The resulting values ($M_p = 53\cdot3$, $M = 40$, $12S = 48$, $12T = 8$) have been used to calculate the bending moments of table 2.11.

EXAMPLES

2.1 A beam of uniform cross-section, full plastic moment M_p, of length $2l$ rests on simple supports at its ends and on a central prop. Equal concentrated loads are applied to the centre of each span. Show that collapse occurs when the loads have the value $6M_p/l$.

2.2 If the loads in example 2.1 were not applied at the centres of the spans but at a distance al from each end of the beam what would be their value at collapse? $\left(Ans.\ \dfrac{(1+a)}{a(1-a)} M_p/l.\right)$

2.3 If the beam of example 2.1 carried four equal loads W, symmetrically arranged about the centre of length, one at the centre of each span and one at a distance $\frac{1}{8}l$ from each end of the beam, what would be the value of W at collapse? ($Ans.\ \frac{24}{5}M_p/l.$)

2.4 If the beam of example 2.1 carried four equal loads, W, symmetrically arranged about the centre of length, one at the centre of each span and one at a distance $3l/8$ from each end of the beam, what would be the value of W at collapse? ($Ans.\ \frac{88}{27}M_p/l.$)

2.5 If the beam of example 2.1 carried four equal loads W, symmetrically arranged about the centre of length, one at the centre of each span and one at a distance al from each end of the beam, what would be the value of a if at collapse the whole length of beam between the loads on each span became plastic? ($Ans.\ \frac{1}{4}$.)

2.6 If the beam of example 2.1 is subjected to a concentrated load at the centre of the left-hand span only, what will be the magnitude of that load at collapse? ($Ans.\ 4M_p/l.$)

2.7 A beam of uniform cross-section, full plastic moment M_p, of length $2l$ is encastré at its ends and also rests on a central prop. Equal concentrated loads are applied to the centre of each span. Show that collapse occurs when the loads have the value $8M_p/l$.

2.8 If the loads in example 2.7 were not applied at the centres of the spans but at a distance al from each of the encastré ends, what would be their value at collapse? $\left(Ans.\ \dfrac{2}{a(1-a)} M_p/l.\right)$

2.9 If the beam of example 2.7 carried four equal loads W, symmetrically arranged about the central prop, one at the centre of each span and one at a distance $\frac{1}{8}l$ from each encastré end, what would be the value of W at collapse?
(*Ans* $\frac{32}{5}M_p/l$.)

2.10 If the beam of example 2.7 is subjected to a concentrated load at the centre of the left-hand span only, what will be the magnitude of that load at collapse?
(*Ans.* $8M_p/l$.)

2.11 An encastré beam length l, of uniform section, full plastic moment M_p, is subjected to a uniformly distributed load W and to a concentrated load $0{\cdot}5W$ at a distance of $\frac{1}{3}l$ from one end. Find the value of W to cause collapse.
(*Ans.* $9M_p/l$.)

2.12 A cantilever of uniform section and length 3 m is propped at its end. It carries a uniformly distributed load of 30 kN/m. When, in addition, a concentrated load of 25 kN is applied, 1 m from the prop, the beam is on the point of collapse. Show that the full plastic moment of the beam is 35 kNm.

2.13 If the concentrated load of 25 kN had been applied 1 m from the encastré end of the propped cantilever of example 2.12, already carrying the uniformly distributed load 30 kN/m, what would have been the load factor?
(*Ans.* 1·13.)

2.14 A beam of uniform section, full plastic moment M_p, length l, is built-in at one end and simply supported at the other. It carries a concentrated load W at a distance a from the built-in end. Show that at collapse W has the value

$$\frac{2l-a}{a(l-a)}M_p.$$

Show that if both ends had been built-in the load at collapse would have increased in the ratio

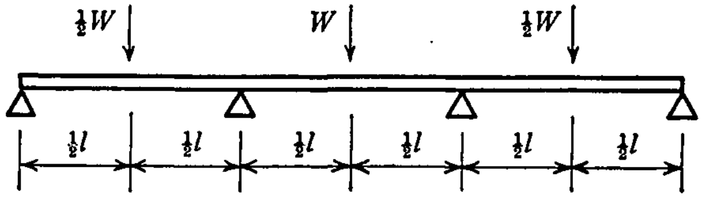
$$\frac{2l}{2l-a}.$$

2.15 A beam of uniform section, full plastic moment M_p, length l, was built in at the left-hand end A, and supported on a prop at the right-hand end B, so that before loading the beam was horizontal. When a uniformly distributed load was applied some settlement of the end A occurred until, when the total load was $8M_p/l$, the restraining moment at A was zero. Find the position of the first plastic hinge and the total load which ultimately produced collapse, assuming that no further settlement occurred.
(*Ans.* Centre of length: $2(3+2\sqrt{2})M_p/l$.)

