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ON THE KINETIC ENERGY DENSITY APPROXIMATION AND ITS  
APPLICATION TO THE NUCLEUS-NUCLEUS POTENTIAL

Fl. Stancu

Institut de Physique, Sart Tilman,  
B-4000 Liège 1, Belgium.

I shall report here some calculations of the real part of the nucleus-nucleus potential starting from the Skyrme interaction density functional and in the sudden approximation. The approach is described in Refs. [1,2]. The new feature here is the use of an approximation for the kinetic energy density introduced by Kirzhnits and others [3] instead of the Thomas-Fermi (TF) approximation we had used previously.

The Skyrme interaction gives the binding energy of the composite system and of the separate nuclei. The potential is defined as the difference between these two quantities; it is a function of the separation distance  $R$  between the centre of the two nuclei. In the sudden approximation the density of the composite system is taken to be equal to the sum of the densities  $\rho_1$  and  $\rho_2$  of each independent nucleus, at any separation distance. For the kinetic energy density of both the composite system and the separate nuclei we have used the TF approximations,

$$\tau_{TF} = \frac{3}{5} (3\pi^2)^{2/3} \rho^{5/3} \quad (1)$$

For the Hartree-Fock densities calculated with the Skyrme interaction SIII [4] this approximation reproduces to a large degree of accuracy the quantity

$$\tau' = - \sum \psi_i^* \nabla^2 \psi_i \quad (2)$$

where  $\psi_i$  are the wave functions of the occupied <sup>neutron or proton</sup> single particle states. It is less good in the centre of light nuclei as  $^{16}\text{O}$  and in the tail region of all magic nuclei because  $\tau'$  takes negative values while the expression (1) gives positive values.

The use of the TF approximation for the composite system has the advantage of including a part of the exchange effects due to antisymmetrization between nuclei as compared to the case

$$\tau' = \tau'_1 + \tau'_2$$

where the antisymmetrization is switched off in the total wave

function. For separation distances larger than the sum of the radii of the interacting nuclei it accounts for more than 75% of the exchange effects, but overestimates them at the barrier [1].

It turns out that the resulting potential is not attractive enough [2]; this may be partly due to the validity of the TF approximation in the surface region of the nucleus.

Here we consider another approximation to the kinetic energy density, introduced by Kirzhnits [3] and others.

(For a detailed list of references see [5, 6].) It consists in adding to  $\tau_{TF}$  of eq. (1) two corrective terms: the quantity  $\tau'$  of eq. (2) is then approximated by

$$\tau'_K = \tau_{TF} + \frac{1}{36} \frac{(\nabla\rho)^2}{\rho} - \frac{1}{6} \nabla^2 \rho. \quad (3)$$

where the second term represents the Weizaecker correction.

We have examined numerically the validity of the approximation (3) for the Skyrme-Hartree-Fock densities [4] of magic nuclei from  $^{160}$  to  $^{208}\text{Pb}$  and found that in the surface region eq. (3) reproduces the exact kinetic energy density  $\tau'$  better than the TF approximation does. In the central region the TF approximation still remains better, as Wong [5] also found for densities built from single particle states in a Woods-Saxon potential. But in the calculation of the tail of the nucleus-nucleus potential this central region does not come into play and therefore eq. (3) is a better approximation for our purposes.

In fig. 1 we show a typical example of the validity of these two approximations. We have plotted the exact neutron kinetic energy density  $\tau'$  (—), the TF approximation  $\tau_{TF}$  (- -) and the Kirzhnits approximation  $\tau'_K$  (· · ·) for  $^{160}$  as a function of the radial distance  $r$  beyond  $r > 3.2$  fm, which is the significant region for the calculation of the potential around the barrier. In the table we present values of the potential for a few pairs of magic nuclei. Column 2 gives the strong absorption radius  $d$  at which the potential is evaluated. For  $^{160} + ^{160}$ ,  $d$  is taken to be  $3 \times A^{1/3}$  while for the other pairs it is calculated from the distance of closest approach  $d = \eta/k (1 + \text{cosec } \theta_c^{\text{exp}}/2)$  where  $\eta$  is the Coulomb parameter,  $k$  the relative wave number and  $\theta_c^{\text{exp}}$  the critical angle. Columns 4 and 5 give the value of the potential  $V^K$  and  $V^{TF}$  derived from  $\tau'_K$  and  $\tau_{TF}$  respectively. In column 3 we present values of the phenomenological potential  $V^{\text{exp}}$  (references, last column) at  $E_{\text{lab}}$  given in the first column.

