Scale efficiency for multi-output cost minimizing producers: the case of the US electricity plants*

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Abstract

To know whether the optimal scale of production has been reached is valuable information for producers. To date, scale efficiency measurements have only been suggested for the entire production process. For multi-output producers, more detailed results are required. Hence, in this paper, we show how to provide such information at the output level. Attractively, our output-specific scale efficiency measurements are nonparametric in nature, they take the economic objective of the producers into account, they can be defined without observing the input prices, and they are easy to interpret and to use in practice. We apply our methodology to a sample of more than 3300 US electricity plants from 1998 to 2012, producing up to 10 types of electricity. We show that, while there is a scale improvement at the total electricity generation level, this is not the case for each of the 10 types of electricity. Also, we demonstrate that, in general, renewable electricity presents better scale of production than non-renewable electricity. Finally, we highlight the importance of multi-output plants in the US electricity sector, and show that this type of plant is preferable for the production of non-renewable electricity, while single-output plants are preferable for renewable electricity.

Keywords: scale efficiency; cost minimizing; multi-output producers; electricity generation.

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1 Introduction

Assessing the optimal scale of a production process is not a new topic in both the economic literature and the production theory. Indeed, the concept of scale efficiency could already be found in the works of Hanoch (1975), Panzar and Willig (1977), Forsund and Hjalmarsson (1979), Banker (1984), Banker, Charnes, and Cooper (1984), Färe and Grosskopf (1985), Banker and Thrall (1992), Forsund (1996), and Golany and Yu (1997). More recent works include those of Simar and Wilson (2002), Forsund and Hjalmarsson (2004), Krivonozhko et al (2004), Zelenyuk (2006, 2016), Podinovski, Forsund, and Krivonozhko (2009), and Peyrache (2013). These works have the investigating of scale efficiency of the entire production process in common. Or in other words, their methods indicate whether optimal scale is reached for the aggregate production level. In this paper, we suggest a technique that also provides scale efficiency results for individual output.

Our motivation to provide output-specific scale efficiency results is two-fold. On the one hand, by considering output-specific indicators, the realism and the discriminatory power of the model are naturally increased. The realism is increased since the links between the inputs and the outputs can be modelled by allocating the inputs to the output-specific production processes.[1] The discriminatory power is increased since output-specific optimization behaviours could be assumed. On the other hand, for multi-output producers, knowing whether the optimal scale is reached for each output separately is clearly additional relevant information; useful when choosing their strategy or when deciding how to allocate the inputs.

Our scale efficiency measurements are specially designed to take the economic objective of the producers into account. In particular, we assume that they are cost minimizers (the following is easily extended to profit or revenue maximizations). Cost minimization fits with many settings and applications, and is, by definition, a necessary condition for profit maximization. Our model is rooted in the nonparametric cost evaluation models initiated by Afriat (1972), Hanoch and Rothschild (1972), Diewert and Parkan (1983) and Varian (1984). That is, we impose very few struc-

[1] For example, employees allocated to specific output production, machines used only to produce certain outputs. For more discussion on the allocation of inputs to outputs, refer, for example, to Färe and Grosskopf (2000), Salerian and Chan (2005), Despic, Despic and Paradi (2007), Färe, Grosskopf and Whittaker (2007), Tone and Tsutsui (2009), and Cherchye, De Rock, and Walheer (2015).

tures on the production process and, therefore, only the following data are required: outputs, inputs, and input prices. The distinguishing feature of our methodology is that by modelling each output separately, we naturally give the option to assess scale efficiency at the output level. Finally, as the observation of the input prices is rather restrictive for some applications, we also provide alternative definitions of our scale efficiency concepts without this assumption.

We apply our methodology to the case of US electricity plants. The Environmental Protection Agency of the US developed a plant-level database for 1998 to 2012. For each plant, the coal, oil, gas, nuclear, other fossil, wind, solar, geothermal, hydro, and biomass electricity generations are specified. As such, by distinguishing between 10 different types of electricity generation, this database offers a unique opportunity to apply our methodology. In particular, the very detailed data allow us to evaluate scale efficiency of both the individual and aggregate electricity generation levels, and to make a distinction between multi- and single-output producers. Therefore, we can investigate whether multi- or single-output producers are preferable for each of the 10 types of electricity generation. This is valuable information for managers, regulators, and policy makers when deciding how to allocate the production of electricity and how to design the plants.

Moreover, our methodology offers two extra advantages in this context. On the one hand, it gives the option to allocate the inputs to each electricity generation type. In particular, renewable electricity is not produced by the use of fuel, while non-renewable electricity generation requires this production factor. As such, our methodology, which recognizes the links between production factors and electricity generation, is particularly useful as it increases the realism of the modelling of the plant production process. On the other hand, while the data for the production factors and electricity generation are available for the plant level, the input prices are only available at the state level. Thus, our methodology that works with partial/without input price data is also very attractive for that reason.

The rest of this paper unfolds as follows. Section 2 presents the methodology. In Section 3, we apply the methodology to the case of the US electricity plants from 1998 to 2012. Section 4 provides conclusions.

2 Methodology

We consider that we observe producers that are cost minimizers. In particular, we assume that they use P inputs, $\mathbf{x} \in \mathbb{R}^P_+$, to produce Q outputs, $\mathbf{y} \in \mathbb{R}^Q_+$. We denote the input price vector by $\mathbf{w} \in \mathbb{R}^P_+$. Firstly, we assume that we observe these input prices. This will be relaxed afterwards.

Output-specific framework. The distinguishing feature of our scale efficiency measurements is that we make a clear distinction between aggregate and individual outputs.[2] In particular, let us denote the q-th entry of $\mathbf{y}$ by y^q. As such, we will define scale efficiency measurements for both $\mathbf{y}$ and y^q. To achieve this goal, we model each output separately by its own production process, captured by input requirement set defined as follows for output q:

$$I^q(y^q) = \{\mathbf{x}^q \in \mathbb{R}^P_+ \mid \mathbf{x}^q \text{ can produce } y^q\}. \tag{1}$$

Cost evaluation does not require us to make strong assumptions about those sets. In fact, we follow Varian (1984) and only assume that those sets are nested: producing less outputs cannot lead to using more inputs.[3] In this context, $\mathbf{x}^q \in \mathbb{R}^P_+$ denote the inputs used to produce the output q. In fact, those inputs are connected to the aggregate inputs (in $\mathbf{x}$). Some inputs could be used to produce certain outputs (for example, employees, machines). That is, these inputs are allocated to specific output production processes. Next, some inputs could be used to produce all the outputs (for example, infrastructure, capital), i.e. these inputs are not allocated to specific output production processes. Formally, we have:

$$(\mathbf{x})_p = (\mathbf{x}^1)_p + \ldots, (\mathbf{x}^Q)_p, \text{ if input } p \text{ is allocated,} \tag{2}$$

$$(\mathbf{x})_p = (\mathbf{x}^q)_p, \text{ if input } p \text{ is not allocated.} \tag{3}$$

Attractively, the distinction between allocated and non-allocated inputs provides a unifying framework that is consistent not only with production models integrating information on the internal production structure, but also with more standard pro-

[2] For more discussion about efficiency analysis in output-specific frameworks, refer to Cherchye et al (2013) for the cost setting, and Cherchye, De Rock, and Walheer for the profit setting.

[3] $I^q(y^q)$ is nested if: $y^q \geq y^{q'} \implies I^q(y^q) \subseteq I^q(y^{q'})$.

duction models (i.e. models that do not consider allocated inputs). As a final remark, note that the non-allocated inputs could also be interpreted as public good (they are non-rival and non-exclusive to the output production processes), and, therefore, they give rise to economies of scope in the production process (See Panzar and Willig (1981) and Nehring and Puppe (2004)).

