

Exercice VI.9

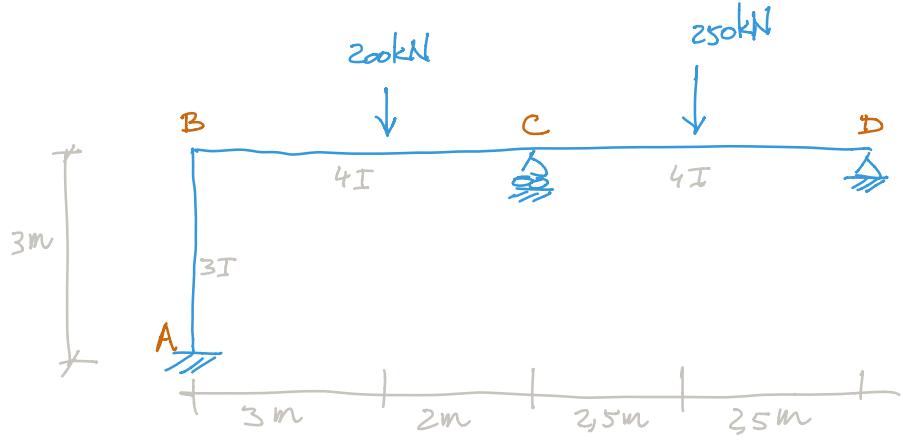


Diagramme des moments ?

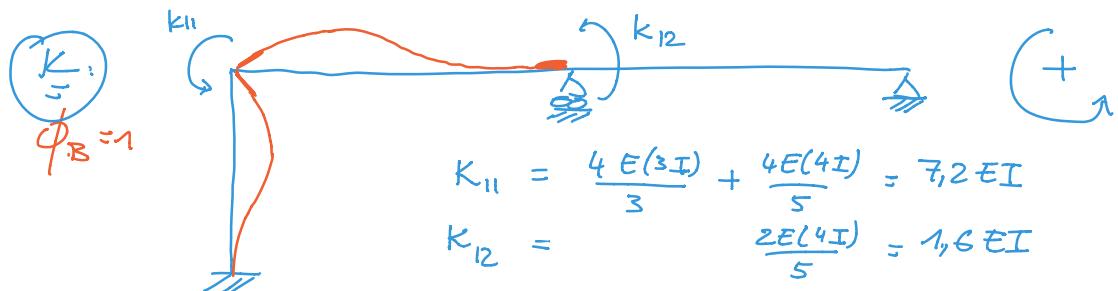
Réaction en D ?

Déformée ? (Esquisse)

Déplacement sous la charge de 250 kN.

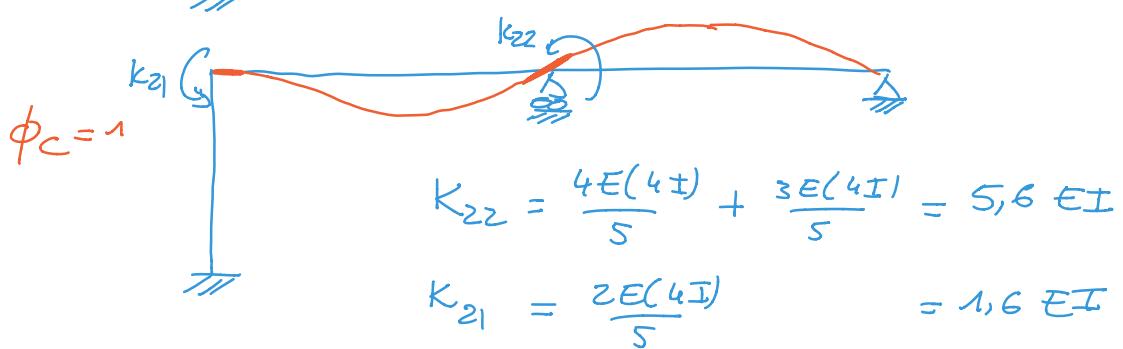
$$\begin{matrix} \underline{k} \underline{u} + \underline{k}_r = \underline{f} & \text{éq. équili. noeuds-} \\ 2 \times 2 \quad (\phi_B) & \left[\begin{array}{l} \text{moments aux noeuds (nuls)} \\ \text{moments sur éléments} \end{array} \right] \end{matrix}$$

