Cost Malmquist productivity index: an output-specific approach for group comparison*

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Abstract

The cost Malmquist productivity index (CMPI) has been proposed to capture the performance change of cost minimizing Decision Making Units (DMUs). Recently, two alternative uses of the CMPI have been suggested: (1) using the CMPI to compare groups of DMUs, and (2) using the CMPI to compare DMUs for each output separately. In this paper, we propose a new CMPI that combines both procedures. The resulting methodology provides group-specific indexes for each output separately, and therefore offers the option to identify the sources of cost performance change. We also define our index when input prices are not observed and establish, in that case, a duality with a new technical productivity index, which takes the form of a Malmquist productivity index. We illustrate our new methodology with a numerical example and an application to the US electricity plant districts.

Keywords: cost Malmquist productivity index; Malmquist productivity index; groups; cost efficiency; electricity.

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1 Introduction

The Malmquist productivity index (MPI) proposed by Caves, Christensen and Diewert (1982), who named it after Malmquist (1953), measures relative performance changes of Decision Making Units (DMUs) between two or more periods.[1] The popularity of this index could be explained by two main reasons.[2] One, only the data of the inputs and the outputs are required. Two, the MPI can be decomposed into different components (as efficiency change and technical change) to better understand the causes of relative performance change (see Färe, Grosskopf, and Norris (1994, 1997) and Ray and Desli (1997)).

Since its initial definition, the MPI has clearly revealed its usefulness as a practical instrument. Nevertheless, in context when DMUs have a specific economic optimization behaviour (such as cost minimization or profit maximization), the usefulness is clearly reduced. Indeed, the initial definition of the MPI neglects those economic optimization behaviours. Hopefully, it is still possible to define an MPI-based index in those contexts. In particular, Maniadakis and Thanassoulis (2004) have proposed an MPI-based index for cost minimizing DMUs. They named it the cost Malmquist Productivity Index (CMPI). Attractively, the CMPI has the same features as the initial MPI, and can be decomposed into different components to better understand the changes in cost performance.[3] However, the CMPI, contrary to the MPI, requires the observation of the input prices. Clearly, this could be a strong assumption for particular settings/applications.

Initially, the MPI and the CMPI were used to compare performances of DMUs over periods, but recent works have highlighted that these indexes could alternatively be used in other contexts. Indeed, they can also be used to compare performances of groups of DMUs, and to compare performances of DMUs for each individual output.

[1] Initially the MPI was interpreted as a productivity change index. Recently, there has been a debate on the validity of the MPI to measure productivity change. Indeed, this is only true under quite restrictive conditions. We refer to O'Donnell (2012) and Peyrache (2014) who highlight this issue. Therefore, in the following, we interpret the MPI as a relative performance index (this is also the interpretation used by Grosskopf (2003))

[2] See, for example, for extensions: Chen (2003), Chen and Ali (2004), Pastor and Lovell (2005), Zelenyuk (2006), Yu (2007), Kao (2010), Oh and Lee (2010), Portela and Thanassoulis (2010), Pastor, Asmild , and Lovell (2011), Wang and Lan (2011), Kao and Hwang (2014), Mayer and Zelenyuk (2014), Yang et al (2015).

[3] See, for example, for extensions: Yang and Huang (2009), Tohidi, and Razavyan, Tohidnia (2012), and Huang and Juo (2015).

Using the indexes to compare performance of groups of DMUs was first discussed by Camanho and Dyson (2006). Their methodology is directly connected to the initial definition of the MPI of Caves, Christensen and Diewert (1982). Indeed, their methodology only requires input and output data, and their MPI can be decomposed into different components. Next, Thanassoulis, Shiraz, Maniadakis (2015) have extended the procedure of Camanho and Dyson (2006) for the CMPI. Their new index also has the same feature as the initial CMPI of Maniadakis and Thanassoulis (2004). Using the CMPI to compare performance of DMUs over periods for each individual output can be attributed to Walheer (2017). This author has showed how to disaggregate the CMPI to obtain output-specific counterparts. As in the context of groups of DMUs, the CMPI in the output-specific context is directly connected to its initial definition. All in all, these two extensions only provide another use of the initial indexes, without altering their definitions and their desirable features.

In this paper, we show how to put together the two recent extensions of Thanassoulis, Shiraz, Maniadakis (2015) and Walheer (2017) to obtain a new CMPI. Attractively, the new CMPI allows group-specific results to be provided for each output separately. We believe that this possibility is very attractive in the group context, since it offers the option to determine which output groups perform better/worse, or in other words, it gives the option to identify the source(s) of cost performance change. As a consequence, the discriminatory power of the new CMPI is high, while keeping the same advantages as those of the two initial extensions. As, for the initial definition of the CMPI, our new CMPI is based on the assumption of observing the input prices. In some settings/applications, this assumption could be seen as too strong or too restrictive. As such, we propose an alternative definition of our new CMPI without assuming observation of the input prices. Interestingly, in that case, we establish a duality with a new technical productivity index that takes the form of an MPI. This new MPI could be seen as a combination of the methods of Camanho and Dyson (2006) and Walheer (2017) when prices are not observed.

The rest of the paper is structured as follows. In Section 2, we start by giving some necessary notations. In Section 3, we define the CMPI in the group context, and show its duality with an MPI when the prices are not observed. In Section 4, we present an illustration of our methodology for US electricity plant districts. In Section 5, we conclude.

2 Preliminaries

Suppose, we observe n DMUs split into two groups: A and B. In group A, each DMU $t \in \{1, \ldots, n_A\}$ uses P inputs, captured by $\mathbf{x}_t^A \in \mathbb{R}_+^P$, to produce Q outputs, captured by $\mathbf{y}_t^A \in \mathbb{R}_+^Q$. Similarly, in group B, each DMU $t \in \{1, \ldots, n_B\}$, uses $\mathbf{x}_t^B \in \mathbb{R}_+^P$ to produce $\mathbf{y}_t^B \in \mathbb{R}_+^Q$. Clearly we have $n_A + n_B = n$. The distinguishing feature of our approach is to model each output separately by its own technology. As such, we can naturally define the cost minimization condition for each output individually. Attractively, by modelling each output separately, we can also naturally allocate the inputs to the output-specific production processes. In this Section, we start by discussing the allocation of inputs to outputs. Next, we define the technology, the prices, and the costs. Finally, we propose an illustration. Below, we give all the necessary notations for a DMU t in group A. It is straightforward to do the same for a DMU t in group B. In fact, it suffices to change A by B in the following.

Allocation of the inputs. We consider that the inputs could be used differently to produce the outputs. Indeed, some inputs could jointly be used to produce all the outputs while others could be allocated to specific output productions. As such, we will use $\mathbf{x}_t^{A,q} \in \mathbb{R}_+^P$ to denote the inputs used to produce the q-th entry of $\mathbf{y}_t^A$ (denoted $y_t^{A,q}$ after). The setting we consider here is very general and provides a unifying framework that is consistent with models integrating information on the internal production structure and also with more standard models.[4] Therefore, the proposed methodology could be used in more standard contexts (i.e. when allocation of inputs do not exist), but also when inputs are allocated to outputs.

Attractively, the output-specific input vectors $\mathbf{x}_t^{A,q}$ can easily be related to the aggregate input vectors $\mathbf{x}_t^A$ using information vectors. In fact, we have that $\mathbf{x}_t^{A,q} = \mathbf{V}_t^{A,q} \odot \mathbf{x}_t^A$, where $\mathbf{V}_t^{A,q} \in \mathbb{R}_+^P$ denotes the information vector that contains the links between output q and the inputs for DMU t in group A. Specifically, $\mathbf{V}_t^{A,q}$ is defined

[4] Refer to Färe and Grosskopf (2000), Salerian and Chan (2005), Despic, Despic, and Paradi (2007), Färe, Grosskopf and Whittaker (2007), Cherchye De Rock, and Vermeulen (2008), Tone and Tsutsui (2009), Cherchye et al (2013), Cherchye, De Rock, and Walheer (2015, 2016), and Walheer (2016a, b, 2017) for related works on the input allocation.

as:

$$(\mathbf{V}_t^{A,q})_p = \begin{cases} 1 & \text{if input } p \text{ is jointly used to produce all the outputs,} \\ a_{pt}^q & \text{if input } p \text{ is allocated to output } q, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

Clearly, we have by definition for each input p: $\sum_{q=1}^{Q} a_{pt}^q = 1$. At this point, it should be clear that we assume, in the following, that the information vectors are observed. This is not a strong assumption for many settings, and this assumption could be relaxed if needed. See Section 3 for more detail.

Define the technology. As we are interested by the cost side of the production process, it is enough to define the technology by means of input requirement sets. Attractively, the previous definition of the input vector $\mathbf{x}_t^{A,q}$ allows us to naturally characterize each output q by its own input requirement set $I_t^{A,q}(y_t^{A,q})$, defined as follows:

$$I_t^{A,q}(y_t^{A,q}) = \{\mathbf{x}^{A,q} \in \mathbb{R}_+^P \mid \mathbf{x}^{A,q} \text{ can produce } y_t^{A,q}\}. \quad (2)$$

The sets $I_t^{A,q}(y_t^{A,q})$ contain the output-specific inputs $\mathbf{x}^{A,q}$ that can produce the output quantity $y_t^{A,q}$. We adopt a nonparametric approach by reconstructing the input set using the data without making specific assumptions in terms of production function.[5] In practice, some general production axioms are imposed to avoid a trivial reconstruction. We assume that the sets are nested, monotone, convex, and satisfy variable returns-to-scale. These production axioms are popular to many nonparametric models and form an attractive minimal set of axioms.[6] As a final remark, note that other estimation methods could be used at this stage. Our methodology is not dependent on the nonparametric approach used here; other nonparametric or parametric approaches could be used.

[5] Refer to Fare, Grosskopf and Lovell (1994), Cooper, Seiford and Zhu (2004), Cooper, Seiford and Tone (2007), Fried, Lovell and Schmidt (2008), and Cook and Seiford (2009) for reviews.

