Is constant returns-to-scale a restrictive assumption for sector-level empirical macroeconomics? The case of Europe*

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Abstract

Assuming constant returns-to-scale is commonly agreed for empirical macroeconomic studies when countries are of interest. Recently, an increasing number of works have started to look at sectors building on the same assumption. In this letter, we question the reliability of this assumption for ten European sectors for the period 1995-2014, for different production factor combinations. We make use of a simple sample-based nonparametric test that does not require any assumptions for any aspect of the production process. Our results suggest that, in general, this assumption is rather acceptable, and that the specification with only capital and labour is the best in this case.

Keywords: macroeconomics; sectors; returns-to-scale; nonparametric; Europe.

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1 Introduction

Since the classic papers of Solow (1956), Baumol (1986), and Barro (1991), much effort has been put into better understanding the economic growth of countries. For theoretical and/or practical reasons, most of the empirical works assume that the country-level technology exhibits constant returns-to-scale (CRS). Recently, more and more attention has been given to study economic growth at the sector-level. See, Miao and Wang (2014), Iscan (2015), Zeira and Zoabi (2015), Walheer (2016a, b, 2018a, b), and Magalhaes and Afonso (2017), to cite only a few. Typically, sector-level empirical works are built on similar or directly inspired methodologies as those used for countries. As a consequence, CRS is also explicitly or implicitly assumed.

While this assumption is commonly agreed for countries, it is perhaps more questionable for sectors. To assess the viability of this assumption, we suggest using a simple sample-based nonparametric test that does not depend on any aspect of the production process, and in particular on the production function. This is particularly attractive in this context as the production function of sectors is typically unknown, and, more importantly, assuming a specific production function could bias the result of the test. Also, to avoid the bias of choosing specific production factors, we consider different combinations. We apply our methodology to the case of ten European sectors for the period 1995-2014.

2 Methodology

We assume that we observe J countries during T periods, and that each country contains I sectors. We consider a very general production process. That is, every sector i in country j at time t uses n production factors, contained in the vector $\mathbf{X}_{ijt}$, to produce output Y_{ijt}. Therefore, the production function is given by $Y(\mathbf{X}_{ijt})$.

To verify the reliability of assuming CRS for sectors, we suggest comparing the production functions when assuming CRS, denoted by $Y_{crs}(\mathbf{X}_{ijt})$, with the production function when no specific returns-to-scale assumption is chosen (i.e. VRS is assumed), denoted by $Y_{vrs}(\mathbf{X}_{ijt})$. If they are equal (or, at least, close enough), we conclude that CRS is not a restrictive assumption for sector i at time t in country j. This test dates to Färe and Grosskopf (1985). The distinguishing feature of the suggested test is that it is based on sector-level technology. In practice, to facilitate the interpretation, it is more convenient to construct a ratio of the two production functions:

$$SR(\mathbf{X}_{ijt}) = \frac{Y_{vrs}(\mathbf{X}_{ijt})}{Y_{crs}(\mathbf{X}_{ijt})}. \tag{1}$$

If $Y_{crs}(\mathbf{X}_{ijt})$ coincides with $Y_{vrs}(\mathbf{X}_{ijt})$, the scale ratio $SR(\mathbf{X}_{ijt})$ is equal to 1. That is, CRS is acceptable. If they do not coincide, $SR(\mathbf{X}_{ijt})$ is smaller than 1, as $Y_{crs}(\mathbf{X}_{ijt}) \geq Y_{vrs}(\mathbf{X}_{ijt})$ since the technology under CRS is always the greatest. This also indicates why it is important to test if CRS is reliable. When $SR(\mathbf{X}_{ijt}) < 1$, decreasing or increasing returns-to-scale should rather be assumed for the technology.[1]

While the previous test is simple, it depends heavily on the production function. Indeed, assuming a specific production function will have a huge impact on the scale ratio $SR(\mathbf{X}_{ijt})$, and thus on the decision of the test. As such, here we adopt a non-parametric approach. That is, we estimate the technology using the observed data. Nevertheless, to avoid a trivial estimation of the production function and to match with previous macroeconomic empirical works, we assume that the production function is quasi-concave, continuous, and strictly increasing. Attractively, in practice, it suffices to solve linear programs to obtain the estimators.

