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We calculate nonstrange baryon resonances pion decay widths. Resonance wavefunctions are described by a semi-relativistic QCD-inspired flux-tube model while for the decay process we assume a quark-antiquark pair creation model. Results are compared with experimental data and previous theoretical results.

1. INTRODUCTION

Some time ago, a so-called "naive" quark pair creation model (QPC) had been introduced to describe strong decay of hadrons [1]. In that work, mesons and baryons were described by gaussian wavefunctions. Attempts to provide a better picture of baryon wavefunctions have led to flux tube variational wavefunctions [2]. Carlson, Kogut and Pandharipande (hereafter abbreviated as CKP) use a semi-relativistic hamiltonian including 2- and 3-body contributions to a confining term. Parameters of the model are found by minimizing the energy. One gluon exchange terms have been added in Ref. [3]. This latter model has been used, together with a pseudoscalar emission one, to describe the decay of baryon resonances.

In the present work, quark pair creation and flux-tube wavefunctions are brought together to calculate resonances decay widths. The quark model is summarized in next section, and so is QPC model in section 3. In that section we also derive the expressions of the transition amplitude. Results and discussions are presented in section 4.

2. FLUX TUBE BARYON WAVEFUNCTIONS

We use the semi-relativistic Hamiltonian of Ref. [2]

$$H = \sum_i (p_i^2 + m_i^2)^{\frac{1}{2}} + V(\vec{r}_1,\vec{r}_2,\vec{r}_3) + E_0 \quad . \tag{2.1}$$

The short range behavior of the potential V is that of the two-body color Coulomb interaction. The long range one is linearly confining

$$V_{LR} = \sum_i \sqrt{\sigma}\, r_{i4} \quad , \tag{2.2}$$

where r_{i4} is determined [2] by the condition that it minimizes the static energy. Using parametrized wavefunctions, one finds their parameters by minimizing the expectation value of (2.1). The baryon wavefunctions have the form

$$\psi_n(\vec{r}_1,\vec{r}_2,\vec{r}_3) = F_{123} \prod_{i<j} f(r_{ij})\, \Phi_n(\vec{r}_1,\vec{r}_2,\vec{r}_3) \quad , \tag{2.3}$$

where f is parametrized as

$$f(r_{ij}) = \exp\left[-W(r_{ij})\,\lambda_1\, r_{ij} - (1 - W(r_{ij}))\,\lambda_{1.5}\, r_{ij}^{1.5}\right]$$

$$W(r_{ij}) = \frac{1 + \exp(-r_0/a)}{1 + \exp((r_{ij} - r_0)/a)} \tag{2.4}$$

$$F_{123} = 1 - \beta\sqrt{\sigma}\,(\sum_i r_{i4} - \tfrac{1}{2}\sum_{i<j} r_{ij}) \quad . \tag{2.5}$$

$\sqrt{\sigma}$ has the conventional value of the string tension constant

$$\sqrt{\sigma} = 1 \text{ GeV/fm} \quad .$$

Results of the variational calculations of the parameters λ_1, $\lambda_{1.5}$, r_o, a, β can be found in ref. [2]. The emitted meson wavefunctions are taken as

$$\psi(r_{ij}) = f(r_{ij})\, r_{ij}^{\delta} \tag{2.6}$$

to deal with the singularity of the L = 0 $q\bar{q}$ system at the origin. Using Jacobi coordinates

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) \quad ; \quad \vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3) \tag{2.7}$$

one can write [4,5] the $\Phi_n = \Phi_{LM}^{sym}(\vec{\rho},\vec{\lambda})$, to be inserted in (2.3).

The various baryon states are described in a SU(6) spin-flavor basis. To split them from one another with respect to their quantum numbers, one includes the hyperfine interaction consisting of a spin-spin and a tensor component [6]. Singularities are removed by a form factor $\exp(-\frac{1}{2}\Lambda^2 q^2)$ where Λ can be viewed as the "size of the quark", in order to cope with the deviation from one gluon exchange due to the cloud of virtual particles surrounding the quark [7].

The states considered in this work are N and Δ's, with up to 2 units of angular momentum or 1 unit of radial excitation (i.e. those pertaining to the $56(0^+,2^+)$, $56'(0^+,2^+)$, $70(0^+,1^-,2^+)$, $20(1^+)$ SU(6) supermultiplets). Mass spectra and mixing angles between states of common J^π can be found in refs. [3,8] for some values of (m,Λ).