As the output-specific inputs $\mathbf{x}^q$ could be different from the inputs $\mathbf{x}$, nothing guarantees that their price should be the same. As such, let us denote the prices of the output-specific inputs by $\mathbf{w}^q \in \mathbb{R}^P_+$. Note that, in general, while the input prices could be observed, the output-specific input prices are not. The relationships between the inputs and the output-specific inputs also imply specific relationships between their prices. These prices coincide with the aggregate prices for allocated inputs. Next, for non-allocated inputs these prices must add up to the aggregate prices. As explained previously non-allocated inputs could be interpreted as public good. As such, the output-specific prices have a similar interpretation as Lindahl prices that, by definition, sum up to the aggregate prices. In that case, the output-specific input prices capture the economies of scope of the production processes. Taking together, we obtain:

$$(\mathbf{w}^q)_p = (\mathbf{w})_p, \text{ if input } p \text{ is allocated,} \tag{4}$$

$$\sum_{q=1}^{Q} (\mathbf{w}^q)_p = (\mathbf{w})_p, \text{ if input } p \text{ is not allocated.} \tag{5}$$

As a final remark, note that the actual cost of the producers could be rewritten exclusively by output-specific counterparts: $\mathbf{w}'\mathbf{x} = \sum_{q=1}^{Q} \mathbf{w}^{q'}\mathbf{x}^q$, where $\mathbf{w}^{q'}\mathbf{x}^q$ represents the cost of output q.

Cost evaluation. The starting point of the scale efficiency evaluation is the minimal cost for each output q:

$$C^q(y^q, \mathbf{w}, \mathbf{w}^q) = \min_{\mathbf{x}^q \in I^q(y^q)} \mathbf{w}^{q'}\mathbf{x}^q. \tag{6}$$

$C^q(y^q, \mathbf{w}, \mathbf{w}^q)$ selects the minimal input vector, in the input requirement set $I^q(y^q)$, to produce the output quantity y^q given the input prices $\mathbf{w}^q$. $C^q(y^q, \mathbf{w}, \mathbf{w}^q) \leq \mathbf{w}^{q'}\mathbf{x}^q$, and $C^q(y^q, \mathbf{w}, \mathbf{w}^q) = \mathbf{w}^{q'}\mathbf{x}^q$ means that output q is produced with minimal cost, revealing cost efficiency, while $C^q(y^q, \mathbf{w}, \mathbf{w}^q) < \mathbf{w}^{q'}\mathbf{x}^q$ reflects potential cost savings. Note that, the minimal costs $C^q(y^q, \mathbf{w}, \mathbf{w}^q)$ depend on the input price $\mathbf{w}$, making them interdependent. This is rather intuitive, since the unobserved output-specific input prices depend on the input prices (see (4) and (5)).

Attractively, as explained before, by summing the output-specific costs, we obtain the cost for the aggregate production level. This also holds for the minimal costs:

$$C(\mathbf{y}, \mathbf{w}, \mathbf{w}^1, \dots, \mathbf{w}^Q) = \sum_{q=1}^{Q} C^q(y^q, \mathbf{w}, \mathbf{w}^q). \tag{7}$$

Clearly, the property of the costs at the aggregate output level are the same as their respective output-specific counterparts. Firstly, $C^q(\mathbf{y}^q, \mathbf{w}, \mathbf{w}^q) \leq \mathbf{w}^{q'}\mathbf{x}^q$, implies that $C(\mathbf{y}, \mathbf{w}, \mathbf{w}^1, \dots, \mathbf{w}^Q) \leq \mathbf{w}'\mathbf{x}$, i.e. at the aggregate output level, the minimal cost is bounded by the actual cost. Next, if each output is produced with minimal costs, i.e. $C^q(y^q, \mathbf{w}, \mathbf{w}^q) = \mathbf{w}^{q'}\mathbf{x}^q$ for all q, then $C(\mathbf{y}, \mathbf{w}, \mathbf{w}^1, \dots, \mathbf{w}^Q) = \mathbf{w}'\mathbf{x}$, i.e. the actual cost coincides with the minimal cost. Finally, if at least one output is produced inefficiently, i.e. $C^q(y^q, \mathbf{w}, \mathbf{w}^q) < \mathbf{w}^{q'}\mathbf{x}^q$ for at least one q, we have $C(\mathbf{y}, \mathbf{w}, \mathbf{w}^1, \dots, \mathbf{w}^Q) < \mathbf{w}'\mathbf{x}$.

In practice, minimal costs can be computed using linear programs. This is particularly attractive since linear programs are easily solved. As noticed previously, the output-specific minimal costs are, by definition, interdependent as the unobserved output-specific input prices depend on the input prices. Attractively, we could compute all the output-specific minimal costs by solving only one program. In fact, it suffices to evaluate the minimal costs for the aggregate output level. In particular, for every producer t operating at $(\mathbf{y}_t, \mathbf{x}_t)$ with input price $\mathbf{w}_t$, the minimal cost C_t is obtained as follows **(LP-1)**:

$$
\begin{aligned}
C_t = & \max_{\substack{C_t^1,\ldots,C_t^Q \in \mathbb{R}_+ \\ \mathbf{w}_t^1,\ldots,\mathbf{w}_t^Q \in \mathbb{R}_+^Q}} \sum_{q=1}^{Q} C_t^q \\
\text{s.t.} \quad & \forall q \in \{1,\ldots,Q\}, \text{ the following holds:} \\
& \textbf{(C-1)} : C_t^q \leq \mathbf{w}_t^{q'} \mathbf{x}_s^q \text{ for all } s : y_s^q \geq y_t^q, \\
& \textbf{(C-2)} : (\mathbf{w}_t^q)_p = (\mathbf{w}_t)_p \text{ for } p \text{ an allocated input}, \\
& \textbf{(C-3)} : \sum_{q=1}^{Q} (\mathbf{w}_t^q)_p = (\mathbf{w}_t)_p \text{ for } p \text{ a non-allocated input}.
\end{aligned}
$$

In words, **(C-1)** picks, for every output q, the minimal cost C_t^q when comparing the evaluated producer t to the dominating producers (i.e. those that produce more outputs than y_t^q). Note that it is why we have to impose that the input requirement sets are nested, otherwise we can only compare producer t to producers that produce exactly the same output quantity. **(C-2)** and **(C-3)** make sure that the unknown output-specific input prices are correctly specified (see (4) and (5)). As a final remark, it could seem counter-intuitive to maximize a cost function. In fact, the maximization selects the most favourable output-specific input prices (notion of shadow prices). See also our discussion below when input prices are assumed unobserved.

Scale efficiency. Our previous definition of the technology, captured by the input requirement sets $I^q(y^q)$, does not assume any particular structure in terms of returns-to-scale. As such, variable returns-to-scale was implicitly assumed. To test formally for scale efficiency, we first have to define minimal costs for both the aggregate and output-specific levels under the hypothetical assumption of constant returns-to-scale.[4] Let us denote the input requirement set satisfying the hypothetical assumption of constant returns-to-scale as $\widehat{I}^q(y^q)$. It is straightforward to define the minimal costs with respect to those sets. In fact, it suffices to use $\widehat{I}^q(y^q)$ instead of $I^q(y^q)$ in (6):

$$
\widehat{C}^q(y^q, \mathbf{w}, \mathbf{w}^q) = \min_{\mathbf{x}^q \in \widehat{I}^q(y^q)} \mathbf{w}^{q'} \mathbf{x}^q. \tag{8}
$$

The interpretation of $\widehat{C}^q(y^q, \mathbf{w}, \mathbf{w}^q)$ is analogous to the interpretation of $C^q(y^q, \mathbf{w}, \mathbf{w}^q)$,

[4] $\widehat{I}^q(y^q)$ satisfies constant returns-to-scale if: $\forall k \in \mathbb{R}_0^+ : \mathbf{x}^q \in \widehat{I}^q(y^q) \implies k\mathbf{x}^q \in \widehat{I}^q(ky^q)$.

the only difference is that the cost (in)efficient behaviour is evaluated when assuming constant returns-to-scale. Note also, that by definition $\widehat{C}^q(y^q, \mathbf{w}, \mathbf{w}^q) \leq C^q(y^q, \mathbf{w}, \mathbf{w}^q)$. It reflects that the input requirement set under constant returns-to-scale is, in general, greater than the input requirement set under variable returns-to-scale, or in other words, that $I^q(y^q)$ is included in $\widehat{I}^q(y^q)$.[5].