When CRS is assumed, the estimator $\widehat{Y}_{crs}(\mathbf{X}_{i_0j_0t_0})$ for $i_0 \in (1,\ldots,I)$, $j_0 \in (1,\ldots,J)$ and $t_0 \in (1,\ldots,T)$ is obtained using the following linear program **(LP-CRS)**:

$$\widehat{Y}_{crs}(\mathbf{X}_{i_0j_0t_0}) = \max_{\lambda_{i_0jt_0}\ (j\in\{1,\ldots J\})} Y$$

$$\text{(C-1) } Y \leq \sum_{j=1}^{J} \lambda_{i_0jt_0} Y_{i_0jt_0},$$

$$\text{(C-2) } \mathbf{X}_{i_0j_0t_0} \geq \sum_{j=1}^{J} \lambda_{i_0jt_0} \mathbf{X}_{i_0jt_0},$$

$$\text{(C-3) } \forall j : \lambda_{i_0jt_0} \geq 0,$$

$$\text{(C-4) } Y \geq 0.$$

When VRS is assumed, the linear program for the estimator $\widehat{Y}_{vrs}(\mathbf{X}_{i_0j_0t_0})$ is very similar to **(LP-CRS)**, in fact **(LP-VRS)** is obtained by adding the following con-

[1] In practice, it is enough to evaluate $Y_{nirs}(\mathbf{X}_{ijt})$ (i.e. when assuming non-increasing returns-to-scale) and compare to $Y_{vrs}(\mathbf{X}_{ijt})$. If they are equal, decreasing returns-to-scale is acceptable. Otherwise, increasing returns-to-scale should be chosen. The linear program to obtain the estimators $\widehat{Y}_{nirs}(\mathbf{X}_{ijt})$ is obtained by a slight modification of **(LP-VRS)**: (C-5)' to $\sum_{j=1}^{J} \lambda_{i_0jt_0} \leq 1$.

straint to **(LP-CRS)**: (C-5) $\sum_{j=1}^{J} \lambda_{i_0 j t_0} = 1$. By plugging-in our nonparametric estimators in (1), we obtain our scale ratio estimator that we interpret in the analogous manner. That is, a value of 1 means that CRS is acceptable, while a value smaller than 1 signifies the opposite.

3 Application

We apply our methodology to the case of the European sectors. As discussed in the Introduction, we consider different sets of production factors. In particular, we consider four production factors: labour L, physical capital K, human capital H, and energy E. We believe that these four production factors represent a starting attractive set as they are used in the majority of macroeconomic empirical works. We consider six different combinations, summarized in Table 1, where we define $\hat{L} = L * H$ (i.e. when human capital enters the technology as a multiplicative augmentation of labour).

Table 1: Production factor specification

Case	# production factors	production factors
1	2	K, L
2	3	$K, \hat{L}$
3	3	K, L, H
4	3	K, L, E
5	4	$K, \hat{L}, E$
6	4	K, L, H, E

We use the OECD Detailed National Accounts database for the data of Y, K, L, and E. For H, we make used of two different constructions: the construction of Hall and Jones (1999) and the construction of Trostel (2004). While both constructions assume that human capital is a nonlinear function; the main difference is that the former views human capital with diminishing returns, while the latter models human capital with increasing returns. We refer to Walheer (2016a, b) and Dias (2017) for more detail about the practical construction of human capital. We select the biggest sample possible, which consists of nineteen European countries, ten sectors, and a period from 1995 to 2014.[2] We compute the estimated production functions using

[2]The countries are: Austria, Belgium, Czech Republic, Denmark, Estonia, Finland, France,

(LP-CRS) and **(LP-VRS)** for the ten sectors in each country for every year, for the six cases. We give, in Table 2, the descriptive statistics of the scale ratios for the six cases grouping all the sectors and all years. Note that for cases 2, 3, 5 and 6, the left column corresponds to the Hall and Jones's (1999) construction, and the right column to the Trostel's (2004) construction. An initial observation is that the

