Multi-output efficiency with good and bad outputs[*]

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Abstract

Cherchye et al. (2013) introduced a DEA methodology that is specially tailored for multi-output efficiency measurement. The methodology accounts for jointly used inputs and incorporates information on how inputs are allocated to outputs. In this paper, we present extensions that render the methodology useful to deal with undesirable (or "bad") outputs in addition to desirable (or "good") outputs. Interestingly, these extensions deal in a natural way with several limitations of existing DEA approaches to treat undesirable outputs. We also demonstrate the practical usefulness of our methodological extensions through an application to US electric utilities.

Keywords: DEA, multi-output production, (sub-)joint inputs, output objectives, undesirable outputs, electric utilities.

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1 Introduction

Data Envelopment Analysis (DEA; after Charnes, Cooper and Rhodes (1978)) evaluates the efficiency of a Decision Making Unit (DMU) by comparing its input-output performance to that of other DMUs operating in a similar technological environment.[1] The method is intrinsically nonparametric as it is avoids using (unverifiable) parametric/functional structure for the production technology. It "lets the data speak for themselves" and directly starts from the observed input-output combinations (associated with the evaluated DMUs). It reconstructs the production possibilities by (only) assuming standard production axioms (such as monotonicity and convexity). DMU efficiency is then measured as the distance of the corresponding input-output combination to the efficient frontier of this empirical production possibility set. By now, DEA has become very popular both as an analytical research instrument and a decision-support tool.

Recently, Cherchye et al. (2013) developed a novel DEA methodology that is specially tailored for multi-output efficiency measurement.[2] The methodology accounts for joint inputs in the production process and incorporates specific information on how inputs are allocated to individual outputs. In what follows, we will present several extensions of this multi-output efficiency measurement methodology, to show its usefulness to deal with undesirable (or "bad") outputs. To this end, we will introduce the new concept of "sub-joint" inputs, and indicate how output objectives can be included in the multi-output efficiency analysis. Interestingly, as we will indicate, these extensions deal in a natural way with several limitations of existing DEA approaches to treat undesirable outputs.

We will demonstrate the practical usefulness of our newly developed methodology through an application to US electric utilities. Obviously, electricity production processes are characterized by not only good but also bad outputs, i.e. greenhouse gas emissions. We remark that electric utilities effectively do have an economic motivation to reduce greenhouse gases. As we will explain more in detail in Section 4, the Acid Rain Program of the Clean Air Act puts limitations on the greenhouse gas emissions, and utilities are penalized if they pollute too much.

[1]See, for example, Fare, Grosskopf and Lovell (1994), Cooper, Seiford and Zhu (2004), Cooper, Seiford and Tone (2007), Fried, Lovell and Schmidt (2008), and Cook and Seiford (2009) for reviews.

[2]See also Cherchye, De Rock and Vermeulen (2008) and Cherchye et al. (2014) for closely related studies.

The rest of this paper unfolds as follows. Section 2 motivates our analysis. Section 3 introduces our methodology for multi-output efficiency evaluation with undesirable outputs, sub-joint inputs and output objectives. Section 4 uses this method to evaluate the efficiency of US electric utilities. Section 5 summarizes our main conclusions.

2 Multi-output efficiency and bad outputs

In this section, we motivate the theoretical and practical relevance of our following analysis. In doing so, we will also position our main contributions in the relevant literature.

2.1 Multi-output efficiency with output objectives

Standard DEA models treat the conversion of inputs into the outputs as a "black box": they do not assume any particular structure on how inputs are linked to outputs. However, in many empirical applications it is possible to allocate particular inputs to specific outputs. The methodology of Cherchye et al. (2013) can account for such information. In particular, the new methodology characterizes each output by its own production technology, while accounting for interdependencies between the different output-specific technologies (through jointly used inputs). An interesting feature of the methodology is that it has more discriminatory power than standard DEA methods, precisely because it uses the available information on the allocation of inputs to outputs and because it explicitly models the economies of scope stemming from joint input use.

More specifically, the starting point of the methodology is that the presence of economies of scope form a prime economic motivation for simultaneously producing multiple outputs. Basically, economies of scope originate from so-called *joint* inputs, which have a "public good" nature in that they simultaneously benefit the production of all the outputs that are produced. Cherchye et al.'s methodology explicitly distinguishes between these joint inputs and *output-specific* inputs, which are allocated to individual outputs. A first extension of the current paper is that we introduce the concept of *sub-joint* inputs, which at the same time contribute to multiple outputs but not to all outputs. In other words, like joint inputs, these sub-joint inputs act as public goods in the production process, but only for a subset of outputs. In a sense, this new category of inputs is situated between the categories of joint inputs (contributing to all outputs) and output-specific inputs (contributing to individual outputs).

In the present paper, we will show that the use of output-specific production technologies characterized by (sub)joint inputs is particularly useful in settings characterized by undesirable outputs. Indeed, inputs that simultaneously generate not only good outputs but also bad outputs are essentially (sub)joint inputs. As such, Cherchye et al.'s methodology for multi-output efficiency measurement provides a natural framework for efficiency analysis with good and bad outputs. This will constitute the basic starting point of our formal argument developed in the following sections.

At this point, we remark that our approach bears a close relationship to several existing approaches in the DEA literature. Firstly, there is a clear connection with network DEA (see Färe and Grosskopf (2000) and Färe, Grosskopf and Whittaker (2007)). Network DEA also makes use of what we call output-specific inputs.[3] However, the crucial difference between our approach and network DEA pertains to our modeling of (sub)joint inputs. As explained above, this type of inputs plays an important role in our approach because it defines the interdependencies between the production processes associated with different outputs. By contrast, to the best of our knowledge, the existing literature on network DEA abstracted from this possibility of jointly used inputs. Secondly, Salerian and Chan (2005) and Despic et al. (2007) present two alternative methods to model inputs that contribute to some outputs but not to others. As such, these models can actually be interpreted as special cases of our model with (sub)joint inputs.

All the above approaches have in common that they try to enhance the realism of the efficiency analysis by integrating information on the internal production structure. We believe that our methodology provides a unifying framework that is consistent with these approaches. This framework should be particularly attractive to empirical researchers who are familiar with standard DEA techniques and interested in the

[3]Both network DEA and our approach assume that the output-specific inputs are observed for each different output. There exist a number of alternative approaches that are not based on observing this information (i.e. the exact decomposition (over outputs) of the output-specific inputs is unknown to the empirical analyst). See, for example, Cook, Habadou and Teunter (2000), Beasley (2003), Lozano and Villa (2004), Li et al. (2009), Yu, Chern, Hsiao (2013) and Du et al. (2014). In principle, these other approaches can be combined with ours but, for compactness, we will not discuss this in the current paper.

analysis of multi-output production characterized by (sub)joint inputs.

