Comparison Between **Differential** and **Variational** Forms of an **Energy**-Based **Hysteresis** Model

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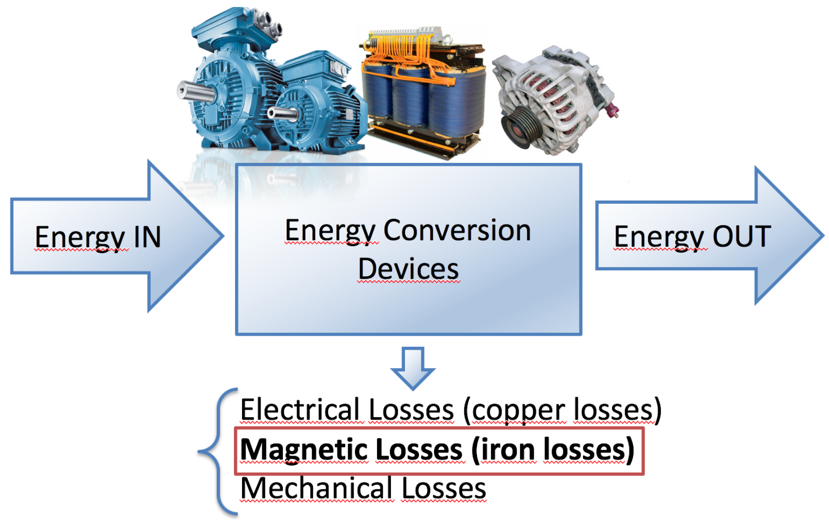
Introduction

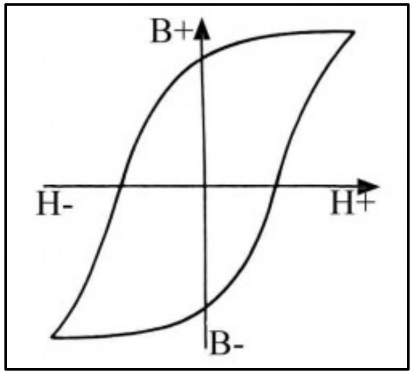


Goal:

Build an **Efficient** Method for calculating **Iron Losses** to improve the **Accuracy** of Simulations of **Energy Conversion Devices**

Main Difficulty:

Modelling **the Hysteresis effect:**
→ very complex *non-linear* and *irreversible* phenomenon



This paper:

1. Focuses on an **Energy**-Based **Hysteresis** Model,
2. Compares two types of implementation in terms of **Efficiency** and **Accuracy**,
3. Deals with its inclusion in **Finite Element** Simulations.

Energy-Based Hysteresis Model

1. **Presentation of the model**
2. **Types of implementations**
   - Differential or Variational Approaches
   - Direct or Inverse Forms
3. **Inclusion in Finite Element Environment (Gmsh/GetDP)**

   Test Cases: Simple square, T-joint, Three-Phases Transformer
4. **Summary of the Results and Conclusion**

Energy-Based Hysteresis Model

Presentation of the model

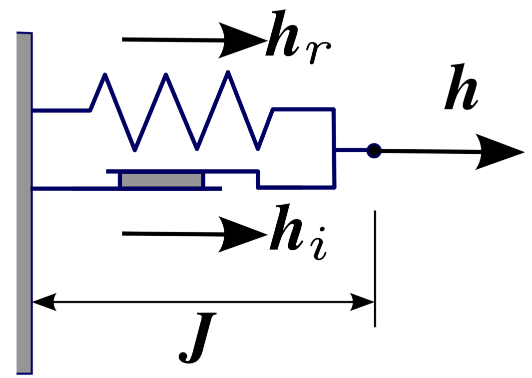
Basic Characteristics

- Based on **Thermodynamic** Principles
- **Dissipation** ≈ **Dry friction** in mechanics
- Naturally driven by $\boldsymbol{h}$ as **input**

Advantages

- ➢ Energy Consistency
- ➢ Naturally vectorial
- ➢ Easy identification of parameters
- ➢ Number of cells can be chosen

Mechanical Analogy



Magnetic Field $\boldsymbol{h}$ ⟷ Force
h_r - reversible part
h_i - irreversible part
Magnetic Polarization $\boldsymbol{J}$ ⟷ Elongation

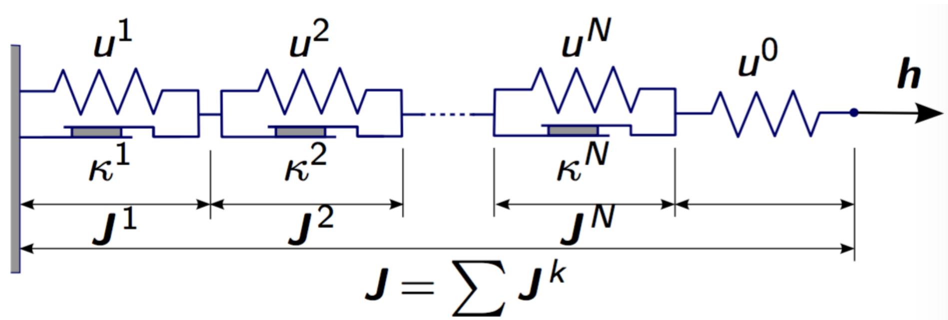
[F.Henrotte & al. 2013]

Energy-Based Hysteresis Model

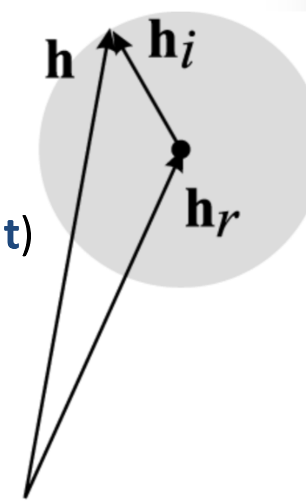
Presentation of the model

PDE coming from Thermodynamic Principles:

$$\boldsymbol{h} - \underbrace{\frac{\partial u^k(|\boldsymbol{J}^k|)}{\partial \boldsymbol{J}^k}}_{\boldsymbol{h}_r^k} - \underbrace{\kappa^k \frac{\dot{\boldsymbol{J}}^k}{|\dot{\boldsymbol{J}}^k|}}_{\boldsymbol{h}_i^k} = \boldsymbol{0}$$



