{g}_{\mathbf{z}_t}^m}{\mathbf{p}_t^m \mathbf{g}_{\mathbf{z}_t}^m} \times PE_t^m(\mathbf{z}_t^m, \mathbf{p}_t^m, \mathbf{g}_{\mathbf{z}_t}^m)} \right)^{-1}, \quad (17)$$

$$MLI_{t+1}(\mathbf{z}_t, \mathbf{z}_{t+1}, \mathbf{p}_t, \mathbf{g}_{\mathbf{z}_t}, \mathbf{g}_{\mathbf{z}_{t+1}}) = \left(\frac{1 + \sum_{m=1}^M \frac{\mathbf{p}_{t+1}^{m'} \mathbf{g}_{\mathbf{z}_{t+1}}^m}{\mathbf{p}_{t+1}^m \mathbf{g}_{\mathbf{z}_{t+1}}^m} \times PE_{t+1}^m(\mathbf{z}_{t+1}^m, \mathbf{p}_{t+1}^m, \mathbf{g}_{\mathbf{z}_{t+1}}^m)}{1 + \sum_{m=1}^M \frac{\mathbf{p}_{t+1}^{m'} \mathbf{g}_{\mathbf{z}_t}^m}{\mathbf{p}_{t+1}^m \mathbf{g}_{\mathbf{z}_t}^m} \times PE_{t+1}^m(\mathbf{z}_t^m, \mathbf{p}_{t+1}^m, \mathbf{g}_{\mathbf{z}_t}^m)} \right)^{-1}. \quad (18)$$

$MLI(\mathbf{z}_t, \mathbf{z}_{t+1}, \mathbf{p}_t, \mathbf{p}_{t+1}, \mathbf{g}_{\mathbf{z}_t}, \mathbf{g}_{\mathbf{z}_{t+1}})$ captures the ratio change in profit efficiency between periods t and $t + 1$, and is not dependent on a specific period for the technology. A value greater than 1 implies profit improvement, a value smaller than 1 profit regress, and a value of 1 similar profit performances between years t and $t + 1$.

Computational aspect and price availability. In practice, it suffices to compute profit efficiency scores to obtain the Luenberger and Malmquist-Luenberger indexes. Those scores require observing the netputs, the allocation of the inputs to the output production processes, and the prices; and choosing specific directional vectors.¹¹ In many contexts, observing the netputs and the allocation of the inputs to the output-specific production processes do not represent a strong requirement.¹² This is often not the case for the prices. Moreover, in our context, it is required to observe both the netput and output-specific netput prices. We provide in Appendix B an extensive discussion about how to compute the profit efficiency scores when the prices are observed, partially observed, and not observed.

3 Application

We apply our methodology to the star-rated hotel industry in China. In what follows, we first discuss the specificities of our set-up in more detail. Subsequently, we present our data

¹¹For discussion about choosing the directional vectors, see, for example, Hampf and Kruger (2014), Atkinson and Tsionas (2016), and Färe, Pasurka, and Vardanyan (2017).

¹²If this information is not (or partially) available, it will only complicate the computation of the profit efficiency scores (by, for example, making the model non-linear; see Walheer (2016b)).

and the results of our empirical analysis.

3.1 Hotel as a multi-activity process

An important aspect of our profit efficiency analysis is to recognize hotels as multi-activity decision makers. Indeed, the majority of hotels not only provide accommodation, but also other supplement services such as catering and entertainment. In our study, we consider that hotels propose three main services: accommodation services (room), food and beverage services (meal), and other services such as entertainment and commodity services (others). A similar model has been considered in Wang, Huang, and Shang (2006) and Yang and Cai (2016). It is straightforward to extend the setting when considering additional activities.

We use the revenue generated to proxy the output of each of the three activities of the hotels. In a sense, we also take the quality of the services into account. Indeed, higher quality services are, in general, more expensive resulting in higher revenue. Refer, for example, to Fukuyama and Weber (2008), Sahoo, Mehdiloozad, and Tone (2014), and Cherchye, De Rock, and Walheer (2016) for more discussion about taking quality into account in profit efficiency analysis, and Hu et al (2010) for the hotel context. Let us denote the output of the room activity by y^1 , of the meal activity by y^2 , and of the others activity by y^3 .

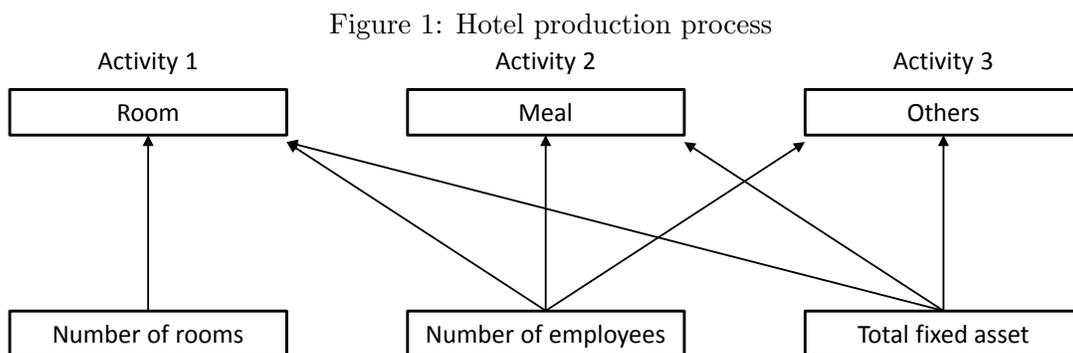
We select three inputs: the number of rooms (x^1), the total fixed asset (x^2), and the number of employees (x^3). These three inputs represent a natural choice in our context, and has been considered in several previous studies as in Barros et al (2011), Assaf (2011a), and Yang, Xia and Cheng (2017). The number of rooms is selected to reflect the operation scale, the total fixed asset is used to reflect the support to its development, and the number of employees is used as the indispensable part and core asset to make the hotels functional to offering all the services available. As discussed in Section 3, an advantage of our methodology is to give the option to link the inputs to the activities. While the number of employees and the total fixed asset are used for the three activities, it is not the case for the number of rooms. As such, this input is completely allocated to the room activity. At this point, we remark that the number of rooms and the total fixed asset may be significantly related (more total fixed asset derive from more rooms). In a sense, we may argue that the total number of rooms is included in the total fixed asset. Putting it differently, the total fixed asset provides a greater measurement of the hotel infrastructure than the number of rooms (think, for example, to gyms, restaurants, swimming pools). This connection between these two inputs form our main motivation to include both inputs. Indeed, while the total asset is used for all activities, the number of rooms is only important for the Room activity. That is, when considering hotels as multi-activity decision-makers, the number of rooms enter rather naturally as an input for the Room activity.