[6] See Cherchye et al (2015) for a formal definition of those axioms in a similar context. Note that imposing monotonicity and convexity does not alter the cost evaluation (see, for example, Varian (1984) and Tulkens (1993) for discussion). We impose these extra technology axioms since they are required to establish the duality of our CMPI with the technical counterpart (i.e. the MPI). Finally, note that the CMPI could be defined under alternative returns-to-scale assumptions.

Prices and costs. We use $\mathbf{w}_t^A \in \mathbb{R}_+^P$ to denote the prices of the inputs $\mathbf{x}_t^A$, and $\mathbf{w}_t^{A,q} \in \mathbb{R}_+^P$ to denote the prices of the output-specific inputs $\mathbf{x}_t^{A,q}$. Our previous definition of the information vector $\mathbf{V}_t^{A,q}$ implies some links between these prices. In fact, they coincide with the aggregate prices for every allocated input, and for jointly used inputs, they must add up to the aggregate DMU-level prices.[7] We obtain the two following relationships:

$$(\mathbf{w}_t^{A,q})_p = (\mathbf{w}_t^A)_p \text{ for } p \text{ an input allocated to the outputs.} \quad (3)$$

$$\sum_{q=1}^{Q}(\mathbf{w}_t^{A,q})_p = (\mathbf{w}_t^A)_p \text{ for } p \text{ an input jointly used to produce all the outputs.} \quad (4)$$

As such, when the output-specific input prices are not observed, they can be recovered using (3) and (4). Refer to Section 3 for more discussions on the practical implementations of those concepts. Also, note that $(\mathbf{w}_t^{A,q})_p$ refers to the $p-$th entry of the vector $\mathbf{w}_t^{A,q}$. Combining all our previous notations, we have that the output-specific costs are given for each output q of DMU t in group A by $\mathbf{w}_t^{A,q\prime}\mathbf{x}_t^{A,q}$. By summing these output-specific costs, we obtain the cost for DMU t in group A: $\sum_{q=1}^{Q} \mathbf{w}_t^{A,q\prime}\mathbf{x}_t^{A,q} = \mathbf{w}_t^{A\prime}\mathbf{x}_t^{A}$.

As a final remark, note that the input prices for group B, denoted $\mathbf{w}_t^B \in \mathbb{R}_+^P$ and $\mathbf{w}_t^{B,q} \in \mathbb{R}_+^P$, are in general different from those in group A. Our methodology does not require the assumption of common input prices between groups.

Illustrative example. To illustrate the output-specific approach, we make use of a fictional example. Let us assume that we observe a specific DMU t in group A that uses 2 inputs ($P = 2$) to produce 2 outputs ($Q = 2$). The output and input levels, and input prices of DMU t are:

$$\mathbf{y}_t^A = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \mathbf{x}_t^A = \begin{bmatrix} 5 \\ 3 \end{bmatrix}, \text{ and } \mathbf{w}_t^A = \begin{bmatrix} 2 \\ 6 \end{bmatrix}. \quad (5)$$

[7] These output-specific prices have a similar interpretation as Lindahl prices for public goods. See Cherchye et al (2008) for more details.

Therefore, the total cost is given by:

$$\mathbf{w}_t^{A\prime}\mathbf{x}_t^A = \begin{bmatrix} 2 \\ 6 \end{bmatrix}^\prime \begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 & 6 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \end{bmatrix} = 10 + 18 = 28. \tag{6}$$

Additionally, let us assume that the first input is jointly used to produce the two outputs, while the second inputs is allocated to each output production process (say 40% is used for output 1 and thus 60% for output 2). We obtain:

$$a_{2t}^1 = 40\% \text{ and } a_{2t}^2 = 60\%. \tag{7}$$

The information vectors for the two outputs are defined as follows:

$$\mathbf{V}_t^{A,1} = \begin{bmatrix} 1 \\ a_{2t}^1 \end{bmatrix} = \begin{bmatrix} 1 \\ 40\% \end{bmatrix} \text{ and } \mathbf{V}_t^{A,2} = \begin{bmatrix} 1 \\ a_{2t}^2 \end{bmatrix} = \begin{bmatrix} 1 \\ 60\% \end{bmatrix}. \tag{8}$$

Building on the concepts of information vectors, we can define the output-specific input vectors:

$$\mathbf{x}_t^{A,1} = \mathbf{V}_t^{A,1} \odot \mathbf{x}_t^A = \begin{bmatrix} 1 \\ 40\% \end{bmatrix} \odot \begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ 1.2 \end{bmatrix}, \text{ and } \mathbf{x}_t^{A,1} = \mathbf{V}_t^{A,1} \odot \mathbf{x}_t^A = \begin{bmatrix} 5 \\ 1.8 \end{bmatrix}. \tag{9}$$

As the first input is jointly used to produce the two outputs, the output-specific input prices have to sum to the aggregate input price for that input. Also, as the second input is allocated to the output production processes, the output-specific input prices for that input have to coincide with the input price for the aggregate input. We obtain the two following relationships:

$$(\mathbf{w}_t^{A,1})_1 + (\mathbf{w}_t^{A,2})_1 = (\mathbf{w}_t^A)_1 = 2. \tag{10}$$

$$(\mathbf{w}_t^{A,1})_2 = (\mathbf{w}_t^{A,2})_2 = (\mathbf{w}_t^A)_2 = 6. \tag{11}$$

While the output-specific prices for the allocated input are directly given by (11), it is not the case for those of the joint input (i.e. (10) gives a relationship, but is not enough to define the prices). In general, when these output-specific prices are not observed, they can be recovered (see Section 3 for more discussions). In the context of our illustration, we assume that those prices are also observed. In particular, we set

$(\mathbf{w}_t^{A,1})_1 = 0.5$ and $(\mathbf{w}_t^{A,2})_1 = 1.5$. Importantly, note that any value could be picked. The only requirement is that (10) is satisfied. The output-specific input price vectors are defined as follows:

$$\mathbf{w}_t^{A,1} = \begin{bmatrix} 0.5 \\ 6 \end{bmatrix}, \text{ and } \mathbf{w}_t^{A,2} = \begin{bmatrix} 1.5 \\ 6 \end{bmatrix}. \tag{12}$$

Therefore, the output-specific costs are given by:

$$\mathbf{w}_t^{A,1'}\mathbf{x}_t^{A,1} = \begin{bmatrix} 0.5 \\ 6 \end{bmatrix}' \begin{bmatrix} 5 \\ 1.2 \end{bmatrix} = 2.5 + 7.2 = 9.7. \tag{13}$$

$$\mathbf{w}_t^{A,2'}\mathbf{x}_t^{A,2} = \begin{bmatrix} 1.5 \\ 6 \end{bmatrix}' \begin{bmatrix} 5 \\ 1.8 \end{bmatrix} = 7.5 + 10.8 = 18.3. \tag{14}$$

Finally, by summing the output-specific costs over the outputs, we recover the total cost:

$$\mathbf{w}_t^{A,1'}\mathbf{x}_t^{A,1} + \mathbf{w}_t^{A,2'}\mathbf{x}_t^{A,2} = 9.7 + 18.3 = 28 = \mathbf{w}_t^{A'}\mathbf{x}_t^{A}. \tag{15}$$

3 Group cost Malmquist productivity index

We define now our indexes for comparing groups. Given our previous output-specific modelling of the production process, it is natural to start by defining output-specific group CMPI. These indexes allow us to benchmark groups of DMUs for each output individually. As such, they offer the option to identify the source(s) of cost performance change. In multi-output settings, it is also important to provide such information at the aggregate production level. Therefore, we will also provide a group CMPI. Attractively, this index can be written as a non-trivial aggregation of the output-specific cost efficiencies, which implies that all the following indexes depend only on output-specific minimization conditions.

Output-specific group cost Malmquist productivity index. The starting point of the CMPI is the minimal cost for each output q. The minimal cost of output q for DMU t in group B taking group A as the reference group is defined by

$$C^{A,q}(y_t^{B,q}, \mathbf{w}_t^{B}, \mathbf{w}_t^{B,q}) = \min_{\mathbf{x}^{B,q} \in I_t^{A,q}(y_t^{B,q})} \mathbf{w}_t^{B,q'}\mathbf{x}^{B,q}. \tag{16}$$

$C^{A,q}(y_t^{B,q}, \mathbf{w}_t^B, \mathbf{w}_t^{B,q})$ gives the minimal cost to produce the output quantity $y_t^{B,q}$ given the input prices $\mathbf{w}_t^B$, the output-specific input prices $\mathbf{w}_t^{B,q}$ and the technology of group A (as such the subscript A on C refers to the referent group for the technology). For each q, we have: $C^{A,q}(y_t^{B,q}, \mathbf{w}_t^B, \mathbf{w}_t^{B,q}) \leq \mathbf{w}_t^{B,q'} \mathbf{x}_t^{B,q}$. Intuitively, it only reflects that the actual cost is bounded by the minimal cost. If the equality holds, output q is produced with the minimal costs. If not, it reflects potential cost savings.

A natural index of cost efficiency, suggested by Farrell (1957), is the ratio of the minimal to the actual cost. Adapting to our specific setting, we obtain for each output q:

$$CE_t^{A,B,q} := CE_t^{A,B,q}(y_t^{B,q}, \mathbf{x}_t^{B,q}, \mathbf{w}_t^B, \mathbf{w}_t^{B,q}) = \frac{C^{A,q}(y_t^{B,q}, \mathbf{w}_t^B, \mathbf{w}_t^{B,q})}{\mathbf{w}_t^{B,q'} \mathbf{x}_t^{B,q}}. \tag{17}$$

The output-specific cost efficiency is situated between 0 and 1, with 1 meaning that output q is produced efficiently, i.e. with the minimal cost. A lower value reflects greater cost inefficiency and thus potential cost savings.