3. QUARK PAIR CREATION AND TRANSITION AMPLITUDE

The modelization of the decay will be based on the creation of a quark-antiquark pair somewhere within the hadronic matter. This process will be described by a purely phenomenological constant γ : dynamics of quarks and gluons inside the hadron is not taken into account here. Other quarks are assumed to be spectators and the quantum numbers of the pair are those of the vacuum, i.e. it is in a ^3P_o state because of parity and charge conjugation conservation [1].

To calculate the transition amplitude, let us define the $\hat{T}$ matrix :

$$S = 1 - 2\pi i\delta(E_f - E_i)\,\hat{T} \quad ; \quad \hat{T} = I_1 \times I_2 \times I_3 \times \hat{T}_{vac} \tag{3.1}$$

$$\langle q_4\bar{q}_5|\hat{T}_{vac}|0\rangle = \delta(\vec{k}_4 + \vec{k}_5)\gamma \sum_m \langle 11m - m|0\,0\rangle\, Y_1^m(\vec{k}_4 - \vec{k}_5)\chi_1^{-m}\,\Phi_o, \tag{3.2}$$

where χ_1^{-m} is the spin triplet wavefunction and Φ_o the flavor singlet one. The transition amplitude is then equal to

$$\langle NM|\hat{T}|R\rangle = \gamma\sum_m \langle 11m{-}m|00\rangle\langle\Phi_N\Phi_M|\Phi_R\,\Phi_{vac}^{-m}\rangle\, I_m(R;N,M) \quad , \tag{3.3}$$

where Φ 's are spin-flavor wavefunctions and

$$I_m(R;N,M) = \frac{1}{3\sqrt{3}}\,\delta(\vec{k}_N + \vec{k}_M)\int d^3k_\rho\, d^3k_\lambda\, Y_1^m(-2(\vec{k}_M + \sqrt{2/3}\,\vec{k}_\lambda))$$

$$\times\, \psi_R(\vec{k}_\rho,\vec{k}_\lambda)\psi_N^*(\vec{k}_\rho,\vec{k}_\lambda + \sqrt{2/3}\,\vec{k}_M)\psi_M^*(\vec{k}_M + 2\sqrt{2/3}\,\vec{k}_\lambda) \ , \tag{3.4}$$

as can be found from momentum conservation, k_ρ, k_λ being momentum Jacobi coordinates.

A Fourier transform brings us back to configuration space coordinates. This will allow us to use the flux tube derived baryon wavefunctions of section 2 :

$$I_m(R;N,M) = -\sqrt{3/4\pi}\,.\,\frac{1}{(2\pi)^{3/2}}\,\gamma\,\delta(\vec{k}_N+\vec{k}_M)\int d^3\rho\, d^3\lambda\, d^3x$$
$$\times\,\psi_R(\vec{\rho},\vec{\lambda}+2\sqrt{2/3}\,\vec{x})\psi_N^*(\vec{\rho},\vec{\lambda})e^{i\vec{k}_M(\sqrt{2/3}\,\vec{\lambda}+\vec{x})}\,\vec{\varepsilon}_m.(\vec{k}_M+i\vec{\nabla}_x)\psi_M^*(2\vec{x}), \quad (3.5)$$

where $\vec{x}$ is the Jacobi coordinate related to the emitted meson.

4. RESULTS AND DISCUSSION

We calculate the 9 dimensional integral (3.5) using a Monte Carlo method. The decay width Γ is obtained from the following expression [3]:

$$\Gamma = \frac{1}{\pi}\,\frac{|\langle NM|T|R\rangle|^2}{2J_R+1}\,\frac{k_M E_N}{m_R}\,(\langle I_N\, I_M\, I_{3N}\, I_{3M}|I_R\, I_{3R}\rangle)^{-2},$$

where J is the total angular momentum, I the isospin and I_3 its third component.

In Table 1, we display the square root of nonstrange baryon widths obtained (i) in ref. [3], using the mass spectrum and mixing angles labelled set II, (ii) in this work, using that same set, without and with configuration mixing, (iii) from experiment [9]. The resonances included in this Table are those classified as two star or more by the Particle Data Group [9].