2.16 A beam $ABCD$ of uniform section throughout, full plastic moment M_p, is pinned to four supports so forming a continuous beam of three equal spans, length l. A load W is applied at the centre of each span. Find the value of W which causes collapse.
(*Ans.* $6M_p/l$.)

2.17 If only one load W had been applied to the centre of the continuous beam of example 2.16, what would have been its value at collapse? If the outer spans had been twice as long as the centre, what would have been the effect on the value of the collapse load? (*Ans.* $8M_p/l$.)

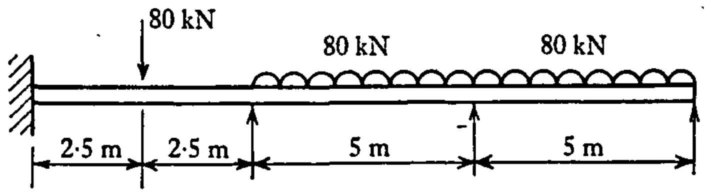
2.18 The continuous beam shown has three equal spans carrying central point loads. There is no change of beam section between supports but the plastic moment of resistance of the outer spans is only two-thirds that of the central span which is M_p. At what value of W does collapse occur? (*Ans.* $\frac{20}{3}M_p/l$.)



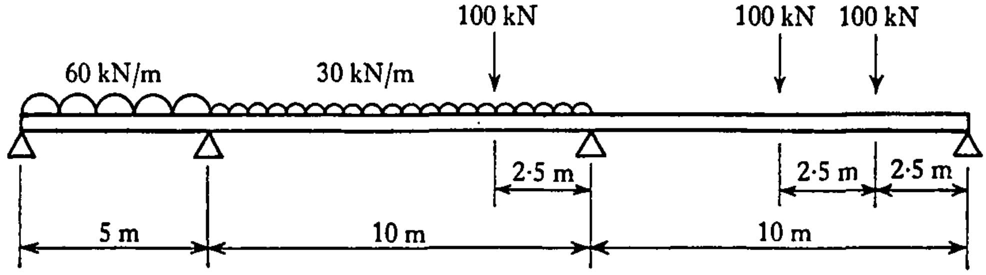
2.19 A uniform continuous beam, of full plastic moment M_p, rests on five simple supports A, B, C, D and E. $AB = 6l$, $BC = CD = 8l$, $DE = 10l$. Each span carries a concentrated load at its midpoint, these loads being W on AB, W on BC, $1{\cdot}4W$ on CD and $0{\cdot}5W$ on DE. Find the value of W which will just cause collapse. (*Ans.* $5M_p/7l$.)

2.20 A continuous beam rests on four supports A, B, C and D; $AB = BC = CD = 4$ m. Each span carries a uniformly distributed load: 25 kN on AB, 50 kN on BC and 30 kN on CD. The beam is to be of uniform section between supports but the section of the centre span is to be heavier than that of either outer span. Find the required value of the full plastic moment for each span if a load factor of 2 at collapse is to be provided. (*Ans.* 17·2, 31·1, 20·6 kNm.)

2.21 The three-span continuous beam shown is to be made of three steel members each of uniform section throughout. If, under the given loads, collapse is to occur in all spans simultaneously, show that possible values for the full plastic moments of the members are 58·3, 25 and 38·3 kNm.



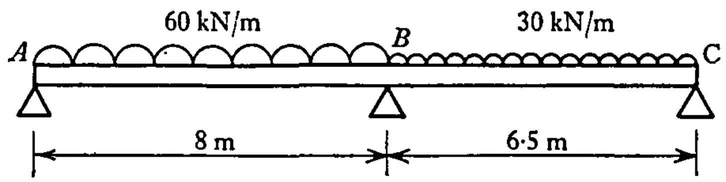
2.22 The continuous beam shown was of uniform section throughout, and collapsed when subjected to the given loads. What was the full plastic moment of the beam? (*Ans.* 255 kNm.)



2.23 If, in example 2.16, the loads W are each 300 kN and the span l is 6 m, show that a possible design for the beam, in a mild steel with a yield stress $\sigma_0 = 250$ N/mm², is a 16 × 6 UB 67 kg.

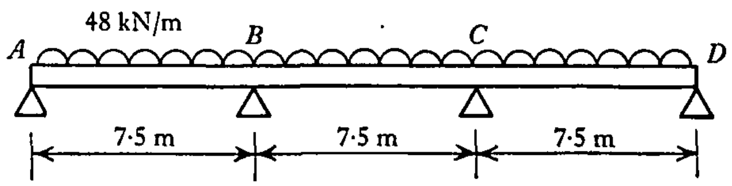
2.24 A beam of uniform section, 10 m long, is to be part of the floor system in a warehouse. It is simply supported at its ends and rests on a central support. The dead load and superimposed working live load together amount to a uniformly distributed load of 65 kN/m. A load factor of 1·75 is to be provided. Find a suitable section for the beam in a steel having $\sigma_0 = 250$ N/mm². (*Ans.* 16 × 7 UB 54 kg.)