All in all, we have three outputs, two non-allocated inputs, and one allocated input.

Using the notations of Section 2, the netput vector and output-specific netput vectors for the hotel production processes are given for period t by:

$$\mathbf{z}_t = \begin{bmatrix} y_t^1 \\ y_t^2 \\ y_t^3 \\ -x_t^1 \\ -x_t^2 \\ -x_t^3 \end{bmatrix}; \mathbf{z}_t^1 = \begin{bmatrix} y_t^1 \\ -x_t^1 \\ -x_t^2 \\ -x_t^3 \end{bmatrix}; \mathbf{z}_t^2 = \begin{bmatrix} y_t^2 \\ 0 \\ -x_t^2 \\ -x_t^3 \end{bmatrix}; \text{ and } \mathbf{z}_t^3 = \begin{bmatrix} y_t^3 \\ 0 \\ -x_t^2 \\ -x_t^3 \end{bmatrix}. \quad (19)$$

Figure 1 summarizes all this and presents a schematic representation of the production process when considering hotels as multi-activity decision-making units.



3.2 Data and descriptive statistics

To obtain the data for our three outputs and three inputs, we make use of three different databases: the China Tourism Statistics Yearbooks, the China Star Hotel Statistics Report, and the Wind Database. The revenue for the room and meal activities are given by these databases. We obtain the revenue for the other activities by subtracting from the total revenue the revenue of the room and meal activities. As such, all other possible activities of the hotels are included in that output. Typically, this includes the entertainment and commodity services.

At this point, we remark that, in our study, we take all star-rated hotels (i.e. from one to five stars), as defined by the China National Tourism Bureau, into account. Indeed, data are given when pooling all star-rated hotels together in the datasets used. Nevertheless, it will be interesting to investigate how the results change when partitioning the star-rated hotels with respect to the number of stars, when these data are be available (see Section 4). Finally, note that the data used are well representative of the Chinese star-rated hotel

sector. For example, in 2017, there were 11,492 star rated hotels in China; 11,226 completed the survey implying a filing rate of 98.03%.

We end with a sample of 30 provinces and a period spanning from 2005 to 2015. Data are provided yearly. As such, this represents a unique opportunity to evaluate profit efficiency of star-rated hotels over a large period of time. Note that we choose not to include Tibet in our analysis given the poor presentation in the databases of star-rated hotels in that province. Also, only the average price of the room is provided by the databases, while we find the average wage from the China National Statistics Bureau. This is not a major issue since, as explained in Section 2, our methodology also works when prices are partially observed.

As an initial step, we present some descriptive statistics to contextualize our profit efficiency analysis. Firstly, we show the proportion of each of the three activities over time in Table 1. Two main lessons can be learned from that Table. On the one hand, it is clear that hotels are multi-activity decision makers. Indeed, on average for the period 2005-2015, accommodation constitutes 46.72% of hotel total revenue while catering service accounted for 40.69%, and 12.59% of the total revenue is generated from other services. On the other hand, we also note that the share of the room service in the total revenue slowly decreases over the period, while the share of the meal service increases. This shows that the hotel industry has gradually shifted its focus from traditional room service to catering. Note that the revenue share of the other services is stable around 14% over the period.

Table 1: Proportion of the activity

Year	Room	Meal	Others
2005	50%	36%	14%
2006	49%	36%	15%
2007	50%	37%	14%
2008	46%	39%	15%
2009	46%	41%	13%
2010	43%	43%	15%
2011	42%	42%	16%
2012	41%	45%	14%
2013	43%	42%	14%
2014	43%	41%	15%
2015	44%	41%	14%

Next, we present in Table 2, the averages and medians of the inputs and outputs. Clearly, the three outputs increase importantly for the period. For the inputs, the number of guest rooms has increased by nearly 10%, while the total fixed asset has almost doubled. This shows that star-rated hotels in China have invested more and more on capital to insure the quality of constructions, facilities and services. The number of employees drop

by 11.21%, while the median increases by 11.27%. This is due to an important decrease in economically strong provinces such as Beijing, Guangdong and Zhejiang, and also to a decrease in Liaoning, Shandong, and Jiangsu. This shows that hotels try to rationalize their input use, revealing that reaching a more profit efficient situation is clearly an objective of the hotel industry.

Table 2: Averages and medians of the inputs and outputs

Year	Room revenue	Meal revenue	Others revenue	Number of employees	Fixed asset	Number of rooms
<i>Averages</i>						
2005	2244312	1610679	625530	50201	12475176	44238
2006	2399814	1777687	756627	52318	13139957	48487
2007	2727417	2004655	747888	55237	14270854	52220
2008	2708414	2281695	877214	55450	14442051	52792
2009	2774213	2467257	803886	55421	14705309	55361
2010	3015970	3011235	1034516	52521	15284921	48893
2011	3266366	3224622	1211296	51268	15221116	48905
2012	3353459	3624014	1103885	52777	15754165	49572
2013	3296780	3231318	1094789	49963	16649178	50996
2014	3107200	2945023	1097192	45241	16584871	49598
2015	3107184	2897195	1001031	44573	18110404	48535
<i>Medians</i>						
2005	1279396	1044082	310432	36936	6831600	35205
2006	1419950	1163389	318305	40354	8587165	41408
2007	1686226	1429049	358841	42167	9091200	44349
2008	1649306	1509608	461123	40611	9128674	48016
2009	1894448	1641027	484838	41123	9476634	46975
2010	1850289	1823821	450222	42435	10315608	44620
2011	2223761	2218437	520762	42138	10122926	42115
2012	2243429	2356113	699669	44710	10677645	43329
2013	2136732	1996885	589728	40452	10834067	44629
2014	2120595	1673621	540083	35416	11591497	45317
2015	2065590	1577900	518695	41098	11334198	44916

3.3 Results

To measure profit efficiency, we first have to specify the directional vectors. A natural choice, given our previous discussion, is to measure profit in both input and output directions. That is, we consider that hotels can modify their input and output to reach a profit efficient situation. Formally, we set $\mathbf{g}_{\mathbf{z}_t} = \mathbf{z}_t$, and $\mathbf{g}_{\mathbf{z}_t^m} = \mathbf{z}_t^m$ for $m = 1, 2, 3$. We present our results in two parts. Firstly, we give the results based on the Luenberger index. Next, we discuss the results using the Malmquist-Luenberger index.