$\underline{f} = 0$



$$K_{11} = \frac{4E(3I)}{3} + \frac{4E(4I)}{5} = 7,2EI$$

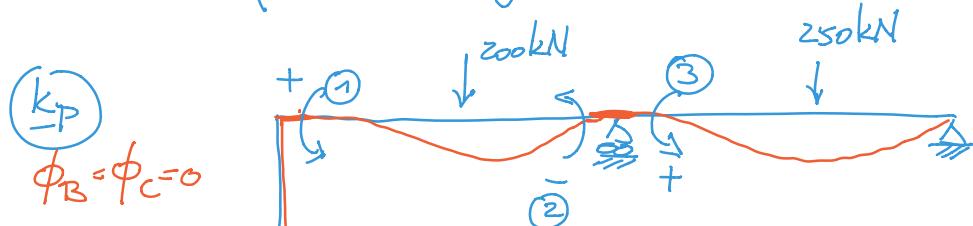
$$K_{12} = \frac{2E(4I)}{5} = 1,6EI$$



$$K_{22} = \frac{4E(4I)}{5} + \frac{3E(4I)}{5} = 5,6EI$$

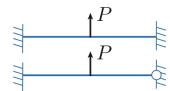
$$K_{21} = \frac{2E(4I)}{5} = 1,6EI$$

$$K = \begin{pmatrix} 7,2 & 1,6 \\ 1,6 & 5,6 \end{pmatrix} EI$$



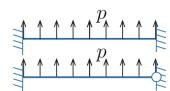
$$\textcircled{1} \quad k_{P,1} = 200 \cdot 5 \cdot 0,6 \cdot 0,4^2 = 96 \text{ kNm}$$

$$\textcircled{2} + \textcircled{3} \quad k_{P,2} = -200 \cdot 5 \cdot 0,4 \cdot 0,6^2 + \frac{3}{16} 250 \cdot 5 \\ = -144 + 234,375 = 90,375 \text{ kNm}$$



$$\begin{pmatrix} 0 \\ -\frac{1}{2}P \\ -\frac{1}{8}PL \\ 0 \\ -\frac{1}{2}P \\ \frac{1}{8}PL \end{pmatrix}$$

$$\textcircled{3} \quad \begin{pmatrix} 0 \\ -\frac{11}{16}P \\ -\frac{3}{16}PL \\ 0 \\ -\frac{5}{16}P \end{pmatrix}$$



$$\begin{pmatrix} 0 \\ -\frac{1}{2}pL \\ -\frac{1}{12}pL^2 \\ 0 \\ -\frac{1}{2}pL \\ \frac{1}{12}pL^2 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ -\frac{5}{8}pL \\ -\frac{3}{8}pL^2 \\ 0 \\ -\frac{3}{8}pL \end{pmatrix}$$

$$k_P = \begin{pmatrix} 96 \\ 90,375 \end{pmatrix} \text{ kNm}$$

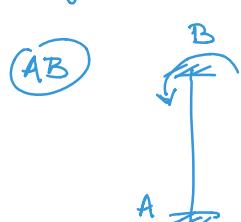
$$\boxed{\begin{array}{c} \textcircled{1} \quad \begin{pmatrix} 0 \\ -P(1-\alpha)^2(1+2\alpha) \\ -PL\alpha(1-\alpha)^2 \\ 0 \\ -P\alpha^2(3-2\alpha) \\ -PL\alpha^2(1-\alpha) \end{pmatrix} \\ \textcircled{2} \quad \begin{pmatrix} 0 \\ -P(1-\alpha)(1+\alpha-\frac{\alpha^2}{2}) \\ -PL\frac{\alpha}{2}(1-\alpha)(2-\alpha) \\ 0 \\ -P\frac{\alpha^2}{2}(3-\alpha) \end{pmatrix} \end{array}}$$

$$\alpha = 0,6$$

$$\begin{pmatrix} 0 \\ -P(1-\alpha)(1+\alpha-\frac{\alpha^2}{2}) \\ -PL\frac{\alpha}{2}(1-\alpha)(2-\alpha) \\ 0 \\ -P\frac{\alpha^2}{2}(3-\alpha) \end{pmatrix}$$

$$K \underline{u} = -k_P \Rightarrow \underline{u} = \begin{pmatrix} \phi_B \\ \phi_C \end{pmatrix} = \frac{1}{EI} \begin{pmatrix} -10,41 \\ -13,16 \end{pmatrix}$$

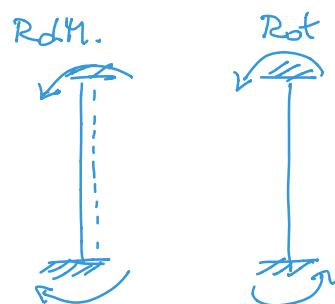
Diagramme des moments.