These results have been obtained using a Monte Carlo integration program. Since computation time grows by a power of 5 while precision increases by a power of 2, our numerical results have been limited here to a 20% precision range for the least favourable cases. More accurate results are expected soon.

Table 2 shows a comparison between these various results using a χ^2 formula :

$$\chi^2 = \Sigma\left(\frac{\Gamma^{1/2}_{theory}-\Gamma^{1/2}_{exp}}{\Delta\,\Gamma^{1/2}_{exp}}\right)^2 .$$

In ref. [3], the two parameters of the decay process model have been fitted by minimizing that χ^2. In the present work, the unique decay parameter γ has been fixed by requiring that resonance P33(1232)(lowest Δ resonance) has its decay width reproduced. Since this resonance is the one known with least uncertainty, our results and theirs can be compared easily. Notice that agreement with experiment would require that $\chi^2 \leq N - 1 = 17$.

These comparisons indicate that the pair creation model describes the decay better than the pseudoscalar emission one, having used identical flux-tube wavefunctions in both cases. The number of parameters has been reduced from 2 to 1 while the results went closer to experiment. (One could also compare them with a prior 4 parameter model [10]).

Still, the agreement is just fair. One shouldn't expect to get perfect agreement with a model about which several questions can arise : is the pion correctly described in such a model where its special character due to chiral symmetry breaking has not been taken into account ? The validity of the model might be tested further by calculating the N + ρ channel decay widths. One should also notice the dependence on hyperfine splitting modelization. Though no fit has been made therewith, this has some impact on our results : one can question how closer agreement between that model and experimental mass spectra and mixing angles could change our decay re-

Column 2 : ref. [3], mixing, set II
Column 3 (4) : this work, without (with) configuration mixing
Column 5 : experiment [9]

Resonance	Ref. [3], mix	No mix	Mix	Experiment [9]
$(7/2^+ N)$ F17(1990	0.5	1.4	1.0	4.2 ± 2.0
$(7/2^+ \Delta)$ F37(1950)	3.6	8.9	6.9	9.8 ± 2.0
$(5/2^+ N)$ F15(1680)	3.6	9.3	6.8	8.7 ± 0.85
$(5/2^+ \Delta)$ F35(1905)	1.1	4.5	2.2	5.8 ± 1.6
$(3/2^+ N)$ P13(1720)	8.6	2.4	7.0	5.4 ± 1.8
$(3/2^+ \Delta)$ P33(1232)	10.9	10.7	10.7	10.7 ± 0.2
$(3/2^+ \Delta)$ P33(1600)	10.6	4.9	7.9	7.0 ± 2.1
$(3/2^+ \Delta)$ P33(1920)	4.1	8.0	7.7	6.5 ± 1.25
$(1/2^+ N)$ P11(1440)	1.0	9.1	6.8	10.9 ± 4.0
$(1/2^+ N)$ P11(1710)	6.0	6.8	4.8	4.0 ± 1.05
$(1/2^+ \Delta)$ P31(1910)	6.4	2.4	5.6	7.0 ± 1.9
$(5/2^- N)$ D15(1675)	3.4	5.7	4.9	7.4 ± 1.25
$(3/2^- N)$ D13(1520)	5.0	10.0	8.5	8.3 ± 1.05
$(3/2^- N)$ D13(1700)	2.3	2.6	3.4	3.2 ± 0.7
$(3/2^- \Delta)$ D33(1700)	2.9	4.9	4.2	6.1 ± 1.65
$(1/2^- N)$ S11(1535)	9.4	9.4	5.9	8.3 ± 2.65
$(1/2^- N)$ S11(1650)	8.9	6.4	10.1	9.5 ± 2.0
$(1/2^- \Delta)$ S31(1620)	3.7	4.9	4.2	6.5 ± 1.0

Table 2. χ^2 over Table 1 resonances

Ref. [3]	No mix	Mix
92	38	31

sults. To learn more about all this, strange and charmed sectors should be investigated as has been done already for pion decay [11, 12].

Finally, one should not forget that the coupling constant γ is taken as a fitted parameter here. Within this framework, we can let it depend on where the pair creation takes place within the hadron [11,12] but ultimately, the answer to what γ is lies in the dynamics of the process, not in any kinematical picture.

The above numerical results should be regarded as preliminary.

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