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THE IMPORTANT CONFIGURATIONS IN SIX-QUARK N-N STATES

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To describe the short range nucleon-nucleon interaction one needs details about the structure of six- and three-quark systems. Until now all calculations from which phase shifts have been derived are based on the cluster model.

In a previous paper [1] we have proposed a classification scheme based on molecular orbitals instead of cluster model states. This classification scheme is general and includes nonrelativistic or semirelativistic constituent quark models as well as relativistic current quark soliton models.

Molecular orbitals are natural to mean field or general independent particle model (IPM) approximations. Among the various quantum numbers characterising an IPM state, we call special attention to parity, since single particle parity is conserved during a collision within the mean field approximation. Cluster model states have mixed parities, but from them, linear combinations of good parity can be built. Let us call Z the separation coordinate. One can define states

$$\psi(\vec{r} - \tfrac{1}{2} Z \hat{z}) = R \quad , \qquad (1)$$

$$\psi(\vec{r} + \tfrac{1}{2} Z \hat{z}) = L \quad ,$$

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which describe quarks located in the right (R) and left (L) clusters, respectively. For simplicity we suppose that the nucleon ground state contains 3 quarks in an s state. Then one can build positive σ and negative π parity states

$$\begin{pmatrix} \sigma \\ \\ \pi \end{pmatrix} = [2(1 \pm \langle R|L\rangle)]^{-\frac{1}{2}} (R \pm L) \quad . \tag{2}$$

These become s and p states in the limit $Z \to 0$. From the σ and π states one can also build pseudo-right and -left states r and ℓ as

$$\begin{pmatrix} r \\ \\ \ell \end{pmatrix} = 2^{-\frac{1}{2}} (\sigma \pm \pi) \quad . \tag{3}$$

For molecular orbitals only the transformation (3) is necessary. In the limit $Z \to \infty$ one has $r \to R$ and $\ell \to L$, otherwise they are different. Moreover r,ℓ are orthogonal functions which greatly simplify the mathematics of totally antisymmetric six-quark states including orbital, isospin-spin and color degrees of freedom. One can work equally well with σ,π or r,ℓ states.

An important result of Ref. 1 is that we demonstrated that certain limiting configurations $p^n s^{6-n}$ are missing in the cluster model basis used at present. They appear automatically in a molecular basis and generate a larger orbital space than the cluster model does, even when both are built from the same R,L states.

Also, in previous calculations the isospin-spin space has been truncated to representations relevant to NN and $\Delta\Delta$ systems. Many other SU(4) representations (see Table 1 of Ref. 2) can be combined to the orbital and color parts to produce totally antisymmetric six-quark states. Taking all these omitted or neglected configurations into account, we have proposed a classification scheme [1] with 16 channels in the T = 0, S = 1 or T = 1, S = 0, 7 in the T = 0, S = 0 and 25 in the T = 1, S = 1 sectors, respectively. Among these, there are 3 physical channels NN, $\Delta\Delta$ and NΔ and the rest couple to the physical states at short separations but the expectation values of their energies become infinite as the separation goes to infinity. The new configurations appearing in our scheme as a result of using molecular orbitals are states with 4 quarks in a bag (or cluster) and 2 in the other or 5 in a bag and 1 in the other. In the cluster model calculations these configurations are omitted.

The aim of the present work is to examine the importance of the previously omitted or neglected configurations. For simplicity, we have considered the case Z = 0. This is an extreme for the configuration mixing and it is relevant for short separations Z. The single particle r,ℓ states are built from cluster model R,L states as indicated above. We diagonalize a six-quark commonly used nonrelativistic hamiltonian containing a confinement $v_c(r_{ij})$ and a spin-spin quark-quark interaction of radial dependence $v_\sigma(r_{ij})$

$$H = \sum_i (m_i c^2 + \frac{\vec{p}_i^{\,2}}{2m_i}) - \frac{1}{2M} \vec{P}^2 + \sum_{i<j} \hat{\lambda}_i.\hat{\lambda}_j \, (v_c(r_{ij}) + \vec{\sigma}_i.\vec{\sigma}_j \, v_\sigma(r_{ij})) \ , \qquad (4)$$

where $M = \Sigma_i m_i$, $\vec{P} = \Sigma_i \vec{p}_i$ and $\hat{\lambda}_i^\alpha$ ($\alpha = 1,\ldots,8$) are the generators of the SU(3) color group. The function $v_c(r_{ij})$ is parametrized in the simple form given by Harvey [2]