$$\phi_B = \frac{-10,41}{EI}$$

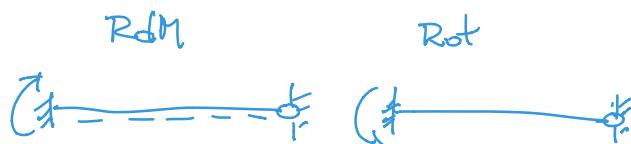
$$M_A = \frac{2E(3I)}{3} \phi_B = -20,8 \text{ kNm}$$

$$M_B = \frac{4E(3I)}{3} \phi_B = -41,6 \text{ kNm}$$



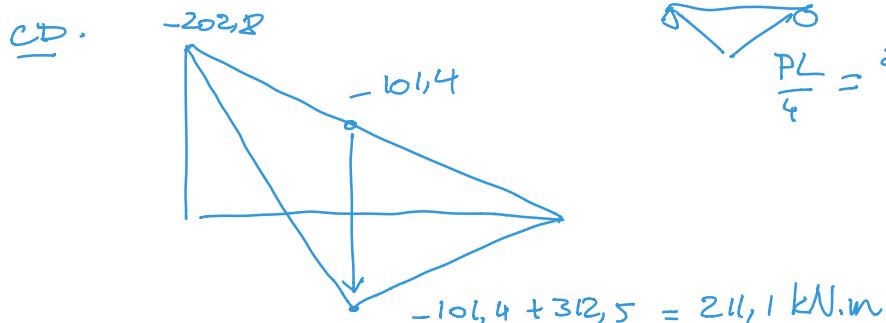
Conv. RdM: $M_A = 20,8 \text{ kN.m}$
 $M_B = -41,6 \text{ kN.m}$

(CD)
 $\phi_C = \frac{-13,16}{EI}$
 $M_C = \frac{3E(4I)}{5} \phi_C + 234,375 = 202,8 \text{ kN.m}$

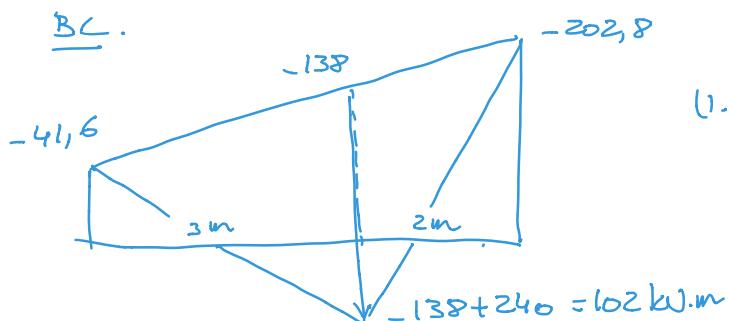


Conv. RdM: $M_C = -202,8 \text{ kN.m}$

moments en travée.



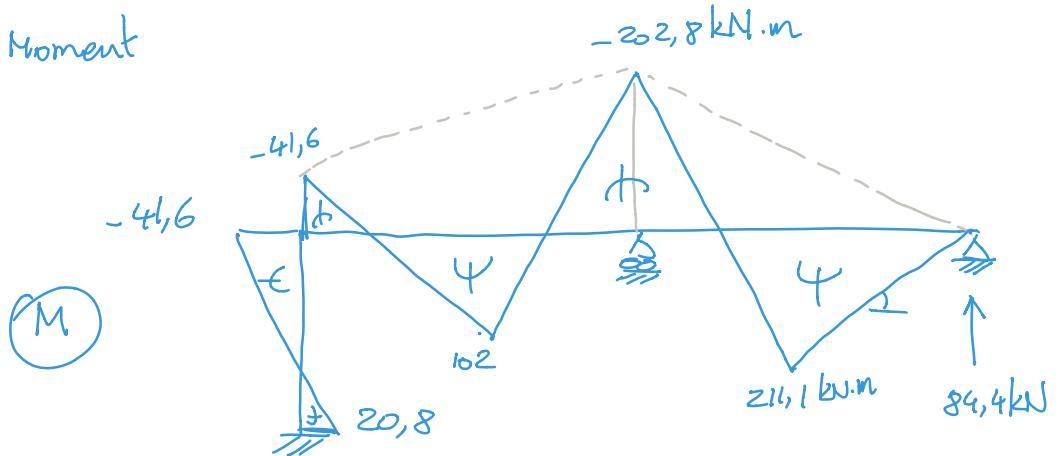
$\frac{PL}{4} = \frac{250 \cdot 5}{4} = 312,5 \text{ kN.m}$