2.25 The beam ABC shown is to be designed to carry the given loads with a load factor of 1·75. Using a steel with a minimum yield stress of 250 N/mm² show that a possible design for the beam is a 21 × 8¼ UB 92 kg, continuous from A to C.



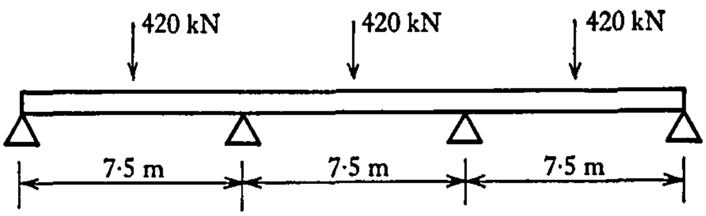
This design is extravagant in weight. It is decided therefore to use a smaller I-section continuous from A to C and to add symmetrical flange reinforcement where necessary in span AB. Show that a basic section 14 × 6¾ UB 45 kg will be satisfactory, and find the size of the flange plates that must be added and their net length. (*Ans.* 300 × 20 mm; 6·5 m.)

2.26 The three-span continuous beam shown carries a working load of 48 kN/m. If a load factor of 1·75 is to be provided by a beam of uniform section in a steel with yield stress 250 N/mm², show that an 18 × 6 UB 74 kg will be needed.



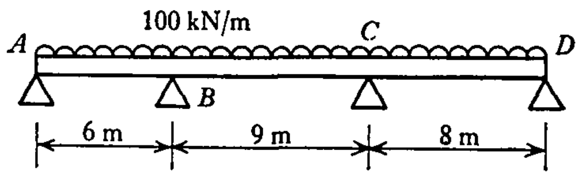
A plated beam would be used more efficiently over span BC. Show that a basic section 16×7 UB 60 kg. would be satisfactory, and find the sizes of the flange plates required. (*Ans.* 190×8 mm.)

2.27 Loads of 420 kN each are to be supported as shown at the centres of the spans of a continuous beam, with a load factor of 1·75. For architectural reasons the overall depth of the beam is limited to 0·5 m. Show that, using steel with $\sigma_0 = 250$ N/mm^2, an $18 \times 7\frac{1}{2}$ UB 98 kg is suitable, plated in the regions under the loads. Find the sizes and lengths of the flange plates.



(*Ans.* 220 mm × 10 mm plates, 1·44 m long in centre span, 280 mm × 16 mm plates, 3·08 m long in end spans.)

2.28 A load of 100 kN/m is to be carried over the three spans shown with a load factor of 1·75. It is decided to use an I-section, in steel with $\sigma_0 = 250$ N/mm^2, running continuously over all spans with added flange plates



running continuously over support C into the spans BC and CD. Find the size of basic section and length and area of flange plates required.

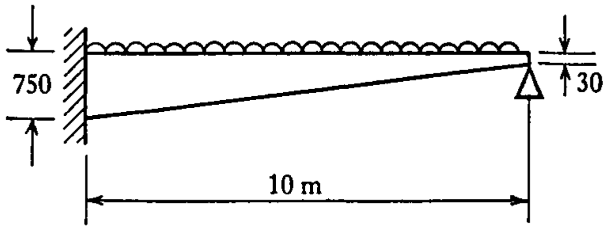
(*Ans.* $21 \times 8\frac{1}{4}$ UB 92 kg; 200 mm × 15 mm, 13·8 m.)

2.29 A continuous beam ABC rests on simple supports at A, B and C, the span AB being l and the span BC being $2l$. The beam is composed of two uniform members AB and BC rigidly jointed at B, the member AB having a fully plastic moment M_p and the member BC having a fully plastic moment KM_p. The span AB carries a central concentrated load W, and the span BC carries a uniformly distributed load $\frac{5}{8}W$.

The beam is to be designed according to the plastic theory, so that for any given value of K the value of M_p is to be such that one or other of the spans fails by plastic collapse. Show that collapse will occur in the span AB rather than in the span BC if K exceeds a value of about 0·8.

It is intended to design the beam ABC so that the total weight of material used is as small as possible. It may be assumed that there is a continuous range of sections available, and that the weight of each member per unit length is proportional to its fully plastic moment. Sketch a curve showing how the total weight of material in the beam varies with K for values of K between 0·5 and 2, and show that the minimum weight of material which can be used is about $0{\cdot}46\lambda Wl^2$, where λ is the weight per unit length of a member with a fully plastic moment of unity. (*M.S.T.* II, 1952.)

2.30 The propped cantilever shown is fabricated from two 300 × 25 mm flange plates, and a thin web plate which may be neglected in determining the plastic moment of resistance. The web depth tapers uniformly from 750 to



30 mm. The yield stress of the flanges is 250 N/mm². Determine the uniformly distributed collapse load. (Imperial College 1964, adapted.)