The second methodological extension that we will present pertains to the fact that the original method of Cherchye et al. (2013) focused exclusively on the minimization of input quantities. In what follows, we will show how to include output objective considerations in the efficiency evaluation, so offering the possibility to simultaneously consider input and output improvements in the efficiency assessment. Again, we will argue that such output objectives can be especially relevant in the context of undesirable outputs. In particular, it allows for explicitly incorporating specific targets regarding the reduction of these bad outputs in the evaluation exercise. At this point, however, we emphasize that the usefulness of this output objective methodology is not restricted to settings with undesirable outputs. Actually, we believe the concept of output objectives can be particularly useful in many alternative contexts where specific (good) output (expansion) objectives are important together with input reduction.

As a concluding remark, by incorporating output objectives in the analysis, we actually do consider simultaneous input and output adjustments in the efficiency evaluation exercise. Interestingly, this falls in line with the existing literature on undesirable outputs, which typically uses non-oriented models that seek simultaneously the reduction of inputs, the increase of good outputs and the decrease of bad outputs. In this sense, our use of output objectives effectively defines a "non-oriented" (or "semi-oriented") version of the method originally proposed by Cherchye et al. (2013).

2.2 Efficiency measurement with undesirable outputs

In the literature, we can distinguish two main approaches to integrate undesirable outputs into DEA efficiency analysis. The first approach, which is the dominant one in the literature, uses specific DEA models to deal with with undesirable outputs (defined by specific production axioms and/or specific efficiency measures). The second approach uses standard DEA models but with a special treatment of the undesirable outputs (i.e. as transformed into desirable outputs or as inputs). Before presenting our own approach, we briefly review each of these existing approaches. This will also help us to highlight the specificities of our novel approach.

The first existing approach makes use of DEA models that are specially tailored to handle undesirable outputs. Here, one possibility is to introduce specific production axioms to reconstruct the production possibilities. The most popular axioms are weak disposability (see Färe, Grosskopf, Lovell and Pasurka (1989)), which implies that bad outputs can only be reduced with a proportional reduction of desirable (or "good") outputs, and null-jointness (see Färe and Grosskopf (2004)), which states that the only way to produce no bad output is to produce no good output. See also Sahoo, Luptacik and Mahlberg (2011), Murty, Russell and Levkoff (2012) and Leleu (2013) for recent contributions. The literature recognized three problems related to this axiomatic approach. Firstly, the analysis of undesirable outputs crucially relies on (non-standard) production axioms that -unfortunately- are usually nonverifiable. Secondly, weak disposability does not exclude positive (instead of negative) shadow prices for the bad outputs, which is counterintuitive (see the debate between Hailu and Veeman (2001), Hailu (2003) and Färe and Grosskopf (2003)). Thirdly, it is often difficult to precisely define the DEA-type production possibility set under the stated axioms (see the exchange between Kuosmanen (2005), Färe and Grosskopf (2009) and Kuosmanen and Podinovski (2009)).

Instead of explicitly altering the production possibility set, one can also use alternative efficiency measures that are specifically defined to account for undesirable outputs. Notable examples are directional distance functions (see Chung, Färe, and Grosskopf (1997) and Färe and Grosskopf (2004)) and hyperbolic efficiency measures (see Färe, Grosskopf, Lovell and Pasurka (1989) and Färe, Grosskopf and Lovell (1994)), among many others. However, it is not a priori clear which of these (non-standard) measures is the "most natural" one to deal with bad outputs. In addition, using these measures often requires extra modeling choices (e.g. defining the direction vector for the directional distance functions), for which clear guidelines are not readily available.

The second existing approach uses standard DEA models but with a special treatment of the undesirable outputs. Here, a first option is to transform the undesirable outputs into desirable outputs. The most common transformations consist of multiplying the bad outputs by -1 (see Golany and Roll (1989)) or taking the reciprocal value of the undesirable output quantities (see Koopmans (1951) and Seiford and Zhu (2002)). Importantly, however, for standard DEA models alternative transformations may significantly change the efficiency results, and the most appropriate transformation is not obvious a priori. See, for example, Scheel (2001) and Zhou, Ang and Poh

(2008) for more discussion.

A directly related procedure, which was adopted by Reinhard, Lovell, and Thijssen (2000) and Hailu and Veeman (2001), consists of treating undesirable outputs as inputs. However, Färe and Grosskopf (2003, 2004) find this procedure inconsistent with physical laws and standard axioms of production theory. Moreover, by definition this approach makes that the link between the inputs and the bad outputs completely disappears.

Because of the above mentioned problems associated with existing approaches, there is currently no consensus on the "best" approach for modeling undesirable outputs. We take this as a starting motivation to propose a novel methodology that avoids these problems, in a very natural way. The main distinguishing feature of our approach is that we characterize bad outputs in terms of their own production technologies (while allowing for interdependencies between bad and good outputs), by suitably adapting the framework for multi-output efficiency measurement that we introduced above. This will have several attractive implications. First, we do not need to resort to production axioms different from the standard ones. We can avoid discussions related to the use of non-standard axioms such as weak disposability and null-jointness. Next, bad outputs are not treated as inputs and the efficiency results are invariant to the specific (bad to good) output transformation that is used. The frontier of the output-specific production sets (and the distance to it) is the same for any transformation. Finally, the approach makes use of a (standard) radial efficiency measure. In our opinion, all this makes our approach particularly attractive to empirical DEA practitioners.

3 Methodology

In what follows, we start by introducing some necessary notation and terminology. Here, we will also define our new concept of sub-joint inputs. Next, we present our efficiency measure and indicate how to compute it in our multi-output setting. Finally, we show how to extend the efficiency measurement methodology in order to account for output objectives.

3.1 Preliminaries

We start by introducing our notation and the concept of input requirement sets. Using a different input requirement set for every individual (good or bad) output will explicitly recognize that each output is characterized by an own production technology. In Section 3.1, we will not explicitly distinguish between good and bad outputs, and implicitly consider all outputs as good outputs. This directly demonstrates that the applicability of our new methodology (with sub-joint inputs and output objectives) is not restricted to settings with undesirable outputs. In Section 3.2, we will discuss the conversion of bad outputs into good outputs, which shows how to use the methodology in case of both good and bad outputs.

Inputs and outputs. We consider a production technology that uses N inputs, captured by the vector $\mathbf{X} = (x^1, \ldots, x^N)' \in \mathbb{R}^N_+$, to produce M outputs, captured by the vector $\mathbf{Y} = (y^1, \ldots, y^M)' \in \mathbb{R}^M_+$. Each individual output is characterized by its own production process and, as indicated in the Introduction, we distinguish between three categories of inputs to capture the interdependence between these output-specific processes.

- *Output-specific* inputs are allocated to individual outputs m since they are only used in the production process of that specific output. We use $\alpha_i^m \in [0, 1]$, with $\sum_{m=1}^{M} \alpha_i^m = 1$, to represent the fraction of the i-th output-specific input quantity that is allocated to output m.

- *Joint* inputs are simultaneously used in the production process of all the outputs and can thus not be allocated to specific outputs. The use of joint inputs makes that output-specific production processes are interdependent.

- *Sub-joint* inputs also figure as joint inputs but only for a subset of outputs. As indicated in the Introduction, these inputs are situated between purely joint inputs and output-specific inputs. Obviously, sub-joint inputs also generate production interdependencies.