Table 2: Descriptive statistics

Statistics	case 1	case 2		case 3		case 4	case 5		case 6	
Average	0.88	0.89	0.91	0.92	0.94	0.90	0.90	0.91	0.92	0.94
Median	0.95	0.95	0.96	0.99	0.98	0.96	0.96	0.97	0.99	0.98
Std	0.16	0.16	0.15	0.15	0.14	0.15	0.15	0.15	0.15	0.14
Min	0.16	0.16	0.17	0.16	0.22	0.18	0.17	0.17	0.18	0.19
Max	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

scale ratios are rather close to unity for all cases, meaning that CRS does not seem a restrictive assumption in this context. To formally test whether there are differences between the six cases and whether CRS is more easily accepted for one case, we make use of the two-sample Kolmogorov-Smirnov test (KS test). This is a nonparametric test that checks if the distributions of two samples are equal or not. Also, as we want to investigate whether one case accepts CRS more easily, we calibrate the test in that sense too, i.e. to check if the distribution is larger. In Table 3, we present the p-values of the two KS tests for our two calibrations, labelled "Equality" and "Larger". If the $p-$value is smaller than 5%, we can reject the null hypothesis, and conclude, that there is a significant difference between the two distributions.

Interesting, the results suggest that there are differences between all the cases, except between cases 1 and 2, and cases 4 and 5. In other words, it shows that modelling human capital as a multiplicative augmentation of labour or as a production factor, has no important impact on the reliability of CRS. Next, the results indicate a ranking between the six cases: $1 \sim 2 > 4 \sim 5 > 3 > 6$. In words, case 1 is the case for which CRS is more easily accepted. It means that the very standard macroeconomic model, present in the paper of Solow (1956), with capital and labour as production factors, is the best when assuming CRS. Also, the specification with energy as an extra input is the second in the ranking. That shows the importance of this extra

Germany, Hungary, Ireland, Italy, Luxembourg, Netherlands, Norway, Poland, Slovakia, Slovenia, Spain, and Sweden. The sectors are: Agriculture, Mining, Manufacturing, Electricity, Gas and Water, Construction, Wholesale, Transport, Public Administration, Education, and Health.

Table 3: KS p-values

Equality	case 2		case 3		case 4	case 5		case 6	
case 1	0.15	0.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00
case 2			0.00	0.00	0.00	0.00	0.00	0.00	0.00
case 3					0.00	0.00	0.00	0.00	0.00
case 4						0.86	0.75	0.00	0.00
case 5								0.00	0.00
Larger	**case 2**		**case 3**		**case 4**	**case 5**		**case 6**	
case 1	0.08	0.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00
case 2			0.00	0.00	0.00	0.00	0.00	0.00	0.00
case 3					0.74	0.71	0.65	0.00	0.00
case 4						0.91	0.87	0.00	0.00
case 5								0.00	0.00

input for some sectors. Finally, the chosen construction for human capital has an impact, but not significant enough.

The previous analysis shows important results, but it does not show how strong the assumption for each sector is individually. In Table 4, we present the averages and medians per sector and case while grouping all the years. The averages and medians per sector reveal that, in general, CRS seems acceptable for the sectors, except for Manufacturing. As done before for the overall scale ratios, we present in Tables 5 and 6, the $p-$values of the two KS tests for every sector and cases. Note that, in these Tables, we rely on the construction of Hall and Jones (1999) for human capital. For compactness, we do not include the results when relying on the construction of Trostel (2004), since the conclusions are very similar. Different important findings come from those tables. Firstly, the connection between cases 1 and 2, and cases 4 and 5 are confirmed for all the sectors. Next, cases 3 and 6 are, in general, the two cases for which CRS is the strongest assumption. Afterwards, cases 1 and 2 are, expect for the Agriculture, Transport, and Electricity, Gas, and Water sectors, the best cases. That is, when only capital and labour (augmented by human capital or not) are the production factors. For the Agriculture, Transport, and Electricity, Gas, and Water sectors, cases 4 and 5 seem more accurate when CRS is chosen. That is, for those sectors adding energy as an extra production factor increase the acceptability of CRS. In other words, energy is an important input for those sectors. This is rather intuitive since energy seems more important for those sectors than, for example, Education, or Public Administration.

