Disaggregation of the Cost Malmquist Productivity Index with joint and output-specific inputs*

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Abstract

In multi-output settings, different types of inputs are simultaneously used in the production process. On the one hand, some inputs are jointly used to produce all (or a subset of) the outputs. These inputs give rise to economies of scope, which constitutes a prime economic motivation to produce multiple outputs. On the other hand, some inputs are allocated to specific output productions. Using nonparametric output-specific modelling of the production process, we propose a new productivity index for cost minimizing producers in these multi-output settings. The new index takes the form of a cost Malmquist productivity index. The output-specific modelling of the production process naturally allows us to define output-specific cost Malmquist productivity indexes and to disaggregate the cost Malmquist productivity index in terms of output-specific cost efficiency measurements. We also tackle the issue of input price availability and explain how to extend the cost Malmquist productivity index with partial input price information or without assuming observation of the input prices. In the latter case, we establish a duality with a technical productivity index that takes the form of a Malmquist productivity index. The new indexes can be used to evaluate cost-productivity and productivity changes or can be fairly easily combined with existing extensions. We propose an application to the electricity plants.

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1 Introduction

Nonparametric efficiency analysis of production activities is a technique used to evaluate a Decision Making Unit (DMU) by comparing its input-output performance to that of other DMUs operating in a similar technological environment. Nonparametric efficiency analysis does not require any parametric/functional specifications of the production technology but rather reconstructs the production possibilities using the observed inputs and outputs and by imposing some technology Axioms (such as monotonicity, convexity, returns-to-scale). Efficiency is therefore measured in technical terms, i.e. as the distance to the reconstructed production possibilities. From an economic perspective, the nonparametric efficiency analysis methodology is rooted in the structural approach to modelling efficient production behaviour that was initiated by Afriat (1972), Hanoch and Rothschild (1972), Diewert and Parkan (1983) and Varian (1984). This approach starts from a structural model of efficient production behavior (such as a cost minimization, a profit maximization) and characterizes inefficiency in terms of deviation from this economic model. Refer to Fare, Grosskopf and Lovell (1994a), Cooper, Seiford and Zhu (2004), Cooper, Seiford and Tone (2007), Fried, Lovell and Schmidt (2008), and Cook and Seiford (2009) for reviews.

While standard nonparametric efficiency models have demonstrated their usefulness in detecting the inefficiency behaviours of DMUs leading to potential cost reductions/profit improvements, these models suffer from a lack of realism in multi-output settings. Indeed, standard nonparametric efficiency models consider that all the inputs simultaneously produce all the outputs (i.e. “black box”modelling) while in multi-output settings different types of inputs are simultaneously used to produce the outputs. On the one hand, some inputs are jointly used to produce all (or a subset of) the outputs (see Salerian and Chan (2005), Despic et al (2007), Cherchye et al (2008)). These inputs give rise to economies of scope that constitute a prime economic motivation to produce multiple outputs. Economies of scope, a term popularized by Panzar and Willig (1981), are present when it is less costly to produce multiple outputs within a firm rather than in several firms. On the other hand, some inputs are allocated to specific output productions (see Fare and Grosskopf (2000), Fare, Grosskopf and Whittaker (2007), Tone and Tsutsui (2009)).

All of above mentioned approaches try to enhance the realism of the nonparametric efficiency analysis by integrating information on the internal production structure.

As a consequence, these approaches have more discriminatory power (i.e. a greater ability to detect inefficiency behaviour) than standard techniques that do not use this information. Cherchye et al (2013, 2014, 2015, 2016) provide a unifying framework that is consistent with all these approaches. They suggest a new nonparametric model that takes the links between inputs and outputs into account. In particular, they model each output separately by its own production technology, while allowing for interdependence between the output-specific technologies. Cherchye et al (2013, 2014, 2015, 2016) consider both technical and economic efficiency measurements.

The Malmquist Productivity index (MPI) proposed by Caves, Christensen and Diewert (1982), who named it after Malmquist (1953), measures productivity changes between two or more periods. The MPI has several desirable features. On the one hand, the MPI is completely nonparametric since it is based on the distances to the reconstructed production possibilities obtained with the nonparametric efficiency analysis. It implies that only the input-output data are required. On the other hand, the MPI can be decomposed into different sources to better understand the causes of productivity change. Several decompositions have been suggested by Fare et al (1994a,b ,1997) and Ray and Desli (1997).

Since the initial definition of the MPI, several theoretical extensions have been proposed. Chen (2003) introduced a non-radial MPI; Chen and Ali (2004) provided a further discussion on its second component; Pastor and Lovell (2005) proposed a global MPI; Camanho and Dyson(2006) suggested using the MPI to compare groups; Zelenyuk (2006) and Mayer and Zelenyuk (2014b) developed aggregate indexes to compare groups; Yu (2007) provided a new decomposition of the MPI that measures capacity productivity change and variable input productivity change; Kao (2010) proposed a common-weight DEA model for the global MPI; Oh and Lee (2010) introduced a metafrontier approach of the MPI when the technologies of the DMUs are not the same; Portela and Thanassoulis (2010) explained how to use MPI with negative data; Pastor et al (2011) introduced a biennial MPI; Wang and Lan (2011) suggested a double frontier MPI; Kao and Hwang (2014) defined a multi-period MPI to capture the productivity change for a large period; Mayer and Zelenyuk (2014a) suggested an aggregate Malmquist productivity index measure that allows inputs to be reallocated within the group, and Yang et al (2015) introduced a factor-specific MPI based on common weights DEA.

All of the above mentioned theoretical extensions and the large numbers of applications clearly reveal the usefulness of the MPI both as a theoretical and a practical instrument. Nevertheless, the MPI is based on the technical formulation of efficiency (i.e. based on the distance to the reconstructed production possibilities) and therefore neglects the economic objective of the DMUs (such as cost minimization behaviour or profit maximization behaviour). Maniadakis and Thanassoulis (2004) filled this gap by proposing a cost Malmquist Productivity Index (CMPI) which has the same feature as the initial MPI (i.e. nonparametric in nature, a decomposition of the cost-productivity into different sources) but has the extra advantage of taking the cost minimization behaviour of the DMUs into account. However, the CMPI, contrary to the MPI, requires the observation of the input prices, which could be difficult to observe or to rely on in practice. Several extensions of the CMPI have also been proposed. Yang and Huang (2009) incorporated variable returns to scale into the decomposition of the CMPI. Tohidi, and Razavyan, Tohidnia (2012) extended the global MPI of Pastor and Lovell (2005) to the cost setting. Thanassoulis, Shiraz, and Maniadakis (2015) explained how to extend the method of Camanho and Dyson (2006), i.e. using the MPI to compare groups, in the cost setting. Huang and Juo (2015) extended the metafrontier MPI approach of Oh and Lee (2010) to the cost setting.

Building on the output-specific modelling of the production process suggested by Cherchye et al (2013, 2014, 2015, 2016), we propose a new productivity index for cost minimizing producers in multi-output settings. The new index takes the form of a CMPI. The output-specific modelling of the production process naturally allows us to define output-specific CMPI and to disaggregate the CMPI in terms of output-specific cost efficiency measurements. The disaggregation procedure also fits with the aggregation procedure introduced by Fare and Zelenyuk (2003, 2005, 2007), Fare, Grosskopf and Zelenyuk (2004), Zelenyuk (2006) and Mayer and Zelenyuk (2014b) which they developed to compare groups.

We also tackle the input price availability issue and explain how to define the CMPI with partial input price information or without assuming observation of the input prices. In the latter case, we establish a duality with a technical productivity index that takes the form of an MPI. The new CMPI and MPI can be used to evaluate cost-productivity and productivity changes or can be fairly easily combined with the extensions of those indexes cited above.

We use the new CMPI technique to study the cost-productivity change of US electricity plants that produce two types of electricity (non-renewable and renewable) using nameplate capacity (used as a proxy for total assets) and the quantity of fuel. It can be reasonably assumed that the electricity generated is exogenously defined, which means that the size of the electricity market (or number of consumers) falls beyond control of the electricity utilities, but the plants can still minimize their costs for a given output production.

The new CMPI technique offers several advantages in this context. Firstly, the two inputs are differently linked to the outputs. Nameplate capacity is used to produce both types of electricity while the quantity of fuel is only used to produce fossil electricity. As such, the new CMPI technique is particularly useful in this context since it recognizes the links between inputs and outputs. Secondly, the new CMPI technique provides cost-productivity results on each output. We believe that it is of particular interest as plants have been producing non-renewable electricity for decades while the production of renewable electricity has started quite recently. Consequently, one can expect different cost-productivity changes for each type of electricity. Moreover, it allows to better understand the changes in cost-productivity at the aggregate level. Finally, the price data for nameplate capacity are not available. Thus, the new CMPI technique that works with/without partial input price data is very attractive for that reason. All in all, it means that the new CMPI model better uses the available information contained in the data and provides more results than standard CMPI techniques (without making extra assumptions on any aspect of the production process).

Outline. The rest of the paper is structured as follows. In Section 2, we define the CMPI in the multi-output context, show its duality with a MPI when the prices are not observed, and explain how to compute the indexes in practice. In Section 3, we present the application to the US electricity plants. In Section 4, we conclude.

2 Methodology

We start by giving some necessary notations. We then define the CMPI in the multi-output context. Next, we establish a duality with a technical productivity index, which takes the form of a MPI, and finally, we explain how to compute the indexes in practice.

2.1 Preliminaries

We assume that DMUs use, at time t, P inputs, captured by the vector $\mathbf{x}_t \in \mathbb{R}^P_+$, to produce Q outputs, captured by the vector $\mathbf{y}_t \in \mathbb{R}^Q_+$. We assume that the input vector $\mathbf{x}_t$ can be allocated to each individual output y^q_t (notation for the q-th entry of $\mathbf{y}_t$) by distinguishing between three categories of inputs. The input allocation to outputs naturally allows us to characterize each output q by its own technology and to define output-specific cost minimization criterion.

Allocation of inputs. We consider three ways to allocate the input vector $\mathbf{x}_t$ to each individual output y^q_t. Firstly, *Output-specific* inputs, introduced by Cherchye et al (2008), that are allocated to individual outputs. Let $(\alpha^q_t)_p \in [0,1]$, with $\sum_{q=1}^{Q}(\alpha^q_t)_p = 1$, represents the fraction of the p-th output-specific input quantity that is allocated to output q at period t. Next, *Joint* inputs, introduced by Cherchye et al (2013), are simultaneously used in the production process of all the outputs. Consequently, these inputs give rise to economies of scope which constitute a prime economic motivation for DMUs to produce more than one output. Finally, *Sub-joint* inputs, introduced by Cherchye et al (2015), also figure as joint inputs but only for a subset of outputs. The sub-joint inputs also give rise, to a more limited extent, to economies of scope.