We obtain our scale efficiency index for output q and for the aggregate production level as follows:

$$SE^q(y^q, \mathbf{w}, \mathbf{w}^q) = \frac{\widehat{C}^q(y^q, \mathbf{w}, \mathbf{w}^q)}{C^q(y^q, \mathbf{w}, \mathbf{w}^q)}. \tag{9}$$

$$SE(\mathbf{y}, \mathbf{w}, \mathbf{w}^1, \ldots, \mathbf{w}^Q) = \frac{\widehat{C}(\mathbf{y}, \mathbf{w}, \mathbf{w}^1, \ldots, \mathbf{w}^Q)}{C(\mathbf{y}, \mathbf{w}, \mathbf{w}^1, \ldots, \mathbf{w}^Q)} = \frac{\sum_{q=1}^{Q} \widehat{C}^q(y^q, \mathbf{w}, \mathbf{w}^q)}{\sum_{q=1}^{Q} C^q(y^q, \mathbf{w}, \mathbf{w}^q)}. \tag{10}$$

As $\widehat{C}^q(y^q, \mathbf{w}, \mathbf{w}^q) \leq C^q(y^q, \mathbf{w}, \mathbf{w}^q)$, $SE^q(y^q, \mathbf{w}, \mathbf{w}^q)$ is, in general, smaller than 1. A value of one indicates scale efficiency behaviour. When $SE^q(y^q, \mathbf{w}, \mathbf{w}^q) < 1$, it reveals scale inefficiency, which could be due to decreasing or increasing returns-to-scale.[6] As for the output-specific value, $SE(\mathbf{y}, \mathbf{w}, \mathbf{w}^1, \ldots, \mathbf{w}^Q) = 1$ reflects scale efficiency, while a value smaller than one implies more scale inefficiency behaviour. Attractively, in that case, the source(s) of inefficiency could be found simply by looking at the values of the $SE^q(y^q, \mathbf{w}, \mathbf{w}^q)$. As a final remark, note that our definition of scale efficiency at the aggregate production level is coherent with the one of Färe and Grosskopf (1985). The only difference is that we base our measurement on output-specific technologies.

The linear program for the minimal costs under the hypothetical assumption of constant returns-to-scale has a structure that is formally analogous to the one of **(LP-1)**. In particular, for every producer t operating at $(\mathbf{y}_t, \mathbf{x}_t)$ with input price $\mathbf{w}_t$, the minimal cost under the hypothetical assumption of constant returns-to-scale is

[5] $\widehat{I}^q(y^q)$ is directly related to $I^q(y^q)$, since $\widehat{I}^q(y^q) = \{\lambda(\mathbf{x}^q) \in I^q(\lambda y^q), \forall \lambda > 0\}$.

[6] In practice, it is enough to evaluate the minimal cost when assuming non-increasing returns-to-scale and compare to $C^q(y^q, \mathbf{w}, \mathbf{w}^q)$. If they are equal, scale inefficiency is due to decreasing returns-to-scale. Otherwise, scale inefficiency is due to increasing returns-to-scale.

obtained as follows **(LP-2)**:

$$
\begin{aligned}
\widehat{C}_t = & \max_{\substack{\widehat{C}_t^1,\dots,\widehat{C}_t^Q \in \mathbb{R}_+ \\ \mathbf{w}_t^1,\dots,\mathbf{w}_t^Q \in \mathbb{R}_+^Q}} \sum_{q=1}^{Q} \widehat{C}_t^q \\
\text{s.t.} \quad & \forall q \in \{1,\dots,Q\}, \text{ the following holds:} \\
& \textbf{(C-1)} : \widehat{C}_t^q \leq \mathbf{w}_t^{q'} (\zeta_s^q \mathbf{x}_s^q) \text{ for all } s : (\zeta_s^q y_s^q) \geq y_t^q, \\
& \textbf{(C-2)} : (\mathbf{w}_t^q)_p = (\mathbf{w}_t)_p \text{ for } p \text{ an allocated input}, \\
& \textbf{(C-3)} : \sum_{q=1}^{Q} (\mathbf{w}_t^q)_p = (\mathbf{w}_t)_p \text{ for } p \text{ a non-allocated input}, \\
& \textbf{(C-4)} : \zeta_s^q = \inf \left\{ \zeta \in \mathbb{R}_0^+ \mid \zeta y_s^q \geq y_t^q \right\}.
\end{aligned}
$$

(LP-2) is very similar to **(LP-1)** except that the input-output of the dominating producers are rescaled by the factor ζ_s^q. In fact, this factor is present exactly to take the hypothetical assumption of constant returns-to-scale into account in the linear program. This is captured by the restriction on the set of the possible value of ζ_s^q. Note that, **(LP-2)** coincides with **(LP-1)** if $\zeta_s^q = 1, \forall q, \forall s$, i.e. no rescaling of the input-output.[7] We end this part by one important remark: there is an interesting relationship between scale efficiency at both levels.[8] In fact, scale efficiency at the aggregate level could be obtained as a weighted sum of the scale efficiencies at the output-specific level where the weights are the output-specific minimal cost shares:

$$
SE(\mathbf{y}, \mathbf{w}, \mathbf{w}^1, \dots, \mathbf{w}^Q) = \sum_{q=1}^{Q} \frac{C^q(\mathbf{y}^q, \mathbf{w}, \mathbf{w}^q)}{\sum_{q=1}^{Q} C^q(\mathbf{y}^q, \mathbf{w}, \mathbf{w}^q)} SE^q(y^q, \mathbf{w}^q, \mathbf{w}^q). \tag{11}
$$

Input price availability. As it is defined, our aggregate and output-specific scale efficiency indicators depend on the observation of the input price vectors ($\mathbf{w}$). In the following we relax this assumption. We believe that it is particularly attractive since we can keep the advantage of basing our indicators on economic objective without

[7] Note that other returns-to-scale assumptions are easily implemented by replacing $\mathbb{R}_0^+$ by $(0,1], [1,\infty)$ for the decreasing or increasing returns to scale assumption, respectively. For more details, see also Petersen (1990) and Bogetoft (1996) in a technical nonparametric setting. Also, note that for **(C-4)**, we assume that, if it is not possible to find such a ζ_s^q (i.e. the solution is the empty set), then $\zeta_s^q = +\infty$, i.e. no constraints are put on the minimal costs.

[8] Zelenyuk (2016) obtains a similar result in a group aggregation context based on revenue maximization.

facing the disadvantage of observing the input prices. As done before for the output-specific input prices, we choose the input prices that maximize the minimal costs (i.e. the shadow prices). As such, we evaluate the producers in the best possible way in the absence of true input price information, which gives the benefit of the doubt to the producers. We obtain:

$$C(\mathbf{y}, \mathbf{w}^1, \ldots, \mathbf{w}^Q) = \sum_{q=1}^{Q} C^q(y^q, \mathbf{w}^q) = \max_{\mathbf{w} \in \mathbb{R}_+^Q} \left\{ \sum_{q=1}^{Q} C^q(y^q, \mathbf{w}, \mathbf{w}^q) \right\}, \tag{12}$$

$$\widehat{C}(\mathbf{y}, \mathbf{w}^1, \ldots, \mathbf{w}^Q) = \sum_{q=1}^{Q} \widehat{C}^q(y^q, \mathbf{w}^q) = \max_{\mathbf{w} \in \mathbb{R}_+^Q} \left\{ \sum_{q=1}^{Q} \widehat{C}^q(y^q, \mathbf{w}, \mathbf{w}^q) \right\}. \tag{13}$$