We summarize the information on how inputs are allocated to outputs by means of a vector $\mathbf{A}^m$ for each output m. Specifically, $\mathbf{A}^m$ is defined as

$$(\mathbf{A}^i)_k = \begin{cases} 1 & \text{if input } i \text{ is joint or sub-joint and used to produce output } m, \\ \alpha_i^m & \text{if input } i \text{ is output-specific and used to produce output } m, \\ 0 & \text{otherwise.} \end{cases}$$

Each $\mathbf{A}^m$ defines then the input vector $\mathbf{X}^m = \mathbf{A}^m \odot \mathbf{X}$, which thus contains the input quantities used in the production process of output m.[4]

As also discussed in Section 2, our use of output-specific and (sub)joint inputs brings together specific features of several existing DEA approaches. For example, network DEA makes use of output-specific inputs but, to our knowledge, does not model jointly used inputs. Next, Salarian and Chan (2005) and Despic et al. (2007) model inputs that contribute to some inputs but not to others, which parallels our notion of subjoint inputs. In a sense, our methodology is a unifying one in that it combines these different elements into a single encompassing framework.

Illustrative example. Consider a firm that produces three outputs. Let x^1 represent the input "building" and assume that this input cannot be allocated to any output since all outputs are produced in the same building. This input is an example of a joint input, meaning that $(A^1)_1 = (A^2)_1 = (A^3)_1 = 1$.

Next, x^2 represents the input "accounting". This input is only used in the production process of the first two outputs, but again it is not possible to allocate it to one of these two outputs. This is an example of a sub-joint input for which $(A^1)_2 = (A^2)_2 = 1$ and $= (A^3)_2 = 0$.

Finally, let x^3 represent "employees" that can be allocated to the production process of the specific outputs. This is an example of an output-specific input. Suppose the allocation of this input is 50% to output 1, 30% to output 2 and 20% to output 3. In terms of our above notation, we get

$$\mathbf{Y} = \begin{bmatrix} y^1 \\ y^2 \\ y^3 \end{bmatrix}, \mathbf{X} = \begin{bmatrix} x^1 \\ x^2 \\ x^3 \end{bmatrix}, \alpha_3^1 = 0.5,\ \alpha_3^2 = 0.3,\ \alpha_3^3 = 0.2, \tag{1}$$

[4]The symbol $\odot$ stands for the Hadamard (or element-by-element) product.

$$\mathbf{A}^1 = \begin{bmatrix} 1 \\ 1 \\ 0.5 \end{bmatrix}, \mathbf{A}^2 = \begin{bmatrix} 1 \\ 1 \\ 0.3 \end{bmatrix}, \mathbf{A}^3 = \begin{bmatrix} 1 \\ 0 \\ 0.2 \end{bmatrix}, \text{ and} \tag{2}$$

$$\mathbf{X}^1 = \mathbf{A}^1 \odot \mathbf{X} = \begin{bmatrix} x^1 \\ x^2 \\ 0.5 * x^3 \end{bmatrix}, \mathbf{X}^2 = \begin{bmatrix} x^1 \\ x^2 \\ 0.3 * x^3 \end{bmatrix}, \mathbf{X}^3 = \begin{bmatrix} x^1 \\ 0 \\ 0.2 * x^3 \end{bmatrix}. \tag{3}$$

Input requirement sets. Above, we defined the input vector $\mathbf{X}^m$ ($= \mathbf{A}^m \odot \mathbf{X}$) used for the production of output m. In turn, this allows us to characterize each output m by its own production technology. Formally, we represent this technology by input requirement sets $I^m(y^m)$, which contain all the combinations of output-specific, joint and sub-joint inputs (in $\mathbf{X}^m$) that can produce the output quantity y^m, i.e.

$$I^m(y^m) = \{\mathbf{X}^m \in \mathbb{R}^N_+ \mid \mathbf{X}^m \text{ can produce } y^m\}. \tag{4}$$

It is useful to emphasize once more the interdependencies between the different output-specific technologies. As mentioned before, joint and sub-joint inputs simultaneously enter the input vector $\mathbf{X}^m$ for multiple outputs m. As such, our definition of input requirement sets $I^m(y^m)$ provides a formal statement of these output-interdependencies.

As indicated above, the use of output-specific input requirement sets characterized by (sub)joint inputs naturally allows for dealing with undesirable outputs. The reason is that undesirable outputs can be considered as side-products of inputs that are used for the production of good outputs, which means that these inputs essentially figure as (sub)joint inputs.

3.2 Efficiency measurement

In what follows, we will first show how to transform bad outputs into good outputs prior to the actual efficiency evaluation exercise. Then, we will define our input efficiency measure. For a given output y^m and associated input $\mathbf{X}^m$, this measure quantifies the distance from $\mathbf{X}^m$ to the isoquant $\text{Isoq}I^m(y^m)$, which defines the technically efficient frontier of the input requirement set $I^m(y^m)$. In practical applications, we typically do not observe the true set $I^m(y^m)$ and so we need to construct an empirical approximation $\widehat{I}^m(y^m)$. As we will explain, we propose an empirical set $\widehat{I}^m(y^m)$ that is based on a number of standard production axioms commonly used in a nonparametric efficiency analysis. To facilitate empirical applications, we will also indicate how to compute our input efficiency measure with respect to $\widehat{I}^m(y^m)$ by means of simple linear programming techniques.

As our input efficiency measure quantifies the distance of some evaluated input vector to the technically efficient frontier, it is essentially a measure of technical efficiency. However, and importantly, it is also possible to interpret the same measure in term of cost efficiency. This follows from an argument of Cherchye et al. (2013). These authors start from a multi-output cost efficiency measure inspired by the structural efficiency measurement approach initiated by Afriat (1972), Hanoch and Rothschild (1972), Diewert and Parkan (1983) and Varian (1984), and obtain as a dual measure the technical efficiency measure that we use here. For compactness, we will not repeat the argument here, but refer to Cherchye et al. for more details.

Transforming bad outputs. In our efficiency analysis we must account for the undesirable feature of bad outputs. We do so by transforming the bad outputs $\mathbf{Y}^B$ into good outputs. Let $g\left(\mathbf{Y}^B\right)$ be the function that represents the bad output transformation. Then, the output vector $\mathbf{Y}$ is given by

$$\mathbf{Y} = (y^1, \ldots, y^M)' = \begin{bmatrix} \mathbf{Y}^G \\ g(\mathbf{Y}^B) \end{bmatrix}. \tag{5}$$

As mentioned above, several alternative transformations $g(\mathbf{Y}^B)$ are possible. For example, we may multiply the bad outputs by -1, or we may take the reciprocal values of the bad output quantities. The specific choice of the transformation is in general rather ad-hoc. However, for standard DEA models, the selection of the transformation function is not necessarily innocuous, as it influences the outcomes of the efficiency analysis. In this respect, a particularly attractive feature of our multi-output efficiency methodology is that the efficiency results it generates are fully independent of the transformation that is used. It is easily verified that any transformation of bad outputs (i.e. less is better) into good outputs (i.e. more is better) will yield exactly the same efficiency results for the linear program that we outline below. This is due to the fact that our methodology only uses information on output orderings (and not on cardinal output levels) when evaluating DMU efficiency.[5]