Following Cherchye et al (2015), we use $\mathbf{A}^q_t \in \mathbb{R}^P_+$ to denote the information vector that contains the links between output q and the inputs at time t. Specifically, $\mathbf{A}^q_t$ is defined as

$$(\mathbf{A}^q_t)_p = \begin{cases} 1 & \text{if input } p \text{ is joint or sub-joint and used to produce output } q, \\ (\alpha^q_t)_p & \text{if input } p \text{ is output-specific and used to produce output } q, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Interestingly, the information vector $\mathbf{A}^q_t$ allows us to define the input vector allocated to output y^q_t, denoted $\mathbf{x}^q_t \in \mathbb{R}^Q_+$. We directly obtain $\mathbf{x}^q_t = \mathbf{A}^q_t \odot \mathbf{x}_t$ (The symbol $\odot$ stands for the element-by-element, or Hadamard, product).

Input requirement sets. We define the technology of period t by means of input requirement sets. Attractively, the previous definition of the input vector $\mathbf{x}^q_t$ allows us to characterize each output q by its own input requirement set $I^q_t(y^q_t)$, defined as follows

$$I_t^q(y_t^q) = \{\mathbf{x}_t^q \in \mathbb{R}_+^P \mid \mathbf{x}_t^q \text{ can produce } y_t^q\}. \tag{2}$$

For each q, the set $I_t^q(y_t^q)$ contains all the combinations of output-specific, joint and sub-joint inputs (in $\mathbf{x}_t^q$) that can produce the output quantity y_t^q.

We assume that the sets $I_t^q(y_t^q)$ are consistent with the following technology Axioms.

Axiom T1 (observability means feasibility): $(\mathbf{y}_t, \mathbf{x}_t^1 \ldots, \mathbf{x}_t^Q)$ is observed $\implies \forall q$: $\mathbf{x}_t^q \in I_t^q(y_t^q)$.

Axiom T2 (nested input sets): $y_t^q \geq y_t^{q'} \implies I_t^q(y_t^q) \subseteq I_t^q(y_t^{q'})$.

Axiom T3 (monotone input sets): $\mathbf{x}_t^q \in I_t^q(y_t^q)$ and $\mathbf{x}_t^{q'} \geq \mathbf{x}_t^q \implies \mathbf{x}_t^{q'} \in I_t^q(y_t^q)$.

Axiom T4 (convex input sets): $\mathbf{x}_t^q \in I_t^q(y_t^q)$ and $\mathbf{x}_t^{q'} \in I_t^q(y_t^q) \implies \forall \lambda \in [0,1]$: $\lambda \mathbf{x}_t^q + (1-\lambda)\mathbf{x}_t^{q'} \in I_t^q(y_t^q)$.

Axiom T5 (constant returns-to-scale technologies): $\forall k \in \mathbb{R}_0^+$: $\mathbf{x}_t^q \in I_t^q(y_t^q) \implies k \times \mathbf{x}_t^q \in I_t^q(k \times y_t^q)$.

These five axioms are common to many popular nonparametric efficiency models and form an empirically attractive minimal set of assumptions. It is important to note that the CMPI could be defined without assuming Axioms *T3* and *T4* since imposing monotonicity and convexity does not alter the cost evaluation (see, for example, Varian (1984) and Tulkens (1993) for discussion). We impose these extra technology Axioms since they are required to establish the duality with the technical counterpart (i.e. the MPI). We refer to Section 2.3 for more details.

We consider a constant returns-to-scale setting since the CMPI (and the MPI) measures productivity correctly under this returns-to-scale assumption, even if the true technology does not exhibit constant returns-to-scale (see Fare et al (1994b ,1997) for the MPI and Maniadakis and Thanassoulis (2004) for the CMPI).[1] Also, under constant returns-to-scale, the indexes could be interpreted as a total factor productivity and decomposed into different sources.[2]

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[1]See also Ray and Desli (1997) and Yang and Huang (2009) which extends the works of Fare et al (1994b, 1997) and Maniadakis and Thanassoulis (2004) by considering variable returns-to-scale in their decomposition. At this point, it should be clear that Ray and Desli (1997) extend Fare et al (1994b, 1997) by considering a different decomposition for the technical change using variable returns-to-scale, and a scale efficiency change, but the Malmquist index is still computed under constant returns-to-scale.

[2]The decomposition has also been popularized by the papers of Kumar and Russell (2002),

At this point, it is important to note that in some cases the CMPI and the MPI cannot always be interpreted as a measure of total factor productivity. On the one hand, it requires that the technology is inversely homothetic (intuitively, it implies separability between inputs and outputs, see Fare et al (1996) for details) which could be a stringent condition. On the other hand, the CMPI and the MPI are not complete (intuitively, it means that they could not be written as a function of aggregate inputs and outputs). We refer to O'Donnell (2012) that highlights this issue and suggests indexes that satisfy this property. We point out that it is fairly easy to adapt the indexes of his paper to the output-specific setting.

Finally, we note that the new indexes suggested in this paper do not crucially depend on the constant returns-to-scale assumption. Indeed, any other returns-to-scale assumption could easily be assumed. In practice, it suffices to restrict the set of the function $\beta^q_{kb}(y^q_c)$. We refer to Section 2.4 for more details.

Input prices. We use $\mathbf{w}_t \in \mathbb{R}^P_+$ to denote the prices of the inputs $\mathbf{x}_t$. In the same vein, we use $\mathbf{w}^q_t \in \mathbb{R}^P_+$ to denote the prices of the output-specific inputs $\mathbf{x}^q_t$. Some important comments must be made about these prices. Firstly, these prices coincide with the aggregate prices for every output-specific inputs. Next, for joint (and sub-joint) inputs these prices must add up to the aggregate DMU-level prices. As explained in detail by Cherchye et al (2008), these output-specific prices have a similar interpretation as Lindahl prices for public goods. Specifically, Pareto efficient provision of public goods equally requires these Lindahl prices to sum up to the aggregate prices. Taking together, we obtain

$$(\mathbf{w}^q_t)_p = (\mathbf{w}_t)_p \text{ for } p \text{ an output-specific input,} \tag{3}$$

$$\sum_{q=1}^{Q} (\mathbf{w}^q_t)_p = (\mathbf{w}_t)_p \text{ for } p \text{ a joint (or sub-joint) input.} \tag{4}$$

Costs. We have now all the necessary notations to define the actual costs at time t. The output-specific costs are given for each output q at time t by $\mathbf{w}^{q'}_t \mathbf{x}^q_t$. By summing these output-specific costs, we obtain the (overall) cost at time t: $\sum_{q=1}^{Q} \mathbf{w}^{q'}_t \mathbf{x}^q_t = \mathbf{w}'_t \mathbf{x}_t$.

Henderson and Russell (2005), and Walheer (2016a, b) to study the growth and the convergence of countries.

2.2 Cost Malmquist productivity index

Using the output-specific modelling of the production process, we naturally start by defining the CMPI for each output q individually. Next, we define the CMPI for the aggregate output production. After, we show how to disaggregate the CMPI in terms of output-specific cost efficiency measurements. Finally, we tackle the input price availability issue.

In the following, we assume that, for each DMU at time t, the output and input vectors $\mathbf{y}_t$ and $\mathbf{x}_t$ are observed. Also, we assume that the allocation of the inputs to the outputs, captured by the vectors $\mathbf{A}_t^1, \ldots, \mathbf{A}_t^Q$, are observed. Therefore, using our previous notation, we observe the input vectors allocated to the outputs $\mathbf{x}_t^1, \ldots, \mathbf{x}_t^Q$. Firstly, we will assume that all the prices are observed, i.e. the input prices $\mathbf{w}_t$ and the output-specific prices $\mathbf{w}_t^1, \ldots, \mathbf{w}_t^Q$. This assumption will be relaxed afterwards.

Output-specific cost Malmquist productivity index. The starting point of the CMPI is the minimal cost for each output q. The minimal cost at time t for a specific output q is defined as follows

$$C_t^q(y_t^q, \mathbf{w}_t, \mathbf{w}_t^q) = \min_{\mathbf{x}^q \in I_t^q(y_t^q)} \mathbf{w}_t^{q'} \mathbf{x}^q \tag{5}$$

$C_t^q(y_t^q, \mathbf{w}_t, \mathbf{w}_t^q)$ gives the minimal cost to produce the output quantity y_t^q given the input prices $\mathbf{w}_t$, the output-specific input prices $\mathbf{w}_t^q$ and the technology at time t (as such the subscript t on C^q refer to the time period of the technology). Clearly, for each q: $C_t^q(y_t^q, \mathbf{w}_t, \mathbf{w}_t^q) \leq \mathbf{w}_t^{q'} \mathbf{x}_t^q$. If $C_t^q(y_t^q, \mathbf{w}_t, \mathbf{w}_t^q) = \mathbf{w}_t^{q'} \mathbf{x}_t^q$, output q is produced with minimal costs. $C_t^q(y_t^q, \mathbf{w}_t, \mathbf{w}_t^q) < \mathbf{w}_t^{q'} \mathbf{x}_t^q$ reflects potential cost savings on output q. The output-specific minimal costs $C_t^q(y_t^q, \mathbf{w}_t, \mathbf{w}_t^q)$ are independent since, at this stage, we assume that all the prices, i.e. the input prices $\mathbf{w}_t$ and the output-specific prices $\mathbf{w}_t^1, \ldots, \mathbf{w}_t^Q$, are observed. This assumption will be relaxed afterwards making the output-specific minimal costs interdependent.

A natural index of cost efficiency, suggested by Farrell (1957), is the ratio of the minimal to the actual cost. Adapting his definition to our multi-output setting, we obtain for each q:

$$CE_t^q(y_t^q, \mathbf{x}_t^q, \mathbf{w}_t, \mathbf{w}_t^q) = \frac{C_t^q(y_t^q, \mathbf{w}_t, \mathbf{w}_t^q)}{\mathbf{w}_t^{q'} \mathbf{x}_t^q}. \tag{6}$$

The output-specific cost efficiency $CE_t^q(y_t^q, \mathbf{x}_t^q, \mathbf{w}_t, \mathbf{w}_t^q)$ is situated between 0 and 1 with

1 meaning that output y_t^q is produced efficiently, i.e. with the minimal cost, at time t. Lower values reflect greater cost inefficiency and thus potential cost savings. Also, the output-specific cost efficiencies $CE_t^q(y_t^q, \mathbf{x}_t^q, \mathbf{w}_t, \mathbf{w}_t^q)$ are independent. As explained previously, this property comes from the assumption of observing all the prices. As such, these output-specific cost efficiencies could be evaluated independently. See Section 2.4 for more details.