As the computed input prices are the most favourable, the minimal cost $C(\mathbf{y}, \mathbf{w}^1, \ldots, \mathbf{w}^Q)$ and $\widehat{C}(\mathbf{y}, \mathbf{w}^1, \ldots, \mathbf{w}^Q)$ provide upper bounds for the minimal costs when the input prices are assumed observed. Importantly, since no input prices are available, the constraints on the output-specific input prices in (4) and (5) are irrelevant. The alternative definitions of scale efficiency are obtained by plugging-in the new definitions of the minimal costs (12) and (13) in (9) and (10). Attractively, we can also evaluate those minimal costs by linear programs. For every producer t operating at $(\mathbf{y}_t, \mathbf{x}_t)$, the minimal cost $\widehat{C}_t$ is obtained by solving **(LP-3)**:

$$\widehat{C}_t = \max_{\substack{\widehat{C}_t^1, \ldots, \widehat{C}_t^Q \in \mathbb{R}_+ \\ \mathbf{w}_t^1, \ldots, \mathbf{w}_t^Q \in \mathbb{R}_+^Q}} \sum_{q=1}^{Q} \widehat{C}_t^q$$

$$\begin{aligned}
\text{s.t.} \quad & \forall q \in \{1, \ldots, Q\}, \text{ the following holds:} \\
& \textbf{(C-1)} : \widehat{C}_t^q \leq \mathbf{w}_t^{q\prime} (\zeta_s^q \mathbf{x}_s^q) \text{ for all } s : (\zeta_s^q y_s^q) \geq y_t^q, \\
& \textbf{(C-2)} : \zeta_s^q = \inf \left\{ \zeta \in \mathbb{R}_0^+ \mid \zeta y_s^q \geq y_t^q \right\}. \\
& \mathbf{w}_t' \mathbf{x}_t = 1.
\end{aligned}$$

As explained previously, when input prices are not observed, no constraints (except that they have to be non-negative) are put on the output-specific input prices. A common way to proceed, that dates from the work of Charnes, Cooper and Rhodes (1978), is to normalize the actual cost to unity (i.e. $\mathbf{w}_t' \mathbf{x}_t = 1$). The normalization is used to make the computed minimal costs comparable, i.e. the benchmark value is

1.[9] The minimal costs C_t are obtained, for every producer t, by setting $\zeta_s^q = 1, \forall q, \forall s$ in **(LP-3)**. Finally, note that extra constraints on the computed prices could easily be added in **(LP-3)**. For example, lower and upper bounds could be included, as it is the case in our Application where we use state-level input prices as lower and upper bounds for the unknown plant-level input prices (See Section 3 for more details). Those extra constraints are added to increase the realism of the computed prices.

3 Application

We apply our methodology to the case of the US electricity plants. Investigating the scale optimality, building on a cost minimization behaviour, of electricity plants has already been considered by several authors. See, for example, Christensen and Greene (1976), Nelson (1985, 1989), Krautmann and Solow (1988), Nemoto et al. (1993), Burns and Weyman-Jones (1996) Filippini (1996), Considine (2000), Filippini and Wild (2001), Kleit and Terrell (2001), Maloney (2001), Rhine (2001), Hiebert (2002), Fraquelli et al. (2005), Kopsakangas-Savolainen and Svento (2008), Akkemik (2009), Arcos and De Toledo (2009), Assaf, Barros and Managi (2011), Kumbhakar et al (2015), Ajayi, Weyman-Jones, and Glass (2017).[10] The cost minimizing framework is mostly chosen in this context as the output side of the production process is rather exogenous to the plant (fixed, in a sense, by the demand), while the plants can still control the input side given the electricity generation. As such, the input side is rather endogenous to the plants. This implies that cost minimization is preferable to profit or revenue maximization in this context.

The distinguishing feature of our methodology is to consider each type of electricity separately, instead of modelling only the aggregate electricity generation. As such, our analysis, while remaining consistent with the previous works, offers the advantage to provide more detailed results, without making extra assumptions on any aspect of the production process. Moreover, our methodology offers two extra advantages in this context. One, it allows us to allocate the inputs to each electricity generation

[9]Note that any value could, in principle, be used for the normalization, 1 is more convenient and commonly used in this case.

[10]Note that in several works other methods are used to compute the cost functions (such as a stochastic frontier model). In principle, these alternative methods could be extended to include output-specific indicators. The advantages of the nonparametric model are its easy use and that no strong assumptions are required about the production process. See our discussion of (1).

type, by explicitly recognizing the links between production factors and electricity generation. In particular, it is clear that renewable electricity is not produced by the use of fuel, while non-renewable electricity generation is. Two, our methodology also works with partial input price data. This is very attractive in this context as, while the data for the production factors and electricity generation are available for the plant-level, the input prices are only available at the state-level.

We tackle two important questions in this empirical part. Firstly, we compute scale efficiency of 10 types of electricity generations: coal, oil, gas, nuclear, other fossil, wind, solar, geothermal, hydro, and biomass. This is attractive since it is not obvious that the scale performances are the same for each type of electricity production. Moreover, our sample consists of more than 3300 plants over a large period (1998-2012), meaning that our results are trustworthy. Next, we investigate whether multi- or single-output producers are preferable for each of the 10 types of electricity generation. We believe that the answers to these two questions are valuable information for managers, regulators, and policy makers when deciding how to allocate the production of electricity and how to design the plants.

To present our empirical application, we first define the production process of the plants. Next, we present our data and highlight the importance of multi-output producers in the US. Afterwards, we present the scale efficiency results. Finally, we provide the shadow prices for the inputs.

Data and plant production process. We use data of the *eGRID* system developed by the Environmental Protection Agency in the US. In particular, we use all the databases available between 1998 and 2012 (there are no databases available after 2012). Unfortunately, the databases are not provided for each year but for nine years: 2012, 2010, 2009, 2007, 2005, 2004, 2000, 1999, and 1998. There are also two more databases in 1996 and 1997, but the input labels are different; thus, we do not take those two extra databases into account. This results in a sample of 3389 plants.

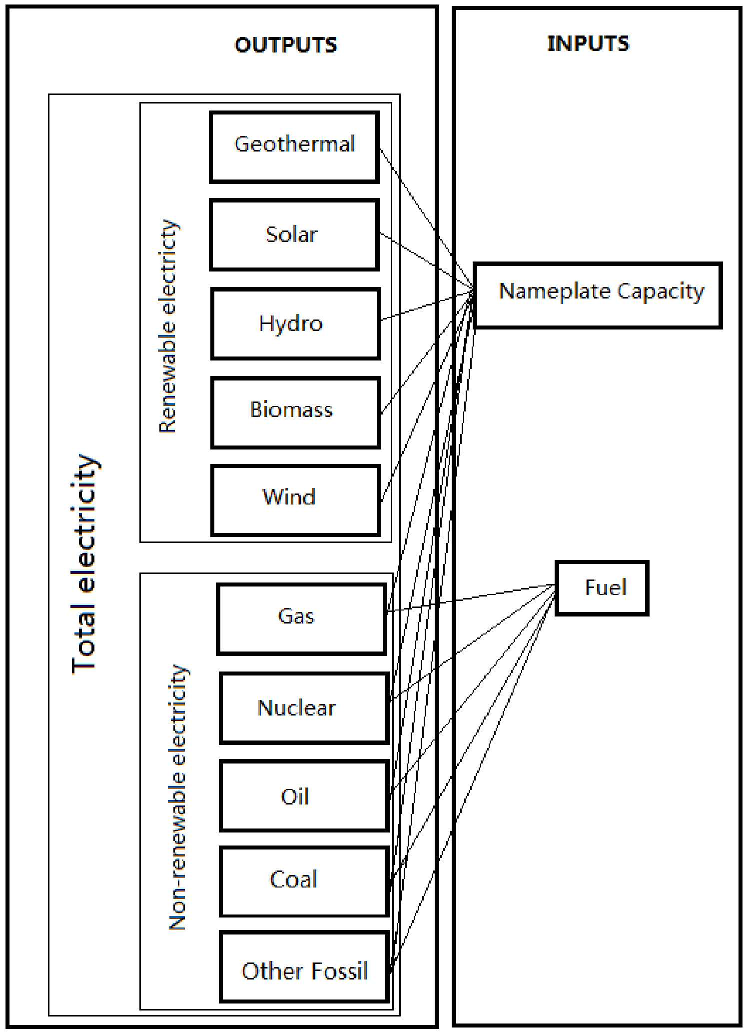
As explained in the Introduction, the first distinguishing feature of our empirical analysis is that we model each type of electricity separately. Each plant can produce up to 10 types of electricity: coal, oil, gas, nuclear, other fossil, wind, solar, geothermal, hydro, and biomass (in fact, the maximum is 10 and the average is 1.40 on the period). We have $Q = 10$. As done by the *eGRID* system, we regroup the ten types of electricity into two categories: renewable (wind, solar, geothermal, hydro, and biomass) and non-renewable (coal, oil, gas, nuclear, other fossil). By summing the ten types of electricity generation, we obtain the total electricity generation. Therefore, we obtain three levels of comparison: ten types of electricity generation (coal, oil, gas, nuclear, other fossil, wind, solar, geothermal, hydro, and biomass), two aggregate categories (renewable and non-renewable), and the total generation.

