INTERNATIONAL WORKSHOP XIV

ON

GROSS PROPERTIES OF NUCLEI AND NUCLEAR EXCITATIONS

HIRSCHEGG, KLEINWALSERTAL, AUSTRIA, JANUARY 13 - 18, 1986

Sponsored by:
BUNDESMINISTER FÜR FORSCHUNG UND TECHNOLOGIE, BONN, FRG

ORGANIZING COMMITTEE:

F. BECK
Institut für Kernphysik, TH Darmstadt, FRG

E.R. HILF
Universität Oldenburg

H. FELDMEIER, W. NÖRENBERG
Gesellschaft für Schwerionenforschung mbH, Darmstadt, FRG

Edited by: HANS FELDMEIER

ISSN 0720-8715

A FLUX TUBE POTENTIAL MODEL OF BARYON SPECTROSCOPY

R. SARTOR and Fl. STANCU,
Institut de Physique B5, Université de Liège,
Sart Tilman, B-4000 Liège 1, Belgium.

At present in the hadron spectroscopy two types of models are most frequently used : soliton (or bag) models and potential (or constituent quark) models. The former have the merit of being relativistic but are less accessible to spectroscopic studies. The latter are at most semi-relativistic but can be more extensively applied to spectroscopy.

The present study deals with a potential type model. It is inspired by the quark-gluon dynamics of QCD which in the static source limit leads to the flux tube picture.[1] In this limit the potential of a quark-antiquark pair is given by the sum of a Coulomb and a linear confinement terms :

$$V_{q\bar{q}} = -\frac{4}{3}\frac{\alpha_s}{|\underset{\sim}{r}_1 - \underset{\sim}{r}_2|} + \sqrt{\sigma}\,|\underset{\sim}{r}_1 - \underset{\sim}{r}_2| \tag{1}$$

where α_s is the strong interaction fine structure constant and $\sqrt{\sigma}$ is the flux tube tension. They can be obtained either from lattice gauge numerical studies or from a fit to meson spectra. The values we are using here are :

$$\frac{4}{3}\frac{\alpha_s}{\hbar c} = 0.5 \quad ; \quad \sqrt{\sigma} = 1 \text{ GeV.fm}^{-1} \quad . \tag{2}$$

For a 3-quark colour singlet system the potential energy can be expressed as a sum of two-body and three-body terms. The two-body potential between two quarks is $\frac{1}{2} V_{q\bar{q}}$ and the three-body term depends on the tension in the flux tube

$$V(\underset{\sim}{r}_1,\underset{\sim}{r}_2,\underset{\sim}{r}_3) = \sqrt{\sigma}\,[\sum_{i=1}^{3} r_{i4} - \frac{1}{2}\sum_{i<j} r_{ij}] \tag{3}$$

where r_{ij} is the interquark distance and r_{i4} is the distance between the quark i and a point $\underset{\sim}{r}_4$ where the flux tubes meet at 120°. For angles larger than 120° this point becomes identical to one at the quark positions $\underset{\sim}{r}_i$.

In Ref. 1, Carlson, Kogut and Pandharipande have added to the potential energy a relativistic kinetic energy

$$T = \sum_i (p_i^2 + m^2)^{\frac{1}{2}} \qquad (4)$$

and provided variational wave functions for mesons and baryons.

Based on such a non-perturbed hamiltonian we calculate the positive- and negative-parity spectra of baryons by diagonalizing the hyperfine interaction[2] in a space spanned by the $56(0^+,2^+)$, $56'(0^+)$, $70(0^+,1^-,2^+)$ and $20(1^+)$ SU(6) multiplets. In other words we allow for up to two units of angular momentum or up to one unit of radial excitations. The resulting highest spin is $J = \frac{7}{2}$ for positive parity and $J = \frac{5}{2}$ for negative parity.