Pair	$E_{lab}$ (MeV)	$d$ (fm)	$v^{exp}$ (MeV)	$v^K$ (MeV)	$v^{TF}$ (MeV)	$V_0$ (MeV)	$R_0$ (fm)	$T$ (fm)	Ref.
$^{160} + ^{160}$	40	7.56	-2.98	-2.12	-1.13	-40.48	5.65	0.660	9
$^{160} + ^{40}Ca$	40	9.59	-0.92	-0.67	-0.29	-46.43	6.8	0.658	10
	60	9.30	-1.63	-1.03	-0.48				11
$^{160} + ^{48}Ca$	40	9.75	-0.96	-0.76	-0.34	-50.25	7.0	0.659	10
	60	9.70	-1.05	-0.82	-0.37				12
$^{160} + ^{208}Pb$	104	12.58	-1.32	-0.99	-0.54	-61.87	9.8	0.675	13
	129.5	12.7	-1.16	-0.83	-0.44				14
	192	12.5	-1.28	-1.11	-0.61				14

One can see that while the ratio  $v^{TF}/v^{exp}$  is of the order 0.3 - 0.4, the ratio  $v^K/v^{exp}$  takes a value around 0.7 - 0.8 which means that  $v^K$  is roughly twice deeper than  $v^{TF}$  at the strong absorption radius. To see better the difference between  $v^{TF}$  and  $v^K$  in fig. 2 we plot the elastic scattering cross section  $\sigma/\sigma_R$  as a function of  $\theta_{c.m.}$  for the pair  $^{160} + ^{48}Ca$  at  $E_{lab} = 40$  MeV. The three curves are obtained with the phenomenological potential (—), the potential  $v^K$  (---) and  $v^{TF}$  (-·-), respectively. In fact the tails of the calculated potentials  $v^K$  and  $v^{TF}$  are parametrized by Woods-Saxon forms (for  $v^K$  see columns 6-8) which are used in the calculation of  $\sigma/\sigma_R$ . The imaginary potential has the same geometry as the real one and a variable strength. It seems that  $v^K$  reduces considerably the discrepancy with respect to the experiment.

In conclusion,  $\tau_K^i$  gives more satisfactory values than  $\tau_{TF}$  for the nucleus-nucleus potential. It would be interesting to look for other effects which might contribute towards an improvement of the potential, for example a polarization effect [7,8].

#### References.

1. D.M. Brink and Fl. Stancu, Nucl. Phys. A243 (1975) 175.
2. Fl. Stancu and D.M. Brink, Nucl. Phys. A270 (1976) 236.
3. D.A. Kirzhnits, Field Theoretical Methods in Many-Body Systems, (Pergamon, Oxford 1967) p. 52.
4. M. Beiner et al., Nucl. Phys. A238 (1975) 29.
5. C.Y. Wong, Phys. Lett. 63B (1976) 395.
6. O. Bohigas et al., Phys. Lett. 64B (1976) 381.
7. D.M. Brink, European Conference on Nuclear Physics with Heavy Ions, Caen (France), 1976.
8. J. Fleckner and U. Mosel, preprint, Giessen University.
9. R.H. Siemssen et al., Phys. Rev. Lett. 25 (1970) 536.
10. K.O. Groeneweld et al., Phys. Rev. C6 (1972) 805.
11. F.D. Becchetti et al., Nucl. Phys. A203 (1973) 1.
12. J.B. Ball et al., Nucl. Phys. A244 (1975) 341.
13. F.D. Becchetti et al., Phys. Rev. C6 (1972) 2215.
14. J.B. Ball et al., Nucl. Phys. A252 (1975) 208.