Luenberger index. The averages, medians, and the number of improvements for the Luenberger index are provided in Table 3. An advantage of our methodology is to provide an index for the overall level, i.e. for all activities together, but also for each of the three activities separately. As such, results for both levels are provided in Table 3.

For the overall level, we find an increase of the Luenberger index for the period, except for the periods 2008-2010 and 2013-2015. Note that the second decrease seems more severe as only the averages are negative for the first decrease, while both the averages and medians are negative for the second decrease. Moreover, the number of improvements drops importantly for the second decrease, but not for the first. The first decrease in 2008-2010 could be attributed to the financial crisis. Indeed, the tourism activity is, in general, more sensitive to economic crisis. As such, in that case, hotels generally face an important fall for their activity (see also Yang and Cai (2016) for related discussion). The second decrease in 2013-2015 could be due to high investments in high-end hotels (four and five star hotels). Indeed, in general, it takes several years to obtain returns from their initial investment. As such, for our profit efficiency evaluation, we only see an important increase of the input levels, but not for the output side of the production process (this is also confirmed from Table 1), resulting in a more profit inefficient behaviour (see also Yang, Xia and Cheng (2017) for more discussion).

The results per activity give some additional valuable information to better understand the profit efficiency results observed for the overall level, and in particular for the important decreases in 2008-2010 and 2013-2015. The overall performance changes are mainly due to the room and meal activities. That is, the others activity has an index close to zero, indicating that no profit improvements are observed for that activity. We see two possible explanations for that result. One, as shown in Table 1, the others activity is a minor activity in terms of revenue meaning that hotels are not putting much effort into reaching a profit efficiency situation for that activity. Two, the output side for that activity could be exogenous for the hotels in the sense that they do not completely control this side of the production process. Also, we see that more progress occurs for the meal activity. This is consistent with the changes in the revenue shares observed in Table 1. Indeed, the meal activity presents increasing revenue shares, accompanied by higher profit efficiency changes. This shows that, on average, hotels are more profit efficient in their traditional room activity.

The analysis per activity also gives useful complementary information regarding the important decreases observed for 2008-2010 and 2013-2015. Indeed, the negative changes are mainly due to the two main activities, i.e. catering and food service, while the other services are not impacted. This gives credit to the exogenous argument of the output side for that activity. Also, the meal activity is more affected than the room activity, revealing once more that the room activity is a more traditional activity for the hotels.

Table 3: Averages, medians, improvements for the Luenberger index

Year	Overall	Room	Meal	Others
<i>Averages</i>				
2005-06	2.02	0.41	1.27	0.34
2006-07	1.07	1.15	0.02	-0.11
2007-08	1.87	0.57	1.35	-0.05
2008-09	-0.11	-0.10	-0.01	-0.01
2009-10	-0.56	-0.23	-0.31	-0.01
2010-11	1.24	0.49	0.75	0.00
2011-12	0.49	0.01	0.44	0.04
2012-13	0.32	0.40	-0.10	0.01
2013-14	-1.55	-0.66	-0.91	0.02
2014-15	-0.54	-0.15	-0.38	0.00
<i>Medians</i>				
2005-06	0.09	0.00	0.02	0.00
2006-07	0.45	0.00	0.00	0.00
2007-08	0.54	0.00	0.13	0.00
2008-09	0.50	0.00	0.09	0.00
2009-10	0.11	0.00	0.00	0.00
2010-11	0.67	0.00	0.29	0.00
2011-12	0.23	0.00	0.02	0.00
2012-13	-0.60	0.00	0.00	0.00
2013-14	-0.72	0.00	0.00	0.00
2014-15	-0.05	0.00	-0.04	0.00
<i>Improvements</i>				
2005-06	21	7	16	4
2006-07	24	8	16	0
2007-08	20	10	15	0
2008-09	21	12	15	4
2009-10	19	14	8	1
2010-11	27	13	18	0
2011-12	21	5	15	1
2012-13	5	3	5	2
2013-14	6	6	1	1
2014-15	12	10	4	1

Next, we present in Table 4, the average of the Luenberger index per province for the period. An initial observation is that the overall performance is 0.48 for the period, due to improvements of the meal activity (0.26) and the room activity (0.20). The average change for the others activity is close to 0, revealing once more the status quo of the profit efficiency performances for that activity.

Zhejiang, Beijing and Shanghai have an average index close to 0 for the period. This shows that no profit efficiency improvement occurs for those provinces. This is explained as those provinces represent the best practice and have thus profit efficiency scores close to 0. This is not a surprise as those provinces are the most advanced economic provinces. Tianjin, Guangdong, Yunnan, Shanxi, and Jilin present a regress of their profit efficient behaviour. The main reason is their poor performance in the catering and food services. The huge progress of Ningxia, Qinghai, Inner Mongolia, and Shaanxi could be attributed to the important investment of the government in the local tourism industry of these economically less developed provinces. Note that the reason for their important increase is mainly due to the meal activity, except for Qinghai where it is due to both the room and meal activities.

Also, we note the important progress of the room activity of several provinces that benefit from unique local tourism resources: Hainan (attracting tourists from north China in the winter because of the special tropical climate, geographical location, the warm sea and island view), Jiangxi (unique mountain views combined with Chinese patriotism education attractions), Heilongjiang (ice and snow tourism with Russian-style buildings and streets), Guizhou and Anhui (unique natural landscapes), Fujian, Gansu and Xinjiang (natural, historical and minority ethnic culture attractions). In a sense, this shows that these provinces have put more effort into rationalizing their input-output use in their traditional activity.

Finally, we point out two extreme cases: Chongqing and Hubei. Chongqing is the only province with poor performance for the other services. Note also that this province is famous for its local food (hotpot, for example), probably explaining the important performance increase for the meal activity. For Hubei, the regress for the meal activity is surprising as this province is also well-known for its local food.