(*Ans.* 820 kN.)

2.31 A propped cantilever of uniform section, length l, full plastic moment M_p, flexural rigidity EI, is initially stress free. It is subjected to a uniformly distributed load which is gradually increased until collapse occurs. Assuming that the moment/curvature relationship is that of fig. 2.1, determine the deflexion at incipient collapse at the cross-section where the final hinge forms.

(*Ans.* $0{\cdot}089 M_p l^2/EI$.)

2.32 An I-beam, having thin flanges and web, is simply supported and subjected to a uniformly distributed load of intensity two-thirds of that which would cause collapse. A rigid prop is inserted at the centre and the load is increased until collapse occurs. Trace the development of the plastic hinges which will be formed. The effects of shear and instability are to be neglected. (*M.S.T.* II, 1950.)

2.33 A steel I-joist is designed to carry, as a simply-supported beam, a uniformly distributed load of intensity w over a span $2l$ with a load factor of 2.

When a load of intensity $2w$ has been applied, so that the beam is about to collapse, a rigid prop is inserted at the centre. Show that more load can be added until yield occurs at sections $3l/5$ from each end. Show further that complete collapse of the propped beam does not take place until the total intensity of load carried is $2(3+2\sqrt{2})w$.

It is to be assumed that the thickness of the flanges and web are negligible in comparison with the depth of the member and that no buckling of the web or other condition of instability occurs. (*M.S.T.* II, 1947.)

2.34 An encastré beam of span L is to be designed for collapse under a single central point load λW. For a distance $\frac{1}{2}kL$ at each end of the span the fully plastic moment is to have r times the value obtaining for the remainder of the span. The weight per unit length of beam is

$$\beta\left[1+\frac{32M}{\lambda WL}\right],$$

where M is the fully plastic moment at the section in question, and β is a constant.

Determine the ratios r and k for minimum weight of the beam, and find the saving in material as compared with a design using a uniform beam. (University of London B.Sc. (Engineering) Part III: Civil, King's College, 1965.)

(*Ans.* $r = 5/3$, $k = 1/4$, 10%.)

2.35 In a fixed-base rectangular portal frame $ABCD$ the column AB is of height 16 and the column DC is of height 24; the beam BC of length 16 is horizontal. All the members of the frame have the same full plastic moment M_p. The beam BC carries a central concentrated vertical load of 70, and a concentrated horizontal load of 28 is applied at C in the direction BC. Find the value of M_p so that collapse just occurs. (*Ans.* 189.)

2.36 In example 2.35, the horizontal load of 28 is reversed in direction. Find M_p. (*Ans.* 176.)

2.37 In example 2.35, the frame is not uniform, but the columns have the same full plastic moment M_1 and the beam has full plastic moment M_2. Find M_1 and M_2 so that collapse just occurs whichever way the horizontal load acts.

(*Ans.* 134·4, 280.)

2.38 A fixed-base rectangular portal frame is of height and span l. The columns each have full plastic moment $2M_p$, and the beam has full plastic moment M_p. One of the columns is subjected to a uniformly distributed horizontal load W. Find the value of W which would cause collapse.

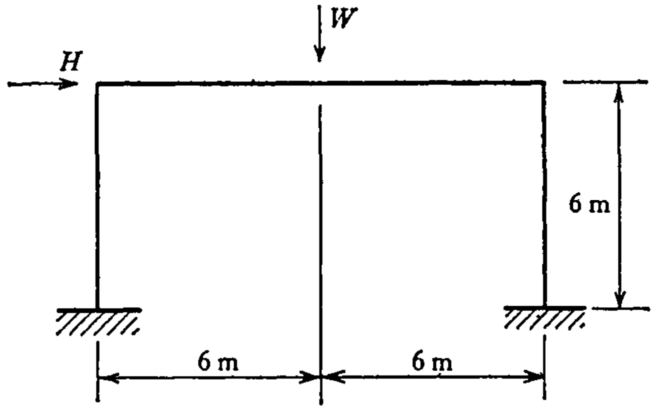
(Answer is too strong a hint.)

2.39 Repeat example 2.38 for a uniform frame of full plastic moment M_p.

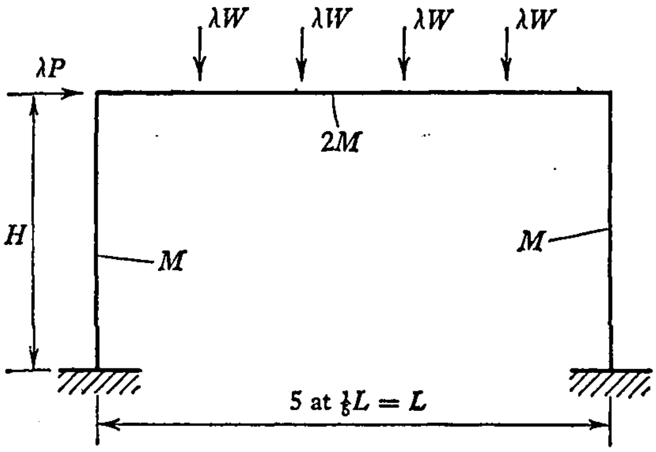
(*Ans.* $2(2+\sqrt{3})M_p/l$.)