By adapting the group comparison index procedure developed by Thanassoulis, Shiraz, and Maniadakis (2015) to our specific context, we obtain, for each output q, the following index to compare group B to group A (i.e. group A represents the technology):

$$CMPI^{A,q} := \frac{\left[\prod_{t=1}^{n_B} CE_t^{A,B,q}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A,q}\right]^{1/n_A}}. \tag{18}$$

Similarly, to compare group A to group B on output q (i.e. group B represents the technology), we obtain the following index:

$$CMPI^{B,q} := \frac{\left[\prod_{t=1}^{n_B} CE_t^{B,B,q}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} CE_t^{B,A,q}\right]^{1/n_A}}. \tag{19}$$

$CMPI^{A,q} > 1$ indicates that the DMUs in group B are, on average, more productive in cost terms than those in group A. Or, in other words, DMUs in group B have less scope for cost savings than DMUs in group A. A value smaller than 1 indicates the opposite, while a value of 1 means that the two groups have the same cost performance. The interpretation of $CMPI^{B,q}$ is analogous to the interpretation of $CMPI^{A,q}$.

To avoid an arbitrary choice of the reference group, it is commonly agreed to take the geometric mean of the two above indexes (see Färe, Grosskopf, and Norris (1994)):

$$CMPI^{q} = \left[CMPI^{A,q} \times CMPI^{B,q}\right]^{1/2}. \tag{20}$$

Again, a value greater than 1 induces a greater cost performance of DMUs in group B than those in group A, and this is true irrespective of the chosen technology, i.e. the reference group.

Group cost Malmquist productivity index. The steps to obtain the group CMPI are very similar to those done previously to obtain the output-specific CMPI but apply, this time, to the overall output production process. We start by defining the minimal cost to produce all the outputs by summing the output-specific minimal costs:

$$C^{A}(\mathbf{y}_t^B, \mathbf{w}_t^B, \mathbf{w}_t^{B,1}, \ldots, \mathbf{w}_t^{B,Q}) = \sum_{q=1}^{Q} C^{A,q}(y_t^{B,q}, \mathbf{w}_t^B, \mathbf{w}_t^{B,q}). \tag{21}$$

As before, we can define the cost efficiency by taking the ratio of the minimal to the actual costs:

$$CE_t^{A,B} := CE_t^{A,B}(\mathbf{y}_t^B, \mathbf{x}_t^B, \mathbf{w}_t^B, \mathbf{w}_t^{B,1}, \ldots, \mathbf{w}_t^{B,Q}) = \frac{C^{A}(\mathbf{y}_t^B, \mathbf{w}_t^B, \mathbf{w}_t^{B,1}, \ldots, \mathbf{w}_t^{B,Q})}{\mathbf{w}_t^{B\prime}\mathbf{x}_t^B}. \tag{22}$$

As shown by Walheer (2017), the overall cost efficiency can be rewritten in terms of the output-specific cost efficiencies by means of linear weights. Adapting his procedure to our group comparison framework, we obtain the following weight:

$$\alpha_t^{A,B,q} := \alpha_t^{A,B,q}(\mathbf{x}_t^B, \mathbf{x}_t^{B,q}, \mathbf{w}_t^B, \mathbf{w}_t^{B,q}) = \frac{\mathbf{w}_t^{B,q\prime}\mathbf{x}_t^{B,q}}{\mathbf{w}_t^{B\prime}\mathbf{x}_t^B}. \tag{23}$$

These weights could be interpreted as the share of the total budget that is allocated to output q for group B taking group A as the reference group for the technology. At this point, it should be clear that the referent technology appears in the right-hand side of (23) in the output-specific input prices (and also in the input prices when they are not observed). Indeed, most of the time, the output-specific input prices are not observed and are thus recovered using the data (see our discussion below about the practical implementation). As such, these prices depend on the DMUs used in the recovery process. In our group context, as DMUs are compared across groups, this makes the output-specific prices dependent on the technology of the other groups (here group A).

Clearly, those weights sum to one over the output. We obtain the following relationship:

$$CE_t^{A,B} = \sum_{q=1}^{Q} \alpha_t^{A,B,q} CE_t^{A,B,q}. \tag{24}$$

As such, the weight allows us to identify which output-specific cost efficiency contributes more to the cost efficiency for group B taking group A as the reference group.

Following the same procedure undertaken previously for the output-specific CMPI, we obtain the overall group specific CMPI

$$CMPI = \left(CMPI^A \times CMPI^B\right)^{1/2}, \tag{25}$$

where

$$CMPI^A := \frac{\left[\prod_{t=1}^{n_B} CE_t^{A,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} CE_t^{A,A}\right]^{1/n_A}} = \frac{\left[\prod_{t=1}^{n_B} \left(\sum_{q=1}^{Q} \alpha_t^{A,B,q} CE_t^{A,B,q}\right)\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} \left(\sum_{q=1}^{Q} \alpha_t^{A,A,q} CE_t^{A,A,q}\right)\right]^{1/n_A}}, \tag{26}$$

$$CMPI^B := \frac{\left[\prod_{t=1}^{n_B} CE_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} CE_t^{B,A}\right]^{1/n_A}} = \frac{\left[\prod_{t=1}^{n_B} \left(\sum_{q=1}^{Q} \alpha_t^{B,B,q} CE_t^{B,B,q}\right)\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} \left(\sum_{q=1}^{Q} \alpha_t^{B,A,q} CE_t^{B,A,q}\right)\right]^{1/n_A}}. \tag{27}$$

The interpretation of those indexes is completely analogous to the interpretation of the output-specific CMPIs but applies here at the aggregate production level ($\mathbf{y}_t^B$). That is, a value greater than 1 indicates that the DMUs in group B have higher performance in cost terms than those in group A on the overall production process, a value smaller than 1 indicates the opposite, and a value of 1 means that the two groups have the same cost-performances for the overall production process.

As a final remark, the right parts of equations (26) and (27) reveal the combination of the procedures of Thanassoulis, Shiraz, and Maniadakis (2015) and Walheer (2017). Indeed, the sum of the output-specific efficiency measurements to obtain the overall cost efficiency measurement is due to Walheer (2017), and the product of the overall cost efficiency measurement to obtain the group cost efficiency is due to Thanassoulis, Shiraz, and Maniadakis (2015).

Practical implementations. Attractively, our CMPIs can be obtained by linear programs. In particular, it suffices to evaluate four cost efficiency measurements: $CE_t^{A,A}$, $CE_t^{A,B}$, $CE_t^{B,A}$, and $CE_t^{B,B}$, for every DMU t in both groups. For the sake of compactness, we only give below the program for $CE_t^{A,B}$. In fact, the other cost measurements are easily obtained by changing A to B and/or B to A in the program.

At this point, it is important to note that, in our linear programs, we assume that the information vectors $\mathbf{V}_t^{A,q}, \ldots, \mathbf{V}_t^{A,Q}$ and $\mathbf{V}_t^{B,q}, \ldots, \mathbf{V}_t^{B,Q}$ are observed. In words, we assume that the distinction between joint and allocated inputs is known, and that the allocation factors are observed (captured by the a_{pt}^{q}'s). In many settings, this information is available, and thus it does not represent a strong assumption. Nevertheless, the following programs could be extended to cases when this information is not or only partially observed. See Cherchye, De Rock, and Walheer (2016) and Walheer (2017) for more discussions. Note that, it is always possible to assume that all the inputs are jointly used, in order to avoid having to specify the allocation factors. In the following, we rather focus our practical implementation discussion on whether the input and output-specific input prices are observed or not. We believe that this is more central to our method.

Let us first assume that we observe the input and output-specific prices for all the DMUs in both groups. In particular, for every DMU t in group B operating at $(\mathbf{y}_t^B, \mathbf{x}_t^B)$ with input prices $\mathbf{w}_t^B$ and output-specific input prices $\mathbf{w}_t^{B,q}$, the cost efficiency score $CE_t^{A,B}$ is obtained as follows **(LP-1)**:

$$
\begin{aligned}
CE_t^{A,B} = & \max_{C_t^{A,1},\ldots,C_t^{A,Q} \in \mathbb{R}_+} \frac{\sum_{q=1}^{Q} C_t^{A,q}}{\mathbf{w}_t^{B'} \mathbf{x}_t^B} \\
\text{s.t.} \quad & \forall q \in \{1, \ldots, Q\} : C_t^{A,q} \leq \mathbf{w}_t^{B,q'} \mathbf{x}_s^{A,q} \text{ for all } s : y_s^{A,q} \geq y_t^{B,q}.
\end{aligned}
$$

In words, the constraint verifies that DMUs producing more output quantity than the evaluated DMU t (i.e. all DMU s such that $y_s^{A,q} \geq y_t^{B,q}$), use higher cost than DMU t (i.e. $C_t^{A,q} \leq \mathbf{w}_t^{B,q'} \mathbf{x}_s^{A,q}$). As such, the distinguishing feature of the program is that there is one constraint for each output q, while for more standard settings, the constraint is defined for the aggregate output level (i.e. $\mathbf{y}_s^{A,q} \geq \mathbf{y}_t^{B,q}$). Note that this type of program dates to Varian (1984). If the minimal cost $\sum_{q=1}^{Q} C_t^{A,q}$ corresponds to the actual cost $\mathbf{w}_t^{B'}\mathbf{x}_t^B$, the program will give a cost efficiency score of 1. Otherwise, the cost efficiency score will be smaller than 1. As a final remark, it could seem counter-intuitive to maximize a cost function. In fact, the maximization reflects that the DMUs are evaluated in the best possible light. That is, the computed minimal costs are the highest possible (this spirit is called the benefit of the doubt). In the technical formulation (see **(LP-4)**), the most favourable prices are referred to as multipliers.