The output-specific CMPI at time t to compare $(y_{t+1}^q, \mathbf{x}_{t+1}^q)$ and $(y_t^q, \mathbf{x}_t^q)$ is defined as the inverse of the ratio of cost efficiencies taking period t as the reference year for the technology

$$\begin{aligned} CMPI_t^q(y_t^q, y_{t+1}^q, \mathbf{x}_t^q, \mathbf{x}_{t+1}^q, \mathbf{w}_t, \mathbf{w}_t^1, \dots, \mathbf{w}_t^Q) &= \left(\frac{CE_t^q(y_{t+1}^q, \mathbf{x}_{t+1}^q, \mathbf{w}_t, \mathbf{w}_t^q)}{CE_t^q(y_t^q, \mathbf{x}_t^q, \mathbf{w}_t, \mathbf{w}_t^q)} \right)^{-1} \\ &= \frac{\mathbf{w}_t^{q'} \mathbf{x}_{t+1}^q / C_t^q(y_{t+1}^q, \mathbf{w}_t, \mathbf{w}_t^q)}{\mathbf{w}_t^{q'} \mathbf{x}_t^q / C_t^q(y_t^q, \mathbf{w}_t, \mathbf{w}_t^q)}. \end{aligned} \tag{7}$$

Similarly, the output-specific CMPI at period $t+1$ to compare $(y_{t+1}^q, \mathbf{x}_{t+1}^q)$ and $(y_t^q, \mathbf{x}_t^q)$ is defined by taking period $t+1$ as the reference year for technology

$$\begin{aligned} CMPI_{t+1}^q(y_t^q, y_{t+1}^q, \mathbf{x}_t^q, \mathbf{x}_{t+1}^q, \mathbf{w}_{t+1}, \mathbf{w}_{t+1}^1, \dots, \mathbf{w}_{t+1}^Q) &= \left(\frac{CE_{t+1}^q(y_{t+1}^q, \mathbf{x}_{t+1}^q, \mathbf{w}_{t+1}, \mathbf{w}_{t+1}^q)}{CE_{t+1}^q(y_t^q, \mathbf{x}_t^q, \mathbf{w}_{t+1}, \mathbf{w}_{t+1}^q)} \right)^{-1} \\ &= \frac{\mathbf{w}_{t+1}^{q'} \mathbf{x}_{t+1}^q / C_{t+1}^q(y_{t+1}^q, \mathbf{w}_{t+1}, \mathbf{w}_{t+1}^q)}{\mathbf{w}_{t+1}^{q'} \mathbf{x}_t^q / C_{t+1}^q(y_t^q, \mathbf{w}_{t+1}, \mathbf{w}_{t+1}^q)}. \end{aligned} \tag{8}$$

To avoid an arbitrary choice of the reference period for technology, the output-specific CMPI is defined as the geometric mean of the cost indexes taking t and $t+1$ as reference years for the technology (see Fare et al (1994b)):

$$\begin{aligned} & CMPI^q(y_t^q, y_{t+1}^q, \mathbf{x}_t^q, \mathbf{x}_{t+1}^q, \mathbf{w}_t, \mathbf{w}_{t+1}, \mathbf{w}_t^1, \dots, \mathbf{w}_t^Q, \mathbf{w}_{t+1}^1, \dots, \mathbf{w}_{t+1}^Q) = \\ & = \left[\left(\frac{CE_t^q(y_{t+1}^q, \mathbf{x}_{t+1}^q, \mathbf{w}_t, \mathbf{w}_t^q)}{CE_t^q(y_t^q, \mathbf{x}_t^q, \mathbf{w}_t, \mathbf{w}_t^q)} \times \frac{CE_{t+1}^q(y_{t+1}^q, \mathbf{x}_{t+1}^q, \mathbf{w}_{t+1}, \mathbf{w}_{t+1}^q)}{CE_{t+1}^q(y_t^q, \mathbf{x}_t^q, \mathbf{w}_{t+1}, \mathbf{w}_{t+1}^q)} \right)^{-1} \right]^{1/2}. \end{aligned} \tag{9}$$

The benchmark value for the output-specific CMPI is 1. An index value less than 1 implies a cost-productivity progress for output q, a value greater than 1 implies a cost-productivity regress for output q and a value of 1 indicates constant cost-productivity for output q.

Cost Malmquist productivity index. The output-specific CMPI provides cost-productivity information for each output q individually. In multi-output settings, it is also important to provide such information at the aggregate production level $\mathbf{y}_t$. As done previously for the output-specific CMPI, we start by defining the minimal costs. Attractively, as explained before, the minimal costs can be obtained by summing the output-specific minimal cost $C_t^q(y_t^q, \mathbf{w}_t, \mathbf{w}_t^q)$ over the Q outputs:

$$C_t(\mathbf{y}_t, \mathbf{w}_t, \mathbf{w}_t^1, \ldots, \mathbf{w}_t^Q) = \sum_{q=1}^{Q} C_t^q(y_t^q, \mathbf{w}_t, \mathbf{w}_t^q). \tag{10}$$

If each output is produced with the minimal costs, i.e. $C_t^q(y_t^q, \mathbf{w}_t, \mathbf{w}_t^q) = \mathbf{w}_t^{q'}\mathbf{x}_t^q$ for all q then $C_t(\mathbf{y}_t, \mathbf{w}_t, \mathbf{w}_t^1, \ldots, \mathbf{w}_t^Q) = \sum_{q=1}^{Q} \mathbf{w}_t^{q'}\mathbf{x}_t^q = \mathbf{w}_t^{'}\mathbf{x}_t$. That is, the minimal cost corresponds to the actual costs. Using the actual and minimal costs, we obtain the Farrell's (1957) index of cost efficiency at the aggregate level $\mathbf{y}_t$ defined as follows:

$$CE_t(\mathbf{y}_t, \mathbf{x}_t, \mathbf{w}_t, \mathbf{w}_t^1, \ldots, \mathbf{w}_t^Q) = \frac{C_t(\mathbf{y}_t, \mathbf{w}_t, \mathbf{w}_t^1, \ldots, \mathbf{w}_t^Q)}{\mathbf{w}_t^{'}\mathbf{x}_t} = \frac{\sum_{q=1}^{Q} C_t^q(y_t^q, \mathbf{w}_t, \mathbf{w}_t^q)}{\sum_{q=1}^{Q} \mathbf{w}_t^{q'}\mathbf{x}_t^q}. \tag{11}$$

The interpretation of the cost efficiency $CE_t(\mathbf{y}_t, \mathbf{x}_t, \mathbf{w}_t, \mathbf{w}_t^1, \ldots, \mathbf{w}_t^Q)$ is analogous to the output-specific efficiency $CE_t(y_t^q, \mathbf{x}_t^q, \mathbf{w}_t, \mathbf{w}_t^q)$. That is, the cost efficiency is situated between 0 and 1 with 1 meaning that each output $\mathbf{y}_t$ is produced efficiently at time t. Also, it is important to note that the cost efficiency $CE_t(\mathbf{y}_t, \mathbf{x}_t, \mathbf{w}_t, \mathbf{w}_t^1, \ldots, \mathbf{w}_t^Q)$ is completely defined with the output-specific minimal and actual costs (see (11)).

The CMPI comparing $(\mathbf{y}_{t+1}, \mathbf{x}_{t+1})$ and $(\mathbf{y}_t, \mathbf{x}_t)$ taking period t as the reference year for the technology is defined as follows:

$$\begin{aligned} CMPI_t(\mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{x}_{t+1}, \mathbf{w}_t, \mathbf{w}_t^1, \ldots, \mathbf{w}_t^Q) &= \left(\frac{CE_t(\mathbf{y}_{t+1}, \mathbf{x}_{t+1}, \mathbf{w}_t, \mathbf{w}_t^1, \ldots, \mathbf{w}_t^Q)}{CE_t(\mathbf{y}_t, \mathbf{x}_t, \mathbf{w}_t, \mathbf{w}_t^1, \ldots, \mathbf{w}_t^Q)} \right)^{-1} \\ &= \frac{\mathbf{w}_t^{'}\mathbf{x}_{t+1}/C_t(\mathbf{y}_{t+1}, \mathbf{w}_t, \mathbf{w}_t^1, \ldots, \mathbf{w}_t^Q)}{\mathbf{w}_t^{'}\mathbf{x}_t/C_t(\mathbf{y}_t, \mathbf{w}_t, \mathbf{w}_t^1, \ldots, \mathbf{w}_t^Q)}. \end{aligned} \tag{12}$$

and taking period $t+1$ as the reference year for the technology is defined as:

$$CMPI_{t+1}(\mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{x}_{t+1}, \mathbf{w}_{t+1}, \mathbf{w}^1_{t+1}, \ldots, \mathbf{w}^Q_{t+1}) = \left(\frac{CE_{t+1}(\mathbf{y}_{t+1}, \mathbf{x}_{t+1}, \mathbf{w}_{t+1}, \mathbf{w}^1_{t+1}, \ldots, \mathbf{w}^Q_{t+1})}{CE_{t+1}(\mathbf{y}_t, \mathbf{x}_t, \mathbf{w}_{t+1}, \mathbf{w}^1_{t+1}, \ldots, \mathbf{w}^Q_{t+1})} \right)^{-1}$$
$$= \frac{\mathbf{w}'_{t+1}\mathbf{x}_{t+1}/C_{t+1}(\mathbf{y}_{t+1}, \mathbf{w}_{t+1}, \mathbf{w}^1_{t+1}, \ldots, \mathbf{w}^Q_{t+1})}{\mathbf{w}'_{t+1}\mathbf{x}_t/C_{t+1}(\mathbf{y}_t, \mathbf{w}_{t+1}, \mathbf{w}^1_{t+1}, \ldots, \mathbf{w}^Q_{t+1})}. \tag{13}$$

Their geometric mean, to avoid an arbitrary choice of the reference year, is given by:

$$CMPI(\mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{x}_{t+1}, \mathbf{w}_t, \mathbf{w}_{t+1}, \mathbf{w}^1_t, \ldots, \mathbf{w}^Q_t, \mathbf{w}^1_{t+1}, \ldots, \mathbf{w}^Q_{t+1}) =$$
$$= \left[\left(\frac{CE_t(\mathbf{y}_{t+1}, \mathbf{x}_{t+1}, \mathbf{w}_t, \mathbf{w}^1_t, \ldots, \mathbf{w}^Q_t)}{CE_t(\mathbf{y}_t, \mathbf{x}_t, \mathbf{w}_t, \mathbf{w}^1_t, \ldots, \mathbf{w}^Q_t)} \times \frac{CE_{t+1}(\mathbf{y}_{t+1}, \mathbf{x}_{t+1}, \mathbf{w}_{t+1}, \mathbf{w}^1_{t+1}, \ldots, \mathbf{w}^Q_{t+1})}{CE_{t+1}(\mathbf{y}_t, \mathbf{x}_t, \mathbf{w}_{t+1}, \mathbf{w}^1_{t+1}, \ldots, \mathbf{w}^Q_{t+1})} \right)^{-1} \right]^{1/2}. \tag{14}$$

Once more, the interpretation is analogous to the output-specific CMPI but for the aggregate output production $\mathbf{y}_t$. That is, an index value less than 1 implies cost-productivity progress for the aggregate output production, a value greater than 1 implies cost-productivity regress for the aggregate output production and a value of 1 indicates constant cost-productivity for the aggregate output production. As a final remark, we would like to point out again that the CMPI solely depends on output-specific minimal and actual costs (see (11)).