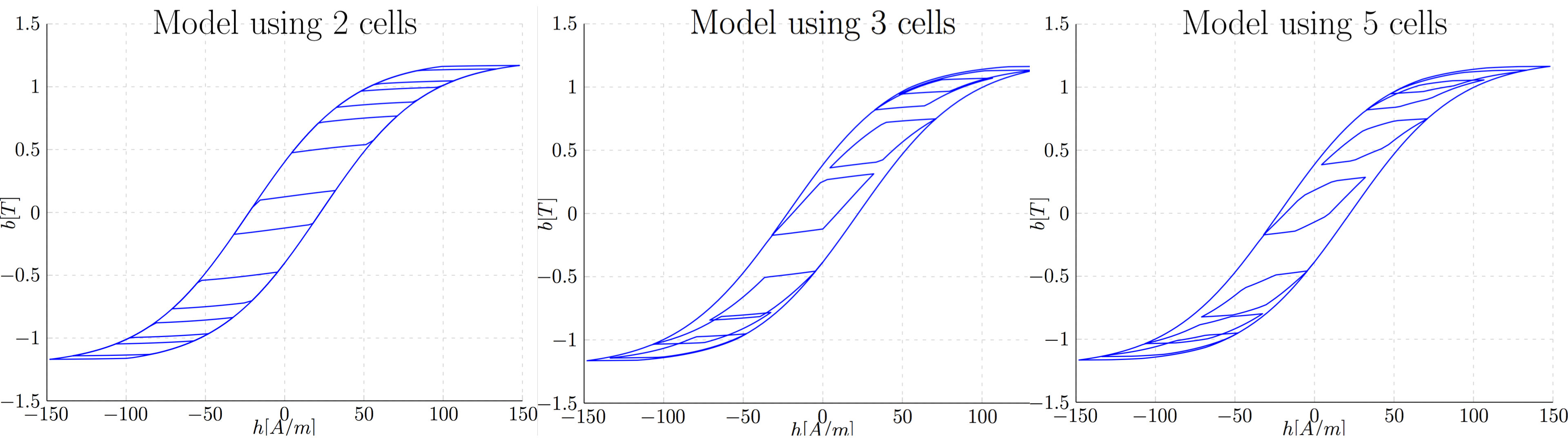
- $\boldsymbol{J} = \sum_k \boldsymbol{J}^k$: Magnetic Polarization $[T]$
- $\boldsymbol{h}$: Magnetic Field $[A/m]$
- u^k : Stored Magnetic Endergy Density $[J/m^3]$ (**Reversible component**)
- κ^k : Pinning Field $[A/m]$ (**Irreversible component**)
- $\boldsymbol{b} = \mu_0 \boldsymbol{h} + \boldsymbol{J}$: Magnetic Induction $[T]$



Energy-Based Hysteresis Model

Presentation of the model

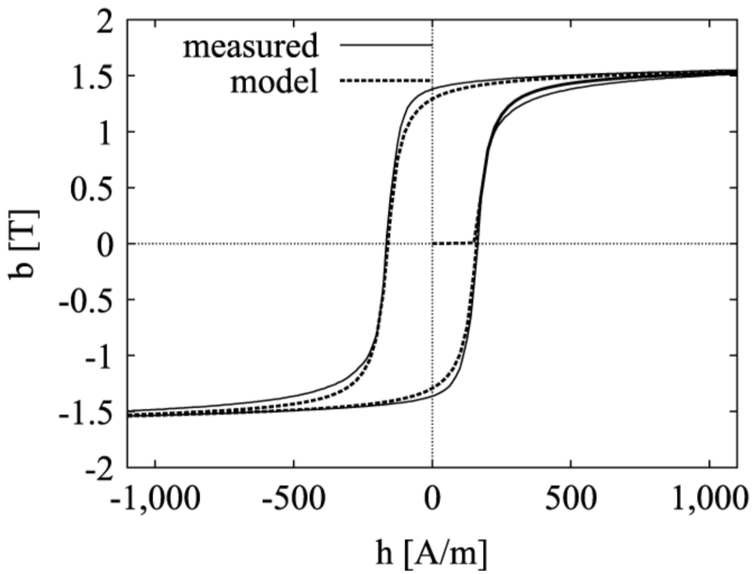
The choice of the number of cells allows for a trade-off between accuracy and complexity.



Energy-Based Hysteresis Model

Presentation of the model

Validation of the model for simple experimental configurations (1D).



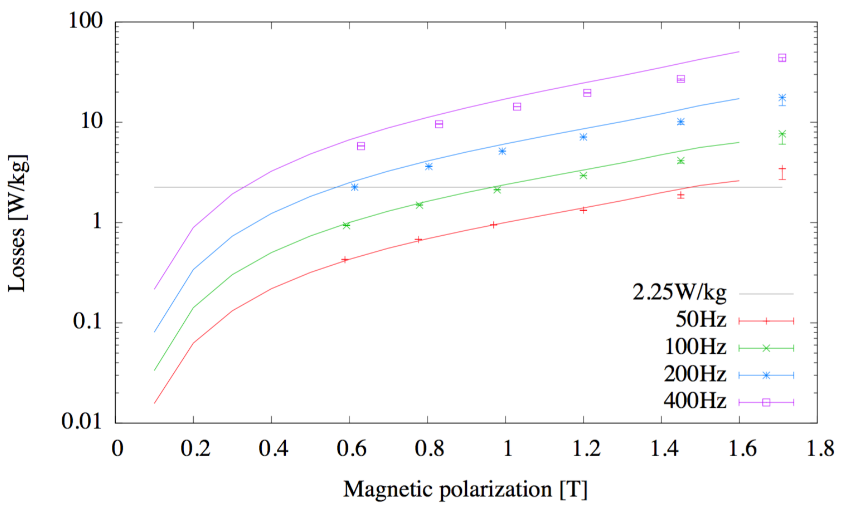


Fig. 11. Comparison between measured data for M23535A at 50Hz, 100Hz, 200Hz and 400Hz (solid lines) and calculated data (points).