While the output side of the production process is described in much detail by the *eGRID* system, the input side is not so detailed. Indeed, only the fuel consumption is provided by *eGRID* system. Fortunately, we could proxy the missing inputs (such as infrastructure, labour, etc.) by the nameplate capacity, provided by the *eGRID* system. This strategy has already been used, for example, by Tone and Tsutsui (2011), Sarkis and Cordeiro (2012), Cherchye , De Rock, and Walheer (2015), and Walheer (2017, 2018) in a similar context. We have two inputs, i.e. $P = 2$. An attractive feature of our technique, as discussed previously, is that it gives the option to allocate the inputs to the outputs. In this context, this is particularly relevant as the fuel input is clearly not used to produce the renewable electricity. We summarize the production process of a typical plant in Figure 1.

We end this part by two remarks. Firstly, no data for the input prices are reported for the plants by the *eGRID* system. Nevertheless, the Environmental Protection Agency provides price data for the states. As such, we use those prices to increase the realism of the computed prices. In particular, we use the plant-level prices as lower and upper bounds for the unknown plant-level prices. The goal of these bounds is to avoid trivial and/or unrealistic prices (as too close to zero or too large). See our discussion at the end of Section 2 for more details. Next, as the data for the number of boilers and generators are also provided by the *eGRID* system, we could incorporate them as extra inputs. This is done in, for example, Sarkis and Cordeiro (2012). In that case, $P = 4$. As the results are very similar and for the sake of simplicity, we present the case with two inputs, i.e. $P = 2$.

Descriptive statistics and importance of the multi-output plants. We present the descriptive statistics in Table 1. The first part of Table 1 (top) gives the averages, medians, and maxima for the total, non-renewable, and renewable electricity generation for the period. Clearly, plants produce more non-renewable than renewable electricity for the period. This is confirmed by both the averages and the medians. Moreover, as highlighted by the maxima, the largest plants are those producing non-

Figure 1: Production process of the electricity plants



renewable electricity. Next, renewable electricity generation represents around 10% of the total electricity production, and this percentage is more or less constant for the period. The second part of Table 1 (bottom) presents the relative importance of each type of electricity. It reveals that coal electricity generation represents around 50% of the total electricity generation for the period, while nuclear electricity generation represents around 25%, gas electricity generation represents around 10%, and hydro electricity generation represents slighlty less than 10%. For non-renewable electricity, the coal and oil electricity generation are decreasing, while the gas and nuclear are increasing. For renewable electricity, they are all more or less stable for the period.

Table 1: Descriptive statistics (MWh) and percentage of electricity generation per type

Year	Total electricity			Non-Renewable electricity			Renewable electricity			
	Average	Median	Max	Average	Median	Max	Average	Median	Max	Percentage
2012	932991	54907	31933916	1421533	72088	31933916	201069	38620	26461174	11.99%
2010	997874	61727	31199935	1545660	82487	31199935	184443	40978	18351775	10.12%
2009	972270	56840	30661851	1494580	74293	30661851	192098	40642	20987193	10.91%
2007	1054755	58641	26782391	1648342	105251	26782391	177627	36096	21632495	9.12%
2005	1057669	66996	25807446	1643632	121436	25807446	191635	41163	20474048	9.84%
2004	1043974	62891	28112609	1625071	102214	28112609	191788	37513	18917123	10.01%
2000	1083721	77744	30380571	1646675	133821	30380571	207732	44034	21765745	10.35%
1999	1063401	81071	30415572	1590314	136600	30415572	234004	46131	24967234	12.04%
1998	1041813	81273	30301045	1574153	144480	30301045	233544	47031	21038360	12.27%

Year	Coal	Oil	Gas	Nuclear	Other Fossil	Hydro	Biomass	Wind	Solar	Geothermal
2012	2697422	23062	392136	11743938	221226	134013	54078	41027	428174	86535
	48.24%	0.84%	15.06%	24.67%	0.39%	8.71%	1.50%	0.09%	0.02%	0.38%
2010	3136804	32581	327569	12174188	200186	128759	54144	41391	425636	79218
	54.14%	1.14%	11.62%	23.47%	0.35%	7.36%	1.36%	0.09%	0.02%	0.35%
2009	2968009	33543	312489	12161972	211640	124572	55482	41114	424032	77486
	53.09%	1.23%	11.37%	24.05%	0.35%	8.00%	1.37%	0.09%	0.02%	0.36%
2007	3394674	51684	328110	12139194	188655	132720	62607	46482	428685	103422
	56.18%	1.74%	10.99%	22.13%	0.51%	6.59%	1.32%	0.09%	0.02%	0.34%
2005	3383420	94413	303306	11761769	208463	130399	52782	45437	440625	119388
	55.79%	3.28%	9.98%	21.37%	0.55%	7.24%	1.28%	0.08%	0.02%	0.34%
2004	3312531	92455	294057	11873207	205628	140100	60177	51926	444484	101018
	55.53%	3.22%	9.71%	21.89%	0.47%	7.23%	1.41%	0.09%	0.02%	0.35%
2000	3233258	74824	376365	11554690	215519	140456	64908	57448	480860	94111
	53.62%	2.77%	13.07%	20.66%	0.50%	7.37%	1.33%	0.09%	0.02%	0.36%
1999	3094710	83327	361269	11203248	248562	142549	58681	56591	476607	94328
	52.45%	3.09%	12.71%	20.44%	0.56%	8.68%	1.37%	0.08%	0.02%	0.39%
1998	3133357	92052	352481	10365985	252520	134758	51457	56211	487747	91118
	53.38%	3.41%	12.45%	19.32%	0.50%	8.99%	1.25%	0.08%	0.02%	0.39%

Table 2 presents the descriptive statistics for the input side of the production process. While the fuel consumption slowly decreases for the period, the nameplate capacity increases. It means that plants are becoming bigger. This is also confirmed by the rise of the number of generators and boilers.

Table 2: Production factors

Year	Boilers (number)	Generators (number)	Nameplate capacity (MW)	Fuel (MMBtu)
2012	1.15	3.68	247	10488370
2010	1.17	3.40	246	11607583
2009	1.16	3.38	245	11103202
2007	0.65	3.38	241	12593262
2005	1.00	3.38	241	12795503
2004	0.99	3.42	241	12894084
2000	0.61	2.75	230	13474571
1999	0.59	2.73	230	13237346
1998	0.58	2.06	229	13249417

As explained in the Introduction, the second distinguishing feature of our empirical analysis is that we differentiate between multi- and single-output plants. To justify the importance of this distinction in this context, we present in Table 3, the proportion of multi-output producers for the total, non-renewable, and renewable electricity generation, and the percentage of electricity produced and production factors used by the multi-output plants. While the percentage of multi-output producers decreases from

Table 3: Importance of the multi-output producers

Year	Total		Non-Renewable		Renewable		Production factors	
	Multi	Prod	Multi	Prod	Multi	Prod	Namplate capacity	Fuel
2012	31%	60%	54%	66%	16%	12%	59%	90%
2010	33%	64%	56%	70%	17%	13%	61%	92%
2009	32%	63%	55%	68%	17%	12%	60%	92%
2007	34%	67%	57%	72%	16%	14%	64%	93%
2005	35%	68%	60%	73%	16%	12%	65%	94%
2004	34%	67%	59%	72%	16%	12%	63%	92%
2000	39%	68%	66%	73%	18%	13%	67%	92%
1999	38%	67%	65%	73%	18%	12%	65%	92%
1998	37%	67%	64%	74%	16%	11%	65%	92%

almost 40% to 30% over the period, they are important as they produce around 60%

of the total electricity, and represent 60% of the nameplate capacity and 90% of the fuel consumption. This importance is higher for non-renewable electricity generation as more than 50% of the producers are multi-output producers, and they represent around 70% of the production of this type of electricity. Both these numbers decrease over the period. For the renewable electricity production, the multi-output producers represent around 16% of the number of producers, and generate around 12% of the electricity. Both these numbers are stable over the period.