2.40 The rigid frame shown has 6×6 UC 37 kg columns and a $10\times 5\frac{3}{4}$ UB 43 kg beam; the yield stress of the steel is 250 N/mm². Neglecting the effects of axial load and instability, determine the ratio of W to H for all possible modes of plastic collapse of the frame. Determine the value of W when $W = 1{\cdot}8H$. (Institution of Structural Engineers, 1962; adapted.)

(*Ans.* 63·6 kN.)



2.41 In the figure the load factor, λ, and the plastic moment, M, are to be regarded as variables, as well as W, P, L and H. Plot, in the plane of the dimensionless variables $(\lambda WL/M)$ and $(\lambda PH/M)$, the collapse locus for the family of loaded frames represented.

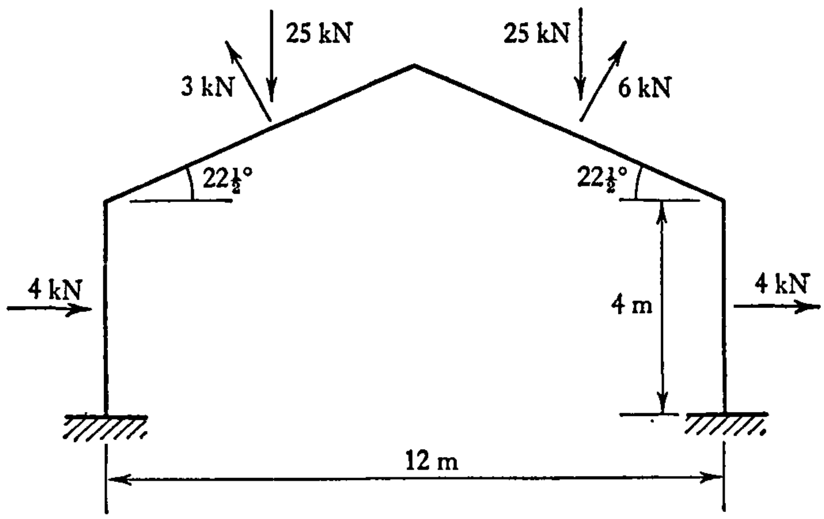


A frame conforming to the figure has $H:L = 4:3$. It is designed to a load factor of 2 under vertical loading only. What is the limiting value of $P:W$ for this frame if under combined loading the load factor may be reduced to 3/2? (University of London B.Sc. (Engineering) Part III: Civil, King's College, 1965.) (*Ans.* 13:20.)

2.42 A single storey building consists of a number of frames, each frame being comprised of several equal bays of pitched-roof portals rigidly connected at all joints. Considering one frame subjected only to a uniformly distributed vertical dead load on the roof, show that collapse of the frame will occur only in the bays at either end, providing that the rafter section is continuous and uniform throughout, and that the two end stanchions have equal sections (different from that of the rafters).

2.43 The frame in example 2.42 is subjected to a wind load, in addition to the dead load. The effect of the wind is to produce a small uniform pressure on the windward stanchion, and an equal suction on the leeward stanchion; all other wind effects are to be ignored. Show that collapse of the frame occurs only in the leeward bay.

2.44 The pitched roof portal frame shown is of the same cross-section and ductile material throughout. All the joints are rigid and the feet are fixed in foundations which can develop the full plastic moment of the members. Loads of the magnitudes shown are uniformly distributed over the members.

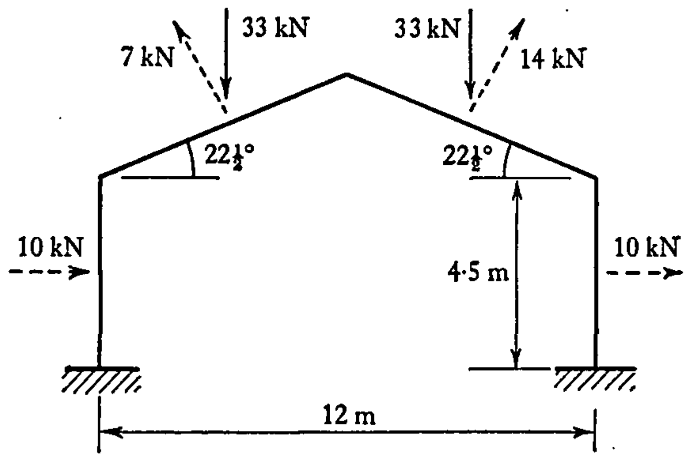


Estimate the required value of the full plastic moment so that the load factor against plastic collapse shall be 1·4. Instability effects and the reduction of plastic moments due to axial forces are to be neglected. (*M.S.T.* II, 1953; adapted.) (*Ans.* 31·5 kNm.)