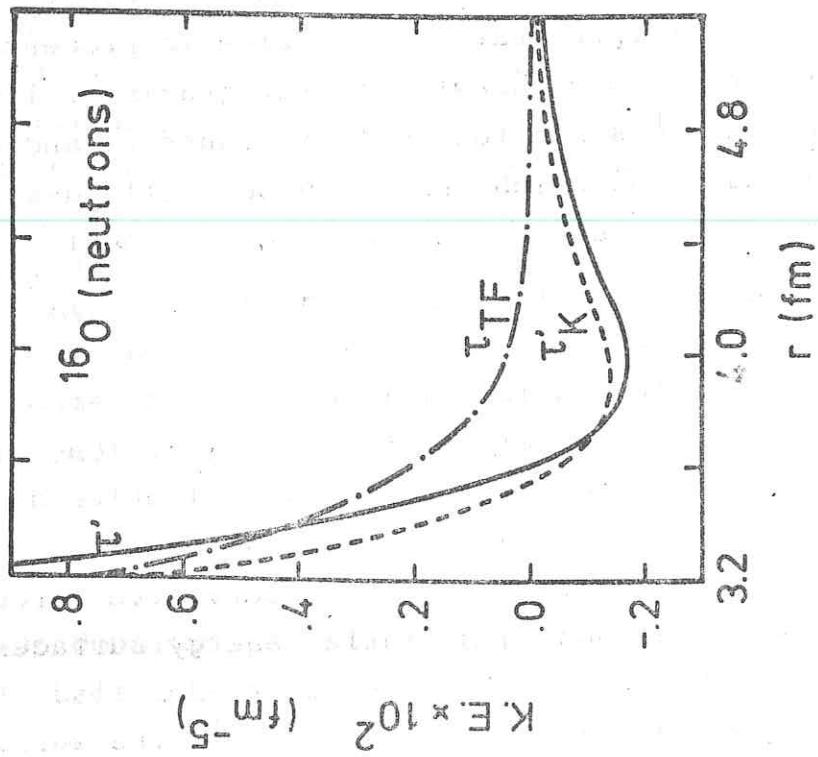


Fig. 1

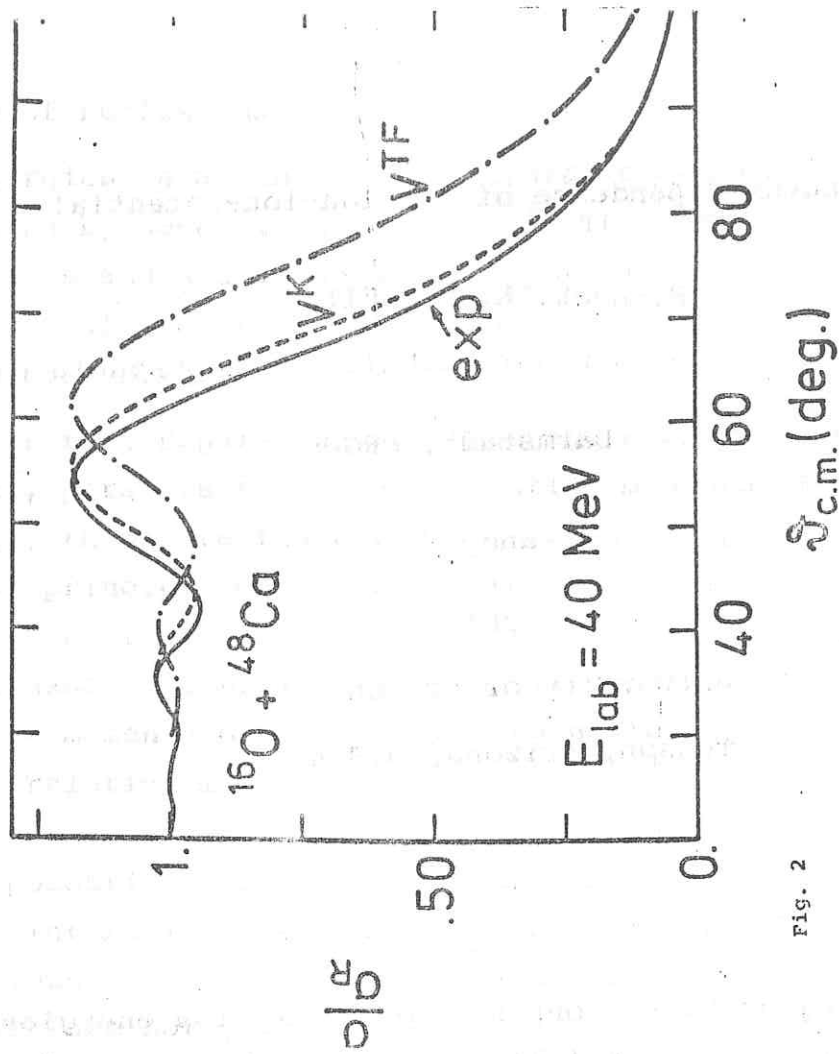


Fig. 2