$(1-d)p$ P $d(1-d)PL$

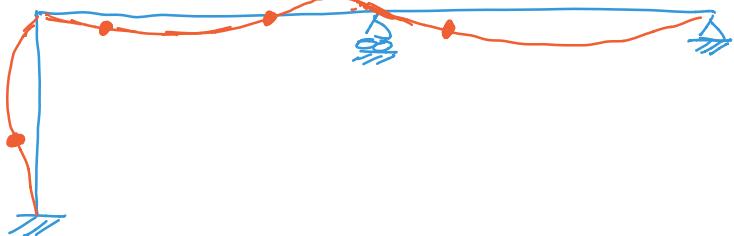
$(1-d)p + d(1-d)PL = 0,6 \cdot 0,4 \cdot 200,5 = 240 \text{ kN.m}$

Moment



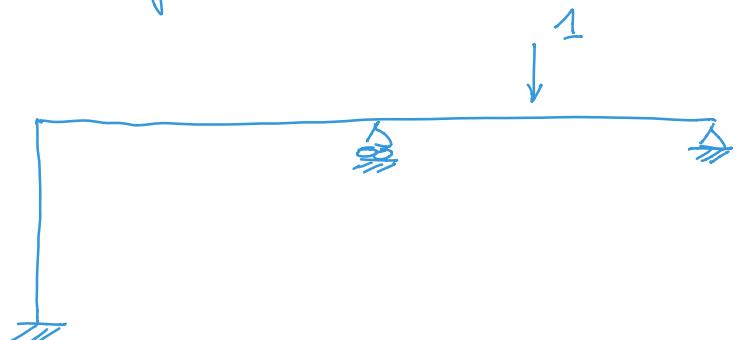
Réaction : $T = \frac{211,1}{2,5} = 84,4 \text{ kN}$

Esquisse



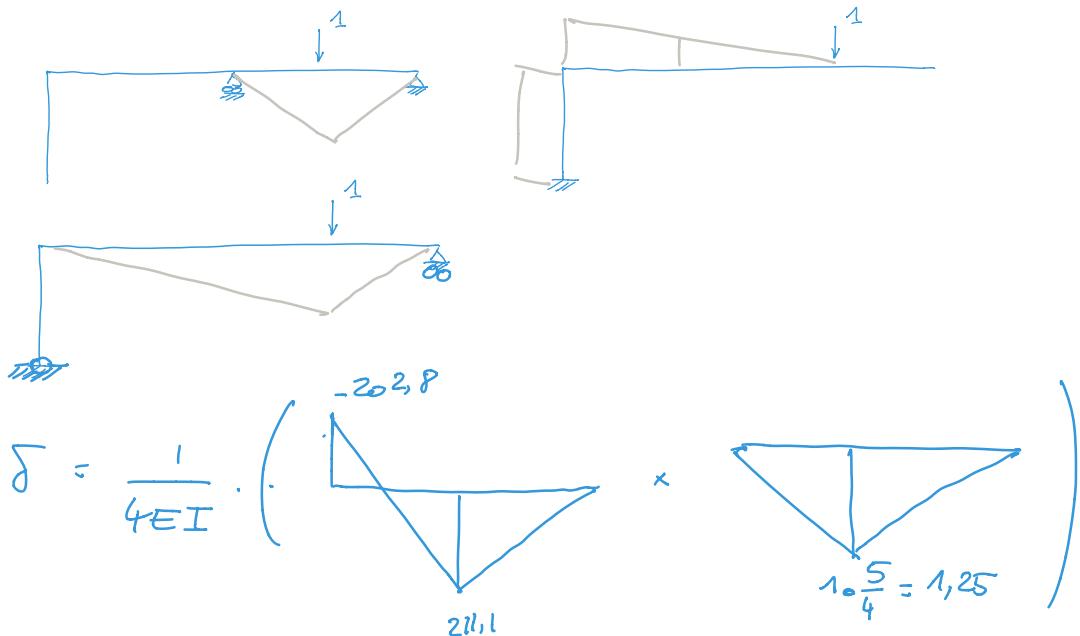
Déplacement sous charge de 250 kN

(M_1)



$$\frac{1}{EI} \int M M_1 ds + \text{Tables de Mohr}$$

Rendre la structure isostatique



$$\begin{aligned} &= \frac{1}{4EI} \left(\frac{1}{6} (-202,8 + 2 \cdot 211,1) \cdot 1,25 \cdot 2,5 \right. \\ &\quad \left. + \frac{1}{3} (211,1) \cdot 1,25 \cdot 2,5 \right) = \frac{83,5}{EI} \end{aligned}$$