2.45 The portal frame shown is of uniform section throughout. It is to be designed to resist

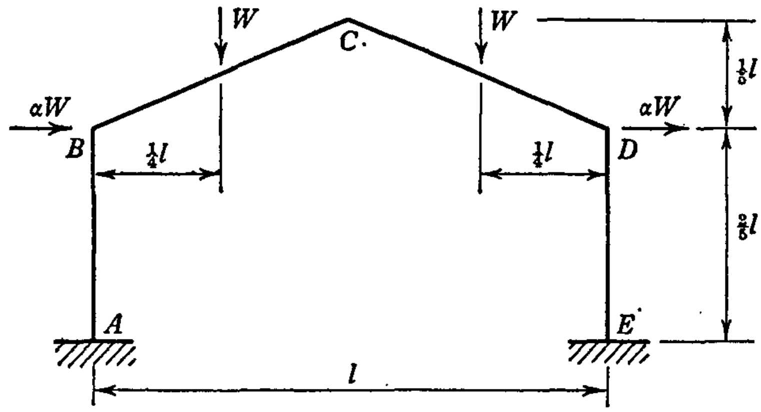
(i) dead plus superimposed loading, load factor 1·75;

(ii) dead plus superimposed plus wind loading, load factor 1·4. The loading is as shown, the dead plus superimposed loading being indicated



by the full arrows and the wind loading by the dotted arrows. All the loads may be regarded as uniformly distributed over the members. Find the required value of M_p. (*Ans.* 57·7 kNm.)

2.46 The continuous symmetrical pitched roof portal frame shown has rigid joints and is rigidly fixed to foundations at A and E. The frame has a uniform full plastic moment of M_p, on which the effect of axial loads may be neglected.

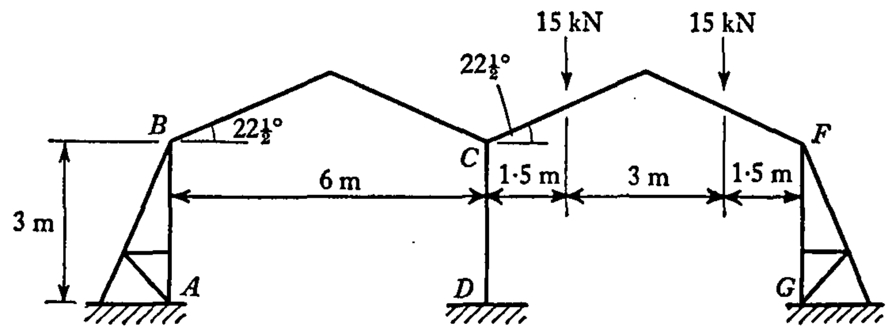


Investigate the value of W at which collapse will just occur for the load system shown. Present the solution in such a way that the mode of collapse and the value of W at collapse are indicated for any value of α. Neglect the effect of the deformation of the structure on the equations of equilibrium. (*M.S.T.* II, 1963.)

2.47 A symmetrical pitched-roof portal has a span of 7·2 m. The height at the ridge is 5·4 m and at the eaves 3·6 m. All joints are rigid and the feet of the stanchions are encastré. The stanchions have a fully plastic moment M_p and the roof members αM_p. The portal is subjected to a vertical load of 20 kN at the ridge and an outwardly directed horizontal load in the plane of the portal of 10 kN at the top of one stanchion.

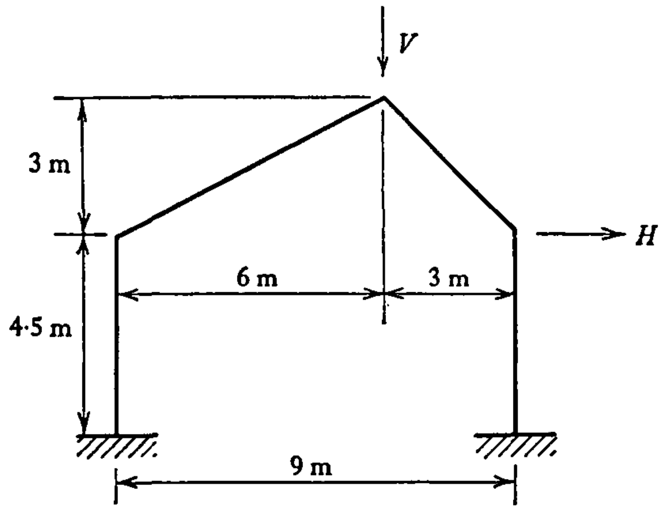
For a given value of α find the least value of M_p which will prevent collapse and plot this value of M_p against α. (*M.S.T.* II, 1954; adapted.)

2.48 The plane portal frame with rigid joints shown is made of steel joist with a full plastic moment of resistance about the axis of bending of 12 kNm.