$$v_c = A \, e^{-r^2/\alpha^2} + B \, r^2 + C \qquad . \qquad (5)$$

The values of the parameters used in v_c, v_σ and the size oscillator parameter are taken from Table 1 (case 1a) of Ref. 3. They are chosen such as to minimize the ground state energy of the nucleon. The results of the diagonalisation are displayed in Tables 1 and 2 for the sectors T = 0, S = 1 and T = 0, S = 0, respectively. In each table the first column gives the state vectors taken into account. The physical states NN and ΔΔ together with the color-color (CC) state are defined in Table 11 of Ref. 2 in terms of symmetry states. For the other vectors we use the notation of Ref. 1. For example in the vector $(42^+[6]\{33\})$, 42 indicates a state with 4 quarks in a bag and 2 in the other, the upper index + specifies an even total parity (even relative angular momentum), the partition [6] gives the permutation symmetry of the orbital part of the wavefunction with respect to S_6 and {33} is the dual SU(4) representation to which the isospin-spin part of the wavefunction belongs. Columns 2 and 4 are the expectation values in the R,L and r,ℓ basis, respectively, for each six-quark state. Column 3 is the result of the diagonalization of a 3 × 3 matrix in the R,L basis. Columns 5 to 8 are results of diagonalization of matrices of various sizes in the r,ℓ basis. For both sectors the 3 × 3 matrix in the r,ℓ basis gives a higher ground state than the R,L basis. But the addition

of states with $42^{\pm}$, $51^{\pm}$ configurations has a dramatic effect in lowering the ground state. One can also see that the total effect on NN of these unphysical states is more important than the combined effect of the $\Delta\Delta$ and CC states through the coupling.

On the contrary, the role of SU(4) states - without asterisk in Harvey's [1] table 1 - is negligible, as proved by the examples exhibited in Tables 1 and 2. These are the {321} states for T = 0, S = 1 and the {222} for T = 0, S = 0. At large separations this kind of states give rise to 3-quark clusters (or bags) which do not have values of T and S compatible with N and Δ. The reason of their negligible effect is that they couple to the other states through the spin-spin interaction only (and tensor if included) and this contribution is much smaller than the contribution of the confining potential which makes large the coupling of the $42^{\pm}$ and $51^{\pm}$ channels to the 33 configuration forming the NN.

As a similar pattern is expected for the T = 1, S = 1 sector, we can conclude that the number of important states is 7 out of 16 for T = 0, S = 1 or T = 1, S = 0 sectors, 5 out of 7 for the T = 0, S = 0 and 11 out of 25 for the T = 1, S = 1 sectors.

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References :

[1] Fl. Stancu and L. Wilets, Phys. Rev. C36, 726 (1987).
[2] M. Harvey, Nucl. Phys. A352, 301 (1981).
[3] M. Harvey and J. Letourneux, Nucl. Phys. A424, 419 (1984).

TABLE 1 : T = 0, S = 1

Channel	R,L (cluster)		r,ℓ (molecular)				
	1x1	3x3	1x1	3x3	6x6	7x7	10x10
NN	905	592	961	952	413	311	309
ΔΔ	1373	1163	1687	1594	1308	1292	1285
CC	1083	1603	1840	1942	1495	1494	1490
$(42^{+}[6]\{33\})$			2099		2158	1578	1573
$(42^{+}[42]\{33\})$			2223		2344	2159	1655
$(42^{+}[42]\{51\})$			1658		2750	2344	2107
$(51^{+}[6]\{33\})$			1613			2903	2162
$(42^{+}[51]\{321\})$			1879				2345
$(42^{+}[42]\{321\})$			2136				2722
(33 [42]{321})			2485				2933

TABLE 2 : T = 0, S = 0

Channel	R,L (cluster)		r,ℓ (molecular)				
	1x1	3x3	1x1	3x3	4x4	5x5	6x6
NN	1140	947	1070	1068	253	214	213
ΔΔ	1289	1256	1644	1606	1604	1155	1154
CC	2020	2245	2226	2266	2266	1605	1594
$(42^{-}[51]\{42\})$			1592		2408	2282	2045
$(51^{-}[51]\{42\})$			1456			2731	2285
(33 [33]{222})			2038				2735