In Table 6 in the Appendix, we give the number of producers for each of the 10 types of electricity generation, and the proportion of multi-output producers. A first observation is that the proportion of multi-output producers is very high for the coal and solar electricity generation, as they represent more than 90% of the producers. Next, they represent around 70% of the producers for the oil, gas, and biomass electricity generation. The majority of producers of nuclear, hydro, and wind electricity generation are single-output producers, while there are no multi-output producers for the geothermal electricity generation. Finally, note that the proportion of multi-output producers is slowly decreasing for every type of electricity generation, except for solar electricity generation.

Scale efficiency results. The previous descriptive analysis has set the stage by revealing the main characteristics of the US production plants, and the importance of the multi-output plants. In this part, we present the results of the scale efficiency scores. Those scores are computed using **(LP-3)** for every plant. We also add extra constraints on the prices as discussed before. We use three tools to present our results without losing too much information. Indeed, we have the scale efficiency scores for the 3389 plants for the 10 outputs and the nine years. In particular, an obvious choice is to give the averages and the medians for each year. While these two statistics give a good first approximation of the change of scale efficiency, relying only on the averages and medians to conclude if the scale efficiency has improved or not could be restrictive in this context. Namely, because of the size of our sample. As such, a more formal way to check the existence of an improvement is to make use of the two-sample Kolmogorov-Smirnov test (KS test). This is a nonparametric test that checks whether the distributions of two samples are equal or not. In our context, we calibrate the test to check whether an improvement exists, i.e. if the distribution has moved to the right. As such, we also provide the $p-$values of the two-sample

KS test. If the $p-$value is smaller than 5%, we can reject the null hypothesis, and conclude that the distribution has moved to the right implying an improvement of scale efficiency. The results for the total, non-renewable, and renewable electricity generations are shown in Table 4. For each type of electricity generation, we consider three cases: all the producers, the multi-output producers, and the single-output producers.

Table 4: Averages, Medians, and Kolmogorov-Smirnov $p-$values

Year	Total			Non-Renewable			Renewable		
	All	Multi	Single	All	Multi	Single	All	Multi	Single
2012	0.58	0.50	0.62	0.49	0.48	0.51	0.67	0.57	0.69
2010	0.61	0.57	0.64	0.51	0.54	0.48	0.72	0.69	0.72
2009	0.53	0.54	0.53	0.47	0.51	0.41	0.61	0.68	0.59
2007	0.46	0.56	0.40	0.54	0.54	0.54	0.37	0.62	0.32
2005	0.60	0.60	0.60	0.57	0.57	0.57	0.63	0.70	0.61
2004	0.55	0.57	0.54	0.52	0.53	0.51	0.58	0.70	0.56
2000	0.58	0.61	0.55	0.58	0.58	0.56	0.58	0.71	0.55
1999	0.51	0.58	0.47	0.57	0.57	0.57	0.45	0.58	0.42
1998	0.52	0.58	0.48	0.54	0.56	0.51	0.49	0.66	0.46
2012	0.62	0.53	0.69	0.48	0.51	0.46	0.71	0.60	0.73
2010	0.70	0.64	0.72	0.53	0.58	0.46	0.76	0.76	0.76
2009	0.57	0.61	0.56	0.49	0.55	0.37	0.62	0.76	0.60
2007	0.46	0.60	0.36	0.54	0.55	0.53	0.28	0.71	0.22
2005	0.63	0.66	0.61	0.62	0.63	0.59	0.63	0.78	0.62
2004	0.57	0.64	0.55	0.55	0.58	0.48	0.57	0.77	0.55
2000	0.58	0.70	0.52	0.64	0.66	0.56	0.54	0.77	0.51
1999	0.51	0.65	0.41	0.63	0.64	0.58	0.40	0.64	0.35
1998	0.51	0.66	0.39	0.63	0.63	0.58	0.39	0.73	0.38
2012 > 1998	0.00	0.01	0.00	0.00	0.02	0.00	0.00	0.99	0.00
2012 > 2010	0.00	0.01	0.00	0.00	0.00	0.00	1.00	0.97	1.00
2010 > 2009	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.61	0.00
2009 > 2007	0.00	0.19	0.00	0.34	0.17	0.90	0.00	0.00	0.00
2007 > 2005	0.94	0.03	0.84	0.00	0.03	0.00	0.82	0.99	0.83
2005 > 2004	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.45	0.00
2004 > 2000	0.00	1.00	0.00	0.96	1.00	0.17	0.00	0.20	0.00
2000 > 1999	0.00	0.00	0.00	0.00	0.00	0.15	0.00	0.00	0.00
1999 > 1998	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.89	0.21

Firstly, we can conclude that there is an improvement of scale efficiency over the period for the total electricity generation. Except for 2007, the averages and medians have increased, and the $p-$values of the KS test are close to zero. The same conclusion holds if we take only the multi- or the single-output producers into account. Note that the averages have decreased for the multi-output producers, but the $p-$values of the KS test are close to zero. This reveals the usefulness of this test in our context. Next, non-renewable electricity generation shows a slight improvement, while the improvement on renewable electricity generation is rather high. This is confirmed by the $p-$values of the KS test. Again, similar results hold when looking only at the multi- and single-output producers. Next, two important rankings seem present in Table 4. One, renewable electricity generation seems to present higher scale efficiency than non-renewable electricity. Two, single-output producers seem less efficient on non-renewable electricity, but more efficient on renewable than multi-output producers. The differences are more pronounced for renewable electricity generation. To formally check those two observations, we again make use of the KS test. The results are available in Table 5.

Table 5: Kolmogorov-Smirnov $p-$values

Year	Renewable > Non-renewable			Multi > Single		
	All	Multi	Single	Total	Non-renewable	Renewable
2012	0.00	0.00	0.00	0.00	0.00	0.00
2010	0.00	0.00	0.00	0.00	0.32	0.07
2009	0.00	0.00	0.00	0.09	0.36	0.20
2007	0.00	0.00	0.00	0.14	0.02	0.74
2005	0.00	0.00	0.00	0.00	0.01	0.13
2004	0.00	0.00	0.00	0.00	0.00	0.32
2000	0.00	0.00	0.00	0.00	0.00	0.38
1999	0.00	0.00	0.00	0.15	0.00	0.26
1998	0.00	0.00	0.00	0.06	0.00	0.24

Our first observation about the relationship between renewable and non-renewable electricity generation is clearly confirmed, as the KS test $p-$values are all close to zero, implying that the distribution of renewable electricity is always larger than that for non-renewable electricity. Our second observation is also confirmed. Indeed, the majority of the KS test $p-$values are close to zero for the total and non-renewable electricity, but not for the renewable electricity. As such, multi-output producers are justified for non-renewable electricity, while single-output producers are preferred for renewable electricity.