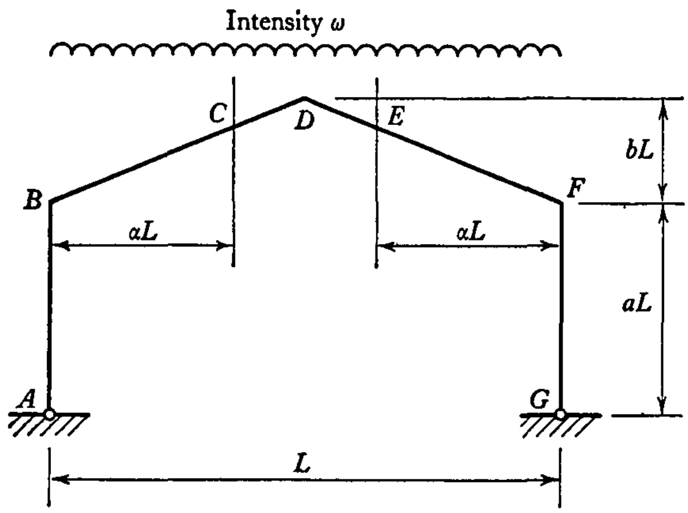


The outer stanchions *AB* and *FG* are rigidly supported and the foot of the centre stanchion *CD* is encastré. By means of the plastic theory, making the usual simplifying assumptions, estimate the factor by which the loading shown would have to be multiplied for collapse just to occur. (*M.S.T.* II, 1958; adapted.) (*Ans.* 1·69.)

2.49 The north-light portal frame shown is composed of uniform members having full plastic moment 90 kNm, and has fixed feet and full strength joints; loads *V* and *H* are applied as shown. Plot a graph (interaction diagram) from which may be read the positive values of *V* and *H* which will just cause collapse of the frame. The usual assumptions of simple plastic theory may be made. (*M.S.T.* II, 1966; adapted.)

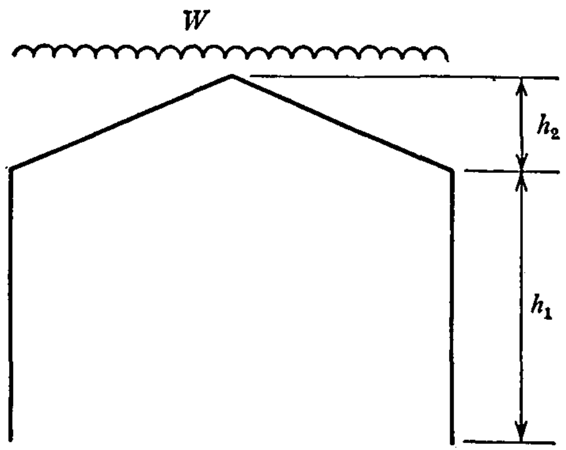


2.50 A pitched portal with hinged feet is subjected to vertical loading of constant intensity ω per unit horizontal distance, as indicated. The frame may be taken to have a plastic moment of resistance *M* throughout. Instability, and the effects on the plastic moment of axial and shear forces may be ignored.

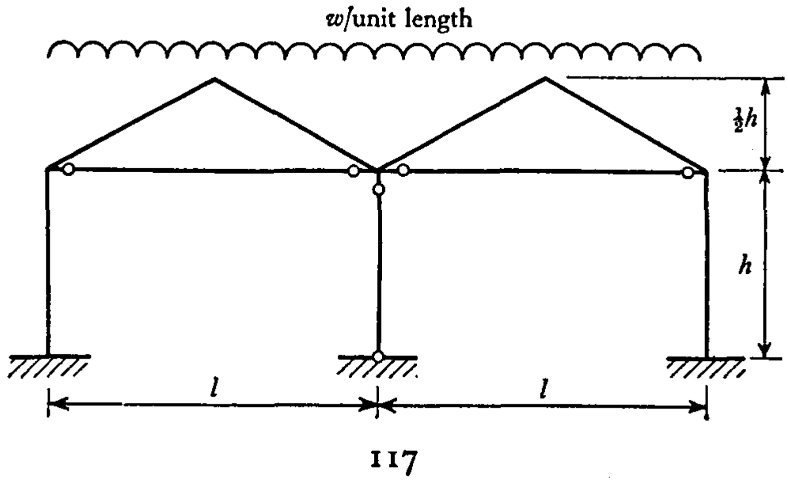


Show that when
$$\omega = \frac{4M(1+Q\alpha)}{L^2\alpha(1-\alpha)},$$
where $Q = b/a$, plastic collapse occurs with plastic hinges formed at B, F, C and E, the last two points being defined by
$$\alpha = \frac{1}{Q}\{\sqrt{(1+Q)}-1\}.$$
(University of London B.Sc. (Engineering) Part III: Civil, King's College, 1964.)