Malmquist-Luenberger index. We present the averages, medians, and the number of improvements of the Malmquist-Luenberger index in Table 5, and the average per province of the Malmquist-Luenberger index for the period in Table 6. Our main conclusions based on the Luenberger index remain true when using the Malmquist-Luenberger index. This gives a robustness feature to our conclusions. In particular, the increasing importance of the meal activity is confirmed. Indeed, the averages, the medians, and the number of improvements are higher for that activity. Next, the status quo of the profit efficiency performances of the others activity is also confirmed: most of the indexes for that activity are equal or

Table 4: Luenberger index per province

Province	Overall	Room	Meal	Others
Anhui	0.50	0.31	0.19	0.00
Beijing	0.02	0.02	0.00	0.00
Chongqing	0.41	-0.12	0.70	-0.17
Fujian	0.45	0.31	0.14	0.00
Gansu	0.30	2.00	-2.40	0.70
Guangdong	-0.03	0.00	-0.02	-0.01
Guangxi	0.10	0.26	-0.16	0.00
Guizhou	0.21	-1.08	1.29	0.00
Hainan	0.54	0.54	0.00	0.00
Hebei	0.15	0.33	-0.17	0.00
Heilongjiang	0.33	0.33	0.00	0.00
Henan	0.19	0.00	0.19	0.00
Hubei	0.11	0.44	-0.47	0.14
Hunan	0.23	0.04	0.13	0.06
InnerMongolia	1.23	0.00	1.23	0.00
Jiangsu	0.10	0.00	0.10	0.00
Jiangxi	0.50	0.66	-0.17	0.00
Jilin	-0.16	0.00	-0.16	0.00
Liaoning	0.01	0.00	0.01	0.00
Ningxia	4.83	0.00	4.83	0.00
Qinghai	4.20	1.50	2.70	0.00
Shaanxi	0.66	-0.16	0.82	0.00
Shandong	0.06	0.02	0.04	0.00
Shanghai	0.04	0.04	0.00	0.00
Shanxi	-0.27	0.09	-0.37	0.00
Sichuan	0.12	0.00	0.12	0.00
Tianjin	-0.44	0.22	-0.80	0.14
Xinjiang	0.23	0.34	-0.11	0.00
Yunnan	-0.13	-0.14	0.00	0.01
Zhejiang	0.02	0.00	0.01	0.00
<i>Average</i>	0.48	0.20	0.26	0.03

very close to 1 (note that 1 is the benchmark value for the Malmquist-Luenberger index, while it is 0 for the Luenberger index, see Section 2 for more discussion). Afterwards, the important decrease in 2013-2015 is confirmed by the averages, the medians, and the number of improvements. Finally, most of our conclusions for the provinces are also confirmed.

Table 5: Averages, medians, improvements for the Malmquist-Luenberger index

Year	Overall	Room	Meal	Others
<i>Averages</i>				
2005-06	3.04	1.23	1.67	1.27
2006-07	1.44	1.71	1.18	0.98
2007-08	2.68	1.67	2.65	0.98
2008-09	1.27	1.13	1.31	1.01
2009-10	1.07	1.29	0.99	1.01
2010-11	1.30	1.18	1.25	1.00
2011-12	1.17	1.10	1.29	1.03
2012-13	1.50	1.52	1.06	1.01
2013-14	0.85	1.10	0.89	1.02
2014-15	0.95	1.10	0.97	1.00
<i>Medians</i>				
2005-06	1.04	1.00	1.01	1.00
2006-07	1.07	1.00	1.00	1.00
2007-08	1.12	1.00	1.04	1.00
2008-09	1.14	1.00	1.03	1.00
2009-10	1.07	1.00	1.00	1.00
2010-11	1.15	1.00	1.10	1.00
2011-12	1.04	1.00	1.01	1.00
2012-13	0.87	1.00	1.00	1.00
2013-14	0.88	1.00	1.00	1.00
2014-15	0.99	1.00	0.99	1.00
<i>Improvements</i>				
2005-06	21	7	16	3
2006-07	22	8	14	0
2007-08	19	8	15	0
2008-09	21	12	15	2
2009-10	19	14	8	1
2010-11	25	12	17	0
2011-12	20	4	15	1
2012-13	5	4	5	1
2013-14	5	5	1	1

Table 6: Malmquist-Luenberger per province

Province	Overall	Room	Meal	Others
Anhui	1.12	1.17	1.03	1.00
Beijing	1.02	1.02	1.00	1.00
Chongqing	1.40	1.04	2.31	0.95
Fujian	1.31	1.30	1.09	1.00
Gansu	1.04	2.97	0.95	1.59
Guangdong	0.98	1.01	0.99	0.99
Guangxi	1.03	1.22	1.03	1.00
Guizhou	1.45	1.36	2.28	1.00
Hainan	1.24	1.24	1.00	1.00
Hebei	1.10	1.33	1.06	1.00
Heilongjiang	1.04	1.04	1.00	1.00
Henan	1.41	1.00	1.41	1.00
Hubei	1.21	1.74	0.93	1.09
Hunan	1.20	1.11	1.32	1.05
InnerMongolia	1.17	1.00	1.17	1.00
Jiangsu	1.09	1.00	1.09	1.00
Jiangxi	1.08	1.29	0.97	1.00
Jilin	1.00	1.00	1.00	1.00
Liaoning	1.01	1.00	1.01	1.00
Ningxia	5.54	1.00	5.54	1.00
Qinghai	8.14	2.94	1.64	1.00
Shaanxi	1.12	0.95	1.24	1.00
Shandong	1.06	1.01	1.04	1.00
Shanghai	1.04	1.04	1.00	1.00
Shanxi	1.05	1.07	0.97	1.00
Sichuan	1.07	1.22	1.46	1.00
Tianjin	1.47	2.17	0.87	1.24
Xinjiang	1.05	1.60	1.27	1.00
Yunnan	1.08	1.22	1.00	1.06
Zhejiang	1.02	1.00	1.01	1.00
<i>Average</i>	1.52	1.30	1.32	1.03

4 Summary and discussion

We summarize the main findings for our empirical study in four points:

- Star-rated hotels are of high importance for the hotel and tourism sector in China. Indeed, they represent the main part of the total revenue of the hotel sector. These hotels have initiated a diversification process of their activities by slowly replacing their traditional room activity by the meal activity. Other activities represent around 15% of the total revenue for the period studied. This proportion is stable for the period.
- Profit maximization behaviour is observed for the star-rated hotels as important new investments have occurred for the sector, while the number of employees has been rationalized. The activity diversification process of the star-rated hotels also reveals their profit maximization behaviour.
- There is a profit performance progress for the Chinese star-rated hotel industry. This is mainly due to room and meal activities. Higher process occurred for the meal activity, revealing once more the willingness of the Chinese star-rated hotels to increase the importance of this activity. We find performance regresses during the economic crisis, and in the case of new important investments. This reveals the sensitivity of the Chinese star-rated hotel industry, and the need for policy interventions.
- The performances are volatile across regions. The leading regions include the most advanced economic provinces, and those with unique local tourism resources.