Disaggregation of the Cost Malmquist productivity index. Following our previous discussion, it is now established that the CMPI for the aggregate output production only depends on output-specific minimal and actual costs (see (11)). Nevertheless, it would be more attractive to relate the CMPI to the output-specific cost efficiency measurements. Indeed, it would provide a disaggregate of the CMPI in terms of the output-specific cost efficiency measurements. Moreover, it would also facilitate the computational aspect (see Section 2.4). In particular, let F be the function that links the output-specific cost efficiency measurements to the CMPI

$$CMPI(\mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{x}_{t+1}, \mathbf{w}_t, \mathbf{w}_{t+1}, \mathbf{w}^1_t, \ldots, \mathbf{w}^Q_t, \mathbf{w}^1_{t+1}, \ldots, \mathbf{w}^Q_{t+1}) =$$
$$= F\left(CE^1_c(y^1_b, \mathbf{x}^1_b, \mathbf{w}_c, \mathbf{w}^1_c), \ldots, CE^Q_c(y^Q_b, \mathbf{x}^Q_b, , \mathbf{w}_c, \mathbf{w}^Q_c) \right), \text{ for } b, c = \{t, t+1\}. \tag{15}$$

The disaggregation is obtained with the following weights:

$$\alpha^q_{c,b}(\mathbf{x}_b, \mathbf{x}^q_b, \mathbf{w}_c, \mathbf{w}^q_c) = \frac{\mathbf{w}^{q\prime}_c \mathbf{x}^q_b}{\mathbf{w}'_c \mathbf{x}_b}, \text{ for } b, c = \{t, t+1\}. \tag{16}$$

A first observation is that the weights $\alpha^q_{c,b}(\mathbf{x}_b, \mathbf{x}^q_b, \mathbf{w}_c, \mathbf{w}^q_c)$ depend on the inputs $\mathbf{x}_b$ and their price $\mathbf{w}_b$, and on the output-specific inputs $\mathbf{x}^q_b$ and their price $\mathbf{w}^q_b$. This is not surprising since the goal is to aggregate output-specific cost efficiencies into a CMPI for the aggregate production process.

Next, the weights $\alpha^q_{c,b}(\mathbf{x}_b, \mathbf{x}^q_b, \mathbf{w}_c, \mathbf{w}^q_c)$ sum to unity:

$$\sum_{q=1}^{Q} \alpha^q_{c,b}(\mathbf{x}^q_b, \mathbf{x}^q_b, \mathbf{w}_c, \mathbf{w}^q_c) = \sum_{q=1}^{Q} \frac{\mathbf{w}^{q\prime}_c \mathbf{x}^q_b}{\mathbf{w}'_c \mathbf{x}_b} = \frac{\sum_{q=1}^{Q} \mathbf{w}^{q\prime}_c \mathbf{x}^q_b}{\mathbf{w}'_c \mathbf{x}_b} = \frac{\mathbf{w}'_c \mathbf{x}_b}{\mathbf{w}'_c \mathbf{x}_b} = 1. \tag{17}$$

Finally, the weights $\alpha^q_{c,b}(\mathbf{x}_b, \mathbf{x}^q_b, \mathbf{w}_c, \mathbf{w}^q_c)$ allow us to obtain the cost efficiency as a weighted sum of the output-specific cost efficiencies:

$$CE_c(\mathbf{y}_b, \mathbf{x}_b, \mathbf{w}_c, \mathbf{w}^1_c, \ldots, \mathbf{w}^Q_c) = \sum_{q=1}^{Q} \alpha^q_{c,b}(\mathbf{x}_b, \mathbf{x}^q_b, \mathbf{w}_c, \mathbf{w}^q_c) CE^q_c(y^q_b, \mathbf{x}^q_b, \mathbf{w}_c, \mathbf{w}^q_c), \text{ for } b, c = \{t, t+1\}. \tag{18}$$

As such the weights $\alpha^q_{c,b}(\mathbf{x}_b, \mathbf{x}^q_b, \mathbf{w}_c, \mathbf{w}^q_c)$ allow us to identify which output-specific cost efficiency contributes more to the cost efficiency at period b taking period c as the reference year for technology. Therefore, the weights could be interpreted as the share of the total budget that is allocated to output q at period b taking period c as the reference year for the technology.

These weights share a close relationship with existing weights in the literature. Firstly, they generalize the weights introduced by Cherchye et al (2013, 2016) to aggregate their output-specific cost/profit efficiency in their static model. Secondly, they are conceptually similar to the weights introduced by Fare and Zelenyuk (2003, 2005, 2007), Fare, Grosskopf and Zelenyuk (2004), Zelenyuk (2006) and Mayer and Zelenyuk (2014b) to aggregate efficiency to group efficiency.

Using the weights $\alpha^q_{c,b}(\mathbf{x}_b, \mathbf{x}^q_b, \mathbf{w}_c, \mathbf{w}^q_c)$ for $b, c = \{t, t+1\}$, we obtain the desired disaggregation of the CMPI

$$
\begin{aligned}
&CMPI(\mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{x}_{t+1}, \mathbf{w}_t, \mathbf{w}_{t+1}, \mathbf{w}_t^1, \ldots, \mathbf{w}_t^Q, \mathbf{w}_{t+1}^1, \ldots, \mathbf{w}_{t+1}^Q) = \\
&= \left[\left(\frac{\sum_{q=1}^Q \alpha_{t,t+1}^q(\mathbf{x}_{t+1}, \mathbf{x}_{t+1}^q, \mathbf{w}_t, \mathbf{w}_t^q)\, CE_t^q(y_{t+1}^q, \mathbf{x}_{t+1}^q, \mathbf{w}_t, \mathbf{w}_t^q)}{\sum_{q=1}^Q \alpha_{t,t}^q(\mathbf{x}_t, \mathbf{x}_t^q, \mathbf{w}_t, \mathbf{w}_t^q)\, CE_t^q(y_t^q, \mathbf{x}_t^q, \mathbf{w}_t, \mathbf{w}_t^q)} \right.\right. \\
&\times \left.\left. \frac{\sum_{q=1}^Q \alpha_{t+1,t+1}^q(\mathbf{x}_{t+1}, \mathbf{x}_{t+1}^q, \mathbf{w}_{t+1}, \mathbf{w}_{t+1}^q)\, CE_{t+1}^q(y_{t+1}^q, \mathbf{x}_{t+1}^q, \mathbf{w}_{t+1}, \mathbf{w}_{t+1}^q)}{\sum_{q=1}^Q \alpha_{t+1,t}^q(\mathbf{x}_t, \mathbf{x}_t^q, \mathbf{w}_{t+1}, \mathbf{w}_{t+1}^q)\, CE_{t+1}^q(y_t^q, \mathbf{x}_t^q, \mathbf{w}_{t+1}, \mathbf{w}_{t+1}^q)} \right)^{-1} \right]^{1/2} . \quad (19)
\end{aligned}
$$

Input price availability issue. As it is defined, the CMPI depends on both the input and output-specific input prices. In the following we relax these two assumptions. We also note that if partial information is available for the input price data, they can be used to increase the realism of the computed prices (as it is the case for many applications and in particular our application in Section 3). Let us first consider that the input prices are observed while the output-specific prices are not. In that case, we evaluate the DMUs in the best possible way, which gives the benefit of the doubt in the absence of true price information. In nonparametric technical efficiency models, the most favourable prices are referred to as multipliers (see, for example, Cooper et al (2007) and Cherchye et al (2007)). As such, evaluating the DMUs under the most favourable prices corresponds to the multiplier formulation of nonparametric technical efficiency models (see also our discussion in Section 2.3).

Therefore in the absence of true output-specific input price information, we choose the output-specific input prices that maximizes the minimal costs for $b, c = \{t, t+1\}$.

$$
\begin{aligned}
&C_c(\mathbf{y}_b, \mathbf{w}_c) = \sum_{q=1}^Q C_c^q(y_b^q, \mathbf{w}_c) = \\
&= \max_{\mathbf{w}_c^1, \ldots, \mathbf{w}_c^Q \in \mathbb{R}_+^P} \left\{ \sum_{q=1}^Q C_c^q(y_b^q, \mathbf{w}_c, \mathbf{w}_c^q) \;\middle|\; \begin{array}{l} (\mathbf{w}_c^q)_p = (\mathbf{w}_c)_p \text{ for } p \text{ an output-specific input,} \\ \sum_{q=1}^Q (\mathbf{w}_c^q)_p = (\mathbf{w}_c^q)_p \text{ for } p \text{ a joint (sub-joint) input.} \end{array} \right\} . \quad (20)
\end{aligned}
$$

Maximizing a cost function could seem counter-intuitive but this maximization reflects exactly the benefit of the doubt spirit of the methodology. Indeed, the minimal cost $C_c(\mathbf{y}_b, \mathbf{w}_c)$ selects the output-specific prices $\mathbf{w}_c^1, \ldots, \mathbf{w}_c^Q$ that maximize the minimal cost at time c, or in other words, the obtained prices can be interpreted as the most favourable prices for evaluating the joint (and sub-joint) inputs, i.e. the shadow prices. (These prices are only required to be strictly positive). As a consequence, the minimal cost $C_c(\mathbf{y}_b, \mathbf{w}_c)$ provides an upper bound for $C_c(\mathbf{y}_b, \mathbf{w}_c, \mathbf{w}_c^1, \ldots, \mathbf{w}_c^Q)$. Also, the output-specific minimal costs $C_c^q(y_b^q, \mathbf{w}_c)$ are interdependent since they depend on the input prices $\mathbf{w}_c$ (see (3) and (4)) that are not observed. Additionally, we also point out that other procedures (for example the one used in Fare and Zelenyuk (2007)) to obtain the minimal costs without knowing the price information could be used at this stage. We use the benefit of the doubt spirit since it allows us to obtain an interesting dual equivalence of our CMPI, as explained in detail in Section 2.3.

The cost efficiency measurements in the absence of true output-specific input price information are given by

$$CE_c^q(y_b^q, \mathbf{x}_b^q, \mathbf{w}_c) = \frac{C_c^q(y_b^q, \mathbf{w}_c)}{\mathbf{w}_c^{q'} \mathbf{x}_b^q}. \tag{21}$$

$$CE_c(\mathbf{y}_b, \mathbf{x}_b, \mathbf{w}_c) = \frac{C_c(\mathbf{y}_b, \mathbf{w}_c)}{\mathbf{w}_c' \mathbf{x}_b} = \frac{\sum_{q=1}^{Q} C_c^q(y_b^q, \mathbf{w}_c)}{\sum_{q=1}^{Q} \mathbf{w}_c^{q'} \mathbf{x}_b^q}. \tag{22}$$

Following the benefit of the doubt spirit, we have $CE_c^q(y_b^q, \mathbf{x}_b^q, \mathbf{w}_c) \geq CE_c^q(y_b^q, \mathbf{x}_b^q, \mathbf{w}_c, \mathbf{w}_c^q)$ and $CE_c(\mathbf{y}_b, \mathbf{x}_b, \mathbf{w}_c) \geq CE_c(\mathbf{y}_b, \mathbf{x}_b, \mathbf{w}_c, \mathbf{w}_c^1, \ldots, \mathbf{w}_c^Q)$. It implies that the cost efficiencies without observing the output-specific input prices are the upper bound of the cost efficiencies when these prices are observed. Also, the output-specific cost efficiencies $CE_c^q(y_b^q, \mathbf{x}_b^q, \mathbf{w}_c)$ are interdependent since, as mentioned previously, the output-specific minimal costs $C_c^q(y_b^q, \mathbf{w}_c)$ are interdependent. As a consequence, these output-cost efficiencies can not be evaluated separately. See Section 2.4 for more details.