Input efficiency. Suppose we observe data for K DMUs. For each DMU $k \in \{1, \ldots, K\}$ we observe the output vector $\mathbf{Y}_k$ (with y_k^m the quantity of output m), the input vector $\mathbf{X}_k$, and the allocation of the inputs as joint, sub-joint and output-specific inputs. Using our notation introduced above, we can decompose $\mathbf{X}_k$ into $\mathbf{A}_k^1 \odot \mathbf{X}_k, \ldots, \mathbf{A}_k^M \odot \mathbf{X}_k$, which yields $\mathbf{X}_k^1, \ldots, \mathbf{X}_k^M$. Taken together, this gives the following data set S:

$$S = \{(\mathbf{Y}_k, \mathbf{X}_k^1, \ldots, \mathbf{X}_k^M) \mid k = 1, \ldots, K\}. \quad (6)$$

We evaluate input efficiency as the distance of the evaluated DMU's input vector to the isoquant $\text{Isoq}I^m(y_k^m)$, which is defined as

$$\text{Isoq}I^m(y_k^m) = \{\mathbf{X}^m \in I^m(y_k^m) \mid \text{for } \beta < 1, \beta\mathbf{X}^m \notin I^m(y_k^m)\}. \quad (7)$$

Thus, $\mathbf{X}^m \in \text{Isoq}I^m(y_k^m)$ means that the inputs $\mathbf{X}^m$ constitute minimal input quantities to produce the output quantity y_k^m and, as such, $\text{Isoq}I^m(y_k^m)$ represents the technically efficient frontier of $I^m(y_k^m)$.

In DEA, the most commonly used technical efficiency measure is the Debreu-Farell input efficiency measure. When adapting this measure to our multi-output setting (with M output-specific sets $I^m(y_k^m)$), we get

$$TE_k = TE_k(\mathbf{Y}_k, \mathbf{X}_k^1, \ldots, \mathbf{X}_k^M) = \min\{\theta \mid \forall m : \theta\mathbf{X}_k^m \in I^m(y_k^m)\}. \quad (8)$$

In words, TE_k defines the maximal equiproportionate/radial input reduction (captured by $\theta(\mathbf{X}_k^1, \ldots, \mathbf{X}_k^M)$) that still allows for producing the output $\mathbf{Y}_k$. Generally, TE_k is situated between 0 and 1, and a lower value of TE_k indicates greater technical inefficiency.[6]

[5] For completeness, we must add that the specific cardinalization that is used may matter when specific output objectives (which we capture by the vector τ below) are included in the analysis. Specifically, if the vector τ differs from the zero vector, alternative transformations of the bad outputs can impact the empirical approximation of the input requirement sets and, through this channel, can have an ("indirect") effect on the value of our efficiency measure.

[6] We remark that the DEA literature has also suggested measures of technical efficiency that are different from the radial Debreu-Farrell measure. It should be clear that our following methodology does not crucially rely on our use of the Debreu-Farrell measure, and so can easily include these alternatives measures. Our principal motivation to focus on the Debreu-Farrell measure is that

Technology axioms. As we defined it above, the measure TE_k does not have direct usefulness in practice. Indeed, it is based on the set $I^m(y_k^m)$, which is typically unknown to the empirical analyst. To solve this problem, we need to construct an empirical approximation $\widehat{I}^m(y_k^m)$ of the input requirement set $I^m(y_k^m)$ on the basis of the "minimum extrapolation" principle. This principle states that the set $\widehat{I}^m(y_k^m)$ must be the smallest empirical construction that is consistent with some given set of technology axioms. In the current paper, we make use of the following axioms.

Axiom 1 (nested input sets): $y^m \geq y^{m'} \implies I^m(y^m) \subseteq I^m(y^{m'})$.

Axiom 2 (monotone input sets): $\mathbf{X}^m \in I^m(y^m)$ and $\mathbf{X}^{m'} \geq \mathbf{X}^m \implies \mathbf{X}^{m'} \in I^m(y^m)$.

Axiom 3 (convex input sets): $\mathbf{X}^m \in I^m(y^m)$ and $\mathbf{X}^{m'} \in I^m(y^m) \implies \forall \lambda \in [0,1]: \lambda \mathbf{X}^m + (1-\lambda)\mathbf{X}^{m'} \in I^m(y^m)$.

Axiom 4 (observability means feasibility): $(\mathbf{Y}_k, \mathbf{X}_k^1 \ldots, \mathbf{X}_k^M) \in S \implies \forall m : \mathbf{X}_k^m \in I^m(y_k^m)$.

These four axioms are common to many popular DEA models and form an empirically attractive minimal set of assumptions. In words, Axiom 1 says that, if $\mathbf{X}^m$ can produce y^m, then it can also produce less output (i.e. $y^{m'}$). Essentially, this axiom of nested input sets implies that outputs are freely disposable. Next, Axiom 2 is equivalent to requiring freely disposable inputs, i.e. more input never reduces the outputs. Axiom 3 states that, if two inputs $\mathbf{X}^m$ and $\mathbf{X}^{m'}$ can produce y^m, then any convex combination $\lambda \mathbf{X}^m + (1-\lambda)\mathbf{X}^{m'}$ can also produce the same output. Finally, Axiom 4 says that what we observe is certainly feasible. Or, if we observe $(\mathbf{Y}_k, \mathbf{X}_k^1 \ldots, \mathbf{X}_k^M)$, then these input vectors can certainly produce the observed output.

Cherchye et al. (2013) have shown that the smallest empirical construction of the input requirement set $I^m(y_k^m)$ that is consistent with Axioms 1-4 is given by (for $s \in \{1, \ldots, K\}$)

$$\widehat{I}^m(y_k^m) = \left(\mathbf{X}^m \;\middle|\; \begin{array}{l} \sum_s \lambda_s^m \mathbf{X}_s^m \leq \mathbf{X}^m; \sum_s \lambda_s^m = 1 \\ \forall s : \lambda_s^m \geq 0 \text{ if } y_s^m \geq y_k^m \text{ and } \lambda_s^m = 0 \text{ otherwise} \end{array} \right). \tag{9}$$

this measure is still the most popular one in applied DEA work. Actually, as explained in the Introduction, we believe that an attractive feature of our methodology is exactly that it can make use of this (standard) radial efficiency measure when dealing with bad outputs.