2.51 The frame of uniform section shown is to be designed by simple plastic theory to carry the uniformly distributed load W. Two designs are made, one for a frame with pinned feet, and one for fixed feet. Show that the ratio of full plastic moments for the two designs is
$$\left[\frac{1+\sqrt{(1+2k)}}{1+\sqrt{(1+k)}}\right]^2. \quad (M.S.T.\ 11,\ 1965.)$$



2.52 The two-bay pitched-roof portal frame shown has members of uniform section of full plastic moment M_p. The rafters are connected to each other and

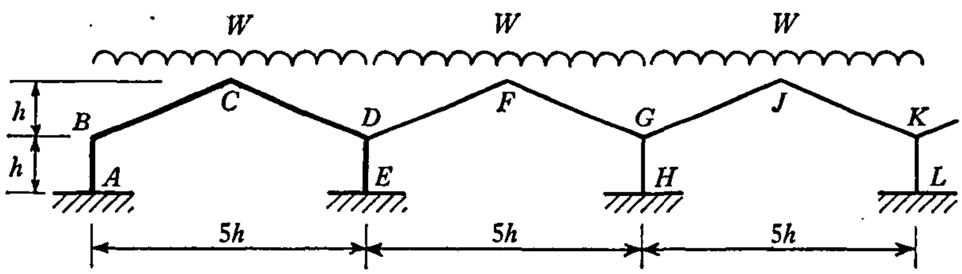


to the two external columns by full strength connexions. The external columns can sustain the full plastic moment at their feet, but the central column is a simple prop, pinned to the ground at one end and to the frame at the other. Light ties are connected across the frames at eaves level, and these are of sufficient section to prevent significant spread of the eaves when the frame is subjected to the factored collapse design load λw per unit length, uniformly distributed.

One of the ties is cut. Show that the load factor against collapse drops to $0{\cdot}364\lambda$.

The usual assumptions of simple plastic theory may be made. (*M.S.T.* II, 1961.)

2.53 The figure shows part of a continuous multi-bay frame of pitched-roof construction in which all the columns are rigidly fixed to the foundations. The dimensions of all bays are the same, the span of each being $5h$, the height to eaves h and the rise of the rafters h. The rafters BC and CD of the outer bay



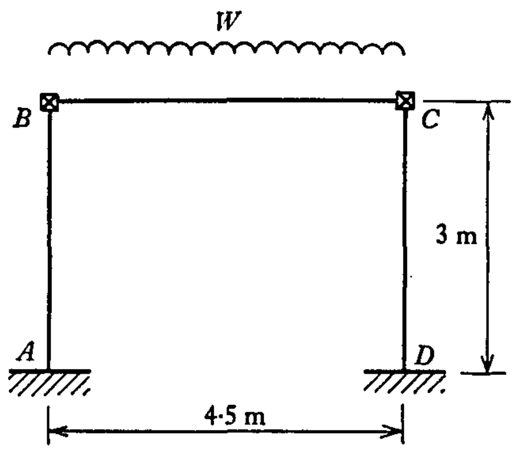
and the columns AB and DE have the same uniform full plastic moment M_1, while the inner rafters DF, FG, GJ, JK, etc., and the columns GH, KL, etc., have a smaller full plastic moment M_2. Each bay sustains a total vertical load W uniformly distributed. In designing the frame to a load factor at plastic collapse of 1·75, the value of M_2 is first chosen, and is made as small as possible. The necessary value of M_1 is then calculated. Obtain expressions for M_1 and M_2, making the approximation that, in the outer bay $ABCDE$, plastic hinges are confined to the joints and foundations.

The effect of elastic deformations on the collapse load and the effect of axial loads on the values of the full plastic moments are to be ignored. (*M.S.T.* II, 1960.) (*Ans.* $M_1 = 1{\cdot}75(\frac{23}{128}Wh)$, $M_2 = 1{\cdot}75(\frac{5}{64}Wh)$.)

2.54 The portal frame shown is composed of three straight uniform members, flexural rigidity 4·7 MNm^2, full plastic moment 66·0 kNm. The base connexions at A and D are rigid; the connexions between the members at B and C are flexible, the moment–rotation characteristic of each connexion being given by the following table:

Moment (kNm)	12·5	25·0	37·5	50·0	62·5	66·0
Rotation (radians)	0·0023	0·0053	0·0091	0·0136	0·0189	0·0205

The portal is subjected to a uniformly distributed vertical load W acting over the full length of the beam. Calculate the central deflexion of the beam as W is increased slowly from zero up to the collapse value, and compare the resulting load-deflexion curve with that for a similar frame with rigid connexions at B and C.



It may be assumed that a cross-section where the bending moment is less than 66·0 kNm remains completely elastic, and that indefinitely large rotations can occur at constant moment at a plastic hinge.

Deflexions need be calculated only at the six loading conditions corresponding to the six sets of values given in the table. (*M.S.T.* II, 1961; adapted.)