The disaggregation of the CMPI without observing the output-specific input prices is therefore given by[3]

$$\begin{aligned} & CMPI(\mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{x}_{t+1}, \mathbf{w}_t, \mathbf{w}_{t+1}) = \\ & = \Bigg[\Bigg(\frac{\sum_{q=1}^{Q} \alpha_{t,t+1}^q(\mathbf{x}_{t+1}, \mathbf{x}_{t+1}^q, \mathbf{w}_t) \, CE_t^q(y_{t+1}, \mathbf{x}_{t+1}, \mathbf{w}_t)}{\sum_{q=1}^{Q} \alpha_{t,t}^q(\mathbf{x}_t, \mathbf{x}_t^q, \mathbf{w}_t) \, CE_t^q(y_t, \mathbf{x}_t, \mathbf{w}_t)} && (23) \\ & \times \frac{\sum_{q=1}^{Q} \alpha_{t+1,t+1}^q(\mathbf{x}_{t+1}, \mathbf{x}_{t+1}^q, \mathbf{w}_{t+1}) \, CE_{t+1}^q(y_{t+1}, \mathbf{x}_{t+1}, \mathbf{w}_{t+1})}{\sum_{q=1}^{Q} \alpha_{t+1,t}^q(\mathbf{x}_t, \mathbf{x}_t^q, \mathbf{w}_{t+1}) \, CE_{t+1}^q(y_t, \mathbf{x}_t, \mathbf{w}_{t+1})} \Bigg)^{-1} \Bigg]^{1/2}. && (24) \end{aligned}$$

[3]It is straightforward to define the output-specific CMPI by using $CE_c^q(y_b^q, \mathbf{x}_b^q, \mathbf{w}_c)$ instead of $CE_c^q(y_b^q, \mathbf{x}_b^q, \mathbf{w}_c, \mathbf{w}_c^q)$ in (9).

Note that, in this case, the weights $\alpha^q_{c,b}(\mathbf{x}_b, \mathbf{x}^q_b, \mathbf{w}_c)$ do not depend on the output-specific input prices $\mathbf{w}^q_c$, contrary to the weights defined before $\alpha^q_{c,b}(\mathbf{x}_b, \mathbf{x}^q_b, , \mathbf{w}_c, \mathbf{w}^q_c)$

Now, let us assume that neither the input prices nor the output-specific input prices are observed. In this case, it is still possible to evaluate the CMPI. In the same manner to that done previously, when the output-specific input prices are not observed, we suggest evaluating the DMUs in the best light by taking the most favourable input and output-specific input prices (i.e. the shadow prices). That is, at time $b, c = \{t, t+1\}$, we choose the input prices that maximize the minimal costs without observing the output-specific input prices

$$C_c(\mathbf{y}_b) = \sum_{q=1}^{Q} C^q_c(y^q_b) = \max_{\mathbf{w}_c \in \mathbb{R}^Q_+} \left\{ \sum_{q=1}^{Q} C^q_c(y^q_b, \mathbf{w}_c) \right\}. \tag{25}$$

Clearly, since no input prices are available, the constraints on the output-specific input prices in (20) are irrelevant. As such, the minimal cost with no price information can also be obtained from the initial minimal cost when all input prices are available

$$C_c(\mathbf{y}_b) = \sum_{q=1}^{Q} C^q_c(y^q_b) = \max_{\mathbf{w}^1_c, \ldots, \mathbf{w}^Q_c \in \mathbb{R}^Q_+} \left\{ \sum_{q=1}^{Q} C^q_c(y^q_b, \mathbf{w}_c, \mathbf{w}^q_c) \right\}. \tag{26}$$

In this definition, no constraints are put on the output-specific prices except that they are strictly positive. As such, the input prices are defined with the output-specific prices as explained in (3) and (4). Also, it is clear that $C_c(\mathbf{y}_b) \geq C_c(\mathbf{y}_b, \mathbf{w}_c)$ following the benefit of the doubt spirit. See also our previous discussion on the benefit of the doubt spirit when defining the minimal cost $C_c(\mathbf{y}_b, \mathbf{w}_c)$. Using the new definition of the minimal cost, we can define the cost efficiency measurements as follows:

$$CE^q_c(y^q_b, \mathbf{x}^q_b) = \frac{C^q_c(y^q_b)}{\mathbf{w}^{q'}_c \mathbf{x}^q_b} = \frac{C^q_c(y^q_b)}{\mathbf{w}^{q'}_c \mathbf{x}^q_b}. \tag{27}$$

$$CE_c(\mathbf{y}_b, \mathbf{x}_b) = \frac{C_c(\mathbf{y}_b)}{\mathbf{w}'_c \mathbf{x}_b} = \frac{\sum_{q=1}^{Q} C^q_c(y^q_b)}{\sum_{q=1}^{Q} \mathbf{w}^{q'}_c \mathbf{x}^q_b}. \tag{28}$$

Clearly, the remark made previously on the interdependence of the output-specific minimal costs and output-specific cost efficiencies when the output-specific input prices are not observed remains true when no input price information is available. As such, the evaluation of the $CE^q_c(y^q_b, \mathbf{x}^q_b)$s cannot be done separately. See Section 2.4

for more details.

Finally, the CMPI can be disaggregated as follows[4]

$$CMPI(\mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{x}_{t+1}) =$$
$$= \left[\left(\frac{\sum_{q=1}^{Q} \alpha_{t,t+1}^q(\mathbf{x}_{t+1}, \mathbf{x}_{t+1}^q) CE_t^q(y_{t+1}^q, \mathbf{x}_{t+1}^q)}{\sum_{q=1}^{Q} \alpha_{t,t}^q(\mathbf{x}_t, \mathbf{x}_t^q) CE_t^q(y_t^q, \mathbf{x}_t^q)} \times \frac{\sum_{q=1}^{Q} \alpha_{t+1,t+1}^q(\mathbf{x}_{t+1}, \mathbf{x}_{t+1}^q) CE_{t+1}^q(y_{t+1}^q, \mathbf{x}_{t+1}^q)}{\sum_{q=1}^{Q} \alpha_{t+1,t}^q(\mathbf{x}_t, \mathbf{x}_t^q) CE_{t+1}^q(y_t^q, \mathbf{x}_t^q)} \right)^{-1} \right]^{1/2}. \tag{29}$$

This establishes a CMPI and its disaggregation when only input-output data are available. We believe that it is particularly attractive since it still allows the consideration of cost minimization behaviour of the DMUs even if no input price data are available. We note that if partial information is available for the price data, they can be used to increase the realism of the computed prices (as is the case for many applications and in particular our application, see Section 3).

2.3 Malmquist productivity index

In this Section, we establish a duality for the CMPI when no price data are available with a technical productivity index that takes the form of an MPI. We start by defining technical efficiency in our multi-output contexts with different types of inputs. Then, we define the MPI. Finally, we explain the duality.

Technical Background When efficiency is evaluated, the boundaries of the output-specific input sets $I_t^q(y_t^q)$ are of interest. In this case, they are defined, for each output q, by the isoquants of the input sets $I_t^q(y_t^q)$

$$\text{Isoq}I_t^q(y_t^q) = \{\mathbf{x}_t^q \in I_t^q(y_t^q) \mid \text{for } \beta < 1, \beta\mathbf{x}_t^q \notin I_t^q(y_t^q)\}. \tag{30}$$

Thus $\mathbf{x}_t^q \in \text{Isoq}I_t^q(y_t^q)$ means that the inputs $\mathbf{x}_t^q$ constitute the minimal input quantities to produce the output quantity y_t^q. As such, $\text{Isoq}I_t^q(y_t^q)$ represents the technically efficient frontier of $I_t^q(y_t^q)$.

We evaluate input efficiency as the distance of the evaluated DMU's input vector

[4] It is easy to obtain the output-specific CMPI by using $CE_c^q(y_b^q, \mathbf{x}_b^q)$ instead of $CE_c^q(y_b^q, \mathbf{x}_b^q, \mathbf{w}_c, \mathbf{w}_c^q)$ in (9).

to the isoquants Isoq$I_t^q(y_t^q)$, which is defined as:

$$D_t(\mathbf{y}_t, \mathbf{x}_t) = \sup \left\{ \theta \mid \forall q : \left(\frac{\mathbf{x}_t^q}{\theta} \right) \in I_t^q(y_t^q) \right\}. \tag{31}$$

This is an adapted version of the standard input distance function introduced by Shephard (1953, 1970) to our multi-output setting. The distance is the reciprocal of an adapted Debreu (1951) $-$ Farrell (1957) input efficiency measurement. In particular, it is defined as:

$$TE_t(\mathbf{y}_t, \mathbf{x}_t) = \frac{1}{D_t(\mathbf{y}_t, \mathbf{x}_t)} = \inf\{\eta \mid \forall q : \eta \mathbf{x}_t^q \in I_t^q(y_t^q)\}. \tag{32}$$

In words, $TE_t(\mathbf{y}_t, \mathbf{x}_t)$ defines the maximal equiproportionate/radial input reduction (captured by $\eta(\mathbf{x}_t^1, \ldots, \mathbf{x}_t^Q)$) that still allows for producing the output $\mathbf{y}_t$. Generally, $TE_t(\mathbf{y}_t, \mathbf{x}_t)$ is situated between 0 and 1, and a lower value of $TE_t(\mathbf{y}_t, \mathbf{x}_t)$ indicates greater technical inefficiency.

Malmquist productivity index. The MPI to compare $(\mathbf{y}_{t+1}, \mathbf{x}_{t+1})$ and $(\mathbf{y}_t, \mathbf{x}_t)$ is the geometric mean of the distance ratios taking period t and $t+1$ as the year reference for the technology

$$MPI(\mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{x}_{t+1}) = \left[\frac{D_t(\mathbf{y}_{t+1}, \mathbf{x}_{t+1})}{D_t(\mathbf{y}_t, \mathbf{x}_t)} \times \frac{D_{t+1}(\mathbf{y}_{t+1}, \mathbf{x}_{t+1})}{D_{t+1}(\mathbf{y}_t, \mathbf{x}_t)} \right]^{1/2} \tag{33}$$

$$= \left[\left(\frac{TE_t(\mathbf{y}_{t+1}, \mathbf{x}_{t+1})}{TE_t(\mathbf{y}_t, \mathbf{x}_t)} \times \frac{TE_{t+1}(\mathbf{y}_{t+1}, \mathbf{x}_{t+1})}{TE_{t+1}(\mathbf{y}_t, \mathbf{x}_t)} \right)^{-1} \right]^{1/2}. \tag{34}$$

As for the CMPI, the benchmark value is 1. If the index is greater than 1, it implies that on average, the input levels $\mathbf{x}_{t+1}$ are further from the efficient boundary than the inputs $\mathbf{x}_t$ for securing the corresponding outputs, which implies a productivity regress. When the index is smaller than 1, it is the opposite, on average, the input levels $\mathbf{x}_t$ are further from the efficient boundary than the inputs $\mathbf{x}_{t+1}$ for securing the corresponding outputs, which implies a productivity progress. if the index equals one, it is the status quo.

The new MPI could be seen as a particular version of the MPI of Caves et al (1982) when the inputs are allocated to the outputs.[5] Clearly, in settings with one output (i.e.