The hyperfine interaction[2] has been modified to include the finite size Λ of the quark. Then both the spin-spin part V_{SS} and the tensor part V_T take the form :

$$V_{SS} = \frac{4\sqrt{2}\,\pi\,\alpha_S}{9\,m^2}\,\frac{1}{(2\pi\Lambda^2)^{3/2}}\,e^{-\frac{\rho^2}{2\Lambda^2}}\,\underset{\sim}{S}_1\cdot\underset{\sim}{S}_2 \qquad (5)$$

$$V_T = \frac{\alpha_S}{\sqrt{2}\,m^2\,\rho^3}\left[\,\mathrm{erf}\left(\frac{\rho}{\sqrt{2}\,\Lambda}\right) - \frac{\sqrt{2}}{3\sqrt{\pi}}\,\frac{\rho^3}{\Lambda^3}\left(1 + 3\,\frac{\Lambda^2}{\rho^2}\right)e^{-\frac{\rho^2}{2\Lambda^2}}\,\right]$$
$$\times\left(\frac{1}{\rho^2}\,\underset{\sim}{S}_1\cdot\underset{\sim}{\rho}\;\underset{\sim}{S}_2\cdot\underset{\sim}{\rho} - \frac{1}{3}\,\underset{\sim}{S}_1\cdot\underset{\sim}{S}_2\right) \qquad (6)$$

where

$$\rho = \frac{1}{\sqrt{2}}\,|\underset{\sim}{r}_1 - \underset{\sim}{r}_2| \;.$$

The quark mass m and its size Λ were left as free parameters. The range of m was taken between 250 MeV and 360 MeV and for Λ we took values in the range 0.0 - 0.325 fm with a size step of 0.065 fm. It turned out[3] that the model can accomodate the whole mass range if Λ was properly chosen for each m. As a general trend the calculated positive parity spectrum was found within the experimental error bars except for those dominated by $L = 0$ and/or $S = \frac{1}{2}$. For example the Roper resonance has too high an energy for all sets m, Λ. For the negative parity states we obtained good agreement with the experimental resonances.

Among the various sets m, Λ we chose those given in Table 1. Set I is common with Ref. 1 and Set II was found to give one of the best overall agreements and a good nucleon mass. These sets were used subsequently in the analysis of photodecay amplitudes of N and Δ resonances. A crucial test of the model is the calculation of the photodecay or strong decay amplitudes because these depend on the configuration mixing obtained from the diagonalization of the hyperfine interaction.

We assume that the photodecay process is due to the emission of a photon by one of the constituent quarks. Then due to the symmetry of the wave function and the identity of the three quarks one can take the quark-photon interaction H as three times the contribution of the third quark. In our calculations we use the standard non-relativistic quark-photon interaction[4]

$$H = -3\,\frac{e\,q^{(3)}}{m}\,(\underset{\sim}{p}^{(3)}\cdot\underset{\sim}{A} + g\,\underset{\sim}{S}^{(3)}\cdot\underset{\sim}{H}) \tag{7}$$

where g is the gyromagnetic factor of the quark, $q^{(3)}$, $\underset{\sim}{p}^{(3)}$ and $\underset{\sim}{S}^{(3)}$ the charge, momentum and spin operators of the third quark, $\underset{\sim}{H}$ the magnetic field and $\underset{\sim}{A}$ the corresponding vector potential. As in Ref. 4 we took :

$$g = 1 \quad ; \quad \mu = 0.13\ \mathrm{GeV}^{-1} \tag{8}$$

where μ is the quark magnetic moment.

From the experimental data one can extract the helicity amplitudes $A_{1/2}$ and $A_{3/2}$. By definition $A_{1/2}$ is the matrix element $\langle f|H|i\rangle$ of (7) where the resonance $|i\rangle$ is in a state of angular momentum projection $J_z^i = \frac{1}{2}$ and the nucleon has $J_z^f = -\frac{1}{2}$ in its ground state $|f\rangle$. The amplitude $A_{3/2}$ corresponds to $J_z^i = \frac{3}{2}$ and $J_z^f = \frac{1}{2}$. Both the resonances and the nucleon ground state contain configuration mixings. The most important components of the nucleon wave function are $|^2N(56,0^+)\frac{1}{2}^+\rangle$,

$|^2N(56',0^+)\frac{1}{2}^+\rangle$ and $|^2N(70,0^+)\frac{1}{2}^+\rangle$. The results for the helicity amplitudes of protons A^p and neutrons A^n are given in Table II. For each resonance the first and second lines are the results obtained with the parameter sets I and II respectively. The third line reproduces the experimental values of Particle Data Group.[5] The resonance masses are in MeV and the helicity amplitudes in 10^{-3} $GeV^{-\frac{1}{2}}$. The sign of the decay amplitudes is well reproduced in most cases and the magnitudes are of the right order. A detailed analysis[6] shows that the configuration mixings play an important role. A further test of the model would be to calculate the strong decays.