For many settings, the input prices for group A and group B ($\mathbf{w}_t^A$ and $\mathbf{w}_t^B$, $\forall t$) could be observed, but it is rarely the case for the output-specific input price ($\mathbf{w}_t^{A,q}$ and $\mathbf{w}_t^{B,q}$, $\forall q, \forall t$). Attractively, in that case, the minimal costs can also be computed by a linear program. For every DMU t in group B operating at $(\mathbf{y}_t^B, \mathbf{x}_t^B)$ with input price $\mathbf{w}_t^B$, the cost efficiency score $CE_t^{A,B}$ is obtained as follows **(LP-2)**:

$$CE_t^{A,B} = \max_{\substack{C_t^{A,1},\ldots,C_t^{A,Q} \in \mathbb{R}_+ \\ \mathbf{w}_t^{B,1},\ldots,\mathbf{w}_t^{B,Q} \in \mathbb{R}_+^Q}} \frac{\sum_{q=1}^{Q} C_t^{A,q}}{\mathbf{w}_t^{B'}\mathbf{x}_t^B}$$

$$\begin{aligned}
\text{s.t.} \quad & \forall q \in \{1,\ldots,Q\}, \text{ the following holds:} \\
& \textbf{(C-1)} : C_t^{A,q} \leq \mathbf{w}_t^{B,q'}\mathbf{x}_s^{A,q} \text{ for all } s : y_s^{A,q} \geq y_t^{B,q}, \\
& \textbf{(C-2)} : (\mathbf{w}_t^{B,q})_p = (\mathbf{w}_t^B)_p \text{ for } p \text{ an input allocated to the outputs,} \\
& \textbf{(C-3)} : \sum_{q=1}^{Q} (\mathbf{w}_t^{B,q})_p = (\mathbf{w}_t^B)_p \text{ for } p \text{ an input jointly used to produce all the outputs.}
\end{aligned}$$

In words, **(C-1)** is similar to the constraint of **(LP-1)**. That is, **(C-1)** picks, for every output q, the minimal cost $C_t^{A,q}$ when comparing the evaluated DMU t in group B to the dominating DMUs in group A (i.e. DMUs that produce more output than $y_t^{B,q}$). **(C-2)** and **(C-3)** make sure that the unknown output-specific input prices respect the constraints explained in Section 2 (see (3) and (4)). Note that $(\mathbf{w}_t^{B,q})_p$ refers to the $p-$th entry of the vector $\mathbf{w}_t^{B,q}$. As such, **(LP-1)** is a lighter version of **(LP-2)**, i.e. without the second and third constraints. This is intuitive, since when the output-specific input prices are observed, the constraints on those prices are by definition satisfied.

Once the four cost efficiency scores are computed, for every DMUs in both groups, using linear programs **(LP-1)** or **(LP-2)**, the group CMPI is directly obtained by plugging in the cost scores. For the output-specific group CMPI, we first have to compute the output-specific cost scores. In fact, those scores are directly obtained when solving the linear programs. Indeed, these measurements depend on the output-specific minimal costs $C_t^{A,q}$ and on the output-specific input prices $\mathbf{w}_t^{B,q}$, which are both given once solving the linear programs. The weights are also obtained from the solutions of the linear programs.

Price availability. In this part, we relax the assumption of observing the input prices ($\mathbf{w}_t^A$ and $\mathbf{w}_t^B$, $\forall t$). That is, we assume that we only observe input-output data. This is particularly attractive since it means that cost performance could still be evaluated without observing input prices. As done for the case when the output-specific input prices were assumed unobserved, we choose the input prices that maximize the minimal costs. Or, in other words, we evaluate the groups in the best possible way, which gives the benefit of the doubt in the absence of true price information. We obtain:

$$C^A(\mathbf{y}_t^B) = \sum_{q=1}^{Q} C^{A,q}(y_t^{B,q}) = \max_{\mathbf{w}_t^{B,1},\ldots,\mathbf{w}_t^{B,Q} \in \mathbb{R}_+^Q} \left\{ \sum_{q=1}^{Q} C^{A,q}(y_t^{B,q}, \mathbf{w}_t^B, \mathbf{w}_t^{B,q}) \right\}. \tag{28}$$

In this definition, no constraints are put on the output-specific prices except that they are strictly positive. As such, the input prices are defined with the output-specific prices as explained in (3) and (4). Also, it is clear that $C^A(\mathbf{y}_t^B) \geq C^A(\mathbf{y}_t^B, \mathbf{w}_t^B, \mathbf{w}_t^{B,1}, \ldots, \mathbf{w}_t^{B,Q})$ following the benefit of the doubt spirit. The output-specific and overall cost efficiency measurements, and the weights are given by

$$\widehat{CE}_t^{A,B,q} := \widehat{CE}_t^{A,B,q}(y_t^{B,q}, \mathbf{x}_t^{B,q}) = \frac{C^{A,q}(y_t^{B,q})}{\mathbf{w}_t^{B,q\prime}\mathbf{x}_t^{B,q}}. \tag{29}$$

$$\widehat{CE}_t^{A,B} := \widehat{CE}_t^{A,B}(\mathbf{y}_t^B, \mathbf{x}_t^B) = \frac{C^A(\mathbf{y}_t^B)}{\mathbf{w}_t^{B\prime}\mathbf{x}_t^B}. \tag{30}$$

$$\widehat{\alpha}_t^{A,B,q} := \widehat{\alpha}_t^{A,B,q}(\mathbf{x}_t^B, \mathbf{x}_t^{B,q}) = \frac{\mathbf{w}_t^{B,q\prime}\mathbf{x}_t^{B,q}}{\mathbf{w}_t^{B\prime}\mathbf{x}_t^B}. \tag{31}$$

Combining all these definitions, as done in (25), (26) and (27), we obtain our

CMPI index without assuming observation of the input prices:

$$\widehat{CMPI} = \left(\frac{\left[\prod_{t=1}^{n_B} \widehat{CE}_t^{A,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} \widehat{CE}_t^{A,A}\right]^{1/n_A}} \times \frac{\left[\prod_{t=1}^{n_B} \widehat{CE}_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} \widehat{CE}_t^{B,A}\right]^{1/n_A}} \right)^{1/2}. \tag{32}$$

$$= \left(\frac{\left[\prod_{t=1}^{n_B} \left(\sum_{q=1}^{Q} \widehat{\alpha}_t^{A,B,q} \widehat{CE}_t^{A,B,q}\right)\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} \left(\sum_{q=1}^{Q} \widehat{\alpha}_t^{A,A,q} \widehat{CE}_t^{A,A,q}\right)\right]^{1/n_A}} \times \frac{\left[\prod_{t=1}^{n_B} \left(\sum_{q=1}^{Q} \widehat{\alpha}_t^{B,B,q} \widehat{CE}_t^{B,B,q}\right)\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} \left(\sum_{q=1}^{Q} \widehat{\alpha}_t^{B,A,q} \widehat{CE}_t^{B,A,q}\right)\right]^{1/n_A}} \right)^{1/2}. \tag{33}$$

Note that $\widehat{CMPI}$ has to be interpreted in an analogous manner as $CMPI$, and that output-specific CMPIs, denoted, for every output q, by $\widehat{CMPI}^q$, can be defined using the output-specific cost efficiency measurements $\widehat{CE}_t^{A,B,q}$, as in (18), (19), and (20). As for our previous definition of the CMPI, based on the assumption of observing the input prices, the CMPI without this assumption can be obtained by solving a program. As before, it is enough to evaluate four cost efficiency measurements: $\widehat{CE}_t^{A,A}$, $\widehat{CE}_t^{A,B}$, $\widehat{CE}_t^{B,A}$, and $\widehat{CE}_t^{B,B}$, for every DMU in both groups. In particular the program for $\widehat{CE}_t^{A,B}$ is given for every DMU t in group B operating at $(\mathbf{y}_t^B, \mathbf{x}_t^B)$ by **(NLP-1)**:

$$\widehat{CE}_t^{A,B} = \max_{\substack{\widehat{C}_t^{A,1},\ldots,\widehat{C}_t^{A,Q} \in \mathbb{R}_+ \\ \widehat{\mathbf{w}}_t^{B,1},\ldots,\widehat{\mathbf{w}}_t^{B,Q} \in \mathbb{R}_+^Q}} \frac{\sum_{q=1}^{Q} \widehat{C}_t^{A,q}}{\widehat{\mathbf{w}}_t^{B'} \mathbf{x}_t^B}$$

$$\text{s.t.} \qquad \forall q \in \{1,\ldots,Q\} : \widehat{C}_t^{A,q} \leq \widehat{\mathbf{w}}_t^{B,q'} \mathbf{x}_s^{A,q} \text{ for all } s : y_s^{A,q} \geq y_t^{B,q}.$$

(NLP-1) looks very similar to **(LP-1)**. The only difference is that in **(NLP-1)**, the input prices are unobserved, while in **(LP-1)**, they are assumed observed. It implies that these prices are also variables in the program. As a result, **(NLP-1)** is a non-linear program as variables appear at both the numerator and denominator of the objective. Fortunately, we can make **(NLP-1)** linear by using a simple transformation. In practice, we set the denominator $(\mathbf{w}_t^{B'} \mathbf{x}_t^B)$ equals unity. This transformation, introduced by Charnes and Cooper (1962), has been made popular by Charnes and Cooper (1978) for nonparametric efficiency methods. As a final remark, note that only the output-specific input prices are variables in the program. In fact, the input prices are fully defined by those prices (see our discussion of (3) and (4)). The linear program is thus given by **(LP-3)**:

$$
\begin{aligned}
CE_t^{A,B} = & \max_{\substack{\widehat{C}_t^{A,1},\ldots,\widehat{C}_t^{A,Q}\in\mathbb{R}_+ \\ \widehat{\mathbf{w}}_t^{B,1},\ldots,\widehat{\mathbf{w}}_t^{B,Q}\in\mathbb{R}_+^Q}} \sum_{q=1}^{Q} \widehat{C}_t^{A,q} \\
\text{s.t.} \quad & \forall q \in \{1,\ldots,Q\} : \widehat{C}_t^{A,q} \leq \widehat{\mathbf{w}}_t^{B,q\prime}\mathbf{x}_s^{A,q} \text{ for all } s : y_s^{A,q} \geq y_t^{B,q}. \\
& \widehat{\mathbf{w}}_t^{B\prime}\mathbf{x}_t^{B} = 1.
\end{aligned}
$$

The other cost scores are easily obtained by changing A to B and/or B to A in the program. As a final remark, note that it is also possible to add extra constraints for the output-specific input prices, as lower and upper bounds, in **(LP-3)** to increase the realism of those computed prices (see, for example, Cherchye, De Rock, and Walheer (2016) for an example in a profit maximization context).