Finally, we present in Tables 7, 8, and 9 (available in the Appendix), the averages, medians, and KS test $p-$values of the scale efficiency scores for each type of electricity generation. We give below the main findings. Firstly , the averages and medians for the nuclear generation are the highest, and, as shown by the KS test $p-$values, they are increasing. The performance of the three other types of non-renewable electricity generation (i.e. coal, oil, and gas) are decreasing; this explains the relative worse performances for non-renewable electricity generation discussed previously. Then, there are high scores in hydro electricity generation (and increasing), biomass electricity generation (and decreasing), and wind electricity generation (and increasing), which explains the better performances for renewable electricity generation highlighted before.

When comparing single- and multi-output producers, it is clear that single-output producers perform better for hydro, biomass, and wind electricity generation, which explains the better performances of this type of producer for renewable electricity. The performances of the single-output plant for geothermal electricity generation are also high, but slowly decreasing. Note that no results are presented for geothermal electricity generation for the multi-output plants as only single-output plants produce that type of electricity. Interestingly, the performances of the multi-output producers are higher for solar electricity generation. For non-renewable electricity, the multi-output producers perform better for nuclear, coal, gas, and oil electricity generation; but as discussed previously, the differences are more pronounced for renewable generation.[11]

Shadow prices. We end our application by providing the shadow prices. As explained previously, one of the advantages of the suggested methodology for scale efficiency is that it works when prices are not or partially available. For our US plant context, this is attractive as only the state-level input prices are provided by the Environmental Protection Agency of the US. We provide in Table 6 the average shadow prices when assuming variable returns-to-scale. Note that similar results could be obtained when assuming constant returns-to-scale, but since the latter assumption is more restrictive, we rely on the former. This also reveals that assumptions about the technology should be well motivated as they have an impact on the results. As done previously for the scale efficiency results, we make a distinction between renewable

[11]Those results are also confirmed by KS tests. For the sake of compactness (i.e. 10 types of electricity for nine years), we do not present those results in the paper, but they are available by request from the author.

and non-renewable electricity generation, and between single- and multi-output producers. The shadow prices for nameplate capacity are given in dollar per MW, and for fuel in dollar per MMBtu.

Table 6: Shadow prices for nameplate capacity and fuel

Year	All	Non-Renewable		Renewable	
		Multi	Single	Multi	Single
2012	6.51	5.52	5.8	6.81	6.83
2010	7.73	6.65	6.81	8.01	7.86
2009	8.08	7.52	7.65	8.56	8.45
2007	8.09	7.52	7.64	8.56	8.45
2005	6.41	6.35	6.21	6.99	6.84
2004	5.42	5.31	5.22	5.68	5.76
2001	5.51	5.45	5.62	5.62	5.71
1999	5.5	5.31	5.54	5.63	5.64
1998	5.81	5.22	5.32	6.01	5.98
2012	3.09	3.01	3.12	-	-
2010	3.85	3.84	3.86	-	-
2009	3.86	3.7	4.01	-	-
2007	4.63	4.4	4.71	-	-
2005	5.63	5.56	5.65	-	-
2004	4.45	4.4	4.48	-	-
2004	5.45	5.32	4.98	-	-
1999	5.64	5.61	5.67	-	-
1998	5.21	5.19	5.23	-	-

As explained in detail in Section 2, the shadow prices represent the most favourable prices in the absence of price information. As such, they are not the estimators of the true unknown prices, and have thus to be interpreted carefully. Nevertheless, they not only provide interesting useful information for the managers of the plants, but also for policy-makers and regulators. At this juncture, we point out that, in general, estimating the production cost of the plants is not an easy task. In the US, the Environmental Protection Agency is putting much effort into estimating these costs.[12] The provided shadow prices could help in that task.

An initial observation is that the price of nameplate capacity for the producers of renewable electricity is, on average, higher than the price for the producers of non-renewable electricity. This could be explained as it is the only input used by

[12]See the following website for more detail: www.eia.gov/outlooks/capitalcost.

the producers of renewable electricity. Next, on average, the prices of the single-output producers are higher for both inputs than those of the multi-output producers. This might be explained by the presence of economies of scope. Indeed, a reason for producing more than one type of electricity is to obtain a decrease of the average total cost (see also our discussion of (3)). The smaller input prices might be interpreted in that sense.

4 Conclusion

In this paper, we have shown how to provide scale efficiency indicators at the output-level. This is particularly attractive for multi-output producers since it represents valuable information. Moreover, our output-specific scale efficiency measurements are nonparametric in nature, they take the economic objective of the producers into account, they can be defined without observing the input prices, and they are easy to interpret and to use in practice.

We have applied our methodology to the case of the US electricity plants. Using the plant-level database developed by the Environmental Protection Agency of the US, we evaluated the scale efficiency of more than 3300 US electricity plants from 1998 to 2012 for 10 different types of electricity generation (coal, oil, gas, nuclear, other fossil, wind, solar, geothermal, hydro, and biomass). We show that, while there is a scale improvement at the total electricity generation level, this is not the case for each of the 10 types of electricity. Also, we demonstrated that, in general, renewable electricity presents better scale of production than non-renewable electricity. Next, we highlighted the importance of the multi-output plants in the US electricity sector, and show that those plants are preferable for the production of non-renewable electricity, while single-output plants are preferable for renewable electricity.

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Appendix

Table 7: Number of producers and percentage of multi-output producers

Year	Coal	Oil	Gas	Nuclear	Other Fossil	Hydro	Biomass	Wind	Solar	Geothermal
2012	538	1032	1144	63	136	1178	339	53	18	27
	92%	73%	63%	8%	99%	2%	69%	9%	94%	0%
2010	581	1095	1167	65	151	1213	357	57	16	28
	93%	75%	65%	9%	99%	2%	69%	11%	94%	0%
2009	586	1075	1149	65	148	1219	358	55	15	28
	92%	74%	64%	6%	97%	3%	69%	7%	88%	0%
2007	588	1133	1171	65	177	1223	354	53	12	28
	91%	75%	67%	11%	98%	3%	69%	2%	92%	0%
2005	589	1196	1160	65	166	1219	351	54	12	28
	92%	76%	70%	11%	98%	3%	68%	4%	92%	0%
2004	591	1174	1142	65	165	1214	355	52	11	28
	92%	76%	69%	11%	97%	3%	66%	2%	91%	0%
2000	602	1330	1258	65	192	1225	345	51	11	27
	95%	79%	76%	14%	94%	3%	74%	0%	91%	0%
1999	602	1299	1252	65	213	1221	342	51	11	29
	95%	78%	74%	14%	94%	3%	74%	0%	91%	0%
1998	593	1264	1228	64	191	1219	323	51	11	28
	95%	78%	73%	14%	94%	1%	72%	0%	91%	0%