Table 4: Averages and medians per sector and case

Averages	A	Mi	Ma	EGW	C	W	T	PA	E	H
case 1	0.93	0.91	0.75	0.87	0.94	0.93	0.89	0.90	0.84	0.90
case 2	0.93	0.91	0.77	0.89	0.94	0.94	0.89	0.90	0.84	0.90
	0.95	0.90	0.76	0.92	0.95	0.94	0.90	0.88	0.83	0.89
case 3	0.91	0.90	0.92	0.90	0.94	0.96	0.94	0.94	0.90	0.89
	0.90	0.89	0.91	0.93	0.95	0.96	0.93	0.95	0.89	0.91
case 4	0.90	0.95	0.79	0.88	0.93	0.95	0.92	0.92	0.87	0.92
case 5	0.90	0.96	0.80	0.89	0.93	0.94	0.92	0.92	0.87	0.91
	0.90	0.95	0.79	0.91	0.94	0.95	0.91	0.91	0.88	0.91
case 6	0.88	0.96	0.91	0.90	0.96	0.96	0.96	0.94	0.90	0.90
	0.89	0.95	0.93	0.92	0.97	0.97	0.97	0.63	0.91	0.91
Medians	**A**	**Mi**	**Ma**	**EGW**	**C**	**W**	**T**	**PA**	**E**	**H**
case 1	0.98	0.97	0.75	0.93	0.98	0.97	0.95	0.95	0.90	0.98
case 2	0.98	0.97	0.77	0.95	0.98	0.98	0.96	0.96	0.92	0.98
	0.97	0.96	0.78	0.96	0.99	0.99	0.94	0.97	0.90	0.99
case 3	0.98	0.99	0.96	0.98	0.99	1.00	0.99	1.00	0.99	0.99
	0.98	0.98	0.98	0.99	0.99	0.99	0.99	1.00	1.00	1.00
case 4	0.96	0.99	0.80	0.95	0.98	0.97	0.97	0.98	0.95	0.99
case 5	0.96	0.99	0.79	0.95	0.98	0.97	0.97	0.98	0.95	0.98
	0.98	0.99	0.81	0.96	0.97	0.98	0.96	0.97	0.96	0.97
case 6	0.96	0.99	0.97	0.98	1.00	1.00	1.00	1.00	1.00	1.00
	0.96	0.98	0.98	0.97	1.00	0.99	0.99	0.99	0.99	1.00

A: Agriculture; Mi: Mining; Ma:Manufacturing; EGW:Electricity, Gas & Water; C: Construction; W: Wholesale; T: Transport; PA: Public Administration; E: Education; H: Healt.