- F. Henrotte, A. Nicolet, K. Hameyer, "An energy-based vector hysteresis model for ferromagnetic materials," *COMPEL*, vol. 25, no. 1, pp. 71–80, 2006.
- F. Henrotte, S. Steentjes, K. Hameyer, C. Geuzaine, "Iron Loss Calculation in Steel Laminations at High Frequencies," *IEEE Trans. Mag.*, vol. 50, no. 2, pp. 333–336, 2014.

Energy-Based Hysteresis Model

Types of Implementation: DIFF vs. VAR

$$h - \underbrace{\frac{\partial u^k(|J^k|)}{\partial J^k}}_{h_r^k} - \underbrace{\kappa^k \frac{\dot{J}^k}{|\dot{J}^k|}}_{h_i^k} = 0 \quad (*)$$

- **Simple Differential Approach (DIFF):**

  Approximation: $\dot{J}^k \parallel \dot{h}_r^k$ → **Approximated explicit** solution of the PDE (*)

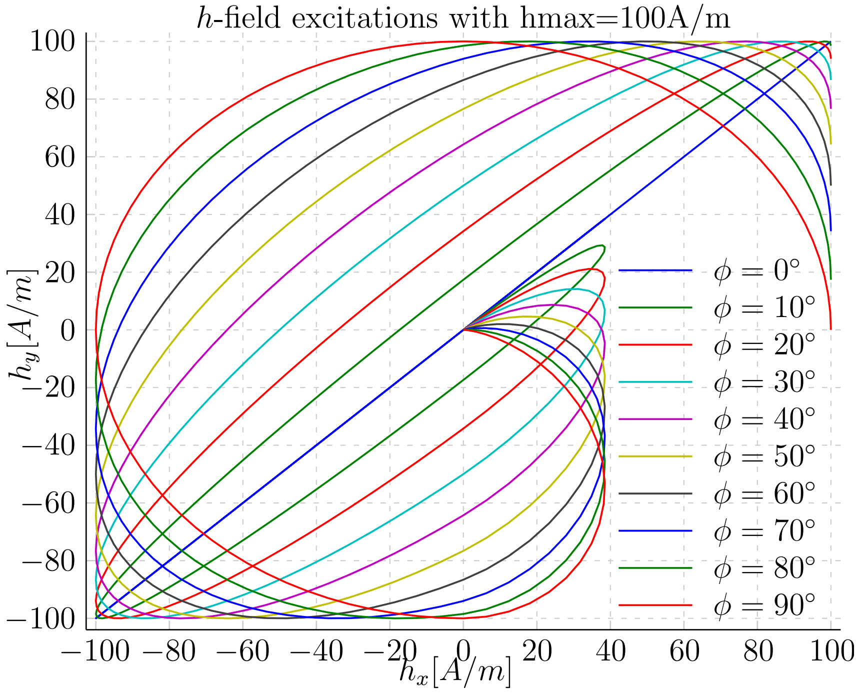
- **Variational Approach (VAR):**

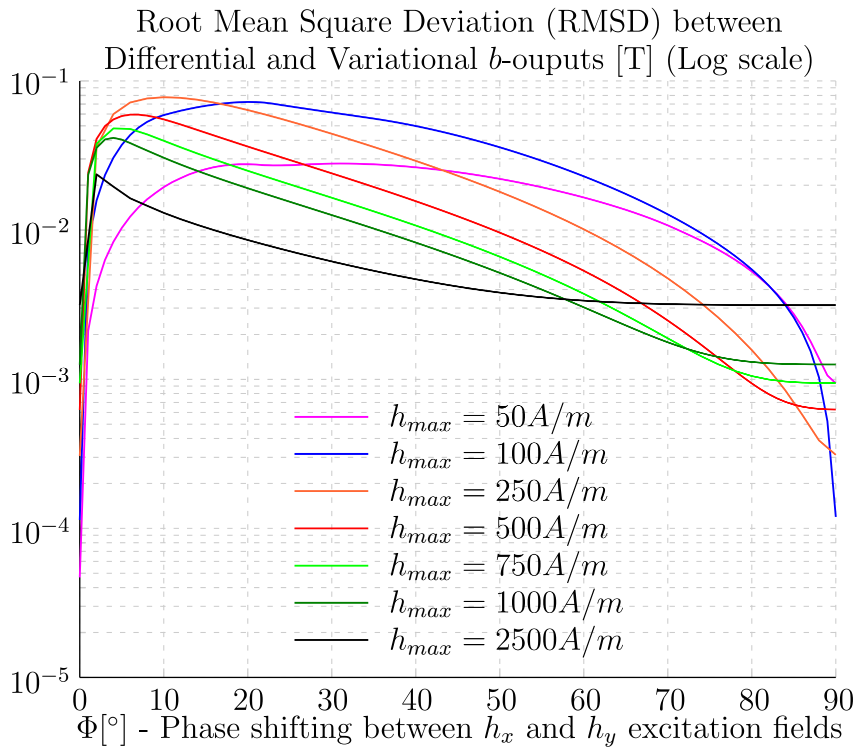
  Borrows from the theory of plasticity a **variational** formulation