Attractively, the CMPI without price information has also an interpretation in terms of technical efficiency index:

$$
\widehat{CMPI} = \left[\left(\frac{\left[\prod_{t=1}^{n_B} D_t^{A,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} D_t^{A,A}\right]^{1/n_A}} \times \frac{\left[\prod_{t=1}^{n_B} D_t^{B,B}\right]^{1/n_B}}{\left[\prod_{t=1}^{n_A} D_t^{B,A}\right]^{1/n_A}} \right)^{-1} \right]^{1/2}, \tag{34}
$$

where

$$
D_t^{A,B} := D_t^{A,B}(\mathbf{y}_t^B, \mathbf{x}_t^B) = \sup \left\{ \theta \mid \forall q : \left(\frac{\mathbf{x}_t^{B,q}}{\theta} \right) \in I_t^{A,q}(y_t^{B,q}) \right\}. \tag{35}
$$

$D_t^{A,B}$ an adapted version of the input distance function introduced by Shephard (1953, 1970) to our output-specific modelling.

To see this equivalence, it is enough to write the dual of the linear program used to obtained the cost efficiency measurement without price information **(LP-4)**. Specifically, let $\lambda_s^{A,q}$ represent the dual variables for the first constraint (for each output q and DMU s in group A) and θ_t^B be the dual variable for the second constraint of that program. Then, the dual can be written as (**LP-4**):

$$
\begin{aligned}
\widehat{CE}_t^{A,B} &= \min_{\substack{\lambda_s^{A,1},\ldots,\lambda_s^{A,Q}\in\mathbb{R}_+\\ \theta_t^B\in\mathbb{R}_+}} \theta_t^B \\
\text{s.t.} \quad & \forall q \in \{1,\ldots,Q\}, \text{ the following holds:} \\
& \textbf{(C-1)}: \sum_{s=1}^{n_A} \lambda_s^{A,q}\mathbf{x}_s^{A,q} \le \theta_t \mathbf{x}_t^{B,q} \text{ for all } s : y_s^{A,q} \ge y_t^{B,q}, \\
& \textbf{(C-2)}: \sum_{s=1}^{n_A} \lambda_s^{A,q} = 1 \text{ for all } s : y_s^{A,q} \ge y_t^{B,q}.
\end{aligned}
$$

In turn $CE_t^{A,B}$ could be equivalently defined as follows:

$$
\widehat{CE}_t^{A,B} = \inf\left\{\theta \mid \forall q : \left(\theta\mathbf{x}_t^{B,q}\right) \in I_t^{A,q}(y_t^{B,q})\right\}. \tag{36}
$$

As such, $CE_t^{A,B}$ can also be interpreted, when no price information is available, as an adapted version of the Debreu (1951) − Farrell (1957) input efficiency measurement: $CE_t^{A,B}$ defines the maximal equiproportionate/radial input reduction that still allows for producing the output. The equivalence between the two definitions of the CMPI, in terms of cost efficiency without price information (32), and in terms of input distance function (34), is reached by using the well-known relationship between the Debreu (1951) − Farrell (1957) efficiency measurement and the distance function of Shephard (1953, 1970). Indeed, the input distance function is the reciprocal of the input technical efficiency measurement:

$$
\widehat{CE}_t^{A,B} = \frac{1}{D_t^{A,B}}. \tag{37}
$$

As such, $\widehat{CMPI}$ has a dual interpretation in technical terms when no price data are available. That is, as a Malmquist productivity index. Attractively, the definition of $\widehat{CMPI}$ with distance functions could be seen as a combination of the MPI of Camanho and Dyson (2006) and the CMPI of Walheer (2017) when prices are assumed to be unobserved. See the next Section for more discussions.

Relationships with existing Malmquist productivity indexes. As discussed previously, our new CMPI is directly connected with existing MPIs and CMPIs. In this part, we show that our index coincides with the CMPI of Thanassoulis, Shiraz, and Maniadakis (2015) when one output is involved in the production process, and with the MPI of Camanho and Dyson (2006) when one output is involved in the production process and input prices are not observed. Finally, we propose a summary table that explicitly position our extension in the MPI and CMPI literature for DMUs and groups of DMUs.

Contrary to our CMPI, the approaches of Camanho and Dyson (2006) and Thanassoulis, Shiraz, and Maniadakis (2015) are not based on an output-specific modelling. Instead, they consider the production process as an overall or aggregate process. As such, they define the technology in terms of input requirement set for the aggregate output quantities ($\mathbf{y}_t^A$). Formally, it is given for DMU t in group A by:

$$I_t^A(\mathbf{y}_t^A) = \{\mathbf{x}^A \in \mathbb{R}_+^P \mid \mathbf{x}^A \text{ can produce } \mathbf{y}_t^A\}. \tag{38}$$

Based on those sets, Camanho and Dyson (2006) define the concept of Shephard's (1953, 1970) distance function in the group context as follows:

$$D_t^{A,B} := D_t^{A,B}(\mathbf{y}_t^B, \mathbf{x}_t^B) = \sup\left\{\theta \mid \left(\frac{\mathbf{x}_t^B}{\theta}\right) \in I_t^A(\mathbf{y}_t^B)\right\}. \tag{39}$$

In a similar vein, Thanassoulis, Shiraz, and Maniadakis (2015) define the concept of cost efficiency a la Farrell (1957) in the group context, given by:

$$CE_t^{A,B} := CE_t^{A,B}(\mathbf{y}_t^B, \mathbf{x}_t^B, \mathbf{w}_t^B) = \frac{C^A(\mathbf{y}_t^B, \mathbf{w}_t^B)}{\mathbf{w}_t^{B\prime}\mathbf{x}_t^B}, \tag{40}$$

where

$$C^A(\mathbf{y}_t^B, \mathbf{w}_t^B) = \min_{\mathbf{x}^B \in I_t^A(\mathbf{y}_t^B)} \mathbf{w}_t^{B\prime}\mathbf{x}^B. \tag{41}$$

The MPI of Camanho and Dyson (2006) and the CMPI of Thanassoulis, Shiraz, and Maniadakis (2015) are obtained by using their definition of distance function and cost efficiency, respectively, in (25) for the CMPI, and in (34) for the MPI. This reveals that our setting is consistent with their approach. In fact, our approach coincides with their approach when only one output is involved in the production process. Indeed, in that case: $I_t^A(\mathbf{y}_t^A)$ coincides with our output-specific technology $I_t^{A,q}(y_t^{A,q})$, simply because $\mathbf{y}_t^A = y_t^{A,q}$, making the definitions of distance function and cost efficiency equal. As such, in one-output setting, *CMPI* is similar to the

CMPI of Thanassoulis, Shiraz, and Maniadakis (2015), and $\widehat{CMPI}$ is similar to the MPI of Camanho and Dyson (2006). When more than one output is involved in the production process, our approach no longer coincides with their approach. The main interest of our CMPI is to provide output-specific results. This is not possible with the CMPI of Thanassoulis, Shiraz, and Maniadakis (2015), and MPI of Camanho and Dyson (2006). All in all, the major benefits of our approach are to provide more detailed results (by computing output-specific group CMPIs), and to increase the realism of the production process (by giving the option to incorporate information about the allocation of inputs to outputs).

We end this part by providing a summary, available in Table 1, of the different MPIs and CMPIs available to compare DMUs (over different periods of time) and groups of DMUs (for the same period). As such, as discussed previously in Section 3, our approach also bears a close relationship with the CMPI/MPI of Walheer (2017) to compare DMUs over periods of time. Indeed, as Walheer (2017), our approach is based on output-specific technologies. The main difference with his approach is that our new MPI/CMPI allows groups of DMUs to be compared (over one period of time), while his approach is designed to compare DMUs (over periods of time)

Table 1: MPIs and CMPIs: a summary

Index	**DMUs (dynamic comparison)**		**Groups of DMUs (static comparison)**	
	Aggregate	Output-specific	Aggregate	Output-specific
MPI	Caves, Christensen, and Diewert (1982)	Walheer (2017)	Camanho and Dyson (2006)	*This paper* ($\widehat{CMPI}$)
CMPI	Maniadakis and Thanassoulis (2004)	Walheer (2017)	Thanassoulis, Shiraz, and Maniadakis (2015)	*This paper* ($CMPI$)

Illustrative example. To illustrate the added value of our approach to the ones of Camanho and Dyson (2006) and Thanassoulis, Shiraz, and Maniadakis (2015) to compare groups of DMUs, we make use of a fictional example taken from Thanassoulis, Shiraz, and Maniadakis (2015). In that example, we observe 11 DMUs: 6 are members of group A and 5 of group B. The DMUs use two inputs to produce two outputs.

The input prices are also observed. The detailed data are displayed in Table 5 in Appendix A.

For that example, we do not have any information about the allocation of inputs to outputs. As such, we assume that the two inputs are jointly used to produce the outputs.[8] This shows once more that the allocation of inputs to outputs is not a necessary condition for our methodology. Moreover, the output-specific input prices are not observed, but the input prices are. As such, we make use of **(LP-2)**. In an illustration purpose, we also compute the CMPI when prices are assumed as not observed. In that case, we use **(LP-4)** (note that similar results could be obtained with **(LP-3)**). We provide in Appendix A, an extensive discussion about how the linear programs are used in practice to compute the cost efficiency scores for that particular example. The results are given in Table 2.

