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PROXIMITY APPROXIMATION FOR HEAVY ION POTENTIALS[x)]

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In Ref. [1] we have calculated the real part of the nucleus-nucleus interaction potential by using a method based on the Skyrme interaction energy density formalism [2]. In this work we study the relation between the method of [1] and the proximity concept [3] and present in a more quantitative way the conditions for the proximity approximation to be valid.

According to Ref. [3] the interaction energy of two spherical nuclei with radii R_1 and R_2 and whose centres are separated by a distance $R = R_1 + R_2 + S_0$ is given by

$$V_p(s) = 2\pi \frac{R_1 R_2}{R_1 + R_2} \int_{S_0}^{\infty} e(s) ds \qquad (1)$$

where $e(s)$ is the interaction energy per unit area and is a universal function. This formula is valid under the assumption that $e(s)$ decrease fast enough with s and for gently curved objects in close proximity. We are studying the quantitative meaning of these conditions.

We start from the expression of the interaction potential defined as the volume integral

$$V(R) = \int [\bar{H}(\rho_1 + \rho_2) - \bar{H}(\rho_1) - \bar{H}(\rho_2)] d\underline{r} \qquad (2)$$

where R is the separation distance between the centres of the nuclei. The quantity $\bar{H}(\rho)$ is the Skyrme interaction energy density with an adequate approximation [4] for the kinetic energy density and ρ_1 and ρ_2 are Skyrme-Hartree-Fock densities parametrized as Fermi type distributions [1].

In order to reduce the expression (2) to a proximity form we perform the following approximations [5] :
1) Write the quantity $F = \bar{H}(\rho_1 + \rho_2) - \bar{H}(\rho_1) - \bar{H}(\rho_2)$ as a universal function : this approximation involves two steps a) assume that the central densities ρ_0 and surface diffuseness parameter a are the same for all nuclei b) replace $\cos \theta_{12}$ from the surface

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[x)] Work done in collaboration with Dr. D.M. Brink.

term $\tilde{\nabla}\rho_1 \cdot \tilde{\nabla}\rho_2$ by -1 which implies an increase of the attraction especially for negative S_0 .

2) Extend the limits of the integral. For spherical nuclei the integral (2) reduces to a two dimensional one which can be expressed in terms of the variables r_1 and r_2 i.e. the distances from the centres of each nucleus to the integration point [6]. The integration limits are fixed by the triangular conditions $r_1+r_2 \geq R$ and $|r_1-r_2| \leq R$. For leptodermous systems the integration limits can be extended by relaxing the restriction $|r_1-r_2| \leq R$.

3) Make another change of variables $u_1 = r_1-R_1$ and $u_2 = r_2-R_2$ and two further approximations : a) $R \approx R_1+R_2$ i.e. assume $S_0 << R_1+R_2$; b) neglect the terms $R_1u_2+R_2u_1+u_1u_2$ in comparison to R_1R_2 .

After all these approximations we obtain the proximity form with

$$e(s) = \int_{-\infty}^{\infty} F(u_1,s-u_1)du_1 \qquad (3)$$

The validity of these approximations was studied numerically on several systems between $^{16}O+^{16}O$ and $^{208}Pb+^{208}Pb$. The conclusions are the following : the approximations 1a, 2, 3a and the neglect of the term u_1u_2 only (see 3b) do not affect significantly the exact result, while 1b and 3b bring large changes in the value of the potential but compensate each other to a large extent as it can be seen in the Table. These results show that for all pairs the proximity approximation is the best at $S_0 \approx 0$ (close proximity) with an error of at most 4-5 % for $^{16}O+^{16}O$. For each other separation distance the approximation improves also gradually from $^{16}O+^{16}O$ till $^{208}Pb+^{208}Pb$. In the barrier region the error varies between 20 % for lighter systems and 10 % for heavier pairs.

The existence of the surface term in the energy density is essential for the validity of the approximation. Otherwise one must include curvature corrections which come from the term $R_1u_2+R_2u_2$ (see 3b) and amount to the following expression of the potential

$$V(R) = 2\pi \frac{R_1R_2}{R_1+R_2} [\int_{S_0}^{\infty} e(s)ds + \frac{R_1+R_2}{2R_1R_2} \int_{S_0}^{\infty} s\, e(s)ds] \quad (4)$$

The additional term in expression (4) is also an universal function.

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Comparison between proximity approximation V_p (Eq. (1)) and exact calculation V_{Ex} in the Eq. (2)

S_0	$^{16}O + ^{16}O$		$^{40}Ca + ^{40}Ca$		$^{90}Zr + ^{90}Zr$		$^{208}Pb + ^{208}Pb$	
	V_p	V_{Ex}	V_p	V_{Ex}	V_p	V_{Ex}	V_p	V_{Ex}
-3.	116.90	102.51	133.95	114.87	130.28	118.87	97.62	91.16
-2.	11.89	19.18	-2.94	4.52	-19.78	-12.49	-53.77	-46.66
-1.	-30.69	-22.32	-47.84	-41.33	-66.23	-60.20	-92.23	-87.54
0.	-24.17	-23.17	-35.85	-35.63	-46.52	-46.53	-61.45	-61.93
1.	-8.59	-9.68	-13.64	-15.16	-17.55	-18.88	-23.74	-25.35
2.	-1.94	-2.50	-3.53	-4.20	-4.41	-5.09	-6.44	-7.15
3.	-.38	-.54	-.82	-1.04	-.98	-1.16	-1.44	-1.71