All in all, our results allow us to better understand the performance for every province by proposing performance indicators for each of the three activities. In other words, they represent additional valuable information about the star-rated hotel performances in China. Those detailed results can also be used by the managers and local governments to better manage and monitor the hotel industry in every province. In particular, for contexts of policy implementations and new investments. More traditional indexes do not give the option to investigate the performance change for each activity separately, but only at the aggregate or overall level, i.e. when pooling all activities together. It turns out that our new indexes can be used in various other tourism contexts. For example, for travel agencies offering package, transport, and accommodation. and for tourist attractions offering ticket, food and catering, recreation entertainment and shopping.

Moreover, our indexes are designed to take the economic objective of the hotels into account. More traditional indexes are based on a technical or engineering perspective (i.e. only the input-output combinations are considered) and thus ignore the economic objective

of the hotels. In practice, hotels have an economic objective, and ignoring this aspect of the hotel production process may create a bias for the empirical analysis. Also, existing indexes that take the economic objective of the hotels into consideration require observing the prices. It is not the case for our approach (see Section 2 and Appendix B).

We point out several limitations of the data used for our empirical study. Firstly, data are only provided by province and yearly. We could expect that more detailed data will have a direct impact on the results. Also, data are only given for two activities of the hotel. While these two activities are the main activities (around 85% of the total revenue) for the period, distinguishing between more than two activities could be of great interest for managers of the hotels, and also for policy makers. Finally, data are given when pooling all star-rated hotels together. We could expect that the number of stars will have a direct impact on the performance change. In a sense, distinguishing between the number of stars would imply taking (part of) the hotel heterogeneity into consideration.

5 Conclusion

Increasing attention has been given to the use of efficiency analysis to evaluate, manage, and monitor the hotel industry in China. Indeed, due to the vigorous economic development of the tourism industry in China, the number of star-rated hotels has rapidly increased in that country, making efficiency analysis more and more important, and, in some cases, vital. In this paper, we developed two indexes to conduct efficiency analysis over time: the profit Luenberger and Malmquist-Luenberger indexes. Our indexes have three desirable features. Firstly, no parametric assumption about the hotel production process is required. That is, our indexes are nonparametric in nature. Next, the indexes take the economic optimization behaviour of the hotels into consideration. We assume that the hotels are profit maximizers. Finally, our indexes take the multi-activity nature of the hotels into account. As a result, our indexes increase the discriminatory power and the realism of the efficiency analysis by providing profit efficiency change results for each activity, and by linking the inputs to each activity.

We applied our technique to the star-rated industry for 30 provinces over the period 2005-2015. Star-rated hotels represent the main portion of the hotel industry in China, and the majority of domestic and international investments occur in this type of hotel. Therefore, efficiency analysis of this industry is clearly of importance for managers of the hotels, and also for policy makers. Our empirical results revealed that the hotel industry as a whole has achieved a large improvement of their profit efficiency in the past decades, but this does not hold true for every activity. Also, we highlighted two particular efficiency changes in the hotel industry and patterns at the province level. These results could be used by managers

and policy makers to better target their investments and policy implementations, as they represent additional valuable information about the star-rated hotel industry performances.

References

- [1] Arbelo-Perez M., Arbelo A., Perez-Gomez P., 2017, "Impact of quality on estimations of hotel efficiency", *Tourism Management* 61, 200-208.
- [2] Assaf A. G., Agbola F. W., 2011a, "Modelling the performance of Australian hotels: a DEA double bootstrap approach", *Tourism Economics* 7(1), 73-89.
- [3] Assaf A. G., Agbola F. W., 2011b, "Total productivity in the Australian hotel industry: estimating and bootstrapping Malmquist indices", *Tourism Analysis* 16(3), 295-304.
- [4] Assaf A., Magnini V., 2012, "Accounting for customer satisfaction in measuring hotel efficiency: Evidence from the US hotel industry", *International Journal of Hospitality Management* 31, 642-647.
- [5] Atkinson S. E., Tsionas M. G., 2016, "Directional distance functions: Optimal endogenous directions", *Journal of Econometrics* 190, 301-314.
- [6] Barros C. P., 2005, "Measuring efficiency in the hotel sector", *Annals of Tourism Research* 32(2), 456-477.
- [7] Barros C. P., Alves F. P., 2004, "Productivity in the tourism industry", *International Advances in Economic Research* 10(3), 215-225.
- [8] Barros C. P., Botti L., Peypoch N., Robinot E., Solonandrasana B., 2011, "Performance of French destinations: Tourism attraction perspectives", *Tourism Management* 32(1), 141-146.
- [9] Barros C. P., Peypoch N., Solonandrasana B., 2009, "Efficiency and Productivity Growth in Hotel Industry", *International Journal of Tourism Research* 11(4), 389-402.
- [10] Botti L., Peypoch N., Robinot E., Solonandrasana B., 2009, "Tourism Destination Competitiveness: the French regions case", *European Journal of Tourism Research* 2, 5-24.
- [11] Boussemart J.-P., Crainich D., Leleu H., 2015, "A decomposition of profit loss under output price uncertainty.", *European Journal of Operational Research* 243(3), 1016-1027.
- [12] Chambers R. G., 2002, "Exact nonradial input, output, and productivity measurement", *Economic Theory* 20, 751-765.

- [13] Chambers R. G., Chung Y., Färe R., 1998, “Profit, directional distance functions, and Nerlovian efficiency”, *Journal of Optimization Theory and Applications* 98(2), 351-364.
- [14] Charnes A., Cooper W. W., Rhodes E., 1978, “Measuring the Efficiency of Decision Making Units”, *European Journal of Operational Research* 2, 429-444.
- [15] Chen C.-F., 2007, “Applying the stochastic frontier approach to measure hotel managerial efficiency in Taiwan”, *Tourism Management* 28(3), 696-702.
- [16] Cherchye L., De Rock B., Dierynck B., Roodhooft F., Sabbe J., 2013, “Opening the Black Box of Efficiency Measurement: Input Allocation in Multi-Output Settings”, *Operations Research* 61, 1148-1165.
- [17] Cherchye L., De Rock B., Walheer B., 2015, “Multi-output efficiency with good and bad outputs”, *European Journal of Operational Research* 240, 872-881.
- [18] Cherchye L., De Rock B., Walheer B., 2016, “Multi-Output Profit Efficiency And Directional Distance Functions”, *Omega* 61, 100-109.
- [19] Chung Y., Färe R., and Grosskopf S., 1997, “Productivity and undesirable outputs: a directional distance function approach”, *Journal of Environmental Management* 51, 229-240.
- [20] Cracolici M. F., Nijkamp P., Cuffaro M., 2007, “Efficiency and productivity of Italian tourist destinations: a quantitative estimation based on data envelopment analysis and the Malmquist method”, *Springer*..
- [21] Despic O., Despic M., Paradi J., 2007, “DEA-R: ratio-based comparative efficiency model, its mathematical relation to DEA and its use in applications”, *Journal of Productivity Analysis* 28, 33-44.
- [22] Ding J., Dong W., Liang L., Zhu J., 2017, “Goal congruence analysis in multi-Division Organizations with shared resources based on data envelopment analysis”, *European Journal of Operational Research* 263, 961-973.
- [23] Färe R., Grosskopf S., 2000, “Network DEA”, *Socio-Economic Planning Sciences* 34, 35-49.
- [24] Färe R., Grosskopf S., Whittaker G., 2007, “Network DEA”, *Modeling Data Irregularities and Structural Complexities in Data Envelopment Analysis*, J. Zhu and W. Cook, Eds. *Springer*.