Table 5: KS p-values per sector and case

sector	**Agriculture**					**Mining**					**Manufacturing**				
equal	case 2	case 3	case 4	case 5	case 6	case 2	case 3	case 4	case 5	case 6	case 2	case 3	case 4	case 5	case 6
case 1	0.73	0.05	0.03	0.04	0.00	1.00	0.03	0.00	0.00	0.00	0.13	0.00	0.00	0.00	0.00
case 2		0.01	0.00	0.02	0.00		0.03	0.00	0.00	0.00		0.00	0.08	0.02	0.00
case 3			0.00	0.00	0.06			0.00	0.00	0.63			0.00	0.00	0.96
case 4				1.00	0.06				0.99	0.00				0.55	0.00
case 5					0.04					0.00					0.00
larger	case 2	case 3	case 4	case 5	case 6	case 2	case 3	case 4	case 5	case 6	case 2	case 3	case 4	case 5	case 6
case 1	0.39	0.03	0.30	0.42	0.01	0.76	0.01	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.00
case 2		0.00	0.23	0.25	0.00		0.02	0.00	0.00	0.00		0.00	0.04	0.01	0.00
case 3			0.79	0.70	0.11			0.00	0.00	0.58			1.00	1.00	0.59
case 4				0.75	0.03				0.69	0.58				0.28	0.00
case 5					0.02					0.40					0.00
sector	**Construction**					**Wholesale**					**Transport**				
equal	case 2	case 3	case 4	case 5	case 6	case 2	case 3	case 4	case 5	case 6	case 2	case 3	case 4	case 5	case 6
case 1	0.25	0.00	0.06	0.02	0.00	0.15	0.00	0.00	0.00	0.00	0.32	0.00	0.07	0.27	0.00
case 2		0.00	0.47	0.20	0.00		0.00	0.00	0.00	0.00		0.00	0.11	0.34	0.00
case 3			0.00	0.00	0.24			0.00	0.00	0.04			0.00	0.00	0.16
case 4				0.95	0.00				0.67	0.00				0.34	0.00
case 5					0.00					0.00					0.00
larger	case 2	case 3	case 4	case 5	case 6	case 2	case 3	case 4	case 5	case 6	case 2	case 3	case 4	case 5	case 6
case 1	0.13	0.00	0.03	0.01	0.00	0.08	0.00	0.00	0.00	0.00	0.16	0.00	0.03	0.13	0.00
case 2		0.00	0.24	0.10	0.00		0.00	0.00	0.00	0.00		0.00	0.06	0.27	0.00
case 3			0.88	0.84	0.12			0.39	0.88	0.02			0.87	0.94	0.08
case 4				0.58	0.00				0.65	0.00				1.00	0.00
case 5					0.00					0.00					0.00

Table 6: KS p-values per sector and case

sector	**Education**					**Health**				
equal	case 2	case 3	case 4	case 5	case 6	case 2	case 3	case 4	case 5	case 6
case 1	0.86	0.00	0.00	0.00	0.00	0.69	0.00	0.01	0.01	0.00
case 2		0.00	0.02	0.02	0.00		0.00	0.04	0.09	0.00
case 3			0.00	0.00	0.16			0.00	0.01	0.78
case 4				0.77	0.00				0.85	0.02
case 5					0.00					0.02
larger	case 2	case 3	case 4	case 5	case 6	case 2	case 3	case 4	case 5	case 6
case 1	0.48	0.00	0.00	0.00	0.00	0.36	0.00	0.00	0.00	0.00
case 2		0.00	0.01	0.01	0.00		0.00	0.02	0.04	0.00
case 3			0.67	0.72	0.08			0.00	0.03	0.42
case 4				0.42	0.00				0.51	0.01
case 5					0.00					0.01

sector	**Public administration**					**Electricity, gas and water**				
equal	case 2	case 3	case 4	case 5	case 6	case 2	case 3	case 4	case 5	case 6
case 1	0.60	0.00	0.00	0.00	0.00	0.92	0.03	0.34	0.09	0.00
case 2		0.00	0.00	0.00	0.00		0.00	0.22	0.12	0.00
case 3			0.00	0.00	0.64			0.01	0.00	0.05
case 4				0.94	0.00				0.91	0.00
case 5					0.00					0.00
larger	case 2	case 3	case 4	case 5	case 6	case 2	case 3	case 4	case 5	case 6
case 1	0.31	0.00	0.00	0.00	0.00	0.86	0.02	0.46	0.48	0.00
case 2		0.00	0.00	0.00	0.00		0.00	0.29	0.37	0.00
case 3			0.99	0.98	0.45			0.90	0.79	0.03
case 4				0.63	0.00				0.55	0.00
case 5					0.00					0.00

4 Conclusion

An increasing number of macroeconomic empirical works are conducted on sectors. For theoretical or practical reasons, constant returns-to-scale is, most of the time, assumed. In this letter, we suggest using a simple sample-based nonparametric test to question the reliability of this assumption. Our test is nonparametric in nature as it does not depend on any assumptions for any aspect of the production process, and in particular, on the production function.

We applied our technique to the case of ten European sectors for the period 1995-2014, for different production factor combinations. We found that in general, constant returns-to-scale is rather acceptable, and that the simplest specification of the production factors, i.e. with only capital and labour, is the best in this case. Also, we pointed out the importance of energy for three sectors: Agriculture, Transport, and Electricity, Gas, and Water.

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