Thus, if Axioms 1-4 hold, then $\widehat{I}^m(y_k^m) \subseteq I^m(y_k^m)$ and $\widehat{I}^m(y_k^m)$ provides a useful inner bound approximation of $I^m(y_k^m)$.[7]

Technical efficiency measurement. Assume we want to evaluate DMU $t \in \{1, \ldots, K\}$. Given the set $\widehat{I}^m(y_t^m)$, the input-oriented technical efficiency measure can be defined as

$$\widehat{TE}_t = \widehat{TE}_t(\mathbf{Y}_t, \mathbf{X}_t^1, \ldots, \mathbf{X}_t^M) = \min\{\theta \mid \forall m : \theta \mathbf{X}_t^m \in \widehat{I}^m(y_t^m)\}. \tag{10}$$

As before, we have that $\widehat{TE}_t$ is situated between 0 and 1 and lower value of $\widehat{TE}_t$ indicates greater technical inefficiency. Since $\widehat{I}^m(y_t^m) \subseteq I^m(y_t^m)$, we also have that $\widehat{TE}_t \geq TE_t$, i.e. $\widehat{TE}_t$ defines an upper bound to TE_t. Cherchye et al. (2013) have shown that that we can compute $\widehat{TE}_t$ by solving a linear program. Actually, this program is obtained as a special case (with $\tau = 0$) of the linear program that we introduce below for input efficiency evaluation with output objectives.

3.3 Output objectives

Besides minimizing the input quantities, DMUs also often pursue specific output objectives (e.g. increases of good outputs and/or reductions of bad outputs). In this section, we modify the above efficiency measure so that it can account for output-specific objectives. This will define a new input efficiency measure that not only seeks to minimize inputs but simultaneously accounts for output-specific objectives. In particular, we use $\tau = (\tau^1, \ldots, \tau^M) \in \mathbb{R}^M$ to denote the output objective vector as $((1 + \tau^1)\, y^1, \ldots, (1 + \tau^M)\, y^M)$. Clearly, choosing $\tau = (0, \ldots, 0)$ will yield the same efficiency criterion as before, whereas τ^m different from 0 for some m can define more stringent criteria. In our opinion, this provides an intuitive method to account for output objectives that, conveniently, does not involve specific assumptions on the reference technology.

[7] We note that the Axioms 1-4 do not include a specific returns-to-scale assumption and so allow for variable returns-to-scale. At this point, it is worth to stress that our methodology is readily adapted to incorporate alternative production axioms (e.g. specific returns-to-scale properties can be based on Petersen (1990) and Bogetoft (1996)). For simplicity, we opted not to focus on these axioms in the current paper.

Input efficiency with output objectives. As before, we start by defining the input requirement set that contains all the input vectors that can produce the output $(1+\tau^m)y_k^m$. In this case, this set is given as

$$I_\tau^m(y_k^m) = \left\{\mathbf{X}^m \in \mathbb{R}_+^N \mid \mathbf{X}^m \text{ can produce } (1+\tau^m)y_k^m\right\}. \tag{11}$$

Clearly, we have that $I_\tau^m(y_k^m) = I^m(y_k^m)$ if $\tau^m = 0$. More generally, given nested input requirement sets (i.e Axiom 1), if τ^m defines an output objective that dominates the output that is actually produced, we will have $I_\tau^m(y_k^m) \subseteq I^m(y_k^m)$. Finally, it may well be that the set $I_\tau^m(y_k^m)$ is empty, which corresponds to a situation where the stated output objectives are not achievable technically.

The radial Debreu-Farrell efficiency measure with output-specific objectives is given by

$$TE_k^\tau = TE_k^\tau(\mathbf{Y}_k, \mathbf{X}_k^1, \ldots, \mathbf{X}_k^M) = \min\{\theta \mid \forall m : \theta\mathbf{X}_k^m \in I_\tau^m(y_k^m)\}, \tag{12}$$

which has a directly similar interpretation as the measure TE_k that we defined above.

Linear programming formulation. Assume we evaluate DMU $t \in \{1, \ldots, K\}$. As before, we construct the empirical approximation $\widehat{I}_\tau^m(y_t^m)$ of the input requirement set $I_\tau^m(y_t^m)$ by imposing Axioms 1-4. We now get (for $s \in \{1, \ldots, K\}$)

$$\widehat{I}_\tau^m(y_t^m) = \left(\mathbf{X}^m \;\middle|\; \begin{array}{l} \sum_s \lambda_s^m \mathbf{X}_s^m \le \mathbf{X}^m; \sum_s \lambda_s^m = 1 \text{ and} \\ \forall s : \lambda_s^m \ge 0 \text{ if } y_s^m \ge (1+\tau^m)y_t^m \text{ and } \lambda_s^m = 0 \text{ otherwise} \end{array} \right). \tag{13}$$

Clearly, by choosing $\tau^m = 0$, we will obtain $\widehat{I}_\tau^m(y_t^m) = \widehat{I}^m(y_t^m)$ for each DMU t. Just like for $I_\tau^m(y_t^m)$, it may well be that $\widehat{I}_\tau^m(y_t^m)$ is empty for some values of τ^m.

Given the set $\widehat{I}_\tau^m(y_t^m)$, the input-oriented technical efficiency measure with output-specific objectives is defined as

$$\widehat{TE}_t^\tau = \widehat{TE}_t^\tau(\mathbf{Y}_t, \mathbf{X}_t^1, \ldots, \mathbf{X}_t^M) = \min\{\theta \mid \forall m : \theta\mathbf{X}_t^m \in \widehat{I}_\tau^m(y_t^m)\}. \tag{14}$$

It directly follows that $\widehat{I}_\tau^m(y_t^m) \subseteq \widehat{I}^m(y_t^m)$ implies that $\widehat{TE}_t^\tau \ge \widehat{TE}_t$. Or, in words, if τ^m defines an output objective that dominates the output that is actually produced, the corresponding efficiency measure will increase. Intuitively, there will be less scope for input reduction (captured by $\widehat{TE}_t^\tau$) if more stringent output objectives are to be realized.

In turn, this defines the linear program

$$
\begin{gathered}
\widehat{TE}_t^{\tau} = \min_{\lambda_s^m \ (m \in \{1,\ldots,M\}, s \in \{1,\ldots,K\})} \theta_t \\
\forall m : \sum_s \lambda_s^m \mathbf{X}_s^m \leq \theta_t \mathbf{X}_t^m \text{ for all } s : y_s^m \geq (1+\tau^m) y_t^m \\
\forall m : \sum_s \lambda_s^m = 1 \text{ for all } s : y_s^m \geq (1+\tau^m) y_t^m \\
\forall s, \forall m : \lambda_s^m \geq 0 \\
\theta_t \geq 0.
\end{gathered}
$$

This linear program computes the (radial) input reduction θ_t that could be realized for DMU t when it is compared to its dominating DMUs s. More precisely, prior to solving the linear program we need to identify for each output m the dominating DMUs s (i.e. the DMUs with $y_s^m \geq (1+\tau^m)y_t^m$).[8] We remark that, for some evaluated DMU t, this may yield a different set of dominating DMUs s for each other output m. Subsequently, the linear program compares, for each output m, the associated inputs for DMU t to convex combinations of the corresponding inputs for the dominating DMUs s. From that comparison, the linear program computes DMU t's efficiency in terms of a single θ_t that is common to all outputs m. This guarantees that the resulting efficiency value (i.e. the minimal value of θ_t) defines a (radial) input contraction that simultaneously accounts for the technological production possibilities associated with all different outputs m.