- [25] Färe R., Pasurka C., Vardanyan M., 2017, “On endogenizing direction vectors in parametric directional distance function-based models”, *European Journal of Operational Research* 262, 361-369.
- [26] Färe R., Primont D., 1995, “Multi-output Production and Duality: Theory and Applications”, *Kluwer Academic Publishers* Boston.
- Agriculture”, *Journal of the American Statistical Association* 97, 432-442.
- [27] Fukuyama H., Weber W. L., 2008, “Profit inefficiency of Japanese securities firms”, *Journal of Applied Economics* 11, 281-303.
- [28] Hampf B., Kruger J., 2014, “Optimal Directions for Directional Distance Functions: An Exploration of Potential Reductions of Greenhouse Gases”, *American Journal of Agricultural Economics* 97, 920-938.
- [29] Hu J.-L., Chiu C.-N., Shieh, H.S., Huang C. H., 2010, “A stochastic cost efficiency analysis of international tourist hotels in Taiwan”, *International Journal of Hospitality Management* 29(1), 99-107.
- [30] Huang Y., Mesak H. I., Hsu M. K., Qu H., 2012, “Dynamic efficiency assessment of the Chinese hotel industry”, *Journal of Business Research* 65(1), 59-67.
- [31] Luenberger D. G., 1992, “New optimality principles for economic efficiency and equilibrium”, *Journal of Optimization Theory and Applications* 75, 221-264.
- [32] Madanoglu M., Ozdemir O., 2016, “Is more better? The relationship between meeting space capacity and hotel operating performance”, *Tourism Management* 52, 74-81.
- [33] Malmquist S., 1953, “Index numbers and indifference surfaces”, *Trabajos de Estadística* 4, 209-242.
- [34] Nehring K., Puppe C., 2004, “Modelling cost complementarities in terms of joint production”, *Journal of Economic Theory* 118, 252-264.
- [35] Nunez-Serrano J. A., Turrion, J., Velazquez F. J., 2014, “Are stars a good indicator of hotel quality? Assymetric information and regulatory heterogeneity in Spain”, *Tourism Management* 42, 77-87.
- [36] Oses N., Gerrikagoitia J. K., Alzua A., 2016, “Modelling and prediction of a destination?s monthly average daily rate and occupancy rate based on hotel room prices offered online”, *Tourism Economics* 22(6), 1380-1403.

- [37] Panzar J. C., Willig R. D., 1981, "Economies of Scope", *American Economic Review* 71(2), 268-272.
- [38] Peypoch N., 2007, "Productivite du Secteur Touristique Francais: Une comparaison inter-regionale", *Economie et Prevision, Programme National Persee* 177(1), 65-76.
- [39] Ratsimbanierana H., Sbai S., Stenger A. 2013, "Moroccan tourist portfolio efficiency with the mean-variance approach", *European Journal of Tourism Research* 6(2), 122-131.
- [40] Rodreguez-Algeciras A., Talon-Ballesteros P., 2017, "An empirical analysis of the effectiveness of hotel Revenue Management in five-star hotels in Barcelona, Spain", *Journal of Hospitality and Tourism Management* 32, 24-34.
- [41] Sahoo B. K., Mehdiloozad M., Tone K., 2014, "Cost, Revenue and Profit efficiency Measurement in DEA: A Directional Distance Function Approach", *European Journal of Operational Research* 237, 921-931.
- [42] Sahoo B. K., Tone K., 2013, "Non-parametric Measurement of Economies of Scale and Scope in a Non-competitive Environment with Price Uncertainty", *Omega* 41, 97-111.
- [43] Salerian J., Chan C., 2005, "Restricting multiple-output multiple-input DEA models by disaggregating the output-input vector", *Journal of Productivity Analysis* 24, 5-29.
- [44] Shyu J., Hung S. C., 2012, "The true managerial efficiency of international tourist hotels in Taiwan: three-stage data envelopment analysis", *The Service Industries Journal* 32(12), 1991-2004.
- [45] Silva M. C. A., 2017, "Output-specific inputs in DEA: An application to courts of justice in Portugal", *Omega*, forthcoming.
- [46] Sun J., Zhang J., Zhang J., Ma J., Zhang Y., 2015, "Total factor productivity assessment of tourism industry: evidence from China", *Asia Pacific Journal of Tourism Research* 20(3), 280-294.
- [47] Tone K., Tsutsui M., 2009, "Network DEA: A slacks-based measure approach", *European Journal of Operational Research* 197, 243-252.
- [48] Walheer B., 2016a, "A multi-sector nonparametric production-frontier analysis of the economic growth and the convergence of the European countries", *Pacific Economic Review*, 21(4), 498-524.