[5]Also, in their paper, Caves et al (1982) do not consider the input set when defining their index

$Q = 1$), the two indexes coincide. When $Q > 1$, the new MPI offers the advantage to better detect (technical) inefficient behaviours (i.e. a more discriminatory power) due to the output-specific modelling. See also the application in Section 3 for a comparison between the two indexes.

Duality with the cost Malmquist productivity index. Attractively, the CMPI with no price information is dually equivalent to the MPI in the multi-output context

$$CMPI(\mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{x}_{t+1}) = MPI(\mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{x}_{t+1}). \tag{35}$$

This defines a specific dual interpretation of our CMPI in terms of an MPI. As discussed before, when no price information is available, we evaluate the cost efficiencies under the most favourable scenario, i.e. with the most favourable prices. These prices have a direct interpretation in terms of multipliers of the technical efficiency measurement. As such, there is an equivalence between the cost efficiency with no price information and the technical efficiency in our multi-output context. As a consequence, the CMPI and MPI are equal when no price information is available. Moreover, it is important to note that, in similar contexts, this duality has also been discussed in, for example, Zelenyuk (2006) and Mayer and Zelenyuk (2014b).

When prices are observed, the equality between the CMPI and the MPI does not hold anymore. Nevertheless, in that case, it is still possible to relate the two measures by introducing the notions of allocative efficiency change (AEC) and price effect (PE); see Maniadakis and Thanassoulis (2004). AEC indicates the extent to which the DMU catches up with the optimum input mix in light of the input prices. PE captures the changes in the inputs needed to produce certain output attributed to changes in relative input prices. We obtain:

$$\begin{aligned} &CMPI(\mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{x}_{t+1}, \mathbf{w}_t, \mathbf{w}_{t+1}, \mathbf{w}_t^1, \ldots, \mathbf{w}_t^Q, \mathbf{w}_{t+1}^1, \ldots, \mathbf{w}_{t+1}^Q) = \\ &= MPI(\mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{x}_{t+1}) \times \text{AEC} \times \text{PE}. \end{aligned} \tag{36}$$

As a consequence, when prices are not observed, the allocative efficiency change and the price effect are equal to unity. It is straightforward to define AEC and PE in our output-specific context given the definitions provided in Maniadakis and Thanassoulis

but instead the production possibility set. It is straightforward to adapt their definition to coincide with our modelling of the production process.

(2004). Attractively, these two new notions could also be disaggregated, as done for the CMPI, into output-specific measures. This disaggregation would be very similar to what Zelenyuk (2006) has done for the revenue efficiency when considering groups. Given the direct similarity and for the sake of compactness, we do not give the details here but refer to their papers for the definition and the disaggregation procedure.

2.4 Practical implementation

Assume we observe K DMUs during T periods of time. Assume also, we observe, for each period t, the output vector $\mathbf{y}_{kt}$, the input vector $\mathbf{x}_{kt}$, and the information vectors for every output $\mathbf{A}^1_{kt}, \ldots, \mathbf{A}^Q_{kt}$ yielding to the output-specific input vectors $\mathbf{x}^1_{kt}, \ldots, \mathbf{x}^Q_{kt}$ (where, for each q: $\mathbf{x}^q_{kt} = \mathbf{A}^q_{kt} \odot \mathbf{x}_{kt}$). As discussed previously, prices could be difficult to find and/or to rely on. As such, we consider three cases: (1) the input and output-specific input prices are observed, (2) only the input prices are observed, and (3) no input prices are observed. The corresponding data sets are given by D_1, D_2 and D_3 respectively.

$$\begin{aligned} D_1 = \{(\mathbf{y}_{kt}, \mathbf{x}^1_{kt}, \ldots, \mathbf{x}^Q_{kt}, \mathbf{w}_{kt}, \mathbf{w}^1_{kt}, \ldots, \mathbf{w}^Q_{kt}) \mid k = 1, \ldots, K; t = 1, \ldots, T\}, \\ D_2 = \{(\mathbf{y}_{kt}, \mathbf{x}^1_{kt}, \ldots, \mathbf{x}^Q_{kt}, \mathbf{w}_{kt}) \mid k = 1, \ldots, K; t = 1, \ldots, T\}, \\ D_3 = \{(\mathbf{y}_{kt}, \mathbf{x}^1_{kt}, \ldots, \mathbf{x}^Q_{kt}) \mid k = 1, \ldots, K; t = 1, \ldots, T\}. \end{aligned} \tag{37}$$

Clearly $D_3 \subset D_2 \subset D_1$ meaning that less data is available.

The CMPIs under the three different data sets (D_1, D_2 and D_3) cannot be computed directly because of their non-linear nature. Indeed, the CMPIs are ratios of unknown cost efficiency measures. In practice, it is difficult to optimize non-linear function. Fortunately, the cost efficiencies that composed the CMPIs can easily be computed using linear programs. Before giving the practical linear programs to compute these cost efficiency scores, we first need to explain how to deal in practice with the returns-to-scale assumption when assuming convexity of the input sets. The basic idea, introduced by Petersen (1990) and Bogetoft (1996), is to define a function that scales the outputs up or down to make two DMUs comparable. Extending their initial definition in our multi-output constant returns-to-scale context, we obtain the following function $\beta^q_{kb}(y^q_c)$:

$$\beta^q_{kb}(y^q_c) = \inf\left\{\beta \in \mathbb{R}^+_0 \mid \beta y^q_{kb} \geq y^q_c\right\}. \tag{38}$$

$\beta^q_{kb}(y^q_c)$ gives the factor by which the value of y^q_{kb} should be scaled to make it comparable with y^q_c. As explained in Section 2.1, the methodology does not crucially depend on the constant returns-to-scale assumption. Other returns-to-scale assumptions are easily implemented by replacing $\mathbb{R}^+_0$ by $(0,1], [1,\infty), \{1\}$ for the decreasing, increasing and variable returns to scale assumption, respectively.

As a final remark, the function $\beta^q_{kb}(y^q_c)$ is not present in most of the papers on MPI/CMPI. This could be explained by two main reasons. On the one hand, some authors prefer to use the production possibility sets, defined in our output-specific setting as $T^q_t = \{(\mathbf{x}^q_t, y^q_t) \in \mathbb{R}^{P+1}_+ \mid \mathbf{x}^q_t \text{ can produce } y^q_t\}$, instead of the input sets $I^q_t(y^q_t)$. In that case, they do not need the functions $\beta^q_{kb}(y^q_c)$. On the other hand, other authors define their concepts with respect to the input sets $I^q_t(y^q_t)$ but prefer to use a shortcut by defining the linear programs with respect to the technology set T^q_t. In that case, they are not really consistent since the linear programs and their definitions are based on two different technologies. All in all, we believe that in nonparametic settings, it is important to rely on a minimal number of assumptions on the technology. Consequently, in the cost setting, it is preferable to define and compute the concepts with respect to the input sets $I^q_t(y^q_t)$ since imposing convexity on these sets is less restrictive than imposing convexity on the technology sets T^q_t.

For every DMU, operating for each q at $(y^q_b, \mathbf{x}^q_b)$, the cost efficiency score is obtained at periods $b, c = \{t, t+1\}$ as follows:

- The data set D_1 is observed: **(LP-1)**

$$CE_c(\mathbf{y}_b, \mathbf{x}_b, \mathbf{w}_c, \mathbf{w}^q_c) = \max_{C^1_c, \ldots, C^Q_c \in \mathbb{R}_+} \frac{\sum_{q=1}^{Q} C^q_c}{\mathbf{w}'_c \mathbf{x}_b}$$
$$\text{s.t.} \qquad \forall q \in \{1, \ldots, Q\} : C^q_c \leq \mathbf{w}^{q'}_c \beta^q_{kb}(y^q_c)\mathbf{x}^q_{kb} \text{ for all } k : \beta^q_{kb}(y^q_c) y^q_{kb} \geq y^q_c.$$

- The data set D_2 is observed: **(LP-2)**

$$CE_c(\mathbf{y}_b, \mathbf{x}_b, \mathbf{w}_c) = \max_{\substack{C_c^1,\dots,C_c^Q \in \mathbb{R}_+ \\ \mathbf{w}_c^1,\dots,\mathbf{w}_c^Q \in \mathbb{R}_+^Q}} \frac{\sum_{q=1}^{Q} C_c^q}{\mathbf{w}_c^{'}\mathbf{x}_b}$$

$$\text{s.t.} \quad \begin{aligned} &\forall q \in \{1,\dots,Q\} : C_c^q \leq \mathbf{w}_c^{q'} \beta_{kb}^q(y_c^q)\mathbf{x}_{kb}^q \text{ for all } k : \beta_{kb}^q(y_c^q) y_{kb}^q \geq y_c^q, \\ &(\mathbf{w}_c^q)_p = (\mathbf{w}_c)_p \text{ for } p \text{ an output-specific input}, \\ &\sum_{q=1}^{Q} (\mathbf{w}_c^q)_p = (\mathbf{w}_c^q)_p \text{ for } p \text{ a joint (or sub-joint) input.} \end{aligned}$$

- The data set D_3 is observed: **(LP-3)**

$$CE_c(\mathbf{y}_b, \mathbf{x}_b) = \max_{\substack{C_c^1,\dots,C_c^Q \in \mathbb{R}_+ \\ \mathbf{w}_c^1,\dots,\mathbf{w}_c^Q \in \mathbb{R}_+^Q}} \sum_{q=1}^{Q} C_c^q$$

$$\text{s.t.} \quad \begin{aligned} &\forall q \in \{1,\dots,Q\} : C_c^q \leq \mathbf{w}_c^{q'} \beta_{kb}^q(y_c^q)\mathbf{x}_{kb}^q \text{ for all } k : \beta_{kb}^q(y_c^q) y_{kb}^q \geq y_c^q, \\ &\mathbf{w}_c^{'}\mathbf{x}_b = 1. \end{aligned}$$

We end this Section by providing a couple of remarks. Firstly, the $\beta_{kb}^q(y_c^q)$s are not obtained by the linear programs since they are computed before using equation (38). Secondly, as explained in Section 2.2, when the input prices and the output-specific prices are observed (i.e. D_1 is observed), the output-specific cost efficiencies could be obtained independently by solving Q linear programs **(LP-4)**

$$CE_c^q(y_b^q, \mathbf{x}_b^q, \mathbf{w}_c, \mathbf{w}_c^q) = \max_{C_c^q \in \mathbb{R}_+} \frac{C_c^q}{\mathbf{w}_c^{q'}\mathbf{x}_b^q}$$

$$\text{s.t.} \quad C_c^q \leq \mathbf{w}_c^{q'} \beta_{kb}^q(y_c^q)\mathbf{x}_{kb}^q \text{ for all } k : \beta_{kb}^q(y_c^q) y_{kb}^q \geq y_c^q.$$

The advantage of **(LP-1)** is that all the cost measures are obtained by solving only one linear program. When price information is missing (i.e. D_2 or D_3 are observed), the output-specific cost efficiencies are interdependent (this is captured by (4)). As such, they cannot be computed separately and are thus given a posteriori by **(LP-2)** and **(LP-3)**. This is also the case for the output-specific CMPI and the weights that are obtained when solving the linear programs in the three cases. We refer to Section 2.2 for more details. Next, the distance functions, the technical efficiency measurements and the MPI are directly obtained by **(LP-3)**, see our discussion on the duality at the end of Section 2.3. Subsequently, the objective in **(LP-3)** could seem strange since the cost efficiency score has no denominator. In fact, the objective in **(LP-3)** is non-linear. We make it linear by using Charnes and Cooper's (1978) normalization, i.e. the denominator ($\mathbf{w}_c^{'}\mathbf{x}_b$) equals unity. As such, the cost efficiency is given directly by the numerator. Finally, lower and/or upper bounds on the input and output-specific input prices can easily be added to increase the realism of the prices obtained in **(LP-2)** and **(LP-3)** since in those programs only strict positivity is imposed on these prices. See our application for an illustration of that possibility.