  → solve **exactly** the **implicit** PDE (*) by the minimization of a functional

Energy-Based Hysteresis Model

Types of Implementation: DIFF vs. VAR

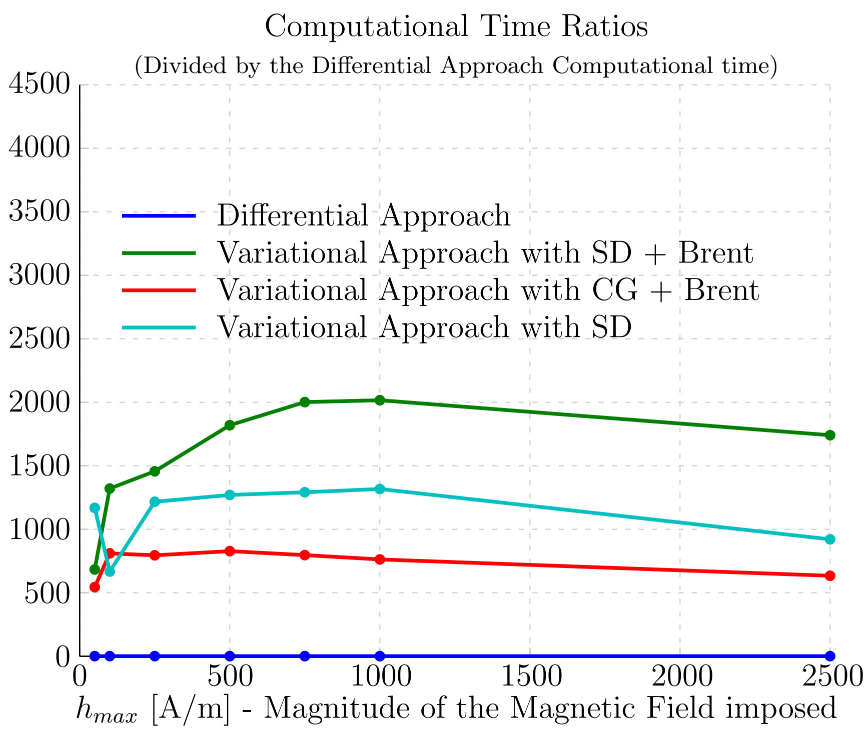


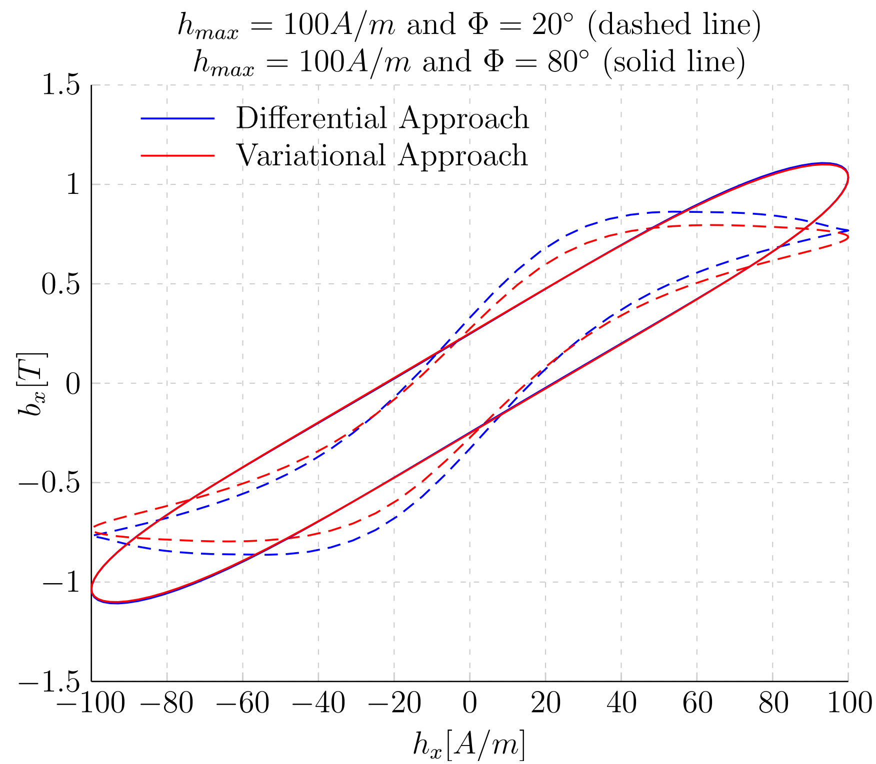


The Simple Differential Approach is a **rather good approximation** ($RMSD < 0.08T$)

Energy-Based Hysteresis Model

Types of Implementation: DIFF vs. VAR





The Variational Approach is **much slower** (at least 700 times !!!).
The Differential one gives similar results in much less time.

Energy-Based Hysteresis Model

Types of Implementation: DIRECT vs. INVERSE

- **Direct Form (DIR):**

  Input: $\boldsymbol{h} \rightarrow$ Output: $\boldsymbol{b}$

- **Inverse Form (INV):**

  Input: $\boldsymbol{b} \rightarrow$ Output: $\boldsymbol{h}$

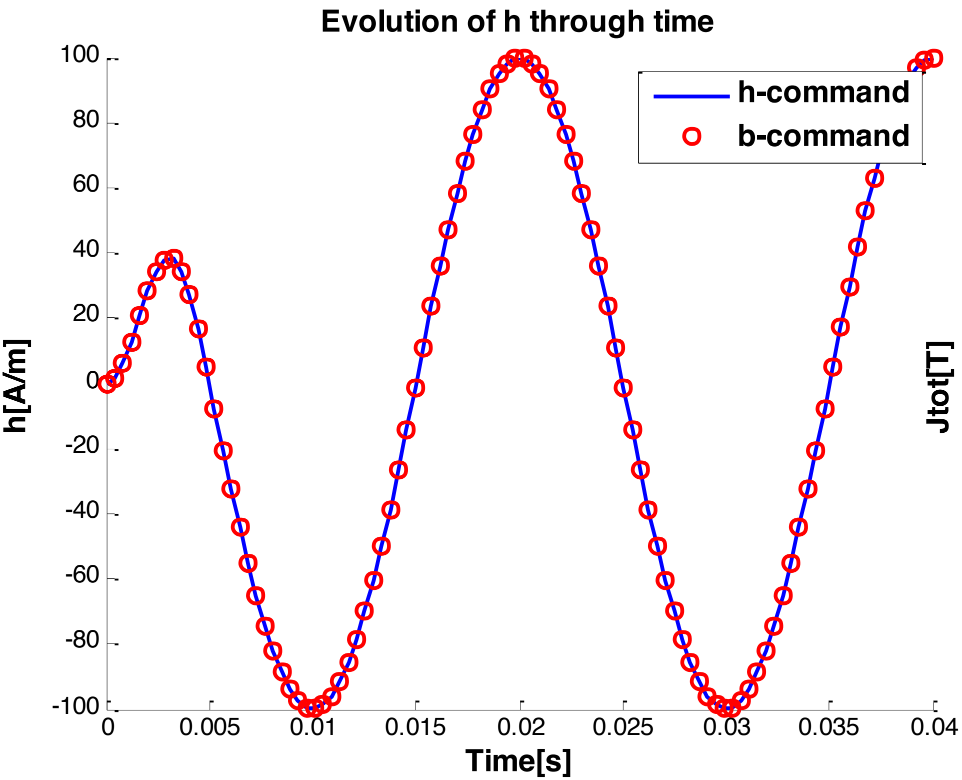
  Inversion Techniques:

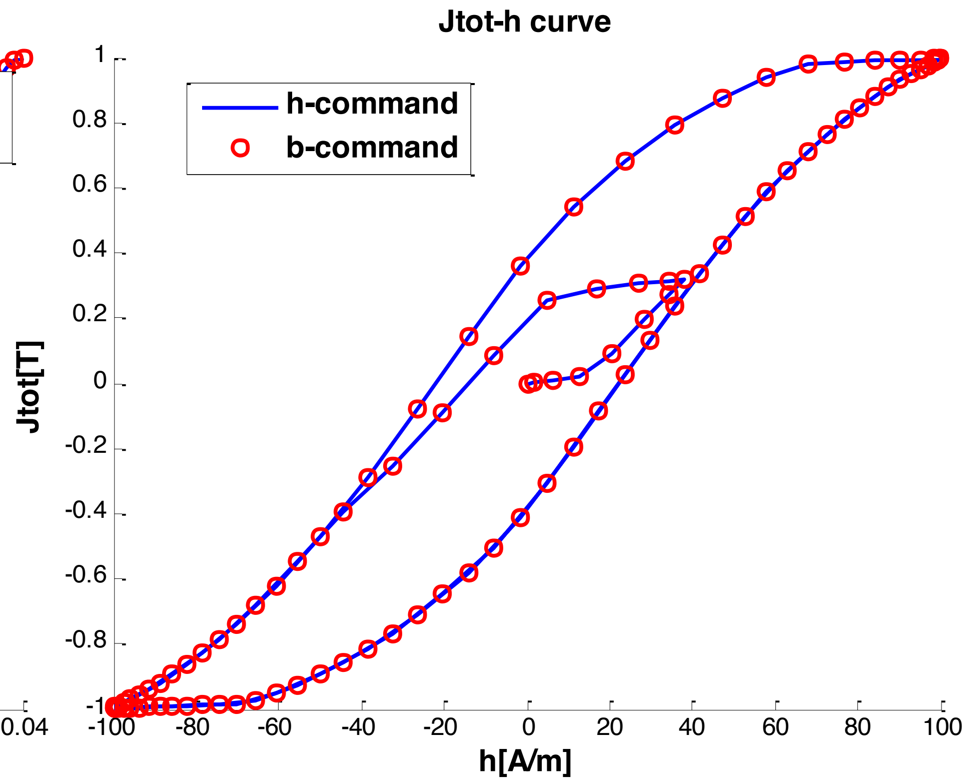
  - Newton-Raphson with analytical Jacobian **(NRana)**
  - Newton-Raphson with numerical Jacobian **(NRnum)**
  - Broyden-Fletcher-Goldfarb-Shanno **(BFGS)**

Energy-Based Hysteresis Model

Types of Implementation: DIRECT vs. INVERSE

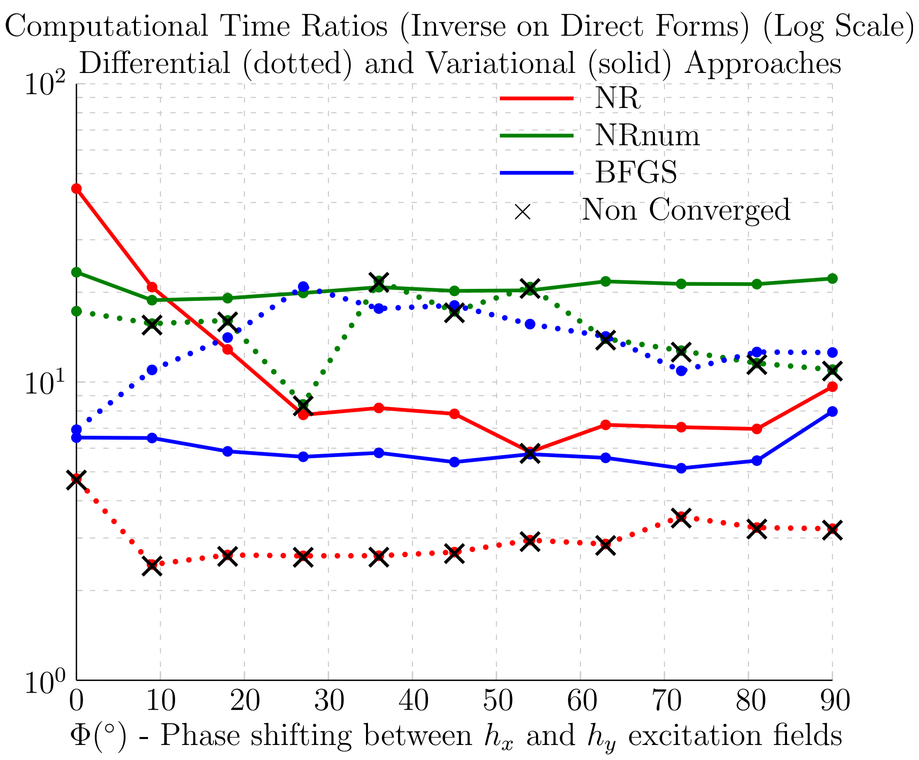
$$h \xrightarrow{direct} b = b \xrightarrow{inverse} h$$





Energy-Based Hysteresis Model

Types of Implementation: DIRECT vs. INVERSE



Inversion of the *DIFF* approach:

NRana – KO

NRnum – KO

BFGS - OK

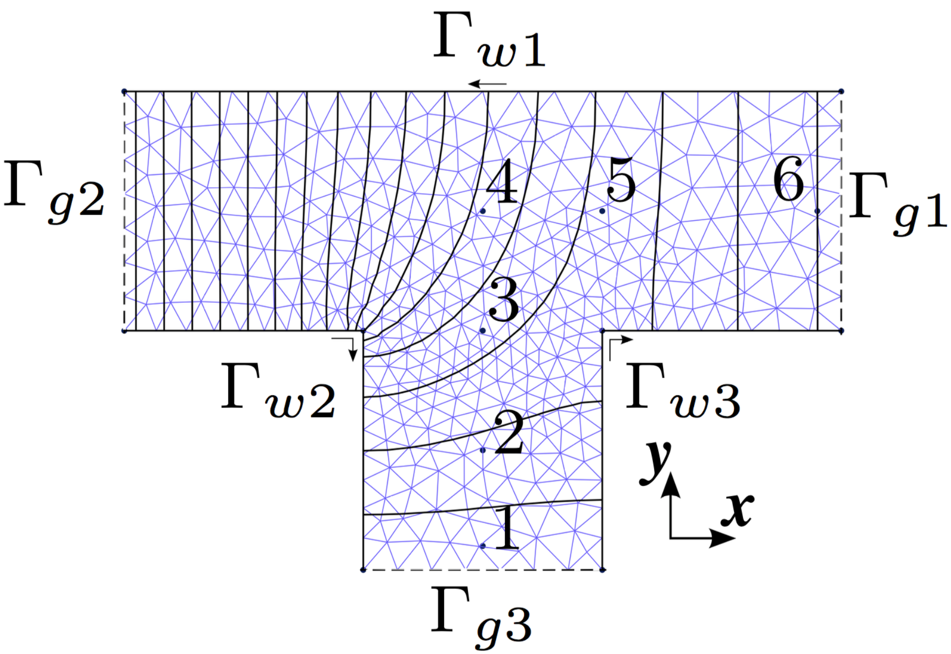
Inversion of the *VAR* approach:

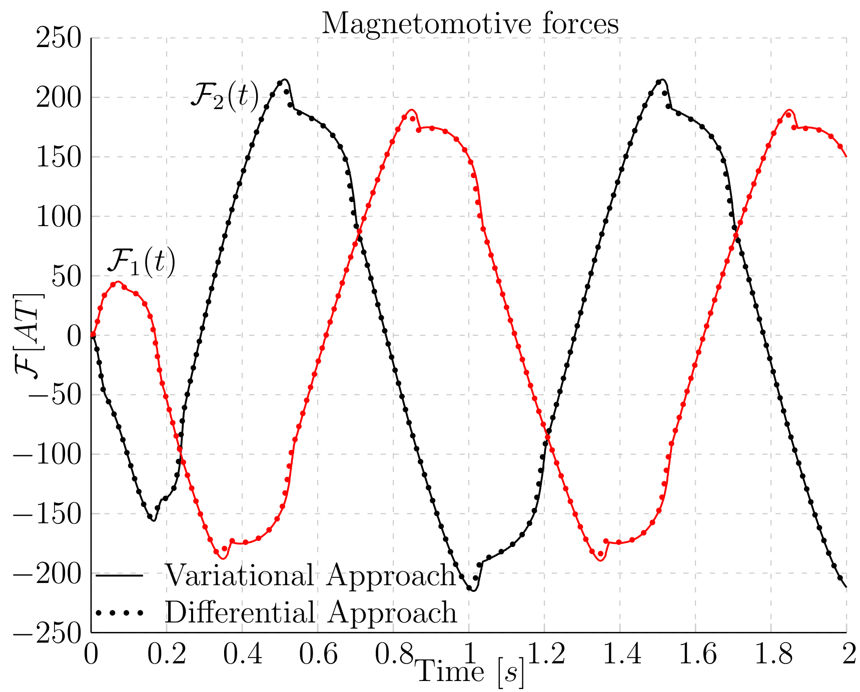
BFGS > NRana > NRnum

Energy-Based Hysteresis Model

Inclusion in Finite Element Environment (Gmsh/GetDP)

T-Joint (magnetostatic ϕ-formulation) [Direct Model]





Very Good Agreement for the Global Quantities from the VAR and DIFF Approaches

Energy-Based Hysteresis Model

Inclusion in Finite Element Environment (Gmsh/GetDP)

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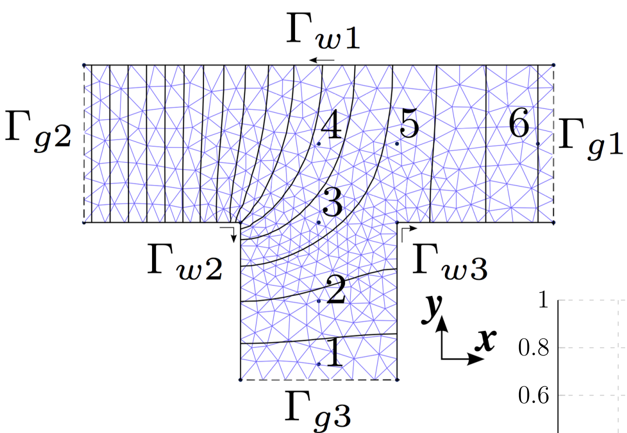
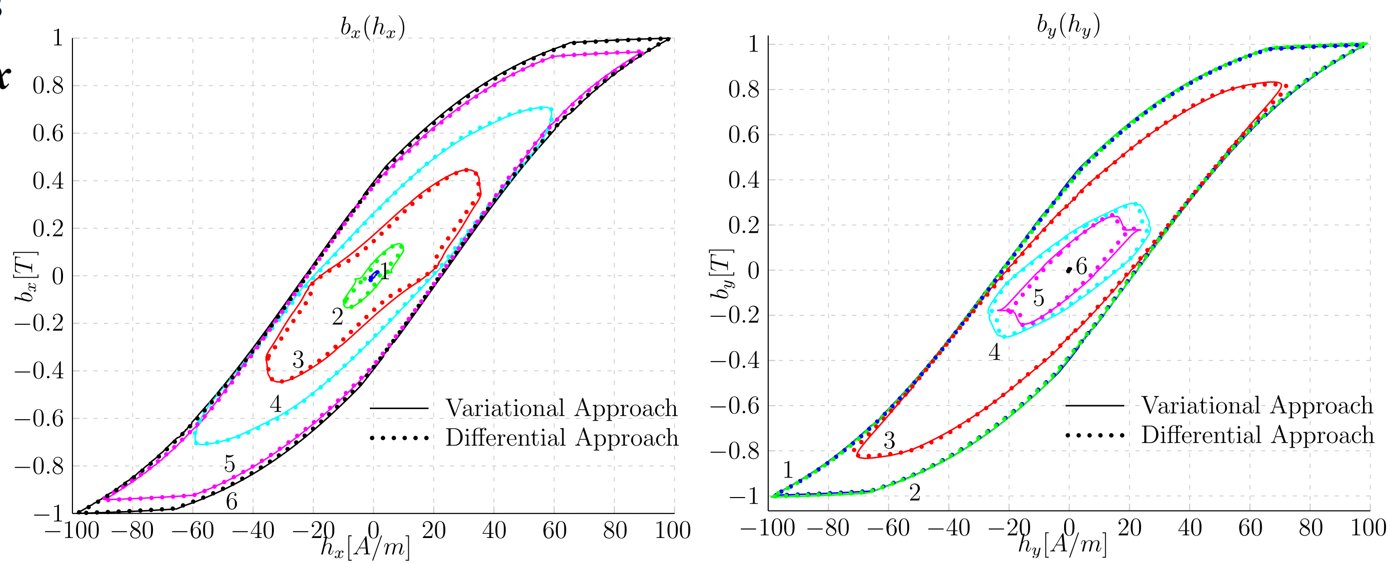