- [49] Walheer B., 2016b, “Growth and Convergence of the OECD countries: A Multi-Sector Production-Frontier Approach”, *European Journal of Operational Research*, 252, 665-675.
- [50] Walheer B., 2018a, “Aggregation of metafrontier technology gap ratios: the case of European sectors in 1995-2015”, *European Journal of Operational Research* doi.org/10.1016/j.ejor.2018.02.048
- [51] Walheer B., 2018b, “Cost Malmquist productivity index: an output-specific approach for group comparison”, *Journal of Productivity Analysis* 49(1), 79-94.
- [52] Walheer B., 2018c, “Disaggregation of the Cost Malmquist Productivity Index with joint and output-specific inputs”, *Omega* 75, 1-12.
- [53] Walheer B., 2018d, “Malmquist productivity index for multi-output producers: An application to electricity generation plants”, *Socio-Economic Planning Sciences* doi.org/10.1016/j.seps.2018.02.003
- [54] Walheer B., 2018e, “Scale efficiency for multi-output cost minimizing producers: the case of the US electricity plants”, *Energy Economics* 70, 26-36.
- [55] Wang F., Huang W.-T., Shang J.-K. 2006, “Measuring the cost efficiency of international tourist hotels in Taiwan ”, *Tourism Economics* 12, 65-85.
- [56] Yang Z.-S., Cai J.-M., 2016, “Do regional factors matter? Determinants of hotel industry performance in China”, *Tourism Management* 52, 242-253.
- [57] Yang Z.-S., Xia L., Cheng Z., 2017, “Performance of Chinese hotel segment markets: Efficiencies measure based on both endogenous and exogenous factors”, *Journal of Hospitality and Tourism Management* 32, 12-23.
- [58] Zhang H., Cheng Z.-D., 2014, “Efficiency Evaluation and Regional Difference Analysis of China Star Hotels Based on Data Envelopment Analysis”, *Resource Development & Market* 30(10), 1207-1212.
- [59] Zhang L.-J., Botti L., Petit S., 2016, “Destination performance: Introducing the utility function in the mean-variance space”, *Tourism Management* 52, 123-132.

Appendix A

Proof of Equation (8):

$$\begin{aligned}
PE_t(\mathbf{z}_t, \mathbf{p}_t, \mathbf{g}_{z_t}) &= \frac{\pi_t(\mathbf{p}_t) - \mathbf{p}'_t \mathbf{z}_t}{\mathbf{p}'_t \mathbf{g}_{z_t}}, \\
&= \frac{\sum_{m=1}^M \pi_t^m(\mathbf{p}_t^m) - \sum_{m=1}^M \mathbf{p}_t^{m'} \mathbf{z}_t^m}{\sum_{m=1}^M \mathbf{p}_t^{m'} \mathbf{g}_{z_t^m}}, \\
&= \frac{\sum_{m=1}^M (\pi_t^m(\mathbf{p}_t^m) - \mathbf{p}_t^{m'} \mathbf{z}_t^m)}{\sum_{m=1}^M \mathbf{p}_t^{m'} \mathbf{g}_{z_t^m}}, \\
&= \sum_{m=1}^M \frac{\pi_t^m(\mathbf{p}_t^m) - \mathbf{p}_t^{m'} \mathbf{z}_t^m}{\sum_{m=1}^M \mathbf{p}_t^{m'} \mathbf{g}_{z_t^m}}, \\
&= \sum_{m=1}^M \frac{\mathbf{p}_t^{m'} \mathbf{g}_{z_t^m}}{\mathbf{p}_t^{m'} \mathbf{g}_{z_t^m}} \times \frac{\pi_t^m(\mathbf{p}_t^m) - \mathbf{p}_t^{m'} \mathbf{z}_t^m}{\sum_{m=1}^M \mathbf{p}_t^{m'} \mathbf{g}_{z_t^m}}, \\
&= \sum_{m=1}^M \frac{\mathbf{p}_t^{m'} \mathbf{g}_{z_t^m}}{\sum_{m=1}^M \mathbf{p}_t^{m'} \mathbf{g}_{z_t^m}} \times \frac{\pi_t^m(\mathbf{p}_t^m) - \mathbf{p}_t^{m'} \mathbf{z}_t^m}{\mathbf{p}_t^{m'} \mathbf{g}_{z_t^m}}, \\
&= \sum_{m=1}^M \frac{\mathbf{p}_t^{m'} \mathbf{g}_{z_t^m}}{\mathbf{p}_t^{m'} \mathbf{g}_{z_t^m}} \times PE_t^m(\mathbf{z}_t^m, \mathbf{p}_t^m, \mathbf{g}_{z_t^m}).
\end{aligned}$$

Appendix B

Assume we observe K DMUs during T periods of time. For each DMU k and period t , we observe the netput vector \mathbf{z}_{kt}^m for every m . As discussed previously, we consider three cases: *Case 1*: the output-specific netput prices are observed, *Case 2*: only the netput prices are observed, and *Case 3*: no prices are observed. The corresponding data sets are given by S_1, S_2 and S_3 , respectively:

$$\begin{aligned}
S_1 &= \{(\mathbf{z}_{kt}^1, \dots, \mathbf{z}_{kt}^M, \mathbf{p}_{kt}^1, \dots, \mathbf{p}_{kt}^M) \mid k = 1, \dots, K; t = 1, \dots, T\}, \\
S_2 &= \{(\mathbf{z}_{kt}^1, \dots, \mathbf{z}_{kt}^M, \mathbf{p}_{kt}) \mid k = 1, \dots, K; t = 1, \dots, T\}, \\
S_3 &= \{(\mathbf{z}_{kt}^1, \dots, \mathbf{z}_{kt}^M) \mid k = 1, \dots, K; t = 1, \dots, T\}.
\end{aligned}$$

Case 1: the output-specific netput prices are observed. In that case, we can evaluate profit efficiency for every output individually. For every DMU operating at \mathbf{z}_b^m with the associated directional vector $\mathbf{g}_{z_b^m}$, the profit efficiency score $PE_c^m(\mathbf{z}_b^m, \mathbf{p}_c^m, \mathbf{g}_{z_b^m})$ for output

m and period $b, c = \{t, t + 1\}$ is obtained using the following linear program (*LP1*):

$$PE_c^m(\mathbf{z}_b^m, \mathbf{p}_c^m, \mathbf{g}_{\mathbf{z}_b^m}) = \min_{\pi_c^m \in \mathbb{R}} \frac{\pi_c^m - \mathbf{p}_c^{m'} \mathbf{z}_b^m}{\mathbf{p}_c^{m'} \mathbf{g}_{\mathbf{z}_b^m}}$$

$$\text{s.t. } \pi_c^m \geq \mathbf{p}_c^{m'} \mathbf{z}_{sb}^m, \text{ for all } s = 1, \dots, K.$$