3 Application

Benchmarking the electricity plants is popular in the nonparametric efficiency literature. See, for example, Yaisawang and Klein (1994), Färe, Grosskopf, Noh and Weber (2005), Sarkis and Cordeiro (2012), Cherchye et al (2015) for analyses of US electric utilities; Goto and Tsutsui (1998), Hattori (2002) and Tone and Tsutsui (2007) for analyses of both Japanese and US electric utilities; and Korhonen and Luptacik (2004) for an analysis of European electric utilities.

All these studies systematically select nameplate capacity (used as a proxy for total assets) and the quantity of fuel used as the two main inputs, and the quantity of electricity generated as an output. This setting ignores the multi-output production profile of the plants. Indeed, to benefit from economies of scope of the joint inputs used, plants produce different types of electricities: renewable (e.g. wind, solar, geothermal) and non-renewable (e.g. coal, oil, gas). Moreover, the fuel quantity is clearly not used to produce renewable electricity. Nameplate capacity is thus a joint input and the fuel quantity is an output-specific input wholly allocated to non-renewable electricity production.[6]

We will investigate the cost-productivity progress/regress of the plants. It can reasonably be assumed that the electricity generated is exogenously defined, which means that the size of the electricity market (or number of consumers) falls beyond

[6] At this point, it is useful to note that additional inputs (such as the total number of employees, the generator capacity and the boiler capacity) and undesirable by-products of the use of fuel as an input (as SO_2, NO_x and CO_2 emissions) are also considered by some studies. For simplicity and to match with all previous studies, we do not consider these additional inputs and undesirable by-products.

control of the electric utilities but the plants can still minimize their costs for given renewable and non-renewable electricity production. The new CMPI technique offers several advantages in this context. Firstly, the two inputs are differently linked to the outputs. Nameplate capacity is used to produce both types of electricity while the quantity of fuel is only used to produce fossil electricity. As such, the new CMPI technique is particularly useful in this context since it recognizes the links between inputs and outputs. Secondly, the new CMPI technique provides cost-productivity results on each output. We believe that it is of particular interest in this context as plants have been producing non-renewable electricity for decades while the production of renewable electricity has started more recently. Consequently, one can expect a different cost-productivity for each type of electricity. Finally, the price data for nameplate capacity are not available. Thus, the new CMPI technique that works with/without partial input price data is very attractive for that reason. All in all, it means that the new CMPI model better uses the available information contained in the data and provides more results than standard CMPI techniques (without making extra assumptions on any aspect of the production process).

To present our empirical application, we first discuss the specificities of our set-up. Subsequently, we present the data and the results.

Input and output selection. Following our discussion above, we have a setting with two outputs ($Q = 2$): non-renewable electricity generated (y^1), and renewable electricity generated (y^2); and two inputs ($P = 2$): nameplate capacity (x^1) and quantity of fuel used (x^2). Nameplate capacity is a joint input and the fuel quantity is an output-specific input wholly allocated to the production of non-renewable electricity. Using the notation of Section 2.1, we have for each plant k at period t

$$\mathbf{y}_{kt} = \begin{bmatrix} y^1_{kt} \\ y^2_{kt} \end{bmatrix}, \mathbf{x}_{kt} = \begin{bmatrix} x^1_{kt} \\ x^2_{kt} \end{bmatrix}, \mathbf{A}^1_{kt} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } \mathbf{A}^2_{kt} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$\mathbf{x}^1_{kt} = \mathbf{A}^1_{kt} \odot \mathbf{x}_{kt} = \begin{bmatrix} x^1_{kt} \\ x^2_{kt} \end{bmatrix} \text{ and, similarly, } \mathbf{x}^2_{kt} = \mathbf{A}^2_{kt} \odot \mathbf{x}_{kt} = \begin{bmatrix} x^1_{kt} \\ 0 \end{bmatrix}. \quad (39)$$

Data and descriptive statistics. We have taken the data from the *eGRID* system developed by the US Environmental Protection Agency (EPA). *eGRID* stands for a comprehensive source of data on the environmental characteristics of all electricity utilities in the US. In particular, we use the two most recent databases: 2009 and 2012. It would be interesting to see how the results change when the data for 2015 is made available. We focus our analysis on plants that are multi-output producers in both years. That is, in this context, plants that produce renewable and non-renewable electricities. We end up with a sample of 277 plants. Tables 5 and 6 in the Appendix report the corresponding descriptive statistics for the different inputs and outputs.

As expected, multi-output plants produce, on average, more non-renewable than renewable electricity for both years. Also, the production of both types of electricity increase, on average, between 2009 and 2012. This is accompanied by an increase in the input quantities/the costs. The question is then whether the increase of the output production is greater than the increase in the costs. Finally, the sample heterogeneity increases between the two years, as shown by the standard deviations.

Unfortunately, no data for the input prices are reported by the *eGRID* system but we can use available information to construct lower and upper bounds to increase the realism of the computed prices. For fuel, the EPA provides prices at the state level. For the nameplate capacity, there is no price available but we can proxy the price by transforming the electricity price (available at the state level too) since nameplate capacity is defined as the maximal electricity generated by the plants during one year.[7] As the output-specific input prices are the share of the input prices borne by each output, we can also put some bounds on these prices by using the relative production share of each type of electricity. The lower and upper bounds for all the prices are found by defining a 50 % confidence interval centred in the proxies used. The results are very similar by using any percentage for the lower and upper bounds. The goal of these bounds is to avoid trivial and/or unrealistic prices (as too close to zero or too large).

Cost Malmquist productivity index results. We compute the cost efficiency scores for the two periods $b, c = \{2009, 2012\}$ using **(LP-3)** with the extra constraints on the input and output-specific input prices as explained previously. Table 1 contains the descriptive statistics for the CMPI at the aggregate and output-specific levels.

At the aggregate level ($CMPI$), on average, the plants have a cost-productivity

[7] The electricity price is given in cents per Kilowatt-hour. As such, the price in Dollars per Megawatt for nameplate capacity is obtained by multiplying by 10 and dividing by the number of hours (cent/kWh=cent/(kW*hr)=cent/((MW/1000)*hr)= 1000*cent/(MW*hr)=1000*(dollar/100)/(MW*hr)=10/hr (Dollar/MW)).

	Min	Mean	Median	Max	St. dev.	# imp	% imp
CMPI	0.23	1.25	1.00	2.45	2.06	138	49.82
$CMPI^1$	0.21	1.22	1.03	2.15	2.15	120	43.32
$CMPI^2$	0.15	1.05	0.99	2.18	2.23	146	52.71

Table 1: CMPI results

regress (index bigger than 1); but the median is close to 1 indicating the status quo between the two years. We also compute the numbers of plants that have a cost-productivity improvement (index smaller than 1). We obtain 138 plants or 49.82% of the sample. Finally, the minimum and the maximum indicate that some plants have huge cost-productivity regress and progress.

At the output-specific level ($CMPI^1$ and $CMPI^2$), on average, plants have a larger cost-productivity regress for non-renewable electricity than for the renewable electricity. The medians indicate that plants have a cost-productivity progress for renewable electricity and a cost-productivity regress on the non-renewable electricity. The number of plants that displays a progression also confirm this tendency: 146 plants (or 52.71% of the sample) on the production of renewable electricity and 120 (or 43.32% of the sample) for non-renewable electricity.

To illustrate the benefit of the output-specific CMPIs, we report in Table 2 the aggregate and output-specific CMPI for selected plants. Plant # 28 has an aggregate CMPI lower than 1 indicating a cost-productivity progress on the period 2009-2012. The output-specific CMPIs contrast this finding since the cost-productivity progress occurs only for renewable electricity. As such, this plant should try to reduce its cost for non-renewable electricity or concentrate its production on renewable electricity since it performs well on that output. For Plant # 59, the opposite holds, the aggregate cost-productivity progress is due to renewable electricity production. Plants # 112 and # 185 present both a cost-productivity regress (index bigger than 1) but the output-specific CMPIS reveal on which output they perform the worse: renewable electricity for Plant # 112 and non-renewable electricity for # 185. The same holds also for Plants # 234 and # 237, which both display a cost-productivity progress (index smaller than 1). Plant # 234 performs better on non-renewable electricity production while plant # 237 performs better on renewable electricity production.

	$CMPI$	$CMPI^1$	$CMPI^2$
Plant # 28	0.7239	1.2260	0.6691
Plant # 59	0.9689	0.7312	1.6072
Plant # 112	1.1666	1.0021	1.2354
Plant # 185	1.1964	1.7476	1.0376
Plant # 234	0.8785	0.7054	0.9125
Plant # 237	0.7547	0.9248	0.6485

Table 2: CMPI for selected plants

Comparison. We propose in the following a comparison between our methodology and the more standard CMPI introduced by Maniadakis and Thanassoulis (2004). We consider two different benchmark indexes: a slightly modified version of their index, and their index using our output-specific modelling of the technology. This comparison will highlight the benefit of our methodology and therefore gives credit to the output-specific setting with allocation of the inputs to the outputs.

The CMPI of Maniadakis and Thanassoulis (2004) does not consider the possibility of not observing the input prices and assumes convexity of the aggregate input set (i.e. $I_t(\mathbf{y}_t) = \{\mathbf{x}_t \in \mathbb{R}^P_+ \mid \mathbf{x}_t \text{ can produce } \mathbf{y}_t\}$). As such, to provide a fair comparison, we will adapt their methodology to give the option of not observing the input prices.[8] Giving the methodology established previously, this is relatively straightforward. We obtain our first benchmark index: $CMPI_{benchmark_1}$.[9]

Using the aggregate input set $I_t(\mathbf{y}_t)$ could also seem unfair. As such, we will consider a second benchmark CMPI by using the definition of Maniadakis and Thanassoulis (2004) with output-specific input sets (i.e. $I^q_t(y^q_t) = \{\mathbf{x}_t \in \mathbb{R}^P_+ \mid \mathbf{x}_t \text{ can produce } y^q_t\}$).

[8]In that case, the modified version (i.e. without assuming the observation of the input prices) of the CMPI of Maniadakis and Thanassoulis (2004) has a direct interpretation in technical terms. In fact, it is dually equivalent to a modified version (i.e. based on the input set) of the MPI of Caves et al (1982). See our discussion in Section 2.3.