Table 2: Illustrative example: productivity index

Approach	Aggregate	Output 1	Output 2
Thanassoulis, Shiraz, and Maniadakis (2015)	1.05	-	-
Camanho and Dyson (2006)	0.95	-	-
This paper ($CMPI$)	1.13	1.24	1.08
This paper ($\widehat{CMPI}$)	0.83	0.78	0.98

An initial observation is that the output-specific indexes are not computed with the approaches of Camanho and Dyson (2006) and Thanassoulis, Shiraz, and Maniadakis (2015). This is, as discussed in detail in the previous part, by construction of their method. The CMPI of Thanassoulis, Shiraz, and Maniadakis (2015) is larger than 1 revealing that, on average, DMUs in Group B are more cost-productive than those in Group A. On average, DMUs in Group A have to reduce their total cost by 1/1.05 in order to attain the same level of cost efficiency as Group B. Also, the MPI of Camanho and Dyson (2006) is smaller than 1, implying better technical performance in group A than in group B. It could seem counter-intuitive that the two indexes give opposite results, but it is simply because they measure different type of performances: cost-based performances for the CMPI and technical-based performances for the MPI.

Our indexes provide similar results for the overall performances. Note that our indexes are more extreme than those of Camanho and Dyson (2006) and Thanassoulis,

[8] An alternative would be to recover the allocation of inputs to outputs. See our discussion in Section 3 (practical implementation).

Shiraz, and Maniadakis (2015). This could reflect that output-specific cost minimization conditions are stronger than overall cost minimization conditions. Next, the output-specific results reveal that DMUs in Group B are more cost-productive than those in Group A for both outputs. The performances are even better for output 1 (i.e. larger output-specific CMPI). For the technical-based index, the output-specific MPIs reveal that, DMUs in Group B are less productive in technical terms than those in Group A for both outputs. They are closer to DMUs in group A for output 2 than for output 1.

All in all, this illustration reveals that our approach, while remaining consistent with the existing indexes for group comparison, gives the advantages to provide results for each output separately. In the context of groups, we believe that it is particularity relevant as, to date, only one index was provided that limits, in a sense, our understanding of the group performance comparison.

4 The case of the US plant districts

We illustrate our methodology by considering the case of US electricity plants. Recently, several authors (see, for example, Tone and Tsutsui (2011), Sarkis and Cordeiro (2012), Cherchye, De Rock, and Walheer (2015), and Walheer (2017)) have studied these plants using the plant-level database provided by the Environmental Protection Agency: the eGRID system. The most recent database dates from 2012. As our methodology is tailored for multi-output producers, we restrict our analysis to plants that generate renewable and non-renewable electricity. This results in a sample of 350 plants. For the inputs, we follow the previous studies and select two inputs: nameplate capacity and the quantity of fuel used. Unfortunately, only data for these two inputs are reported by the eGRID system (the obvious missing input is labour). As such, we could interpret nameplate capacity as an aggregate of all the inputs except fuel consumption. No data for the input prices are reported by the eGRID system but we follow Walheer (2017), and use available information to construct lower and upper bounds to increase the realism of the computed prices.[9]

The plants are usually regrouped into five main PADDs.[10] Historically, these

[9]For fuel, the EPA provides prices at the state level. For the nameplate capacity, we can proxy the price by transforming the electricity price (available at the state level too) since nameplate capacity is defined as the maximal electricity generated by the plants during one year.

[10]PADD I (East Coast): Connecticut, Delaware, District of Columbia, Florida, Georgia, Maine,

districts were created during World War II to help organize the allocation of fuels derived from petroleum products. Today, these regions are still used to track regional movements of crude oil and petroleum products in the US and also for data collection purposes. We will investigate the cost-performance progress/regress of the districts. The new CMPI technique offers several advantages in this context. Firstly, the new CMPI recognizes the links between inputs and outputs by allocating the inputs to each electricity generation process. This is particularly useful in this context since the two inputs are differently linked to the outputs: nameplate capacity is used to produce both types of electricity, while the quantity of fuel is only used to produce non-renewable electricity.[11] Secondly, the new CMPI technique provides group-specific cost-performance results for each output. This feature is very attractive in our district context, since it offers the option to know for which outputs districts perform better/worse, or in other words, to identify the source of cost performance change. Moreover, there is no reason why the performance should be the same for each type of electricity. Indeed, plants have produced non-renewable electricity for decades, while the production of renewable electricity is a more recent activity. All in all, it means that the new CMPI model better uses the available information contained in the data and provides more results.

Input and output selection. Following our discussion above, we have a setting with two outputs ($Q = 2$): non-renewable electricity generated (y^1), and renewable electricity generated (y^2); and two inputs ($P = 2$): nameplate capacity (x^1) and quantity of fuel used (x^2). Nameplate capacity is jointly used to produce the two types of electricity, while the fuel quantity is completely allocated to the production of non-renewable electricity. Using the notation of Section 2, we have for each plant

Massachusetts, Maryland, New Hampshire, New Jersey, New York, North Carolina, Pennsylvania, Rhode Island, South Carolina, Virginia, Vermont, and West Virginia. PADD II (Midwest): Illinois, Indiana, Iowa, Kansas, Kentucky, Michigan, Minnesota, Missouri, Nebraska, North Dakota, South Dakota, Ohio, Oklahoma, Tennessee, and Wisconsin. PADD III (Gulf Coast): Alabama, Arkansas, Louisiana, Mississippi, New Mexico, and Texas. PADD IV (Rocky Mountain): Colorado, Idaho, Montana, Utah, and Wyoming. PADD V (West Coast): Alaska, Arizona, California, Hawaii, Nevada, Oregon, and Washington.

[11] Note that if data for the number of employees are available, we could allocate the employees for each electricity generation process.

t in group A (clearly, the same applies to DMUs in group B):

$$\mathbf{y}_t^A = \begin{bmatrix} y_t^{A,1} \\ y_t^{A,2} \end{bmatrix}, \mathbf{x}_t^A = \begin{bmatrix} x_t^{A,1} \\ x_t^{A,2} \end{bmatrix}, a_{2t}^1 = 1, a_{2t}^2 = 0, \mathbf{V}_t^{A,1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } \mathbf{V}_t^{A,2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$\mathbf{x}_t^{A,1} = \mathbf{V}_t^{A,1} \odot \mathbf{x}_t^A = \begin{bmatrix} x_t^{A,1} \\ x_t^{A,2} \end{bmatrix} \text{ and, similarly, } \mathbf{x}_t^{A,2} = \mathbf{V}_t^{A,2} \odot \mathbf{x}_t^A = \begin{bmatrix} x_t^{A,1} \\ 0 \end{bmatrix}. \quad (42)$$

Descriptive analysis. To contextualize our empirical illustration, we provide some descriptive tables of our sample. These tables reveal the importance of multi-output plants in the US, and show that the districts have different profiles in terms of electricity generation and cost use. We start by giving, in Table 6, the proportion of multi-output plants in the US. There were 350 multi-output plants in the US in 2012, which represents less than 5% of the total numbers of plants. Clearly, the distribution of multi-output plants across districts is not the same (see the second column), simply because the size of the districts are different (see the first column). A good indicator to compare the distribution of multi-output plants across districts is thus the percentage of multi-output plants (see the third column). These percentages reveal that *PADD I* has the higher share, while *PADD IV* has a share less than 1%. The three other districts have shares close to the US level. As such, it should be more important for *PADD I* to present better cost performance for multi-output plants than for *PADD IV*.

We complete our investigation of the importance of the multi-output producers in the US by giving their share in terms of inputs and outputs. In Table 7, we give the share with respect to all the plants, and in Table 8 with respect to multi-output plants only. While the multi-output plants represent less than 5% of the US plants (see Table 6), they generate almost 9% of the renewable electricity generation, but only 2% of the renewable electricity generation at the country level. Next, on the cost side, those plants represent around 2.5 % of the nameplate capacity (used as a proxy for total assets), and around 4% of the fuel consumption. As such, those plants are important since they contribute to greener electricity production, while their cost shares are relatively low. District-level shares also reveal important patterns. Except in *PADD V*, multi-output plants produce, on average, more renewable than non-renewable electricity. Note that in *PADD V*, they produce more than 6% and represent more than 10% of the fuel consumption. The importance of the multi-output producers for renewable electricity generation is clearly high in *PADD I* and *PADD III*. Indeed, they represent, respectively, almost 30% and 20% of the non-renewable electricity in those districts. As such, these multi-output plants are particularly important for those districts since they represent, in general, a high part of the renewable electricity generation, while their cost side is relatively low. The input and output shares of the districts with respect to the multi-output plants in Table 8 are coherent with the conclusions of Table 6. That is, districts with more multi-output producers have higher shares of inputs and outputs. That said, we note that *PADD I* represents more than 50% of the inputs and outputs of all multi-output plant producers in the US. Again, it reveals the importance of presenting good cost performances for that district in particular.

Finally, we present in Table 9, the averages for the inputs and outputs for all plants and multi-output plants at both the country (top) and the district level (bottom). The first observation is that multi-output plants produce more renewable electricity than average plants, but less non-renewable electricity. This is true at the country-level, and for all the districts expect *PADD V*. This confirmed, as highlighted in the previous tables, the greener profile of multi-output plants. Next, on the cost side, multi-output plants use, in general, less nameplate capicity and consume a bit less of fuel. This is confirmed at the district level, except for *PADD V*, since, as discussed previously, multi-output plants produce more non-renewable electricity in that district, which requires more fuel use. All in all, this table reveals, once more, the importance of the multi-output producers, and that providing cost performance results for each type of electricity is clearly useful and will give additional valuable information.

Cost performance changes. While the previous descriptive analysis reveals interesting patterns in terms of importance and profiles of the multi-output plants in the US, they do not give any information about the cost performances. For this task, we use our new CMPI. As explained in Section 3, it is enough to compute the cost efficiency scores for each plant in every district using **(LP-3)** to obtain the CMPI at the aggregate production level (i.e. for both types of electricity). As discussed previously, we also add extra constraints on the input prices to increase their realism. The methodology allows us to compare two groups, thus we will compare the PADDs two by two. Table 3 contains the results for the CMPI at the aggregate production level.