Table 8: Averages, Medians, and Kolmogorov-Smirnov $p-$values: all producers

Year	Coal	Oil	Gas	Nuclear	Other Fossil	Hydro	Biomass	Wind	Solar	Geothermal
2012	0.57	0.33	0.45	0.89	0.42	0.69	0.60	0.67	0.47	0.70
2010	0.74	0.35	0.37	0.89	0.45	0.72	0.73	0.62	0.54	0.78
2009	0.71	0.24	0.37	0.84	0.50	0.56	0.75	0.67	0.55	0.64
2007	0.67	0.38	0.40	0.87	0.54	0.26	0.70	0.65	0.61	0.66
2005	0.75	0.42	0.41	0.89	0.51	0.58	0.77	0.71	0.59	0.67
2004	0.71	0.35	0.41	0.91	0.48	0.52	0.77	0.68	0.60	0.72
2000	0.77	0.37	0.47	0.84	0.54	0.51	0.77	0.65	0.43	0.62
1999	0.74	0.37	0.45	0.85	0.50	0.37	0.64	0.68	0.48	0.63
1998	0.66	0.26	0.56	0.81	0.53	0.42	0.70	0.65	0.47	0.70
2012	0.59	0.33	0.42	0.90	0.42	0.74	0.61	0.63	0.46	0.74
2010	0.76	0.33	0.32	0.91	0.44	0.76	0.78	0.60	0.43	0.78
2009	0.74	0.19	0.34	0.83	0.48	0.57	0.78	0.68	0.50	0.64
2007	0.72	0.38	0.37	0.87	0.52	0.17	0.73	0.64	0.71	0.64
2005	0.78	0.45	0.42	0.90	0.53	0.60	0.81	0.72	0.72	0.69
2004	0.74	0.33	0.38	0.92	0.48	0.54	0.81	0.70	0.60	0.72
2000	0.80	0.34	0.51	0.90	0.62	0.48	0.82	0.71	0.42	0.66
1999	0.76	0.36	0.45	0.91	0.58	0.33	0.68	0.70	0.46	0.69
1998	0.70	0.20	0.66	0.87	0.57	0.37	0.75	0.70	0.38	0.78
2012 > 1998	0.97	0.00	0.00	0.00	0.31	0.00	0.83	0.71	0.27	0.61
2012 > 2010	1.00	0.02	0.00	0.79	0.44	0.99	0.94	0.13	0.77	0.96
2010 > 2009	0.00	0.00	0.05	0.00	0.63	0.00	0.98	0.91	0.74	0.00
2009 > 2007	0.00	0.97	0.98	0.86	0.89	0.00	0.00	0.20	0.60	0.54
2007 > 2005	1.00	0.89	0.00	0.94	0.02	0.65	0.99	0.93	0.43	0.86
2005 > 2004	0.00	0.00	0.08	0.98	0.09	0.00	0.38	0.40	0.80	0.86
2004 > 2000	0.88	0.59	0.28	0.11	0.41	0.00	0.16	0.56	0.13	0.32
2000 > 1999	0.00	0.00	0.00	0.12	0.15	0.00	0.00	0.69	0.86	0.69
1999 > 1998	0.00	0.00	0.15	0.00	0.59	0.23	0.68	0.39	0.51	0.98

Table 9: Averages, Medians, and Kolmogorov-Smirnov $p-$values: multi-output producers

Year	Coal	Oil	Gas	Nuclear	Other Fossil	Hydro	Biomass	Wind	Solar	Geothermal
2012	0.59	0.41	0.52	0.89	1.00	0.51	0.58	0.40	0.49	-
2010	0.81	0.40	0.43	0.91	0.55	0.57	0.70	0.47	0.54	-
2009	0.77	0.20	0.43	0.84	0.86	0.40	0.72	0.25	0.55	-
2007	0.68	0.52	0.48	0.87	0.77	0.14	0.68	0.14	0.61	-
2005	0.79	0.54	0.51	0.89	0.66	0.47	0.74	0.44	0.59	-
2004	0.75	0.38	0.51	0.91	0.73	0.40	0.73	0.65	0.61	-
2000	0.82	0.39	0.61	0.91	0.79	0.35	0.74	-	0.43	-
1999	0.77	0.49	0.56	0.91	0.56	0.30	0.60	-	0.48	-
1998	0.61	0.23	0.68	0.87	0.76	0.24	0.68	-	0.49	-
2012	0.61	0.37	0.49	0.90	1.00	0.57	0.60	0.42	0.46	-
2010	0.85	0.40	0.43	0.91	0.55	0.70	0.76	0.54	0.42	-
2009	0.80	0.10	0.41	0.83	0.86	0.49	0.77	0.20	0.56	-
2007	0.73	0.51	0.46	0.87	0.75	0.07	0.72	0.14	0.71	-
2005	0.83	0.54	0.61	0.91	0.66	0.55	0.80	0.44	0.72	-
2004	0.78	0.33	0.55	0.92	0.77	0.50	0.79	0.65	0.67	-
2000	0.83	0.32	0.67	0.92	0.83	0.41	0.78	-	0.43	-
1999	0.80	0.50	0.60	0.92	0.49	0.29	0.65	-	0.47	-
1998	0.63	0.12	0.77	0.88	0.84	0.21	0.73	-	0.54	-
2012 > 1998	0.87	0.00	0.00	0.00	0.06	0.01	0.97	-	0.61	-
2012 > 2010	0.97	0.00	0.00	0.98	0.06	0.84	0.93	0.78	0.77	-
2010 > 2009	0.13	0.00	0.05	0.00	0.85	0.00	0.93	0.13	0.72	-
2009 > 2007	0.00	0.67	0.64	0.93	0.06	0.00	0.00	1.00	0.59	-
2007 > 2005	0.98	0.02	0.00	0.93	0.07	1.00	0.98	0.14	0.43	-
2005 > 2004	0.06	0.00	0.05	0.98	0.37	0.06	0.40	0.56	0.80	-
2004 > 2000	0.77	0.00	0.10	0.10	0.38	0.31	0.14	-	0.37	-
2000 > 1999	0.01	0.01	0.00	0.15	0.15	0.07	0.00	-	0.93	-
1999 > 1998	0.00	0.00	0.67	0.00	0.49	0.96	0.88	-	0.48	-

Table 10: Averages, Medians, and Kolmogorov-Smirnov $p-$values: single-output producers

Year	Coal	Oil	Gas	Nuclear	Other Fossil	Hydro	Biomass	Wind	Solar	Geothermal
2012	0.57	0.31	0.42	0.88	0.41	0.69	0.63	0.69	0.04	0.70
2010	0.73	0.33	0.34	0.71	0.45	0.72	0.78	0.64	0.45	0.78
2009	0.70	0.25	0.33	0.84	0.49	0.57	0.81	0.71	0.50	0.64
2007	0.67	0.33	0.37	0.84	0.54	0.26	0.75	0.66	0.54	0.66
2005	0.75	0.38	0.37	0.87	0.51	0.59	0.84	0.72	0.55	0.67
2004	0.71	0.34	0.36	0.89	0.47	0.52	0.84	0.68	0.43	0.72
2000	0.77	0.36	0.42	0.53	0.53	0.51	0.85	0.71	0.41	0.74
1999	0.74	0.34	0.41	0.61	0.50	0.38	0.74	0.73	0.39	0.72
1998	0.66	0.27	0.51	0.58	0.51	0.42	0.75	0.71	0.28	0.80
2012	0.59	0.30	0.36	0.89	0.42	0.74	0.65	0.64	0.04	0.74
2010	0.76	0.31	0.29	0.83	0.44	0.76	0.82	0.61	0.45	0.78
2009	0.74	0.23	0.31	0.83	0.48	0.57	0.82	0.70	0.50	0.64
2007	0.71	0.32	0.34	0.87	0.52	0.17	0.76	0.65	0.54	0.64
2005	0.78	0.39	0.35	0.89	0.53	0.60	0.84	0.72	0.55	0.69
2004	0.73	0.33	0.31	0.93	0.47	0.54	0.83	0.70	0.43	0.72
2000	0.79	0.35	0.45	0.82	0.62	0.48	0.88	0.72	0.41	0.69
1999	0.76	0.32	0.38	0.86	0.59	0.33	0.75	0.73	0.39	0.70
1998	0.70	0.23	0.62	0.80	0.55	0.37	0.82	0.71	0.28	0.80
2012 > 1998	0.99	0.00	0.01	0.05	0.30	0.00	0.42	0.67	0.23	0.87
2012 > 2010	1.00	0.99	0.00	0.18	0.57	0.99	0.93	0.06	1.00	0.96
2010 > 2009	0.00	0.00	0.03	0.44	0.53	0.00	0.55	0.98	1.00	0.00
2009 > 2007	0.00	1.00	0.99	0.05	0.92	0.00	0.00	0.13	1.00	0.54
2007 > 2005	1.00	0.97	0.00	0.50	0.04	0.64	0.96	0.93	0.56	0.86
2005 > 2004	0.00	0.00	0.32	0.84	0.10	0.00	0.71	0.38	0.14	0.86
2004 > 2000	0.91	0.90	0.63	0.32	0.29	0.00	0.33	0.74	0.08	0.56
2000 > 1999	0.00	0.03	0.06	0.73	0.22	0.00	0.00	0.60	0.14	0.64
1999 > 1998	0.00	0.00	0.17	0.25	0.39	0.22	0.67	0.36	0.14	0.98