As a final note, in our following application we will use $\widehat{TE}_t^{\tau} = 1$ in case $\widehat{I}^m(y_t^m)$ turns out to be empty. This means that we choose to label a DMU as efficient if the associated output objectives appear to be overly ambitious, i.e. they are not achievable for the given state of technology (and the empirical approximation $\widehat{I}^m(y_t^m)$ that is used). The underlying reasoning is that too severe objectives disable the potential for input reduction, which we capture by $\widehat{TE}_t^{\tau} = 1$.

[8]Essentially, as compared to the linear program in the original study of Cherchye et al. (2013), the use of $(1+\tau^m)y_t^m$ instead of y_t^m guarantees that our efficiency evaluation takes the output-specific objectives (captured by τ^m) into account.

4 An application to US electric utilities

The efficiency of electric utilities has been a popular subject of analysis in the efficiency measurement literature. See, for example, Yaisawang and Klein (1994), Färe, Grosskopf, Noh and Weber (2005) and Sarkis and Cordeiro (2012), for analyses of US electric utilities, Goto and Tsutsui (1998), Hattori (2002) and Tone and Tsutsui (2007) for analyses of both Japanese and US electric utilities, and Korhonen and Luptacik (2004) for an analysis of European electric utilities.

A common feature of these studies is that they systematically select nameplate generation (used as a proxy for total assets) and the quantity of fuel used as two main inputs, and quantity of electricity generated as a (good) output.[9] This set-up implicitly assumes that all electricity is produced by the use of fuel. In our application, we will consider a somewhat refined setting by explicitly distinguishing between electricity generated by fossil energies (e.g. coal, oil, gas) and electricity generated by non-fossil energies (e.g. wind, solar, geothermal). Next, we consider SO_2, NO_x, CO_2 emissions as bad outputs of the electricity production process.

For the given input-output selection, we may reasonably assume the good output (electricity generated) is exogenously defined, which means that the size of the electricity market (or number of consumers) falls beyond control of the electric utilities. As such, we can measure the efficiency of our DMUs in terms of input (or cost) reduction for the given level of the good output. Next, apart form minimizing inputs, electric utilities typically also pursue reduction of greenhouse gases (because of the Acid Rain Program). In our application, we will account for this by additionally including objectives for the undesirable outputs.

In what follows, we first discuss the specificities of our set-up in more detail. Subsequently, we present our data and the results of our empirical analysis.

4.1 Set-up

In this section we motivate the input and output selection that we use in our efficiency evaluation, and we discuss some methodological issues that are specific to our DEA

[9] Some studies use total number of employees, the generator capacity and the boiler capacity as additional inputs. Obviously, these inputs could readily be included in our application provided the required data were available for the DMUs under study. However, the eGRID database that we use does not contain these data.

assessment. Here, we will also indicate how the methodology outlined above can naturally deal with bad outputs.

Input and output selection. We have taken our data from the eGRID system that is developed by the Environmental Protection Agency (EPA) of the US. eGRID stands for a comprehensive source of data on the environmental characteristics of all electric power generated in the US. In particular, we use the eGRID 2012 version 1.0, and concentrate on the year 2009, which is the most recent year for which data are available.

Following the standard approach in this type of applications, our two inputs are nameplate generation (used as a proxy for total assets) and the quantity of fuel that is used. We remark that the total number of employees could also be seen as an important input. However, these data are not available in our database for the DMUs that we evaluate, and so we cannot incorporate this input in our efficiency assessment. As such, the implicit assumption is that the effect of employees on DMU efficiency is adequately captured by the other inputs that we do include. Next, in principle, generator capacity and boiler capacity can also be considered as inputs, but these two inputs are aggregated into nameplate generation and we choose not to include them separately in order to keep our analysis as simple as possible. All this yields to a production setting with two inputs (i.e. $N = 2$).

The production process of electric utilities is characterized by desirable as well as undesirable outputs. Formally, we distinguish between good outputs $\mathbf{Y}^G \in \mathbb{R}_+^{M_{\text{good}}}$ and bad outputs $\mathbf{Y}^B \in \mathbb{R}_+^{M_{\text{bad}}}$, where $M_{\text{good}} + M_{\text{bad}} = M$. As argued in the Introduction, our analysis differs from more standard ones by not treating total electricity production as the only good output. By contrast, we explicitly distinguish between electricity generated by fossil energies (i.e. coal, oil, gas, nuclear) and electricity generated by non-fossil energies (i.e. hydro, biomass, wind, solar and geothermal). The undesirable outputs we consider are the emissions of the greenhouse gases SO_2, NO_x, CO_2. In the end, this defines $M_{\text{good}} = 2$, $M_{\text{bad}} = 3$ and $M = 5$.

As discussed above, our method takes into account that the production processes of the bad and good outputs are linked to each other. More precisely, by considering our inputs as (sub)joint inputs, we incorporate that it is impossible to produce electricity without producing greenhouse gases. Moreover, by treating fuel consumption as a sub-joint input, we also acknowledge that electricity generated by non-fossil energies does not use fuel. Figure **??** summarizes all this and presents a schematic comparison between the "more standard" setting and our approach.

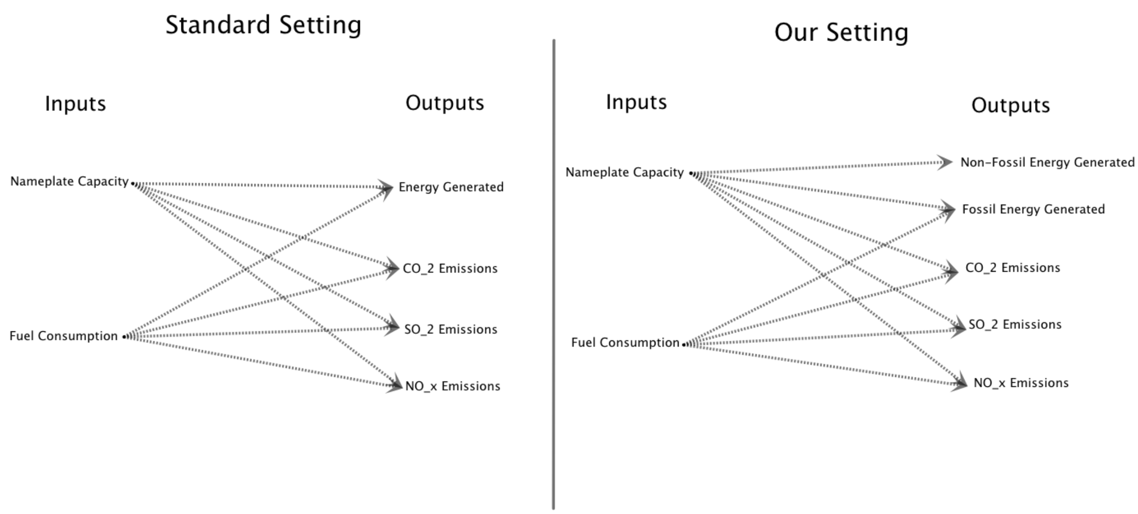

Figure 1: Sub-joint inputs in production

Bad outputs and output objectives. Summarizing, we obtain a setting with two good outputs (non-fossil electricity generated, y_1^G, and fossil electricity generated, y_2^G), three bad outputs (CO_2, y_1^B, SO_2, y_2^B, and NO_x, y_3^B), one joint input (nameplate capacity, x^1), and one sub-joint input (fuel consumption, x^2). To transform our bad outputs into good outputs, we choose the function $g(\mathbf{Y}^B) = -\mathbf{Y}^B$ in our empirical application. Adopting the above notation, we get for each DMU k:

$$\mathbf{Y}_k = \begin{bmatrix} y_{1,k}^G \\ y_{2,k}^G \\ -y_{1,k}^B \\ -y_{2,k}^B \\ -y_{3,k}^B \end{bmatrix}, \mathbf{X}_k = \begin{bmatrix} x_k^1 \\ x_k^2 \end{bmatrix}, \mathbf{A}_k^1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } \mathbf{A}_k^2 = \mathbf{A}_k^3 = \mathbf{A}_k^4 = \mathbf{A}_k^5 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \tag{15}$$

$$\mathbf{X}_k^1 = \mathbf{A}_k^1 \odot \mathbf{X}_k = \begin{bmatrix} x_k^1 \\ 0 \end{bmatrix} \text{ and, similarly, } \mathbf{X}_k^2 = \mathbf{X}_k^3 = \mathbf{X}_k^4 = \mathbf{X}_k^5 = \begin{bmatrix} x_k^1 \\ x_k^2 \end{bmatrix}. \tag{16}$$

Next, our method allows us to set a specific objective for each of our 5 outputs.

Formally, we do this through specifying the vector $\tau = (\tau^1, \tau^2, \tau^3, \tau^4, \tau^5)$, where τ^1 and τ^2 correspond to the good outputs non-fossil and fossil electricity, which take positive values, and τ^3, τ^4 and τ^5 are associated with the bad outputs CO_2, SO_2 and NO_x emissions, which take negative values (for our transformation function $g(\mathbf{Y}^B) = -\mathbf{Y}^B$). Given the specific focus of our analysis, our following empirical analysis will not include specific objectives for the good outputs (i.e. $\tau^1 = \tau^2 = 0$) and, thus, we will exclusively concentrate on reductions of our last three outputs (by appropriately specifying $-\tau^3, -\tau^4$ and $-\tau^5$).

4.2 Data and results

We start by presenting some descriptive statistics of our data. Subsequently, we present the results of our efficiency analysis with and without output objectives.[10]

The data. The original eGRID database covers 5492 electricity plants. Importantly, however, for a DEA analysis to produce reliable results, we need that the different DMUs are sufficiently homogeneous/comparable. To guarantee such homogeneity, we follow Sarkis and Cordeiro (2012) and concentrate on utilities that generated at least 1,000,000 MWh in 2009. For the same reason, we exclude firms that only produce electricity by using non-fossil energies, as these firms exhibit too much heterogeneity. The resulting sample contains 573 plants. Table **??** reports the corresponding descriptive statistics for the different inputs and outputs taken up in our analysis.

	Outputs					Inputs	
	Non-Fossil Energy (MWh)	Fossil Energy (MWh)	CO_2 (tons)	SO_2 (tons)	NO_x (tons)	Nameplate Capacity (MW)	Fuel (MMBtu)
Min	0	71	345	3	1	136.9	4,267
Mean	73,634	4,334,300	3,842,800	3,328	9,593	1,026	41,351,000
Max	19,649,257	22,977,980	24,895,000	42,511	113,140	4,393	242,640,000
Std	996,390	3,829,900	4,223,400	4,671	16,598	697	39,203,000

Table 1: Descriptive statistics for our 573 plants

[10] All our results have been computed in Matlab, by using the Linprog module to solve our linear programming problems.

Efficiency without output objectives. We start by computing efficiency scores without explicitly considering output objectives (i.e. we solve the linear program in Section 3.2, which coincides with the linear program in Section 3.3 for τ a zero vector). Table **??** summarizes the results for our sample. We find that 162 out of 573 electric utilities (i.e. about 30% of all DMUs) are labeled as efficient. Next, the mean efficiency equals 0.90. This suggests that, on average, the electricity plants can save up to 10% of their inputs while still producing the same quantity of electricity and without increasing the greenhouse gas emissions. But there is also quite some heterogeneity across firms. For example, the standard deviation amounts to 0.12 and the minimum efficiency value is no more than 0.40, which suggest a potential input reduction of as much as 60%.

All in all, we believe the numbers in Table **??** usefully reveal the substantial potential of input/cost reduction in the US electricity sector. However, as indicated before, these efficiency results do not take into account the possibility of bad input reductions. From this perspective, it seems useful to evaluate the potential of input reduction when explicitly incorporating objectives on greenhouse gas reductions. This is what we explore next.

Min $\widehat{TE}$	Mean $\widehat{TE}$	Median $\widehat{TE}$	Max $\widehat{TE}$	St. dev.	$\sharp$ efficient	% efficient
0.40	0.90	0.94	1	0.12	162	28.27%

Table 2: Efficiency scores without output objectives

Efficiency with output objectives. To evaluate the effect of output-specific objectives, we consider four different scenarios for $\tau = (\tau^1, \tau^2, \tau^3, \tau^4, \tau^5)$ (where we use $\tau^1 = \tau^2 = 0$ for the good outputs). Essentially, the different scenarios correspond to a different weighting of our three undesirable outputs. For each scenario, we will consider different degrees of stringency for the bad output objectives. See Table **??**, in which the parameter σ figures as our parameter of objective stringency (i.e. higher values of σ indicate more ambitious environmental objectives). In that table, the first scenario is a "naive" one that accords exactly the same weight to CO_2, SO_2 and NO_x emissions. The second scenario is somewhat more sophisticated and uses "intensity-based" objectives, which take as a weight for each greenhouse gas its share relative to CO_2 emissions. See also Table **??**, which summarizes the information that underlies the construction of these shares.

Finally, our last two scenarios are directly related to the Acid Rain Program of the Clean Air Act. The goal of this program is to reduce the annual SO_2 and NO_x emissions, which are the primary causes of acid rain. This program requires a reduction of SO_2 emissions by 10 million tons and a reduction of NO_x emissions by 2 million tons (starting from 1980 levels). The program is split in two phases. Phase I, which began in 1995 and ended in 1999, affected 445 electricity units and only included SO_2 reduction, while Phase II, which began in 2000, impacted more than 2000 units and emphasized NO_x reduction in addition to SO_2 reduction. For more details on this program, we refer to the website of EPA (www.epa.gov).[11]

Scenario	Explanation	τ
1	Naive objectives	(0,0,-σ%,-σ%,-σ%)
2	Intensity-based objectives	(0,0,-σ%,-0.000867σ%,-0.0025σ%)
3	Acid Rain Program objectives (SO_2)	(0,0,0,-σ%, 0)
4	Acid Rain Program objectives (NO_x)	(0,0,0,0,-σ%)

Table 3: Alternative output objective scenarios

	CO_2	SO_2	NO_x	Total
Mean	3,842,800	3,328	9,593	3,855,721
Share	99.66%	0.0864%	0.25%	100%
Relative Share	1%	0.000867%	0.0025%	

Table 4: Scenario 2 - bad output weights

Figure 2 presents a compact summary of our results. For our four scenarios, it displays the percentage of efficient plants as a function of the parameter value σ, which ranges from 1 (least stringent objectives) to 20 (most stringent objectives). Here, we recall from Section 3.3 that more severe output objectives generally imply less potential for input reduction. As such, we may also expect that the number of efficient DMUs will increase when the parameter σ increases. This clearly appears from Figure 2, for each of the objective scenarios that we study. For scenarios 1 and 2 we can even conclude that there is no scope for input reduction at all (i.e. all DMUs are input efficient) when σ is set sufficiently high.