In words, the constraint verifies whether more profit is reachable for the DMU under evaluation when comparing to the other DMUs. Note that (*LP1*) could be seen as a natural extension of the program of Varian (1984) for cost efficiency with one output. If the maximal profit π_c^m corresponds to the actual profit $\mathbf{p}_c^{m'} \mathbf{z}_b^m$, the program will give a profit efficiency score of 0. When it is not the case, the profit efficiency score will be greater than 0. The profit efficiency score $PE_c(\mathbf{z}_b, \mathbf{p}_c, \mathbf{g}_{\mathbf{z}_b})$ for period $b, c = \{t, t + 1\}$ is obtained as explained in (21). Note that, we can equivalently obtain $PE_c(\mathbf{z}_b, \mathbf{p}_c, \mathbf{g}_{\mathbf{z}_b})$ for period $b, c = \{t, t + 1\}$ by solving the following linear program (*LP2*):

$$PE_c(\mathbf{z}_b, \mathbf{p}_c, \mathbf{g}_{\mathbf{z}_b}) = \min_{\pi_c^m \in \mathbb{R} \ (m=1, \dots, M)} \frac{\sum_{m=1}^M \pi_c^m - \sum_{m=1}^M \mathbf{p}_c^{m'} \mathbf{z}_b^m}{\sum_{m=1}^M \mathbf{p}_c^{m'} \mathbf{g}_{\mathbf{z}_b^m}}$$

$$\text{s.t. } \text{For every output } m = 1, \dots, M, \text{ the following holds:}$$

$$\pi_c^m \geq \mathbf{p}_c^{m'} \mathbf{z}_{sb}^m, \text{ for all } s = 1, \dots, K.$$

In (*LP2*), profit efficiency is verified for each output separately, as we have one constraint for each output. As such, the output-specific efficiency scores $PE_c^m(\mathbf{z}_b^m, \mathbf{p}_c^m, \mathbf{g}_{\mathbf{z}_b^m})$, for $m = 1, \dots, M$, can also be recovered after solving (*LP2*). Therefore, solving (*LP1*) for every output is similar than solving (*LP2*). The choice between the two options depends on the sample considered (i.e. number of inputs, outputs, DMUs).

Case 2: only the netput prices are observed. In that case, given the dependence between the unobserved output-specific netput prices, we can no longer evaluate profit efficiency of each output separately (see (3) and (4)). Nevertheless, we can evaluate profit efficiency for the overall production process by solving a linear program. For every DMU, operating for each m at \mathbf{z}_b^m with the associated directional vector $\mathbf{g}_{\mathbf{z}_b^m}$, the profit efficiency score $PE_c(\mathbf{z}_b, \mathbf{p}_c, \mathbf{g}_{\mathbf{z}_b})$ for period $b, c = \{t, t + 1\}$ is obtained using the following linear program

(LP3):

$$\begin{aligned}
PE_c(\mathbf{z}_b, \mathbf{p}_c, \mathbf{g}_{\mathbf{z}_b}) &= \min_{\substack{\pi_c^m \in \mathbb{R} \quad (m=1, \dots, M), \\ \mathbf{p}_c^m \in \mathbb{R}_+^{1+N} \quad (m=1, \dots, M)}} \frac{\sum_{m=1}^M \pi_c^m - \sum_{m=1}^M \mathbf{p}_c^{m'} \mathbf{z}_b^m}{\sum_{m=1}^M \mathbf{p}_c^{m'} \mathbf{g}_{\mathbf{z}_b}^m} \\
\text{s.t.} \quad &\text{For every output } m = 1, \dots, M, \text{ the following holds:} \\
&\pi_c^m \geq \mathbf{p}_c^{m'} \mathbf{z}_{sb}^m, \text{ for all } s = 1, \dots, K, \\
&(\mathbf{p}_{x,c})_n = (\mathbf{p}_{x,c}^m)_n, \text{ if input } n \text{ is allocated,} \\
&(\mathbf{p}_{x,c})_n = \sum_{m=1}^M (\mathbf{p}_{x,c}^m)_n, \text{ if input } n \text{ is not allocated.}
\end{aligned}$$

(LP3) is similar to (LP2); the only difference is that the netput prices are also variables in that program. As such, the relationship between the netput and the output-specific netput prices are added in (LP3) to make sure that the computed output-specific netput prices fulfill the conditions discussed in (3) and (4). Note that extra constraints can be added in (LP3) to increase the realism of the computed prices (see, for example, Cherchye et al (2013) and Cherchye, De Rock, and Walheer (2016) for more discussion). Finally, note that, as pointed out for (LP2), the output-specific efficiency scores $PE_c^m(\mathbf{z}_b^m, \mathbf{p}_c^m, \mathbf{g}_{\mathbf{z}_b}^m)$, for $m = 1, \dots, M$, can also be recovered after solving (LP3).

Case 3: no input prices are observed. Attractively, in that case, the profit efficiency scores can also be obtained by a linear program. For every DMU, operating for every m at \mathbf{z}_b^m with the associated directional vector $\mathbf{g}_{\mathbf{z}_b}^m$, the profit efficiency score $PE_c(\mathbf{z}_b, \mathbf{p}_c, \mathbf{g}_{\mathbf{z}_b})$ for period $b, c = \{t, t+1\}$ is obtained using the following linear program (LP4):

$$\begin{aligned}
PE_c(\mathbf{z}_b, \mathbf{p}_c, \mathbf{g}_{\mathbf{z}_b}) &= \min_{\substack{\pi_c^m \in \mathbb{R} \quad (m=1, \dots, M), \\ \mathbf{p}_c^m \in \mathbb{R}_+^{1+N} \quad (m=1, \dots, M)}} \sum_{m=1}^M \pi_c^m - \sum_{m=1}^M \mathbf{p}_c^{m'} \mathbf{z}_b^m \\
\text{s.t.} \quad &\text{For every output } m = 1, \dots, M, \text{ the following holds:} \\
&\pi_c^m \geq \mathbf{p}_c^{m'} \mathbf{z}_{sb}^m, \text{ for all } s = 1, \dots, K, \\
&(\mathbf{p}_{x,c})_n = (\mathbf{p}_{x,c}^m)_n, \text{ if input } n \text{ is allocated,} \\
&(\mathbf{p}_{x,c})_n = \sum_{m=1}^M (\mathbf{p}_{x,c}^m)_n, \text{ if input } n \text{ is not allocated,} \\
&\sum_{m=1}^M \mathbf{p}_t^{m'} \mathbf{g}_{\mathbf{z}_t}^m = 1.
\end{aligned}$$

(LP4) looks similar to (LP3), but with an extra constraint and no denominator for the objective function. In fact, when the netput prices are not observed, variables appear at

both the numerator and denominator of the objective function, making the program non-linear. As solving non-linear programs is, in general, a difficult task, we make the program linear by setting the denominator to 1. This trick has been made popular by Charnes, Cooper, and Rhodes (1978) for nonparametric efficiency methods. Intuitively, when the netput prices are not observed, using any transformations will only rescale the prices, but has no impact on the objective function. The remark made for *(LP3)* about adding extra constraints to increase the realism of the computed prices also clearly applies here.