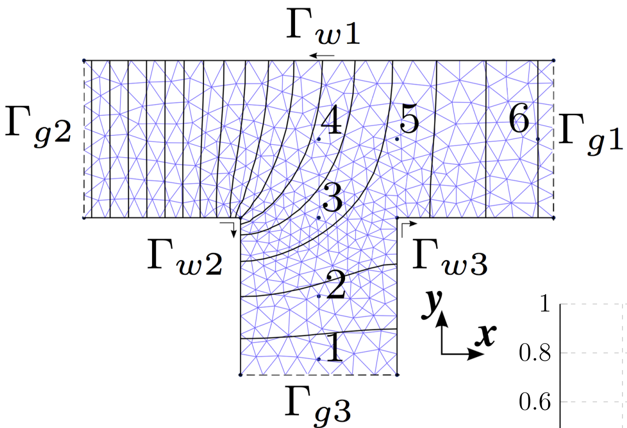
For the local fields, both VAR & DIFF approaches produce outputs that are also very similar.



Energy-Based Hysteresis Model

Inclusion in Finite Element Environment (Gmsh/GetDP)

T-Joint (magnetostatic ϕ-formulation) [Direct Model]



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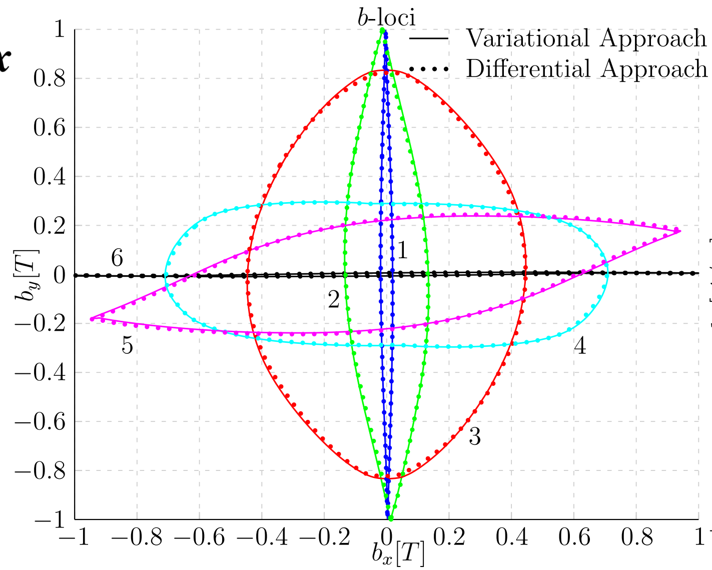
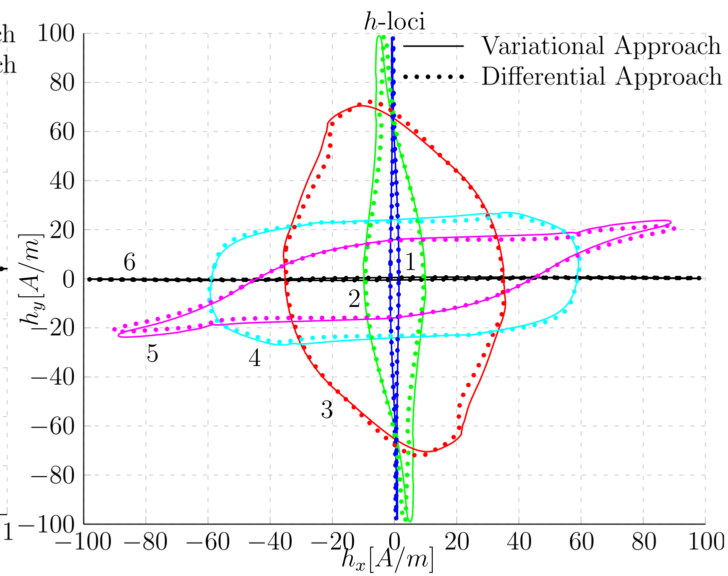