District	*PADD I*	*PADD II*	*PADD III*	*PADD IV*	*PADD V*
PADD I	1	0.9915	0.9274	0.9541	0.9424
PADD II	1.0086	1	0.9353	0.9623	0.9505
PADD III	1.0783	1.0691	1	1.0288	1.0162
PADD IV	1.0481	1.0392	0.9720	1	0.9877
PADD V	1.0611	1.0521	0.9840	1.0124	1

Table 3: CMPI results

Importantly, it is worth noting that the CMPI between district j and i is equal to the inverse of the CMPI between district i and j. This follows directly from the definition of the index, see (15) and (16). For example, CMPI between *PADD II* and *PADD I* = $\frac{1}{\text{CMPI between } PADD\ I \text{ and } PADD\ II} = \frac{1}{0.9915} = 1.0086$. The value of 0.9915 means that plants in *PADD II* are less cost-productive than those in *PADD I*. Clearly, if we take the inverse, i.e. 1.0086, we have the same conclusion. In fact, they have to reduce the aggregate costs by 0.86% to become as efficient as the plants in *PADD II*.

The results reveal that plants in *PADD I* are, on average, more cost-productive than in all the other districts, which is good news since this district is the largest in terms of number of multi-output producers (see Table 6), and inputs and outputs (see Tables 7 and 8). Then, *PADD II* is more cost-productive than all other districts. Again, it is good news since this district is the second largest one in terms of number of multi-output producers see (see Table 6) and has important shares of inputs and outputs (see Tables 7 and 8). *PADD III* is never more cost-productive than another district. As shown in our descriptive analysis, multi-output plants represent an important share of the renewable electricity generation in *PADD III*. As such, the relatively bad performances of this district is rather bad news. Finally, *PADD IV* is more cost-productive than *PADDs III* and *V*. *PADD V* performs better than *PADD III*.

Similar results could be obtained with the CMPI to compare the groups of DMUs of Thanassoulis, Shiraz, and Maniadakis (2015), even if their CMPI does not give the option of partial or unobserved input price data, and of allocating inputs to outputs. The distinguishing feature of our methodology is to provide output-specific results. This is particularity attractive in this context, since, as shown previously with our descriptive analysis, districts have different profiles. In particular, multi-output plants in *PADD V* produce, in general, more non-renewable electricity, while in *PADD I* and *PADD III*, they produce, in general, more renewable electricity. As such, knowing how *PADD V* performs for non-renewable electricity generation, and how *PADD I* and *PADD III* perform for renewable electricity generation would be valuable information in this context.

Using the output-specific cost efficiency scores, given, as explained previously in Section 3, also by **(LP-3)**, we can also compute the output-specific CMPI. Table 3 contains the results for renewable (output 1) and non-renewable (output 2) electricity production.

Output 1	*PADD I*	*PADD II*	*PADD III*	*PADD IV*	*PADD V*
PADD I	1	0.9871	0.8809	0.9150	0.9329
PADD II	1.0131	1	0.7994	0.8303	0.8466
PADD III	1.1352	1.2510	1	1.0387	1.0590
PADD IV	1.0929	1.2044	0.9627	1	1.0196
PADD V	1.0719	1.1812	0.9443	0.9808	1
Output 2	*PADD I*	*PADD II*	*PADD III*	*PADD IV*	*PADD V*
PADD I	1	1.0135	1.0816	1.0378	1.0649
PADD II	0.9867	1	1.0672	1.0240	1.0507
PADD III	0.9246	0.9370	1	0.9595	0.9845
PADD IV	0.9636	0.9765	1.0422	1	0.9784
PADD V	0.9391	0.9517	1.0157	1.0221	1

Table 4: CMPI results for outputs 1 and 2

These results allow us to develop the previous results based on the aggregate production process, and especially demonstrate which type of electricity performs better in each district. The better performance observed for all districts for the overall production process is mainly due to a better performance for renewable electricity production, except for *PADD IV* when comparing with *PADD V*. This is good news since, as show with our descriptive statistics, multi-output plants produce, in general, more renewable electricity. As such, we have identified, using our output-specific CMPIs, the source of cost efficiency. We provide below more detailed results for each electricity generation for each district.

For renewable electricity production, *PADD I* performs better than all districts. These results imply that the larger district, in terms of renewable electricity generation is the most cost efficient district. Next, *PADD III* is always dominated by the other districts, which is rather bad news. Afterwards, *PADD II* and *PADD IV*, which have relatively important shares of renewable electricity production, present quite good performances. *PADD II* dominates *PADDs III*, *IV*, and *V*, while *PADD IV* dominates *PADD III*. Finally, *PADD V*, which is more oriented to non-renewable electricity production, dominates *PADD III* and *PADD IV*. For non-renewable electricity production, *PADD I* is always dominated by the other districts. *PADD III* performs better than all the districts. *PADD IV* performs better than *PADDs I*, *II* and *IV*. *PADD V* has a larger cost-performance progress than *PADDs I* and *II*. As shown previously, *PADD V* is the only district with larger non-renewable than renewable electricity production, but the performances of this district could clearly be increased.

We believe that the output-specific results give extra valuable information for this application. Firstly, they allow us to investigate which output districts perform better. Next, they also allow us to better explain the results at the aggregate level. Finally, our results can also be used to allocate the inputs in each district with regard to the individual performance for each output.

5 Conclusion

We proposed a new cost Malmquist productivity index (CMPI) when considering groups. The distinguishing feature of our new methodology is that it gives results on each output separately. The new CMPI is inspired directly by two recent extensions and, in fact, combine those two extensions. We have also established a duality between our new CMPI and a new technical productivity index, which takes the form of a Malmquist productivity index, when the input prices are not observed.

We illustrated our new methodology with both a numerical example and a application for the case of the US electricity plant districts. We found that the two largest districts are the most cost-productive, but these results are in clear contrast to by output-specific results.

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Appendix A

Table 5: Illustrative example: data (source: Thanassoulis, Shiraz, and Maniadakis (2015))

Group	DMU	Input 1	Input 2	Input price 1	Input price 2	Output 1	Output 2
A	1	6	5	3	2	8	11
A	2	4	7	2	2.5	9	12
A	3	3	6	4	2	8	15
A	4	5	2	3	2.5	7	10
A	5	4	7	4	2	6	13
A	6	8	5	2	1	5	14
B	1	4	3	4	4.5	4	7
B	2	5	6	3	5	2	10
B	3	7	4	5	4.5	5	6
B	4	3	3	4	5	10	14
B	5	2	2	4	6	8	6

In this Appendix, we show how the linear programs work for the illustrative example. In particular, we distinguish between two cases: when the input prices are observed and when they are not. Note that the output-specific input prices are not observed in this example.

Practical implementation when the input prices are observed

Step 1: Solve **(LP-2)** for each member of group A taking group A as the reference group for the technology.

For example for DMU 1 in group A, it is given as:

$$CE_1^{A,A} = \max_{\substack{C_1^{A,1}, C_1^{A,2} \in \mathbb{R}_+ \\ \mathbf{w}_1^{A,1}, \mathbf{w}_1^{A,2} \in \mathbb{R}^2_+}} \frac{C_1^{A,1} + C_1^{A,2}}{28}$$

$$\textbf{(C-1)} : C_1^{A,1} \leq \mathbf{w}_1^{A,1'} \mathbf{x}_s^{A,1} \text{ for all } s : y_s^{A,1} \geq 8,$$

$$\textbf{(C-2)} : C_2^{A,2} \leq \mathbf{w}_1^{A,2'} \mathbf{x}_s^{A,2} \text{ for all } s : y_s^{A,2} \geq 11,$$

$$\textbf{(C-3)} : (\mathbf{w}_1^{A,1})_1 + (\mathbf{w}_1^{A,2})_1 = 3,$$

$$\textbf{(C-4)} : (\mathbf{w}_1^{A,1})_2 + (\mathbf{w}_1^{A,2})_2 = 2.$$

Note that the denominator is the actual cost of DMU 1 in group A: $\mathbf{w}_1^{A'} \mathbf{x}_1^A =$

$6 * 3 + 5 * 2 = 18 + 10 = 28$.

Step 2: Solve **(LP-2)** for each member of group A taking group B as the reference group for the technology.

For example for DMU 1 in group A, it is given as:

$$CE_1^{B,A} = \max_{\substack{C_1^{B,1}, C_1^{B,2} \in \mathbb{R}_+ \\ \mathbf{w}_1^{A,1}, \mathbf{w}_1^{A,2} \in \mathbb{R}_+^2}} \frac{C_1^{B,1} + C_1^{B,2}}{28}$$

$$\textbf{(C-1)} : C_1^{B,1} \leq \mathbf{w}_1^{A,1'} \mathbf{x}_s^{B,1} \text{ for all } s : y_s^{B,1} \geq 8,$$
$$\textbf{(C-2)} : C_2^{B,2} \leq \mathbf{w}_1^{A,2'} \mathbf{x}_s^{B,2} \text{ for all } s : y_s^{B,2} \geq 11,$$
$$\textbf{(C-3)} : (\mathbf{w}_1^{A,1})_1 + (\mathbf{w}_1^{A,2})_1 = 3,$$
$$\textbf{(C-4)} : (\mathbf{w}_1^{A,1})_2 + (\mathbf{w}_1^{A,2})_2 = 2.$$

Step 3: Solve **(LP-2)** for each member of group B taking group B as the reference group for the technology.

For example for DMU 1 in group B, it is given as:

$$CE_1^{B,B} = \max_{\substack{C_1^{B,1}, C_1^{B,2} \in \mathbb{R}_+ \\ \mathbf{w}_1^{B,1}, \mathbf{w}_1^{B,2} \in \mathbb{R}_+^2}} \frac{C_1^{B,1} + C_1^{B,2}}{29.5}$$

$$\textbf{(C-1)} : C_1^{B,1} \leq \mathbf{w}_1^{B,1'} \mathbf{x}_s^{B,1} \text{ for all } s : y_s^{B,1} \geq 4,$$
$$\textbf{(C-2)} : C_2^{B,2} \leq \mathbf{w}_1^{B,2'} \mathbf{x}_s^{B,2} \text{ for all } s : y_s^{B,2} \geq 7,$$
$$\textbf{(C-3)} : (\mathbf{w}_1^{B,1})_1 + (\mathbf{w}_1^{B,2})_1 = 4,$$
$$\textbf{(C-4)} : (\mathbf{w}_1^{B,1})_2 + (\mathbf{w}_1^{B,2})_2 = 4.5.$$

Note that the denominator is the actual cost of DMU 1 in group B: $\mathbf{w}_1^{B'} \mathbf{x}_1^B = 4 * 4 + 3 * 4.5 = 16 + 13.5 = 29.5$.