[9]Each cost efficiency $CE_c(\mathbf{y}_b, \mathbf{x}_b)$ at periods $b, c = \{t, t+1\}$, composing the index, can be computed using the following linear program: **(LP-5)**

$$\begin{aligned} CE_c(\mathbf{y}_b, \mathbf{x}_b) = & \max_{\substack{C_c \in \mathbb{R}_+ \\ \mathbf{w}_c \in \mathbb{R}^Q_+}} C_c \\ \text{s.t.} \quad & C_c \leq \mathbf{w}_c^{'} \beta_{kb}(\mathbf{y}_c)\mathbf{x}_{kb} \text{ for all } k : \beta_{kb}(\mathbf{y}_c)\mathbf{y}_{kb} \geq \mathbf{y}_c, \\ & \mathbf{w}_c^{'}\mathbf{x}_b = 1. \end{aligned}$$

In that case, the function $\beta_{kb}(\mathbf{y}_c)$, which is similar to the definition given by Petersen (1990) and Bogetoft (1996), compare output vectors ($\mathbf{y}_c$ and $\mathbf{y}_{kb}$) instead of individual outputs (y_c and y_{kb}) as the function $\beta^q_{kb}(y_c)$. See our discussion before.

Then, it means that no input allocation is assumed, or in other words, all the inputs are joint. We obtain our second benchmark index: $CMPI_{benchmark_2}$.[10]

The results of the two benchmark CMPIs are presented in Table 3. Given that the first benchmark CMPI is based on the aggregate input set, we cannot compute the output-specific CMPIs for that index. For the second benchmark CMPI, based on the output-specific setting, the output-specific CMPI can be computed.

	Min	Mean	Median	Max	St. dev.	# imp	% imp
$CMPI_{benchmark_1}$	0.38	1.12	1.08	1.59	1.67	132	47.65
$CMPI_{benchmark_2}$	0.27	1.18	1.09	2.79	2.54	133	48.01
$CMPI^1_{benchmark_2}$	0.21	1.12	1.02	2.14	1.56	128	46.21
$CMPI^2_{benchmark_2}$	0.18	1.04	1.00	2.45	1.89	137	49.46

Table 3: Benchmark CMPI results

The results of the $CMPI_{benchmark_1}$ are consistent with what we have found previously for the new *CMPI* in Table 1. The main difference is that, as explained before, the output-specific CMPIs are not given for indexes defined with respect to aggregate technology sets. Next, the results of the $CMPI_{benchmark_2}$ are also consistent with what we found previously. The difference between the two types of electricity (captured by the output-specific CMPIs) is less pronounced, which could be due to the non-allocation of the inputs. In conclusion, the differences between the three CMPIs reflect the importance of taking into account the output-specific setting and the input allocation.

As a final remark, we want to emphasize that these indexes cannot be ranked since they are defined as ratios of cost efficiencies. Nevertheless, as proven by Cherchye et al (2013), cost efficiencies when considering an output-specific setting and the allocation

[10]Each cost efficiency $CE_c(\mathbf{y}_b, \mathbf{x}_b)$, composing the index, can be computed at periods $b, c = \{t, t+1\}$ using the following linear program **(LP-6)**

$$CE_c(\mathbf{y}_b, \mathbf{x}_b) = \max_{\substack{C_c^1,\ldots,C_c^Q \in \mathbb{R}_+ \\ \mathbf{w}_c^1,\ldots,\mathbf{w}_c^Q \in \mathbb{R}_+^Q}} \sum_{q=1}^{Q} C_c^q$$

$$\text{s.t.} \quad \forall q \in \{1,\ldots,Q\} : C_c^q \leq \mathbf{w}_c^{q'} \beta_{kb}^q (y_c^q) \mathbf{x}_{kb} \text{ for all } k : \beta_{kb}^q (y_c^q) y_{kb}^q \geq y_c^q,$$

$$\mathbf{w}_c^{'} \mathbf{x}_b = 1.$$

The only difference between the second benchmark CMPI and our index is that $\mathbf{x}_{kb}^q$ is replaced by $\mathbf{x}_{kb}$ in the constraint. It implies that the inputs are not allocated or, in other words, that all the inputs are joint.

of the inputs to outputs, are always smaller than in the case when they are not being considered. This highlights the greater ability of the methodology to detect inefficient behaviour (i.e. more discriminatory power).

Robustness checks. In this last part, we check if our previous conclusions are robust. Indeed, the CMPI is computed using all the plants and is therefore sensitive to outliers. The impact on the resulting efficiency analysis could be huge since outliers disproportionately, and perhaps misleadingly, influence the evaluation of the performance of the plants. Moreover, under constant returns-to-scale, the consequences can be accentuated. To solve that issue, robust efficiency measurements have been suggested: the order$-m$ efficiency measurement (where m can be viewed as a trimming parameter); and the order$-\alpha$ efficiency measurement (analogous to traditional quantile functions).[11] These measures use a sub-sample of the data set to compute the cost efficiencies. As such, they are less sensitive to outliers. We apply the order$-\alpha$ procedure to our data (using different values of α as suggested by Simar (2003) to detect the outliers). We found 17 outliers (6.14%) so this leaves us with with 260 plants remaining. We recalculate the computations using the new sample. The results are given in Table 4. Clearly, the descriptive statistics of the CMPIs change with the new sample but the previous conclusion remains valid. That is, the plants have on average at the aggregate level a cost-productivity regress (first row of Table 4) while on the output level, they perform on average better on renewable electricity (second and third rows of Table 4). The medians and the numbers of plants with cost-productivity improvement confirm these results.

	Min	Mean	Median	Max	St. dev.	# imp	% imp
$CMPI$	0.23	1.08	1.02	2.04	1.78	127	48.84
$CMPI^1$	0.18	1.21	1.08	2.02	1.85	111	42.69
$CMPI^2$	0.14	1.02	0.98	2.04	1.14	135	51.92
$CMPI_{benchmark_1}$	0.38	1.05	1.04	1.44	1.26	126	47.19
$CMPI_{benchmark_2}$	0.31	1.07	1.05	2.03	2.44	127	47.92
$CMPI^1_{benchmark_2}$	0.23	1.11	1.01	1.98	1.32	120	45.28
$CMPI^2_{benchmark_2}$	0.18	1.05	1.00	2.32	1.50	132	49.81

Table 4: CMPI results without outliers

[11]See, for example, Daraio and Simar (2007) for a large literature survey on these robust nonparametric efficiency measurements.

We also investigate for the presence of outliers when considering our two benchmark indexes. We found 10 outliers for our first benchmark index and 12 for the second one. The results in Table 4 (lines four to seven) confirm our previous findings., i.e. the differences between the three CMPIs reflect the importance of taking into account the output-specific setting and the input allocation.

Testing the differences. Our previous results highlight interesting differences between the performance of the plants in the production of renewable and the non-renewable electricity. In this last part, we formally test if these differences are statistically verified. We will make use of the two-sample Kolmogorov-Smirnov test (the null hypothesis states that the two samples are from the same continuous distribution) to check if there are differences across the whole distribution. We consider two cases: (1) the standard case where all observations are used, and (2) the robust case where outliers have been removed (see Table 4). The $p-$values of the Kolmogorov-Smirnov tests are available in the first line of Table 5. Clearly, these results confirm our previous findings, i.e. there is a performance difference between the two output productions. Removing the outliers does not affect the conclusion.

	Standard	Robust
$CMPI^1$ vs $CMPI^2$	0.000	0.000
$CMPI$ vs $CMPI_{benchmark_1}$	0.011	0.011
$CMPI$ vs $CMPI_{benchmark_2}$	0.021	0.019
$CMPI^1$ vs $CMPI^1_{benchmark_2}$	0.044	0.045
$CMPI^2$ vs $CMPI^2_{benchmark_2}$	0.045	0.051

Table 5: Kolmogorov-Smirnov $p-$values

We also will made use of the same test to check if the CMPIs based on the output-specific modelling were statistically different from the benchmark CMPIs. The $p-$values are given in lines two to five of Table 5. They reveal that there is a statistical difference between the indexes at the aggregate level and at the output-specific levels. This reveals that the output-specific modelling and the input allocation are important for this application.

At a general level, we believe that this empirical analysis convincingly demonstrates the usefulness of our methodology. Firstly, our methodology is based on a more realistic modelling of the production process by taking the links between inputs and outputs into account. Indeed, while nameplate capacity is used to produce both types of electricity, the quantity of fuel is only used to produce fossil electricity. Secondly, the output-specific CMPIs give extra valuable information on cost-productivity and allow us to better understand the change in cost-productivity at the aggregate level. Indeed, at the aggregate level, we found that the plants have, on average, a cost-productivity regress that could be explained by a worse performance for renewable electricity. Finally, prices for nameplate capacity are difficult to find. Thus, our CMPI method that also works with/without partial price data is very useful in this context. All in all, it means that our model uses the available information contained in the data more effectively and provides more results than standard CMPI techniques (without making extra assumptions on any aspect of the production process).

4 Conclusion

In this paper, we presented a new productivity index for cost minimizing producers in multi-output settings. The new index takes the form of a cost Malmquist productivity index (CMPI). The distinguished feature of the new CMPI is that it is based on a nonparametric output-specific modelling of the production process that recognizes the different types of inputs present in multi-output production processes. On the one hand, some inputs are jointly used to produce all (or a subset of) the outputs. These inputs give rise to economies of scope that form a prime economic motivation to produce multiple outputs. On the other hand, some inputs are allocated to specific output productions. Attractively, nonparametric output-specific modelling of the production process naturally allows us to define output-specific CMPIs and to disaggregate the CMPI in terms of output-specific cost efficiency measurements. We also tackled the issue of input price availability and explained how to extend the CMPI with partial price information or without assuming observation of the input prices. In the latter case, we established a duality with a technical productivity index, which takes the form of a Malmquist productivity index. The new indexes can be used to evaluate cost-productivity and productivity changes or can be fairly easily combined with existing extensions.

We proposed an application for electricity plants. The new CMPI technique offers several advantages in this context. Firstly, the two inputs are linked differently to the outputs. Nameplate capacity is used to produce both types of electricity while the quantity of fuel is only used to produce fossil electricity. As such, the new CMPI technique is particularly useful in this context since it recognizes the links between inputs and outputs. Secondly, the output-specific CMPIs give extra valuable information on the cost-productivity and allow to better understand the change in cost-productivity at the aggregate level. Finally, the price data for nameplate capacity are not available. Thus, the new CMPI technique that also works with partial price information or without input prices is very attractive for this reason. All in all, it means that our model uses the available information contained in the data more effectively and provides more results than standard CMPI techniques.

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Appendix

	Outputs		Inputs	
	Non-Renewable Energy (MWh)	Renewable Energy (MWh)	Nameplate Capacity (MW)	Fuel (MMBtu)
Min	1.29	6.73	1.2	49
Mean	157,640	131,490	76.51	2,815,800
Max	8,474,234	607,280	1,755	82,691,000
Std	648,870	136,650	159.19	6,599,800

Table 6: Descriptive statistics for the 277 plants in 2009

	Outputs		Inputs	
	Non-Renewable Energy (MWh)	Renewable Energy (MWh)	Nameplate Capacity (MW)	Fuel (MMBtu)
Min	0.80	5.2	1.2	31.52
Mean	163,770	134,100	78.5	2,931,400
Max	10,149,000	642,230	1,759	99,047,000
Std	729,750	143,560	168.80	7,178,400

Table 7: Descriptive statistics for the 277 plants in 2012