[11] Here, it is worth to add that Färe, Grosskopf, Noh and Weber (2005) and Sarkis and Cordeiro (2012) already studied the impact of this program on the efficiency of the US electricity plants. In a sense, our study is complementary to these earlier studies because we explicitly take up SO_2 reduction (scenario 3) and NO_x reduction (scenario 4) as output objectives in our efficiency assessment.

At a more detailed level, we find for scenario 1 that reducing all three greenhouse gases by 2% still allows input reduction for 90 electricity plans (i.e. 15% of the sample). This last number drops quite dramatically, to 15 plants (i.e. 2.5% of the sample), if we target a 4% reduction of CO_2, SO_2 and NO_x. Finally, input reduction is possible for only a single plant if we set the stringency parameter σ equal to as much as 8 (i.e. 8% reduction). The results for the second scenario in Figure 2 are quite close to the ones for scenario 1 and, correspondingly, have a readily similar interpretation.

Let us then turn to our last two scenarios, which are directly related to the Acid Rain Program. For our third scenario, we find that there is substantial potential to decrease SO_2 emissions in combination with input reduction. For example, such a combination is feasible for 36% of the DMUs when $\sigma = 2$, 22% of the DMUs when $\sigma = 10$, and 19% of the DMUs when $\sigma = 20$. A similar conclusion holds for our final scenario, but now the reduction possibilities are even more pronounced. In particular, we find that simultaneous NO_x and inputs reduction is possible for 50% of the DMUs when $\sigma = 2$, 35% of the DMUs when $\sigma = 10$, and 30% of the DMUs when $\sigma = 20$.

At a general level, we believe that this empirical analysis convincingly demonstrates the usefulness of our methodology for multi-output efficiency measurement with output objectives in the case of undesirable outputs. For example, for our specific application it allows us to draw at least two main conclusions. Firstly, our scenarios 3 and 4 reveal higher numbers of inefficient plants than our scenarios 1 and 2. Probably, this can at least partly be explained by the higher production of CO_2 emissions when compared to SO_2 and NO_x emissions (see Table 1). From the perspective of the Acid Rain Program, however, our observation that there is considerable scope to reduce SO_2 and NO_x may actually be seen as a quite encouraging finding, as these greenhouse gases are primarily responsible for acid rain. Secondly, and directly related to our first conclusion, it appears that US electric utilities have some more potential (and thus can more easily put more effort) to reduce SO_2 than to decrease NO_x.

5 Conclusion

We have extended the DEA approach for multi-output efficiency measurement that was recently introduced by Cherchye et al. (2013). At the methodological level, we

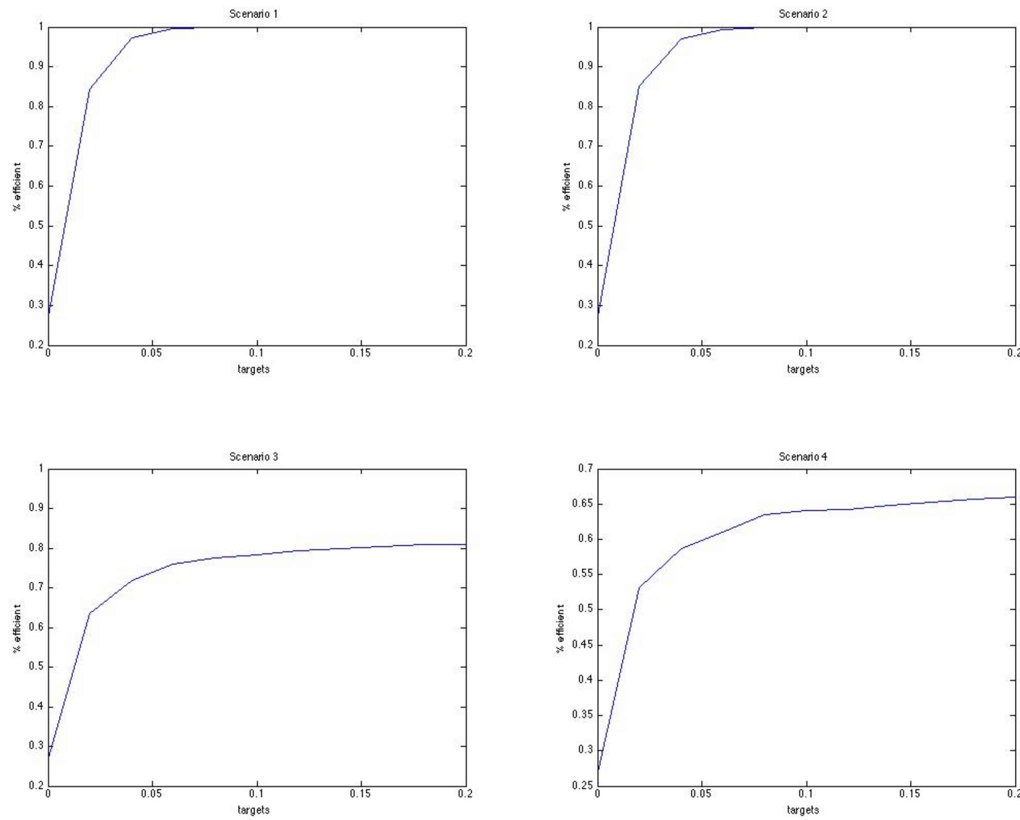

Figure 2: Efficient firms (percentage) with varying output objectives; four scenarios

have introduced the concept of sub-joint inputs, and we have shown how to deal with output objectives in the efficiency evaluation exercise. At the practical level, we have argued that these extensions make the methodology particularly well-suited for assessing a production process characterized by bad outputs. Interestingly, it avoids in a natural way some modeling issues that are specific to existing approaches for handling undesirable outputs in a DEA analysis.

We also demonstrated the empirical usefulness of our novel methodology by conducting an efficiency analysis of US electric utilities. For this application, our concept of sub-joint inputs made it possible to take the specific use of the inputs into account. More precisely, we treat both nameplate capacity and fuel consumption as inputs for our good output fossil electricity production and all our three bad outputs (CO_2, NO_x and SO_2 emissions), while nameplate capacity figured as our only input for the good output non-fossil electricity production. Next, our use of output objectives was directly instrumental to account for DMU objectives regarding the emission of greenhouse gases. Our empirical findings clearly suggest that US electric utilities have substantial potential to reduce both inputs and greenhouse gases (including SO_2 and NO_x, as requested by the Acid Rain Program of the Clean Air Act).

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