Step 4: Solve **(LP-2)** for each member of group B taking group A as the reference group for the technology.

For example for DMU 1 in group B, it is given as:

$$CE_1^{A,B} = \max_{\substack{C_1^{A,1}, C_1^{A,2} \in \mathbb{R}_+ \\ \mathbf{w}_1^{B,1}, \mathbf{w}_1^{B,2} \in \mathbb{R}_+^2}} \frac{C_1^{A,1} + C_1^{A,2}}{29.5}$$

$$\textbf{(C-1)} : C_1^{A,1} \leq \mathbf{w}_1^{B,1'} \mathbf{x}_s^{A,1} \text{ for all } s : y_s^{A,1} \geq 4,$$

$$\textbf{(C-2)} : C_2^{A,2} \leq \mathbf{w}_1^{B,2'} \mathbf{x}_s^{A,2} \text{ for all } s : y_s^{A,2} \geq 7,$$

$$\textbf{(C-3)} : (\mathbf{w}_1^{B,1})_1 + (\mathbf{w}_1^{B,2})_1 = 4,$$

$$\textbf{(C-4)} : (\mathbf{w}_1^{B,1})_2 + (\mathbf{w}_1^{B,2})_2 = 4.5.$$

Step 5: Compute *CMPI* for the overall output and $CMPI^1$ and $CMPI^2$ for the two individual outputs by plugging-in the computed cost efficiency scores in (25), (26), and (27) for *CMPI*, and in (18), (19), and (20) for $CMPI^1$ and $CMPI^2$. As a final remark, note that tout output-specific minimal costs are also obtained when solving the linear programs, as explained in detail in Section 3.

Practical implementation when the input prices are not observed

Step 1: Solve **(LP-4)** for each member of group A taking group A as the reference group for the technology.

For example for DMU 1 in group A, it is given as:

$$CE_1^{A,A} = \min_{\substack{\lambda_s^{A,1}, \lambda_s^{A,2} \in \mathbb{R}_+ \\ \theta_1^A \in \mathbb{R}_+}} \theta_1^A$$

$$\textbf{(C-1)} : \sum_{s=1}^{6} \lambda_s^{A,1} \mathbf{x}_s^{A,1} \leq \theta_1^A \begin{bmatrix} 6 \\ 5 \end{bmatrix} \text{ for all } s : y_s^{A,1} \geq 8,$$

$$\textbf{(C-2)} : \sum_{s=1}^{6} \lambda_s^{A,2} \mathbf{x}_s^{A,2} \leq \theta_1^A \begin{bmatrix} 6 \\ 5 \end{bmatrix} \text{ for all } s : y_s^{A,2} \geq 11,$$

$$\textbf{(C-3)} : \sum_{s=1}^{6} \lambda_s^{A,1} = 1 \text{ for all } s : y_s^{A,1} \geq 8,$$

$$\textbf{(C-4)} : \sum_{s=1}^{6} \lambda_s^{A,2} = 1 \text{ for all } s : y_s^{A,2} \geq 11.$$

Step 2: Solve **(LP-4)** for each member of group A taking group B as the reference group for the technology.

For example for DMU 1 in group A, it is given as:

$$CE_1^{B,A} = \min_{\substack{\lambda_s^{B,1},\lambda_s^{B,2}\in\mathbb{R}_+\\ \theta_1^A\in\mathbb{R}_+}} \theta_1^A$$

$$\textbf{(C-1)}: \sum_{s=1}^{6} \lambda_s^{B,1}\mathbf{x}_s^{B,1} \leq \theta_1^A \begin{bmatrix} 6 \\ 5 \end{bmatrix} \text{ for all } s: y_s^{B,1} \geq 8,$$

$$\textbf{(C-2)}: \sum_{s=1}^{6} \lambda_s^{B,2}\mathbf{x}_s^{B,2} \leq \theta_1^A \begin{bmatrix} 6 \\ 5 \end{bmatrix} \text{ for all } s: y_s^{B,2} \geq 11,$$

$$\textbf{(C-3)}: \sum_{s=1}^{6} \lambda_s^{B,1} = 1 \text{ for all } s: y_s^{B,1} \geq 8,$$

$$\textbf{(C-4)}: \sum_{s=1}^{6} \lambda_s^{B,2} = 1 \text{ for all } s: y_s^{B,2} \geq 11.$$

Step 3: Solve **(LP-4)** for each members of group B taking group B as the reference group for the technology.

For example for DMU 1 in group B, it is given as:

$$CE_1^{B,B} = \min_{\substack{\lambda_s^{B,1},\lambda_s^{B,2}\in\mathbb{R}_+\\ \theta_1^B\in\mathbb{R}_+}} \theta_1^B$$

$$\textbf{(C-1)}: \sum_{s=1}^{6} \lambda_s^{B,1}\mathbf{x}_s^{B,1} \leq \theta_1^B \begin{bmatrix} 4 \\ 3 \end{bmatrix} \text{ for all } s: y_s^{B,1} \geq 4,$$

$$\textbf{(C-2)}: \sum_{s=1}^{6} \lambda_s^{B,2}\mathbf{x}_s^{B,2} \leq \theta_1^B \begin{bmatrix} 4 \\ 3 \end{bmatrix} \text{ for all } s: y_s^{B,2} \geq 7,$$

$$\textbf{(C-3)}: \sum_{s=1}^{6} \lambda_s^{B,1} = 1 \text{ for all } s: y_s^{B,1} \geq 4,$$

$$\textbf{(C-4)}: \sum_{s=1}^{6} \lambda_s^{B,2} = 1 \text{ for all } s: y_s^{B,2} \geq 7.$$

Step 4: Solve **(LP-4)** for each member of group B taking group A as the reference group for the technology.

For example for DMU 1 in group B, it is given as:

$$CE_1^{A,B} = \min_{\substack{\lambda_s^{A,1},\lambda_s^{A,2}\in\mathbb{R}_+\\ \theta_1^B\in\mathbb{R}_+}} \theta_t^B$$

$$\textbf{(C-1)} : \sum_{s=1}^{6} \lambda_s^{A,1}\mathbf{x}_s^{A,1} \leq \theta_1^B \begin{bmatrix} 4 \\ 3 \end{bmatrix} \text{ for all } s : y_s^{A,1} \geq 4,$$

$$\textbf{(C-2)} : \sum_{s=1}^{6} \lambda_s^{A,2}\mathbf{x}_s^{A,2} \leq \theta_1^B \begin{bmatrix} 4 \\ 3 \end{bmatrix} \text{ for all } s : y_s^{A,2} \geq 7,$$

$$\textbf{(C-3)} : \sum_{s=1}^{6} \lambda_s^{A,1} = 1 \text{ for all } s : y_s^{A,1} \geq 4,$$

$$\textbf{(C-4)} : \sum_{s=1}^{6} \lambda_s^{A,2} = 1 \text{ for all } s : y_s^{A,2} \geq 7.$$

Step 5: Compute $\widehat{CMPI}$ for the overall output and $\widehat{CMPI}^1$ and $\widehat{CMPI}^2$ for the two individual outputs by plugging-in the computed cost efficiency scores in (32) and (33) or (34). Note that similar results could be obtained using **(LP-3)**. We provide here the technical formulation in an illustrative purpose.

6 Appendix B

Table 6: Multi-output plants

District	# Plants	# Multi-output Plants	% Multi-output Plants
US	7886	350	4.44
PADD I	2163	146	6.75
PADD II	2059	82	3.98
PADD III	767	39	5.08
PADD IV	470	4	0.85
PADD V	1827	79	4.32

Table 7: Shares with respect to all plants

District	Inputs		Outputs	
	Nameplate Capacity	Fuel	Non-Renewable Energy	Renewable Energy
US	2.47	4.20	2.06	8.88
PADD I	3.82	7.88	3.31	27.83
PADD II	0.94	1.21	0.57	6.31
PADD III	1.51	2.09	0.65	17.13
PADD IV	0.80	1.32	1.00	1.06
PADD V	3.89	10.88	6.33	2.46

Table 8: Shares with respect to multi-output plants

District	Inputs		Outputs	
	Nameplate Capacity	Fuel	Non-Renewable Energy	Renewable Energy
PADD I	51.08	54.77	54.56	50.53
PADD II	10.81	9.70	8.47	13.90
PADD III	11.70	11.35	6.94	22.25
PADD IV	1.20	1.79	1.98	0.99
PADD V	25.20	22.39	28.06	12.34

Table 9: Averages at the country and district level

District	Inputs: Nameplate Capacity (MW)	Inputs: Fuel (MMBtu)	Outputs: Non-Renewable Energy (MWh)	Outputs: Renewable Energy (MWh)
US	179.71	3681042.25	488178.60	67066.79
PADD I	199.74	3620508.41	557937.49	36422.70
PADD II	179.83	4377090.77	531832.78	46416.83
PADD III	326.02	7996458.14	1025062.26	73486.15
PADD IV	102.47	3238580.71	307367.26	85967.37
PADD V	114.32	1270425.77	177515.83	119061.61
US	92.22	3220508.28	208883.07	123987.35
PADD I	112.93	4228405.28	273198.22	150181.26
PADD II	42.56	1332708.95	75478.85	73563.04
PADD III	96.87	3281273.48	130057.85	247538.13
PADD IV	96.85	5039823.59	361639.30	107418.30
PADD V	102.97	3195185.25	259671.56	67763.04