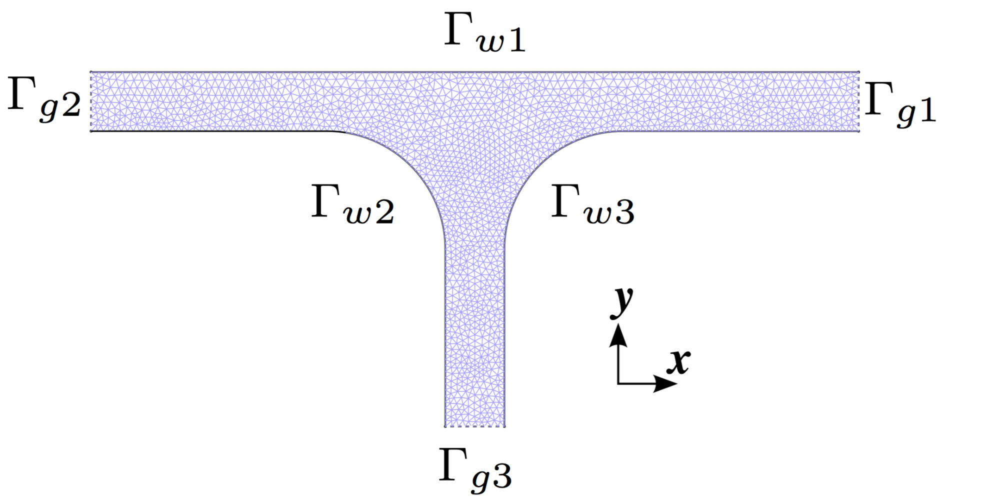


Energy-Based Hysteresis Model

Inclusion in Finite Element Environment (Gmsh/GetDP)

T-Joint (magneto**dynamic** $\mathrm{h}-\phi$-formulation) [Direct Model]

Eddy Current Effects are now taken into account



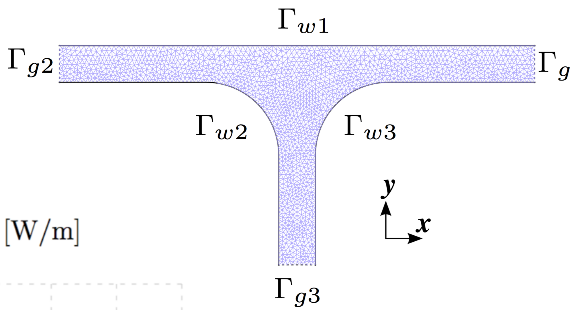
Mesh of a perfectly flux-confining T-joint slab

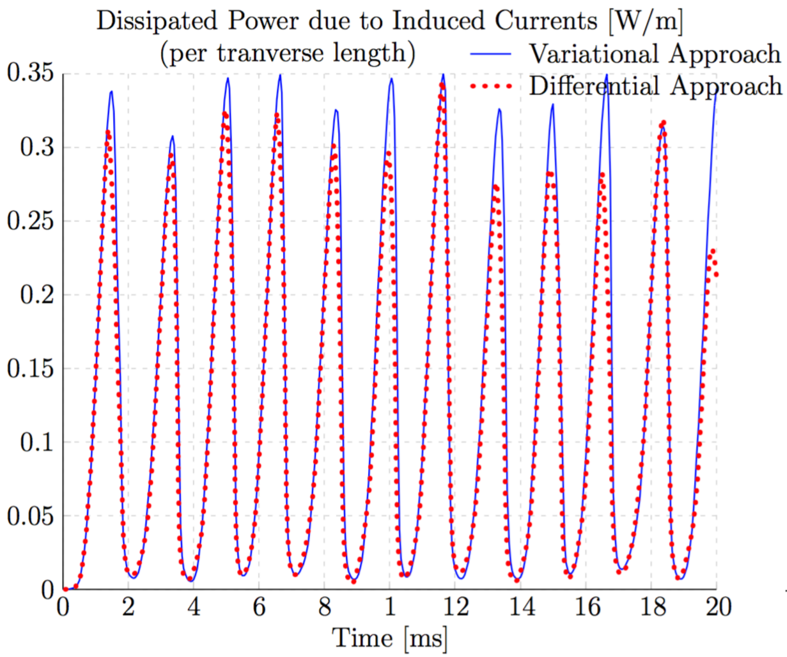
Energy-Based Hysteresis Model

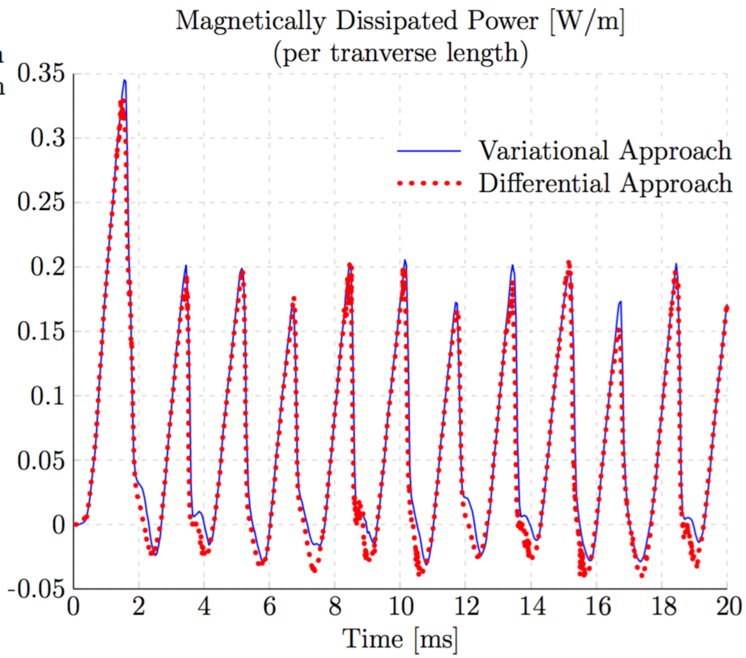
Inclusion in Finite Element Environment (Gmsh/GetDP)

T-Joint (magneto**dynamic** $h - \phi$-formulation) [Direct Model]

Same global evolution behaviour for VAR & DIFF approaches (some significant differences near extrema)







Energy-Based Hysteresis Model

Summary of the Results

At the material level:

- DIFF is much faster than VAR
- Both give similar results in most cases
- Inversion of DIFF is more complicated

Whitin a FE context:

- The overal computational gain of DIFF is less marked
- Results from both approaches were very similar locally and globally

(Correspondance was a bit less good for the magneto**dynamic** case)

Thank you for your attention

Perspectives

Improvements to the Energy-Based Hysteresis Model:

- Stabilize the Inverse Model (If possible)
- Investigate the differential approach without simplification
- Consider anisotropy and magnetostriction
- Extend to 3D test cases
- Compare simulations with measurements in real practical cases
- Clarifying the parameters identification strategy
- …