References

1. J. Carlson, J. Kogut and V.R. Pandharipande, Phys. Rev. D27, 233 (1983).
2. A. De Rujula, H. Georgi and S.L. Glashow, Phys. Rev. D12, 147 (1975).
3. R. Sartor and Fl. Stancu, Phys. Rev. D31, 128 (1985).
4. L.A. Copley, G. Karl and E. Obryk, Nucl. Phys. B13, 303 (1969).
5. Particle Data Group, Rev. mod. phys. 56, S126 (1984).
6. R. Sartor and Fl. Stancu, Phys. Rev. D, ~~to appear~~. D 33 (1986) 727.

TABLE I

Set	m (MeV)	Λ (fm)
I	360	0.13
II	324	0.09

TABLE II

Resonance	Main component	Mass	$A^p_{3/2}$	$A^p_{1/2}$	$A^n_{3/2}$	$A^n_{1/2}$
		1712		-30		+19
P_{11} (1440)	$^2N(56',0^+)1/2^+$	1607		-31		+19
		1400-1480		-69±7		+37±19
		1579	+213	+57	-211	-98
D_{13} (1520)	$^2N(70,1^-)3/2^-$	1496	+202	+45	-201	-88
		1510-1530	+167±10	-22±10	-144±14	-65±13
		1568		+210		-188
S_{11} (1535)	$^2N(70,1^-)1/2^-$	1475		+203		-182
		1520-1560		+73±14		-76±32
		1673		+68		-41
S_{11} (1650)	$^4N(70,1^-)1/2^-$	1627		+77		-48
		1620-1680		+48±16		-17±37
		1690	+4	+3	-35	-25
D_{15} (1675)	$^4N(70,1^-)5/2^-$	1653	+7	+5	-39	-27
		1660-1690	+19±12	+19±12	-69±19	-47±23
		1860	+96	+23	-28	+5
F_{15} (1680)	$^2N(56,2^+)5/2^+$	1754	+94	+22	-30	+2
		1670-1690	+127±12	-17±10	-30±14	+31±13
		1726	+17	+2	-58	-14
D_{13} (1700)	$^4N(70,1^-)3/2^-$	1714	+21	+1	-64	-14
		1670-1730	0±19	-22±13	-2±44	0±56
		1868		-40		+25
P_{11} (1710)	$^2N(70,0^+)1/2^+$	1795		-48		+35
		1680-1740		+5±16		-5±23
		1859	+47	-120	-12	+45
P_{13} (1720)	$^2N(56,2^+)3/2^+$	1752	+45	-118	-12	+47
		1690-1800	-35±24	+52±39	-43±94	-2±26
		2028	-3	-2	-10	-8
F_{17} (1990)	$^4N(70,2^+)7/2^+$	1980	-5	-4	-8	-6
		1950-2050	+31±55	+24±30	-122±55	-49±45
		1308	-159	-90		
P_{33} (1232)	$^4\Delta(56,0^+)3/2^+$	1285	-181	-101		
		1230-1234	-258±11	-141±5		
		1893	-80	-44		
P_{33} (1600)	$^4\Delta(56',0^+)3/2^+$	1904	-106	-38		
		1500-1900	+1±22	-20±29		
		1657		+154		
S_{31} (1620)	$^2\Delta(70,1^-)1/2^-$	1631		+141		
		1600-1650		19±16		
		1657	+212	+149		
D_{33} (1700)	$^2\Delta(70,1^-)3/2^-$	1631	+201	+149		
		1630-1740	+77±28	+116±17		
		1997	+28	+33		
F_{35} (1905)	$^2\Delta(70,2^+)5/2^+$	1962	+41	+36		
		1890-1920	-47±19	+27±13		
		1934		+26		
P_{31} (1910)	$^2\Delta(70,0^+)1/2^+$	1910		+35		
		1850-1950		-12±30		
		1986	+13	-45		
P_{33} (1920)	$^4\Delta(56,2^+)3/2^+$	1964	-39	-78		
		1860-2160	+23±?	+40±?		
		1985	-22	-17		
F_{37} (1950)	$^4\Delta(56,2^+)7/2^+$	1952	-21	-17		
		1910-1960	-90±13	-73±14		